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Supersymmetry, attractors and cosmic censorship

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Abstract

We show that requiring unbroken supersymmetry *everywhere* in black-hole-type solutions of $N = 2, d = 4$ supergravity coupled to vector supermultiplets ensures in most cases absence of naked singularities. We formulate three specific conditions which we argue are equivalent to the requirement of global supersymmetry. These three conditions can be related to the absence of sources for NUT charge, angular momentum, scalar hair and negative energy, although the solutions can still have globally defined angular momentum and non-trivial scalar fields, as we show in an explicit example. Furthermore, only the solutions satisfying these requirements seem to have a microscopic interpretation in String Theory since only they have supersymmetric sources. These conditions exclude, for instance, singular solutions such as the Kerr-Newman with $M = |q|$, which fails to be everywhere supersymmetric.

We also present a re-derivation of several results concerning attractors in $N = 2, d = 4$ theories based on the explicit knowledge of the most general solutions in the timelike class.

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1 Introduction

In spite of the impressive progress made during the last few years in the study of supersymmetric black-hole solutions, there are important questions that remain unanswered or whose answer is unclear. For instance, we know how to construct many supersymmetric black-hole-type solutions, but many of them are singular. Some of these become regular when string corrections are taken into account and for all the regular black hole solutions we seem to have a String Theory model that accounts for its entropy. How are the other singular solutions to be understood? How can it be that they are supersymmetric and yet there is no String Theory model for them? Or, if there is, why are they singular?

The main goal of this paper is to try to answer this question by giving a set of conditions that supersymmetric black-hole-type solutions must satisfy in order to be admissible in the context of $N = 2, d = 4$ supergravity coupled to vector supermultiplets. Admissible solutions will be regular and will describe one or several black holes in static equilibrium, even though the system may have a finite global angular momentum, as is for example the case in the solution constructed in Ref. [1]. Furthermore, we expect only admissible solutions to have a miscroscopic String Theory model. We will argue that the non-admissible solutions are, in general, not truly supersymmetric in the sense that will be explained later on and the conditions of admissibility can be seen as conditions for a solution to be everywhere supersymmetric. For instance: the Kerr-Newman solution with equal charge and mass, which is singular but nevertheless commonly believed to be supersymmetric, is non-admissible according to our criteria. We will show that it fails to be supersymmetric at the singularity, where the sources might be located. Equivalently we can say that the Kerr-Newman field with $M = |q|$ is caused by non-supersymmetric sources. This explains why it is not described by any supersymmetric String Theory model. We will also show that, generically, rotating sources are not allowed by supersymmetry and that regular, supersymmetric solutions with angular momentum are always composite objects made out of several static black holes in equilibrium. The angular momentum has its origin in the dipole momenta of the electromagnetic fields corresponding to the distribution of charged black holes. Something similar happens for scalar fields: supersymmetric configurations satisfying our conditions can have non-trivial scalar fields but cannot have sources.

In order to prove these results, we will make use of the explicit knowledge of the most general solutions of $N = 2, d = 4$ supergravity coupled to vector multiplets, which have recently been classified in Ref. [2][4] . All the asymptotically flat supersymmetric black hole solutions seem to belong to the timelike class, and, although they coincide with the solutions found in Ref. [4], the general formalism will allow us to make further progress in their understanding. In particular, we will use the *Killing Spinor Identities* (KSIs) [5, 6], which can be understood as integrability conditions for the Killing spinor equations, in order to study supersymmetry at the singular points where the sources of these solutions should be located.

The final ingredient will be the attractor equations of $N = 2, d = 4$ supergravity [7, 8, 10, 11]: these provide us with information about the sources thought of as being placed at the attractor points. In fact, we will find interesting relations between KSIs and attractor equations, the former showing explicitly that

1. supersymmetry always requires the absence of the kind of scalar hair called *primary* in Ref. [12], and that

2. when the attractor equations are satisfied there are no sources whatsoever for scalar hair.[5]

These results can be viewed as an extension of those of Ref. [14] in which it was observed that supersymmetry seems to act as a cosmic censor for static black-hole-type configurations but not for the stationary ones, such as the Kerr-Newman $M = |q|$ solution.

This paper is organized as follows: in Section 2 we review the timelike class of supersymmetric solutions of $N = 2, d = 4$ supergravity coupled to vector multiplets. First, we review how all the solutions in this class can be constructed from a symplectic vector of real harmonic functions and then in Section 2.1 we derive the KSIs that, by assumption of supersymmetry, all solutions must satisfy. In Section 2.2 a re-derivation of some of the main

[4]In this paper we will not consider the coupling to hypermultiplets. The classification of the supersymmetric solutions with both vector multiplets and hypermultiplets is considered in Ref. [3].

[5] If there is more than one basin of attraction, contrary to what is assumed in this article, this last conclusion might change due to the *area codes* [13].

results involving $N = 2, d = 4$ supersymmetric black-hole attractors, taking advantage of the actual and explicit knowledge of all the solutions of this kind, which has helped us to improve some of the presentations existing in the literature and prove some new results.

In Section 3 we study how the KSIs constrain the possible sources and singularities of black-hole-type solutions and the interplay with the attractor equations in a general way. The main result of this section will be the formulation of three conditions that express the existence of supersymmetry everywhere in the solutions, including, particularly, the locations of the sources. These conditions should ensure the regularity of the admissible solutions and we study in very close detail several examples in Section 4. Section 5 contains our conclusions.

2 Timelike BPS solutions of $N = 2, d = 4$ SUEGRA

It was recently shown in Ref. [2] that all the supersymmetric solutions in the timelike class of $N = 2, d = 4$ supergravity coupled to n vector multiplets[6] can be constructed by setting the $2\bar{n} = 2(n + 1)$ components of a real, symplectic vector $\mathcal{I} = (\mathcal{I}^\Lambda, \mathcal{I}_\Lambda)$ equal to $2\bar{n} = 2(n + 1)$ real functions harmonic on 3-dimensional Euclidean space[7]

$$\mathcal{I} \equiv \begin{pmatrix} \mathcal{I}^\Lambda \\ \mathcal{I}_\Lambda \end{pmatrix}, \qquad \partial_m\partial_m\mathcal{I}^\Lambda = \partial_m\partial_m\mathcal{I}_\Lambda = 0\,, \qquad \Lambda = 0, 1, \cdots, n\,. \tag{2.1}$$

This real section $\mathcal{I}$ enters the theory as the imaginary part of the section $\mathcal{V}/X$, where $\mathcal{V}$ is the covariantly-holomorphic canonical section defining special geometry:

$$\mathcal{V} = \begin{pmatrix} \mathcal{L}^\Lambda \\ \mathcal{M}_\Sigma \end{pmatrix} \quad \rightarrow \quad \begin{cases} \langle \mathcal{V} \mid \mathcal{V}^* \rangle & \equiv \; \mathcal{L}^{*\Lambda}\mathcal{M}_\Lambda - \mathcal{L}^\Lambda\mathcal{M}^*_\Lambda = -i\,, \\ \mathfrak{D}_{i^*}\mathcal{V} & = \; (\partial_{i^*} - \tfrac{1}{2}\partial_{i^*}\mathcal{K})\mathcal{V} = 0\,, \\ \langle \mathfrak{D}_i\mathcal{V} \mid \mathcal{V} \rangle & = \; 0\,. \end{cases} \tag{2.2}$$

X on the other hand is proportional to the complex, scalar bilinear constructed out of the Killing spinors: supersymmetry and consistency of the solutions imply that it can be expressed in terms of $\mathcal{I}$, see e.g. Ref. [2] or Eq. (2.7).

Eqs. (2.1) are sometimes known as the *generalized stabilization equations*, the standard stabilization equations having the same form but with the harmonic functions $(\mathcal{I}^\Lambda, \mathcal{I}_\Lambda)$ replaced by magnetic and electric charges, *e.g.* (p^Λ, q_Λ).

The real part of $\mathcal{V}/X$, denoted by $\mathcal{R} \equiv (\mathcal{R}^\Lambda, \mathcal{R}_\Lambda)$ can, in principle, be written in terms of the real harmonic functions, which is usually referred to as "solving the stabilization equations". In theories with a prepotential, the homogeneity properties of the prepotential allow us to write

[6] These solutions were first found in slightly different form in Ref. [4] and the procedure followed in Ref [2] shows that they are the only solutions in this class.

[7] If the functions are not harmonic, the field configurations are still supersymmetric, but are *not* solutions of the equations of motion.

$$\mathcal{M}_\Lambda/X = \frac{\partial\mathcal{F}(\mathcal{L}^\cdot/X)}{\partial(\mathcal{L}^\Lambda/X)}\,. \tag{2.3}$$

Taking the imaginary part of this equation, we have

$$\mathcal{I}_\Lambda(\mathcal{R}^\cdot,\mathcal{I}^\cdot) = \mathcal{I}_\Lambda\,, \tag{2.4}$$

which implicitly defines $\mathcal{R}^\Lambda(\mathcal{I}^\cdot,\mathcal{I}_\cdot)$, although solving these equations can be extremely hard and in general the explicit solution is unknown.

The real part of Eqs. (2.3) and the above solutions give straightforwardly the functions $R_\Lambda(R^\cdot(\mathcal{I}^\cdot,\mathcal{I}_\cdot),\mathcal{I}^\cdot)$.

Having the complete symplectic section $\mathcal{V}/X$ entirely given in terms of the real harmonic functions, one can construct the fields of the solutions as follows:

1. The n complex scalar fields Z^i are given by the quotients

$$Z^i = \frac{\mathcal{L}^i/X}{\mathcal{L}^0/X} = \frac{\mathcal{R}^i + i\mathcal{I}^i}{\mathcal{R}^0 + i\mathcal{I}^0}\,. \tag{2.5}$$

2. The metric has the form

$$ds^2 = 2|X|^2(dt+\omega)^2 - \frac{1}{2|X|^2}dx^i dx^i\,, \qquad i,j = 1,2,3\,, \tag{2.6}$$

where

$$\frac{1}{2|X|^2} = \langle\,\mathcal{R}\mid\mathcal{I}\,\rangle\,, \tag{2.7}$$

and ω is a time-independent 1-form on Euclidean 3-dimensional space satisfying the equation

$$(d\omega)_{mn} = 2\epsilon_{mnp}\langle\,\mathcal{I}\mid\partial_p\mathcal{I}\,\rangle\,. \tag{2.8}$$

3. The symplectic vector of field strengths and their duals $F = (F^\Lambda, \tilde{F}_\Lambda)$ is given by

$$F = -\tfrac{1}{2}\{d[\mathcal{R}\hat{V}] - {}^\star[d\mathcal{I}\wedge\hat{V}]\}\,, \qquad \hat{V} = 2\sqrt{2}|X|^2(dt+\omega)\,. \tag{2.9}$$

The Killing spinors of these solutions have the form

$$\epsilon_I = X^{1/2}\epsilon_{I\,0}\,, \qquad \partial_\mu\epsilon_{I\,0} = 0\,, \qquad \epsilon_{I\,0} + i\gamma_0\epsilon_{IJ}\epsilon^J{}_0 = 0\,, \tag{2.10}$$

which implies

$$\epsilon_I + i\gamma_0 e^{i\alpha}\epsilon_{IJ}\epsilon^J = 0\,, \qquad e^{i\alpha} = (X/X^*)^{1/2}\,. \tag{2.11}$$

Observe that we can write

$$X = \frac{\mathcal{L}^{\Lambda}(Z, Z^*)}{\mathcal{R}^{\Lambda} + i\mathcal{I}^{\Lambda}}\,, \tag{2.12}$$

for any Λ.

2.1 Killing Spinor Identities

All supersymmetric configurations satisfy the *Killing Spinor Identities* relating the Einstein equations $\mathcal{E}^{\mu\nu}$, the Maxwell equations $\mathcal{E}_{\Lambda}{}^{\mu}$, the Bianchi identities $\mathcal{B}^{\Lambda\,\mu}$ and the scalar equations of motion $\mathcal{E}^{i}$ [5, 6, 2]

$$\mathcal{E}_a{}^{\mu}\gamma^a\epsilon_I - 4i\langle\, \mathcal{E}^{\mu} \mid \mathcal{V}\,\rangle\epsilon_{IJ}\epsilon^J \;=\; 0\,, \tag{2.13}$$

$$\mathcal{E}^{i}\epsilon^I + 2i\langle\, \not{\mathcal{E}} \mid \mathcal{U}^{*\,i}\,\rangle\epsilon^{IJ}\epsilon_J \;=\; 0\,, \tag{2.14}$$

where $\mathcal{E}^{\mu}$ is the symplectic vector $(\mathcal{B}^{\Lambda\,\mu}, \mathcal{E}_{\Lambda}{}^{\mu})$.

In the timelike case, they lead to the following identities in an orthonormal frame

$$\mathcal{E}^{ab} \;=\; \eta^a{}_0\eta^b{}_0\mathcal{E}^{00}\,, \tag{2.15}$$

$$\langle\, \mathcal{V}/X \mid \mathcal{E}^a\,\rangle \;=\; \tfrac{1}{4}|X|^{-1}\mathcal{E}^{00}\delta^a{}_0\,, \tag{2.16}$$

$$\langle \mathcal{U}^*_{i^*} \mid \mathcal{E}^a\,\rangle \;=\; \tfrac{1}{2}e^{-i\alpha}\mathcal{E}_{i^*}\delta^a{}_0\,. \tag{2.17}$$

These equations imply directly

$$\mathcal{E}^{0m} = 0\,, \quad \mathcal{E}^{mn} = 0\,, \quad \langle\, \mathcal{V} \mid \mathcal{E}^m\,\rangle = 0\,, \quad \langle \mathcal{U}^*_{i^*} \mid \mathcal{E}^m\,\rangle = 0\,. \tag{2.18}$$

Further, the r.h.s. of Eq. (2.15) is real, and this leads to two important identities:

$$\langle\, \mathcal{I} \mid \mathcal{E}^0\,\rangle \;=\; 0\,, \tag{2.19}$$

$$\mathcal{E}^{00} \;=\; \pm 4|\langle\, \mathcal{V} \mid \mathcal{E}^0\,\rangle|\,. \tag{2.20}$$

2.2 Attractor equations

It is well-known that, in general, the scalar fields of the black-hole solutions of these theories have certain attractor values that depend solely on the electric and magnetic charges and which are attained at the event horizons irrespectively of the chosen asymptotic values

[7, 8].[8] The attractor values are those which extremize a specific function; furthermore, the absolute value squared of the central charge for the attractor values is essentially the horizon area [10, 11]. Here we are going to rederive these results using our notation and to relate them to the KSIs. We also want to improve the previous derivations by making explicit use of the knowledge of all the supersymmetric configurations.

Let us consider single, static, asymptotically flat, spherically symmetric, black-hole-type solutions of $N = 2, d = 4$ supergravity coupled to vector multiplets: they are given by real harmonic functions of the form

$$\mathcal{I} = \mathcal{I}_\infty + \frac{q}{r}\,, \tag{2.21}$$

which is the general choice compatible with the assumptions. The metric can be conveniently written in spherical coordinates as

$$ds^2 = 2|X|^2 dt^2 - \frac{1}{2|X|^2}[dr^2 + r^2 d\Omega^2_{(2)}]\,. \tag{2.22}$$

This metric describes black holes if

$$-\,g_{rr} = \frac{1}{2|X|^2} \overset{r\to\infty}{\longrightarrow} 1 + \frac{2M}{r}\,, \tag{2.23}$$

is always finite for finite r, whence M, which is the mass, must be positive. Further, we have to require

$$\frac{1}{2|X|^2} \overset{r\to 0}{\longrightarrow} \frac{A}{4\pi r^2} > 0\,, \tag{2.24}$$

which imposes the existence of an event horizon with area $A > 0$ at $r = 0$ instead of a naked singularity.

The existence of attractors (fixed points) of the scalar fields follows from the fact that in supersymmetric configurations, the scalars satisfy first-order differential equations, as follows immediately from the Killing spinor equations associated to the gaugino supersymmetry transformation rule:

$$\delta_\epsilon \lambda^{Ii} = i\, \not{\partial} Z^i \epsilon^I + \epsilon^{IJ}\, \not{G}^{i\,+} \epsilon_J = 0\,. \tag{2.25}$$

To derive the needed first-order equations, we first use the time-independence of the solutions

$$i\gamma^m \partial_m Z^i \epsilon^I - 4\epsilon^{IJ} G^{i\,+}{}_{0m} \gamma^m \gamma^0 \epsilon_J = 0\,, \tag{2.26}$$

and then the known constraint Eq. (2.11) as to obtain

$$(\partial_m Z^i - 4e^{i\alpha} G^{i\,+}{}_{0m})\gamma^m \epsilon^I = 0\,, \quad \Rightarrow \partial_m Z^i = 4e^{i\alpha} G^{i\,+}{}_{0m}\,. \tag{2.27}$$

[8] If there are multiple attractor regions, it might happen that there is some residual dependency on the asymptotic values. Here we assume there to be only one attractor region.

Going over to curved indices, the equation takes the form

$$\frac{dZ^i}{dr} = 2\sqrt{2} G^{i\,+}{}_{tr}/X^* \,. \tag{2.28}$$

The self-duality of $G^{i\,+}$ allows us to express the $G^{i\,+}{}_{tr}$ component in terms of the $G^{i\,+}{}_{\theta\phi}$:

$$G^{i\,+}{}_{tr} = i({}^\star G^{i\,+})_{\theta\phi} = -i\frac{2|X|^2}{r^2 \sin\theta} G^{i\,+}{}_{\theta\phi}\,, \tag{2.29}$$

which combined with

$$G^{i\,+} = \mathcal{T}^i{}_\Lambda F^{\Lambda\,+} = \tfrac{i}{2}\mathcal{G}^{ij^*}\langle\, \mathfrak{D}_{j^*}\mathcal{V}^* \mid F\,\rangle = \tfrac{i}{2}\mathcal{G}^{ij^*}\mathfrak{D}_{j^*}\langle\, \mathcal{V}^* \mid F\,\rangle\,, \tag{2.30}$$

leads to

$$\frac{dZ^i}{dr} = 2\sqrt{2}\frac{X}{r^2\sin\theta}\mathcal{G}^{ij^*}\mathfrak{D}_{j^*}\langle\, \mathcal{V}^* \mid F_{\theta\phi}\,\rangle\,. \tag{2.31}$$

Since the form of all the fields in terms of $\mathcal{I}(r)$ is in principle known, we can try to find a more explicit form for this equation: using the general form of the vector fields Eq. (2.9) and of $\mathcal{I}(r)$, Eq. (2.21), we find

$$F_{\theta\phi} = \tfrac{1}{\sqrt{2}} r^2 \sin\theta \frac{d\mathcal{I}}{dr} = -\frac{q}{\sqrt{2}}\sin\theta\,. \tag{2.32}$$

After substituting this into Eq. (2.31), one ends up with

$$\frac{dZ^i}{d\rho} = 2X\mathcal{G}^{ij^*}\mathfrak{D}_{j^*}\mathcal{Z}^*\,, \tag{2.33}$$

where $\rho \equiv 1/r$ and where

$$\mathcal{Z}(Z,q) \equiv \langle\, \mathcal{V} \mid q\,\rangle\,, \tag{2.34}$$

is the *central charge of the theory* [15]. Observe that the presence of the factor X in the r.h.s. is crucial for it to have zero global Kähler weight, just as the l.h.s. Further observe that the r-dependence is only through the scalars $Z^i(r)$!

The r.h.s. of this system of differential equations depends only on the scalar fields Z^i, and, thus, it is an autonomous system of ordinary differential equations[9] that has fixed points $Z^i_{\rm fix}$ at the values at which the r.h.s. vanishes

$$\mathfrak{D}_i\mathcal{Z}|_{Z^i=Z^i_{\rm fix}} = 0\,. \tag{2.35}$$

If the solution of this system of equations exists, it gives the fixed values of the scalars $Z^i_{\rm fix}$ as functions of the electric and magnetic charges only

[9]The use of the variable $\rho = 1/r$ is essential in this argument. it is easy to see that the derivatives of the scalar fields of typical black-hole solutions w.r.t. to r do not vanish at $r = 0$, while their derivatives w.r.t. ρ do..

$$Z^i_{\rm fix} = Z^i_{\rm fix}(q)\,, \tag{2.36}$$

since the asymptotic values (moduli) Z^i_∞ do not occur in the above differential equation. The fixed values are reached by the scalars at the value $\rho = \infty$, i.e. $r = 0$, which is where the event horizon would be, as discussed at the beginning of this section and in what follows.

The fixed values may or may not be admissible, i.e. they may or may not belong to the definition domain of the complex coordinates Z^i. If the asymptotic values Z^i_∞ are admissible and the fixed values $Z^i_{\rm fix}(q)$ are not, there must be a singularity between $r = \infty$ and $r = 0$, which will induce a curvature singularity. We will require both the asymptotic and the fixed values to be admissible. These aspects will be discussed in Section 3.

Black-hole solutions whose scalars take the asymptotic values $Z^i_\infty = Z^i_{\rm fix}$ have constant scalar fields, and are called *doubly extreme black holes*. These values are the ones that extremize, not the central charge, but the zero-Kähler-weight combination $e^{\mathcal{K}/2}\mathcal{Z}$:

$$\mathfrak{D}_i \mathcal{Z}|_{Z^i = Z^i_{\rm fix}} = e^{-\mathcal{K}/2}\partial_i \left(e^{\mathcal{K}/2}\mathcal{Z}\right)\Big|_{Z^i=Z^i_{\rm fix}} = 0\,. \tag{2.37}$$

2.2.1 Consequences of the existence of attractors

There are no more scalar fields in the theory, but in the timelike supersymmetric solutions there is another scalar object[10] that satisfies a first-order differential equation: X. From the Killing spinor equation associated to the gravitino supersymmetry transformation rule it is possible to derive [2]

$$\mathfrak{D}_\mu X = -iT^+{}_{\mu\nu}V^\nu\,, \tag{2.38}$$

where V^μ is the timelike Killing vector constructed from the Killing spinor. The graviphoton field strength can be written in the form

$$T^+ = \langle\, \mathcal{V} \mid F\,\rangle\,, \tag{2.39}$$

and, together with

$$V^\nu F_{\nu\mu} = 2\nabla_\mu (|X|^2\mathcal{R})\,, \tag{2.40}$$

the equation for X becomes

$$\mathfrak{D}_\mu X = 2i\langle\, \mathcal{V} \mid \nabla_\mu(|X|^2\mathcal{R})\,\rangle\,. \tag{2.41}$$

Dividing both sides by X and expanding the r.h.s. using $\mathcal{V}/X = \mathcal{R} + i\mathcal{I}$ we get

$$\frac{\mathfrak{D}_\mu X}{X} = 2i|X|^2\langle\, \mathcal{R} \mid \nabla_\mu\mathcal{R}\,\rangle - 2\nabla_\mu|X|^2\langle\, \mathcal{I} \mid \mathcal{R}\,\rangle - 2|X|^2\langle\, \mathcal{I} \mid \nabla_\mu\mathcal{R}\,\rangle\,. \tag{2.42}$$

[10] In previous derivations in the literature the absolute value $|X| = e^U$ is considered, but then the Kähler weights and the reality properties of the two sides of the equations derived are different.

Now, from Eq. (2.7)

$$-2\langle \mathcal{I} \mid \nabla_\mu \mathcal{R} \rangle = \nabla_\mu |X|^{-2} - 2\langle \mathcal{R} \mid \nabla_\mu \mathcal{I} \rangle\,, \tag{2.43}$$

and we get

$$\frac{\mathfrak{D}_\mu X}{X} = 2i|X|^2\langle \mathcal{R} \mid \nabla_\mu \mathcal{R} \rangle - 2|X|^2\langle \mathcal{R} \mid \nabla_\mu \mathcal{I} \rangle\,. \tag{2.44}$$

Finally, using

$$\langle \mathcal{R} \mid \nabla_\mu \mathcal{R} \rangle = \langle \mathcal{I} \mid \nabla_\mu \mathcal{I} \rangle\,, \tag{2.45}$$

which is proved in Appendix A, we arrive at[11]

$$\mathfrak{D}_\mu X^{-1} = 2\langle \mathcal{V}^* \mid \nabla_\mu \mathcal{I} \rangle\,. \tag{2.47}$$

This equation is valid for all supersymmetric configurations in the timelike class. For those considered in this section we arrive at the equation we were looking for:

$$\mathfrak{D}_\rho X^{-1} = 2\mathcal{Z}^*\,. \tag{2.48}$$

The real and imaginary parts of this equation are

$$\frac{d(-g_{rr})}{d\rho} = 2\Re\mathrm{e}(\mathcal{Z}^*/X^*) = 2\langle \mathcal{R} \mid q \rangle\,, \tag{2.49}$$

$$\frac{d\alpha}{d\rho} + \mathcal{Q}_\rho = |X|^2 - 2\Im\mathrm{m}(\mathcal{Z}^*/X^*) = 2\langle \mathcal{I} \mid q \rangle = 2\langle \mathcal{I}_\infty \mid q \rangle\,. \tag{2.50}$$

For the spherically symmetric solutions under consideration ω vanishes and this requires the phase of X to be covariantly constant, i.e.

$$\langle \mathcal{I} \mid q \rangle = \langle \mathcal{I}_\infty \mid q \rangle = 0\,. \tag{2.51}$$

We will later show that this is equivalent to the requirement that the NUT charge vanishes. Since there is only dependence on ρ, the phase of X can simply be gauged away by means of a Kähler transformations. The phase of $\mathcal{Z}$ is then also constant, whence $\mathcal{Z}/X$ is real, which can be used to write

$$\frac{d|X|^{-1}}{d\rho} = \pm 2|\mathcal{Z}|\,. \tag{2.52}$$

[11] Observe that the compatibility between Eq. (2.7) and the following equations requires the identity

$$\langle \nabla_\mu \mathcal{R} \mid \mathcal{I} \rangle = \langle \mathcal{R} \mid \nabla_\mu \mathcal{I} \rangle\,, \tag{2.46}$$

to hold. For theories admitting a prepotential, this is done in Appendix A.

The $\pm$ sign is the sign of $\langle\, \mathcal{R} \mid q \,\rangle$ and we can argue that it has to be positive if the mass is going to be positive: if we take Eq. (2.49) at $\rho = 0$ ($r = \infty$), we find that the mass of the solution is given by the linear combination of charges and moduli

$$M = \langle\, \mathcal{R}_\infty \mid q \,\rangle\,. \tag{2.53}$$

Observe that there is no *a priori* guarantee that $M > 0$: this is a condition that has to be imposed independently as to avoid singularities. We will do so and will only consider the positive sign above; Eq. (2.52) is then the expression found in the literature.

If we take another derivative of Eq. (2.49) and use Eq. (2.52), we find

$$\frac{d^2(-g_{rr})}{d\rho^2} = 2\frac{d|X^{-1}|}{d\rho}|\mathcal{Z}| + 2|X|^{-1}\frac{d|\mathcal{Z}|}{d\rho} = 4|\mathcal{Z}|^2 + 2|X|^{-1}\left(\frac{dZ^i}{d\rho}\partial_i|\mathcal{Z}| + \text{c.c.}\right)\,. \tag{2.54}$$

Now, at $\rho = \rho_{\text{fix}} = 0$ we have $Z^i = Z_{\text{fix}}$ and $dZ^i/d\rho = 0$, and the above equation takes on the form

$$\frac{A}{2\pi} = 4|\mathcal{Z}_{\text{fix}}|^2\,. \tag{2.55}$$

Again, there is no *a priori* guarantee that $|\mathcal{Z}_{\text{fix}}| \neq 0$, which therefore is another condition that has to be imposed independently as to avoid singularities. Actually, even though in this expression A is basically an absolute value, the positivity of A is only guaranteed if the scalar fields take admissible values, the mass is positive etc.

These identities allow us to find two interesting expressions for $|\mathcal{Z}_{\text{fix}}|$. Expanding the two sides of Eq. (2.49) as a power series in ρ we find

$$\frac{A}{2\pi} = 2\langle\, \left.\frac{d\mathcal{R}}{d\rho}\right|_{\rho=0} \mid q \,\rangle\,. \tag{2.56}$$

Using the expressions in Appendix A we get [9, 10]

$$|\mathcal{Z}_{\text{fix}}|^2 = \tfrac{1}{2}\langle\, \left.\frac{d\mathcal{R}}{d\rho}\right|_{\rho=0} \mid q \,\rangle = -\tfrac{1}{2}q^T\mathcal{M}(\mathcal{F}_{\text{fix}})q\,, \tag{2.57}$$

where

$$\mathcal{M}(\mathcal{F}) \equiv \begin{pmatrix} \Im\text{m}\mathcal{F} + \Re\text{e}\mathcal{F}\Im\text{m}\mathcal{F}^{-1}\Re\text{e}\mathcal{F} & -\Im\text{m}\mathcal{F}^{-1}\Re\text{e}\mathcal{F} \\ -\Re\text{e}\mathcal{F}\Im\text{m}\mathcal{F}^{-1} & \Im\text{m}\mathcal{F}^{-1} \end{pmatrix}\,. \tag{2.58}$$

A direct computation of $|\mathcal{Z}_{\text{fix}}|^2$ gives

$$|\mathcal{Z}_{\text{fix}}|^2 = |\langle\, \mathcal{V}_{\text{fix}} \mid q \,\rangle|^2 = -\langle\, q \mid \mathcal{V}_{\text{fix}} \,\rangle\langle\, \mathcal{V}^*_{\text{fix}} \mid q \,\rangle\,. \tag{2.59}$$

The matrix of this bilinear is

$$| \mathcal{V}_{\rm fix} \rangle \langle \mathcal{V}^*_{\rm fix} | = - \left(\begin{array}{cc} \mathcal{M}_\Lambda \mathcal{M}^*_\Sigma & -\mathcal{M}_\Lambda \mathcal{L}^{*\,\Sigma} \\ & \\ -\mathcal{L}^\Lambda \mathcal{M}^*_\Sigma & \mathcal{L}^\Lambda \mathcal{L}^{*\,\Sigma} \end{array} \right)_{\rm fix} . \tag{2.60}$$

We can use the relation

$$\mathcal{L}^{*\,\Lambda} \mathcal{L}^\Sigma = -\tfrac{1}{2} \Im{\rm m}(\mathcal{N})^{-1|\Lambda\Sigma} - f^\Lambda{}_i \mathcal{G}^{ii^*} f^{*\,\Sigma}{}_{i^*} , \tag{2.61}$$

taking into account that at the fixed point the second term in the r.h.s. will not contribute, and that only its symmetric part will contribute, to get [9, 10]

$$|\mathcal{Z}_{\rm fix}|^2 = -\tfrac{1}{2} q^T \mathcal{M}(\mathcal{N}_{\rm fix}) q . \tag{2.62}$$

So far we have checked that the coefficient of the ρ^2 term of $-g_{rr}$ is given by the value of the central charge at the fixed point but, if there are terms of higher order in ρ in $-g_{rr}$ there will not be a regular horizon. We can, however, see that taking another derivative of $-g_{rr}$ w.r.t. ρ at $\rho = 0$ will give zero if the attractor equations (2.35) are satisfied and the same will happen for higher derivatives.

Summarizing we can say that the attractor equations (plus the positivity of the mass, which is not guaranteed) seem to be sufficient conditions to have regular, static, spherically symmetric black holes.

Finally, observe that Eq. (2.53) plus the identification, which will be established later on, between the NUT charge and the linear expression of the charges

$$N = \langle \mathcal{I}_\infty \mid q \rangle , \tag{2.63}$$

lead to a complex BPS relation

$$M + iN = \langle (\mathcal{V}/X)_\infty \mid q \rangle . \tag{2.64}$$

We will argue that supersymmetry requires N to vanish, whence the above relation reads

$$M = \pm\sqrt{2} |\mathcal{Z}_\infty| , \tag{2.65}$$

which is the standard BPS relation between mass and central charge. Of course, only the positive sign will be admissible.

3 Relations between the $N = 2, d = 4$ KSIs, attractors and sources

The equations of motion[12] for supersymmetric configurations of supergravity theories satisfy certain relations known as *Killing spinor identities* (*KSI*s), which can also be derived

[12]By *equation of motion* $\mathcal{E}(\phi)$ of a given field ϕ we will mean here the l.h.s. of the equation of motion $\delta S/\delta\phi = \mathcal{E}(\phi) = 0$. This slight abuse of language should lead to no confusions.

from the integrability conditions of the Killing spinor equations [5, 6]. We have unbroken supersymmetry wherever the Killing spinors exist, and these exist, locally, wherever the KSIs are satisfied. Thus, if we are to have unbroken supersymmetry everywhere we must demand the KSIs to be satisfied everywhere. In this section we are going to study the consequences of demanding the black-hole solutions of $N = 2, d = 4$ supergravity to be everywhere supersymmetric.

The KSIs of $N = 2, d = 4$ supergravity are given in Eqs. (2.13) and (2.14) and they lead to Eqs. (2.15)-(2.20) for configurations in the timelike class. Since we are going to consider configurations that solve the equations of motion, it may seem that the KSIs are automatically satisfied. However, most solutions have singularities at which the equations of motion are not satisfied, i.e. one has $\mathcal{E}(\phi) = \mathcal{J}(\phi)$. The r.h.s. of the equations of motion at the singularities can be associated to sources for the corresponding fields and the KSIs are then understood as relations between the possible sources of supersymmetric solutions: the KSIs put constraints on possible sources of supersymmetric solutions.

Let us consider from this point of view the KSIs Eqs. (2.15)-(2.20): the first of them, Eq. (2.15), tells us that the components $\mathcal{E}^{0m}$ and $\mathcal{E}^{mn}$ of the Einstein equations must vanish automatically for supersymmetric configurations and they must do so everywhere if the solutions are everywhere supersymmetric. This means that the sources $\mathcal{J}^{0m}$ and $\mathcal{J}^{mn}$ of the Einstein equation must vanish identically everywhere

$$\mathcal{J}^{0m} = \mathcal{J}^{mn} = 0\,. \tag{3.1}$$

Hence, singular (delta-like) sources are not allowed, and in particular this means that no localized sources of angular momentum are allowed.

Any singular contributions to $\mathcal{J}^{0m}$ and $\mathcal{J}^{mn}$ must originate in the R^{0m} components of the Ricci tensor; more precisely, they come from the term $\partial_{\underline{m}}(d\omega)_{\underline{mn}}$, where ω is the 1-form that appears off-diagonally in the metric of the timelike supersymmetric solutions of $N = 2, d = 4$ supergravity Eq. (2.6). Therefore, using Eq. (2.8) and defining the complex 3-dimensional vector $\vec{\mathcal{W}}$

$$\vec{\mathcal{W}} = (\mathcal{W}_{\underline{m}}) \equiv (\langle\, \mathcal{V}/X \mid \partial_{\underline{m}}\mathcal{I}\,\rangle)\,, \qquad \Im\mathrm{m}\,(\mathcal{W}_{\underline{m}}) = \tfrac{1}{4}\epsilon_{mnp}(d\omega)_{\underline{np}} = \langle\, \mathcal{I} \mid \partial_{\underline{m}}\mathcal{I}\,\rangle\,, \tag{3.2}$$

we can translate the above KSIs, Eqs. (3.1), to the condition

$$\Im\mathrm{m}\,(\vec{\nabla} \times \vec{\mathcal{W}}) = 0\,, \tag{3.3}$$

which has to be imposed everywhere. Actually, only the singular parts of this equation have to be taken into account since, dealing with solutions, the finite parts must be canceled in the equations of motion by other finite contributions. Therefore, from now on we will ignore all finite contributions to this equation.

Let us consider the real and imaginary parts of Eq. (2.16), namely Eq. (2.20) and (2.19). The real part gives us two important pieces of information: first, it tells us that the component $\mathcal{J}^{00}$ of the source of the Einstein equation is related to component $\mathcal{J}^{0}$ of the source of the combined Maxwell and Bianchi equations $\mathcal{E}^a$. If the electromagnetic fields have only one static point-like source at $r=0$, $\mathcal{E}^t \sim \frac{1}{\sqrt{2}} q\delta^{(3)}(\vec{x})/\sqrt{|g|}$, then using the fact that $\mathcal{Z}/X$ is real (see Eq. (2.51) and the previous discussion)

$$\mathcal{E}^{0t} = \pm 2\sqrt{2}\,|\mathcal{Z}||_{r=0}\,\delta^{(3)}(\vec{x})/\sqrt{|g|}\,, \tag{3.4}$$

which shows that, if the attractor equations are satisfied, the source for the Einstein equations is just $\pm|\mathcal{Z}_{\rm fix}(q)|$. The sign is related to the positivity of $\langle\,\mathcal{R}\mid q\,\rangle$, which is, as was discussed before, associated to the positivity of the mass etc. This is the only value admissible by supersymmetry, since we can understand this source as a source of energy. However, if the scalars take non-admissible values we will find the wrong sign or a zero at $r=0$ and supersymmetry will be broken at the source: we will have to require that the attractor equations are solved by admissible values of the scalars.

The second piece of information we can obtain from the real part concerns the spacelike components of the electromagnetic sources. Combined with the spacelike components of the imaginary part, Eq. (2.19), we get the condition

$$\langle\,\mathcal{V}/X\mid\mathcal{J}^{\underline{m}}\,\rangle = 0\,. \tag{3.5}$$

Let us now consider the time component of the imaginary part of the KSI Eq. (2.16), Eq. (2.19):

$$\langle\,\mathcal{I}\mid\mathcal{J}^{t}\,\rangle = 0\,. \tag{3.6}$$

To find the physical meaning of this condition we use the explicit form of the symplectic vector of vector field strengths F for timelike BPS solutions Eq. (2.9):

$$\mathcal{J}^{\mu} = \mathcal{E}^{\mu} = -({}^{\star}dF)^{\mu} = |X|^2\,(\partial_{\underline{m}}\partial_{\underline{m}}\mathcal{I})\,V^{\mu} = \frac{\delta^{\mu}{}_{t}}{\sqrt{2}}\frac{\partial_{\underline{m}}\partial_{\underline{m}}\mathcal{I}}{\sqrt{|g|}}\,. \tag{3.7}$$

This result tells us that the KSIs Eq. (3.5) are always satisfied and that the KSI Eq. (3.6) is equivalent to the condition

$$\langle\,\mathcal{I}\mid\partial_{\underline{m}}\partial_{\underline{m}}\mathcal{I}\,\rangle = \Im{\rm m}\,(\partial_{\underline{m}}\mathcal{W}_{\underline{m}}) = 0\,, \tag{3.8}$$

which is nothing but the integrability condition for the equation determining ω, which now has to be satisfied everywhere as a consequence of demanding unbroken supersymmetry *everywhere*. For the point-like sources considered above, these equations take the form

$$\sum_{A}\langle\,\mathcal{I}\mid q_A\,\rangle\delta^{(3)}(\vec{x}-\vec{x}^{A})/\sqrt{|g|} = 0\,. \tag{3.9}$$

The consequences of imposing this condition were first studied by Denef and Bates in Refs. [16, 17] in the context of general $N=2, d=4$ supergravity, but was studied earlier by Hartle and Hawking in Ref. [18] in the context of Israel-Wilson-Perjés (IWP) solutions of the Einstein-Maxwell theory. As shown by Tod in Ref. [19] these are precisely the timelike solutions of pure $N = 2, d = 4$ supergravity and a special case of the general problem that we are going to study. Hartle and Hawking were motivated, not by supersymmetry, but rather by the prospect of finding regular solutions describing more than one black hole. They were, in particular, worried about possible string singularities related to NUT charges. These singularities can be eliminated by compactifying the time coordinate with certain period [20], but at the price of losing asymptotic flatness. Let us consider a possible string singularity parametrized by z and choose polar coordinates ρ, ϕ around it. If one considers the integral of the 1-form ω that appears in the metric along a loop of radius R enclosing the possible string singularity at two different points z_1 and z_2, denoted by $I(R, z_{1,2})$, one can use Stokes' theorem to derive

$$I(R, z_1) - I(R, z_2) = \int_{\Sigma^2} d\omega = 2 \int_{\Sigma^2} \star_3 \Im\mathrm{m}\mathcal{W}\,, \tag{3.10}$$

where Σ^2 is a surfaces whose boundaries are the loops of radius R at $z_{1,2}$. In the zero radius limit Σ^2 is a closed surface that crosses the possible string singularity at z_1 and z_2 and we have

$$2\pi \lim_{R\to 0} R[\omega_\phi(R, z_1) - \omega_\phi(R, z_2)] = 2 \int_{\Sigma^2} \star_3 \Im\mathrm{m}\mathcal{W} = \int_{\Sigma^3} d\star_3 \Im\mathrm{m}\mathcal{W} = 2 \int_{\Sigma^3} d^3x \Im\mathrm{m}\,(\partial_{\underline{m}}\mathcal{W}_{\underline{m}})\,, \tag{3.11}$$

where $\partial\Sigma^3 = \Sigma^2$. Thus, $\Im\mathrm{m}\,(\partial_{\underline{m}}\mathcal{W}_{\underline{m}}) \neq 0$ implies that ω_ϕ is singular on the string somewhere between z_1 and z_2. These singularities are related to the presence of NUT sources, since we can define the NUT charge contained in Σ^3 as the integral of $d\omega$ over $\Sigma^2 = \partial\Sigma^3$:

$$-8\pi N_\Sigma = \int_{\Sigma^2} d\omega = \int_{\Sigma^3} d^2\omega = 2 \int_{\Sigma^3} d^3x \Im\mathrm{m}\,(\partial_{\underline{m}}\mathcal{W}_{\underline{m}})\,. \tag{3.12}$$

Thus, the condition $\Im\mathrm{m}\,(\partial_{\underline{m}}\mathcal{W}_{\underline{m}}) = 0$, required by supersymmetry, is equivalent to the absence of sources of NUT charge.

Hartle and Hawking argued that the only solutions in the IWP class with no NUT charge (and no singularities) were the Majumdar-Papapetrou solutions [21, 22] which are regular and static. We will review their arguments in Section 4.1.4 and show that there are indeed non-trivial solutions that satisfy the KSIs and have no NUT charges, apart from the Majumdar-Papapetrou ones; they all have negative total mass, which causes other naked singularities to appear.

Thus, if we include positivity of all masses among the requirements necessary to have supersymmetry, the only supersymmetric black-holes-type solutions of pure $N = 2, d = 4$ supergravity will indeed be the Majumdar-Papapetrou solutions. We will have to consider more general $N = 2, d = 4$ theories in order to be able to have stationary solutions such as the one found in Ref. [1], that satisfy the KSIs and have positive mass. This will be done in Section 4.2.

Next, let us consider the KSI Eq. (2.17) which relates the sources of the scalar fields with those of the vector fields. If we consider only point-like sources and call Σ_A the scalar charge at $\vec{x}_A$, this equation implies, at each sources

$$\Sigma_A = 2e^{-i\alpha}\, \mathfrak{D}_i \mathcal{Z}|_{\vec{x}_A}\ . \tag{3.13}$$

As mentioned before, the scalar sources are completely determined by the electric and magnetic charges and the asymptotic values of the scalar fields. This is known as *secondary scalar hair* [12]. Primary scalar hair correspond to completely free parameters as in the Einstein-scalar solutions of Ref. [23] or in the solutions of Ref. [24] which may be embedded in $N = 4, d = 4$ supergravity. Neither of these solutions is supersymmetric (nor regular) and the above KSI explains just why.

But there is more to the above KSI: it shows that the existence of attractors at the sources implies total absence of scalar sources, either of primary or secondary type. Since this seems to be necessary in order to have regular event horizons, this KSI implies that there will not be supersymmetric black holes with scalar hair in these theories. Unfortunately, it seems possible to have singular supersymmetric solutions with primary scalar hair.

We can summarize the results obtained in this section as follows: we have identified a series of requirements necessary to avoid singularities in supersymmetric black-hole-type solutions of $N = 2, d = 4$ supergravity coupled to vector multiplets, which can be associated to having unbroken supersymmetry everywhere (including the sources).

I The conditions

$$\Im\mathrm{m}\,(\vec{\nabla} \times \vec{\mathcal{W}}) = 0\,, \tag{3.14}$$

$$\Im\mathrm{m}\,(\vec{\nabla} \cdot \vec{\mathcal{W}}) = 0\,, \tag{3.15}$$

have to be satisfied everywhere in order to have supersymmetry everywhere. They ensure the absence of string singularities associated to source of NUT charge and other singularities associated to sources of angular momentum We stress that, when dealing with solutions, all finite contributions to the first equation should be ignored and the second equation can only have singular terms in the l.h.s.

II The mass has to be positive. Actually, the masses of each of the sources of the solutions should be positive. They cannot be rigorously defined in general (for multi-black-hole solutions), but they can be identified with certain confidence in the supersymmetric configurations at hands [25].

III The attractor equations (2.35) must be satisfied at each of the sources for admissible values of the scalars and the value of the central charge at each of them must be finite. As we have seen, the first condition is equivalent to the total absence of scalar sources.

The last two conditions are associated to the finiteness and positivity of $-g_{rr}$ outside the sources. Since $-g_{rr} \sim e^{-\mathcal{K}}$, it would be finite and positive as long as the scalar fields take admissible values within their domain of definition. All the zeroes of $-g_{rr}$ can be related to singularities of the scalar fields. Imposing that the scalar fields take admissible values everywhere is too strong a condition, since it is almost equivalent to directly impose absence of singularities in the metric.

The conditions that we have imposed are, however, heuristically equivalent: for a single black-hole solution the conditions of asymptotic flatness and positivity of the masses ensure positivity of $-g_{rr}$ in the limit $r \to \infty$. The third condition ensures positivity in the $r \to 0$ limit and, furthermore, ensures that there will be a horizon of finite area. Since there are no reasons to expect singularities at finite values of r, the positivity and finiteness should hold for all finite values of r. The same should happen in multi-black-hole solutions.

4 $N = 2, d = 4$ attractors, KSIs and BPS black-hole sources

Now we want to apply the results of the previous sections to several examples of black-hole-type solutions of $N = 2, d = 4$ supergravity theories, demanding the three conditions formulated in the introduction and checking the regularity of those solutions that satisfy them. We are going to start with the simplest theory.

4.1 Pure $N = 2, d = 4$ supergravity

This theory has $\bar{n} = 1$, no scalar fields, and it is given by the prepotential

$$\mathcal{F} = -\tfrac{i}{2}(\mathcal{X}^0)^2\,, \ \Rightarrow \mathcal{F}_0 = -i\mathcal{X}^0\,. \tag{4.1}$$

This implies that the components of the symplectic section $\mathcal{V}$ are constant

$$\mathcal{L}^0 = i\mathcal{M}_0 = e^{i\gamma}/\sqrt{2}\,, \tag{4.2}$$

and X is not related to any Kähler potential, but

$$X = \frac{e^{i\gamma}}{\sqrt{2}}(\mathcal{L}^0/X)^{-1} = \frac{e^{i\gamma}}{\sqrt{2}(\mathcal{R}^0 + i\mathcal{I}^0)}\,. \tag{4.3}$$

The central charge is constant and given by

$$\mathcal{Z} = -\frac{ie^{i\gamma}}{\sqrt{2}}(p^0 - iq_0) \equiv -\frac{ie^{i\gamma}}{\sqrt{2}}\tilde{q}\,. \tag{4.4}$$

The attractor equations do not make sense because $\mathcal{Z}$ is already moduli-independent.

The timelike supersymmetric configurations of this theory were first found by Tod in his pioneering paper Ref. [19], belong to the family of solutions found by Perjés, Israel and

Wilson (IWP) [26, 27]; they are completely determined by the choice of a single complex, harmonic function that we denote by $\tilde{\mathcal{I}}$. In the framework of general $N = 2, d = 4$ theories, the solutions of pure $N = 2, d = 4$ supergravity are given by just two real harmonic functions $\mathcal{I}^0$ and $\mathcal{I}_0$, the components of the real symplectic vector $\mathcal{I}$. The relation between $\mathcal{I}$ and $\tilde{\mathcal{I}}$ is

$$\tilde{\mathcal{I}} = \mathcal{I}^0 - i\mathcal{I}_0\,. \tag{4.5}$$

Observe that

$$X = -\frac{ie^{i\gamma}}{\sqrt{2}\tilde{I}}\,, \tag{4.6}$$

and therefore $\sqrt{2}X$ coincides with the function V of Ref. [19] and is the inverse of the complex harmonic function.

It is convenient to use the complex formulation of this theory. In it, the symplectic product of two real symplectic vectors x, y can be written in the form $\langle\, x \mid y\,\rangle = \Im\mathrm{m}\,(\tilde{x}^*\tilde{y})$ where the tilde indicates complexification ($\tilde{x} = x^0 - ix_0$ etc.). Further, electric-magnetic duality rotations of the symplectic vectors is equivalent to multiplication by a global phase $\tilde{x}' = e^{i\gamma}\tilde{x}$. We would like to stress that the metric is invariant under these transformations.

Using Eq. (4.1) one finds that $\mathcal{R}$, the real part of $\mathcal{V}/X$ is the symplectic vector

$$\mathcal{R} = \begin{pmatrix} -\mathcal{I}_0 \\ \mathcal{I}^0 \end{pmatrix}\,, \;\Rightarrow \tilde{\mathcal{R}} = -i\tilde{\mathcal{I}}\,,\; \Rightarrow -g_{rr} = \frac{1}{2|X|^2} = \langle\, \mathcal{R} \mid \mathcal{I}\,\rangle = |\tilde{\mathcal{I}}|^2\,. \tag{4.7}$$

Finally,

$$\vec{\mathcal{W}} = \tilde{\mathcal{I}}^*\vec{\nabla}\tilde{\mathcal{I}}\,. \tag{4.8}$$

It was argued by Hartle and Hawking [18] that the only regular black hole solutions in the IWP family are the static Majumdar-Papapetrou solutions that describe several charged black holes in static equilibrium. We are going to see that these are in fact the only solutions which are everywhere supersymmetric (condition I) and that demanding positivity of the masses of the components (condition II) is enough to have regular black holes (condition III plays no rôle here).

4.1.1 Single, static black hole solutions

The complex harmonic function $\tilde{\mathcal{I}}$ adequate to describe a static, spherically symmetric, extreme black hole with magnetic and electric charges p^0 and q_0 is

$$\tilde{\mathcal{I}} = \tilde{\mathcal{I}}_\infty + \frac{\tilde{q}}{r}\,, \qquad \tilde{q} \equiv p^0 - iq_0\,, \tag{4.9}$$

and asymptotic flatness requires $|\tilde{\mathcal{I}}_\infty| = 1$. Since $\tilde{\mathcal{I}}_\infty$ is just a phase that can be taken to be unity by an electric-magnetic duality rotation. Then,

$$-g_{rr} = |\tilde{\mathcal{I}}|^2 = 1 + \frac{2\Re\mathrm{e}(\tilde{\mathcal{I}}^*_\infty \tilde{q})}{r} + \frac{|\tilde{q}|^2}{r^2}\,. \tag{4.10}$$

The mass is given by

$$M = \Re\mathrm{e}(\tilde{\mathcal{I}}^*_\infty \tilde{q}) = \langle\, \mathcal{R}_\infty \mid q\,\rangle\,, \tag{4.11}$$

and the equations of motion and supersymmetry seem to allow for it to be positive or negative. When M is negative $|\tilde{I}|^2$ will vanish for some finite value of r, giving rise to a naked singularity. In the limit $r \to 0$, which makes sense if M is positive, we find that the area of the 2-spheres of constant t and r is finite and equal to

$$A = 4\pi|\tilde{q}|^2 = 8\pi|\mathcal{Z}|^2\,. \tag{4.12}$$

Observe that, in general,

$$|M| \neq \sqrt{2}|\mathcal{Z}|\,, \tag{4.13}$$

even though these solutions are usually understood to be supersymmetric.

For this solution Eq. (3.14) is automatically satisfied, while Eq. (3.15) takes the form

$$\Im\mathrm{m}\,(\vec{\nabla}\cdot\vec{\mathcal{W}}) = -4\pi\Im\mathrm{m}\,(\tilde{\mathcal{I}}^*_\infty \tilde{q})\,\delta^{(3)}(\vec{x}) = 0\,. \tag{4.14}$$

We can, either

1. Adopt the point of view proposed in this paper that the integrability condition has to be satisfied everywhere (condition I), whence impose the condition

   $$\Im\mathrm{m}\,(\tilde{\mathcal{I}}^*_\infty \tilde{q}) = \langle\, \mathcal{I}_\infty \mid q\,\rangle = 0\,. \tag{4.15}$$

   $\tilde{\mathcal{I}}_\infty$ is just a phase and this condition determines it: $\tilde{\mathcal{I}}_\infty = \pm\tilde{q}/|\tilde{q}| \equiv e^{i\beta}$. The complex harmonic function becomes

   $$\tilde{\mathcal{I}} = e^{i\beta}\left(1 \pm \frac{|\tilde{q}|}{r}\right)\,, \tag{4.16}$$

   The overall phase $e^{i\beta}$ is irrelevant for our problem (it can always be eliminated by an electric-magnetic duality rotation that does not change the metric), but the relative sign between the two terms, which is the sign of the mass,

   $$M = \pm|\tilde{q}| = \pm|\mathcal{Z}|\,, \tag{4.17}$$

   is important since the minus sign leads to naked singularities. We take the positive sign as to comply with condition II. We can the integrate the equation for ω everywhere. The above condition, however, implies the vanishing of the r.h.s. of the equation and, therefore, also that of ω. Thus, after imposing conditions I and II we obtain a solution which is static and spherically symmetric and has a regular horizon if $M > 0$; Or

2. We can accept this singularity, ignoring condition I, arguing that, after all, the harmonic functions are already singular at that point[13] and proceed to integrate the equation and obtain ω which, in spherical coordinates, takes the form

$$\omega = 2N \cos\theta d\phi \,, \tag{4.18}$$

where N is NUT charge and it is given by

$$N = \Im\mathrm{m}\,(\tilde{\mathcal{I}}^*_\infty \tilde{q}) = \langle\, \mathcal{I}_\infty \mid q \,\rangle\,, \;\; \Rightarrow |M + iN| = \sqrt{2}|\mathcal{Z}_\infty|\,. \tag{4.19}$$

The metric is no longer static, but stationary, and contains either wire singularities or closed timelike curves plus Taub-NUT asymptotics.

It is clear that by imposing conditions I and II, these pathologies are avoided. Furthermore, in the microscopic models of black holes constructed in the framework of String Theory there seem to be no configurations that give rise to macroscopic NUT charge (nor to negative masses). The agreement between spacetime supersymmetry and the microscopic String Theory models on this point, together with the elimination of pathologies is encouraging and we will see that it applies to more cases.

4.1.2 Single black hole solutions with a dipole term

Let us now consider harmonic functions adequate to describe rotating supersymmetric black holes. We can add angular momentum to the previous solution by adding a dipole term to its complex harmonic function which becomes:

$$\tilde{\mathcal{I}} = \tilde{\mathcal{I}}_\infty + \frac{\tilde{q}}{r} + (\vec{\tilde{m}} \cdot \vec{\nabla})\frac{1}{r}\,, \tag{4.20}$$

where $\vec{m} = (\vec{m}^0, \vec{m}_0)$ is a symplectic vector of dipole magnetic and electric momenta. When they are parallel we can take them to have only z component and, then, in spherical coordinates

$$\tilde{\mathcal{I}} = \tilde{\mathcal{I}}_\infty + \frac{\tilde{q}}{r} - \frac{\tilde{m}\cos\theta}{r^2}\,. \tag{4.21}$$

The corresponding ω (which exists except at the singularities of $\tilde{\mathcal{I}}$) is

$$\omega = \left[2N\cos\theta + 2J\frac{\sin^2\theta}{r^2} + \Im\mathrm{m}(\tilde{q}^*\tilde{m})\frac{\sin^2\theta}{r^3}\right] d\phi\,. \tag{4.22}$$

[13] We have seen that the solution can, nevertheless, be regular at that point, which is the event horizon.

N is the NUT charge and is given again by Eq. (4.19). The new features are J, the z component of the angular momentum, given by

$$J = \Im\mathrm{m}\,(\tilde{\mathcal{I}}^{*}_{\infty}\tilde{m}) = \langle\, \mathcal{I}_{\infty} \mid m, \rangle\,, \tag{4.23}$$

and $\Im\mathrm{m}(\tilde{q}^{*}\tilde{m})$ which does not have a conventional name but vanishes when $N = J = 0$.

Let us now analyze the KSIs Eqs. (3.14) and (3.15) (condition I). In the general case they take, respectively, the form

$$2\left[\Im\mathrm{m}\,(\tilde{q}^{*}\nabla_{m})\vec{\nabla}\frac{1}{r}\right]\times\vec{\nabla}\frac{1}{r} - i\left(\nabla_{m^{*}}\vec{\nabla}\frac{1}{r}\right)\times\left(\nabla_{m}\vec{\nabla}\frac{1}{r}\right) = 0\,, \tag{4.24}$$

$$\begin{aligned}\Im\mathrm{m}\,(\tilde{\mathcal{I}}^{*}_{\infty}\tilde{q})\,\delta^{(3)}(\vec{x}) + \Im\mathrm{m}\,(\tilde{\mathcal{I}}^{*}_{\infty}\nabla_{m})\,\delta^{(3)}(\vec{x}) + \frac{1}{r}\Im\mathrm{m}\,(\tilde{q}^{*}\nabla_{m})\,\delta^{(3)}(\vec{x}) + \\ +\delta^{(3)}(\vec{x})\Im\mathrm{m}\,(\tilde{q}\nabla_{m^{*}})\frac{1}{r} + \Im\mathrm{m}\,\left\{\left(\nabla_{m^{*}}\frac{1}{r}\right)\left(\nabla_{m}\delta^{(3)}(\vec{x})\right)\right\} = 0\,,\end{aligned} \tag{4.25}$$

and are satisfied if

$$N = \Im\mathrm{m}\,(\tilde{\mathcal{I}}^{*}_{\infty}\tilde{q}) = \langle\, \mathcal{I}_{\infty} \mid q\,\rangle = 0\,, \tag{4.26}$$

$$\vec{J} = \Im\mathrm{m}\,(\tilde{\mathcal{I}}^{*}_{\infty}\vec{\tilde{m}}) = \langle\, \mathcal{I}_{\infty} \mid \vec{m}\,\rangle = 0\,, \tag{4.27}$$

$$\Im\mathrm{m}\,(\tilde{q}^{*}\vec{\tilde{m}}) = \langle\, q \mid \vec{m}\,\rangle = 0\,, \tag{4.28}$$

$$\Im\mathrm{m}\,(\tilde{m}^{*}_{[\underline{m}}\tilde{m}_{\underline{n}]}) = \langle\, m_{[\underline{m}} \mid m_{\underline{n}]}\,\rangle = 0\,, \tag{4.29}$$

where we have defined the differential operator $\nabla_{m} \equiv \vec{\tilde{m}}\cdot\vec{\nabla}$ and where we have taken into account Eq. (4.23) to identify the angular momentum.

The first condition is, again, the absence of sources of NUT charge. The second condition is the absence of sources of angular momentum. The third and fourth conditions are automatically satisfied in this theory if the first two are.

In this case, these conditions are not enough to eliminate all the singularities introduced by the dipole term since the above conditions do not cancel terms like $|\vec{\tilde{m}}\cdot\vec{\nabla}\frac{1}{r}|^{2}$ in the g_{rr} component of the metric and we no longer find a regular 2-sphere in the $r \to 0$ limit. However, we are going to argue that, although technically possible, dipole terms should not be allowed in $\mathcal{I}$ because their only possible origin is a distribution of point-like charges and it is the fundamental distribution of point-like charges that we have to consider in the above equations and not the field they produce at distances larger than its size. It is in these conditions that imposing supersymmetry everywhere is equivalent to cosmic censorship.

Indeed, from the point of view of the electromagnetic fields, the magnetic dipole momenta, for instance, can have two fundamental origins: dipole momenta in a distribution of magnetic monopoles or fundamental dipole momenta that can be seen as stationary electric currents. In standard electrodynamics the first possibility is experimentally excluded (see, e.g. Ref. [28]) but in $N = 2, d = 4$ supersymmetric configurations it is the only one allowed (see Eq. (3.7)).

4.1.3 The supersymmetric Kerr-Newman solution

Therefore we must only consider distributions of static point-like charges. We will do so in a moment, but there is an interesting example of rotating black-hole-type solution which must be considered before: it is given by the complex harmonic function

$$\tilde{\mathcal{I}} = \tilde{\mathcal{I}}_\infty + \frac{\tilde{q}}{\tilde{r}}\,, \qquad \tilde{r} \equiv \sqrt{x^2 + y^2 + (z - i\alpha)^2}\,, \tag{4.30}$$

which is known to lead to the ("ultra-extreme") supersymmetric Kerr-Newman solution with angular momentum around the z axis; as is known it has naked singularities, as all 4-dimensional supersymmetric rotating "black-holes" [29]. This is the prototype of solution for which supersymmetry does not act as a "cosmic censor" as proposed in [14]. Generalizations of this solution in some other $N = 2, d = 4$ theories have been constructed in Ref. [4].

The asymptotic expansion of $\tilde{I}$

$$\tilde{\mathcal{I}} \sim \tilde{\mathcal{I}}_\infty + \frac{\tilde{q}}{r} - \frac{i\alpha\tilde{q}z}{r^3} + \cdots\,, \tag{4.31}$$

corresponds to a charge distribution with only two independent parameters: α and $\tilde{q}$. The magnetic (electric) dipole momentum is equal to the product of α and the electric (magnetic) charge and the infinite number of non-vanishing higher momenta depend also on these few parameters.

According to the point of view advocated here this solution should not be considered because it corresponds to the far field of a very charge distribution. As we are going to see, condition I is enough to exclude it.

Finding the sources of the solution associated to the above complex harmonic function is very complicated. To start with, $\tilde{\mathcal{I}}$ is singular on the ring $x^2 + y^2 = \alpha^2\,,\ \ z = 0$ but it is also discontinuous on a disk bounded by the ring (see e.g. [30], whose results we are going to use here. See also Refs. [31, 32].).

Eqs. (3.14) and (3.15), which express condition I, take, respectively, the form

$$\Im\mathrm{m}\,(\tilde{\mathcal{I}}^*_\infty\tilde{q})\Re\mathrm{e}\,(\vec{\nabla}\times\vec{C}) + \Re\mathrm{e}\,(\tilde{\mathcal{I}}^*_\infty\tilde{q})\Im\mathrm{m}\,(\vec{\nabla}\times\vec{C}) + |\tilde{q}|^2\Im\mathrm{m}\left(\frac{1}{\tilde{r}^*}\vec{\nabla}\times\vec{C}\right) = 0\,, \tag{4.32}$$

$$\Im\mathrm{m}\,(\tilde{\mathcal{I}}^*_\infty\tilde{q})\Re\mathrm{e}\,(\vec{\nabla}\cdot\vec{C}) + \Re\mathrm{e}\,(\tilde{\mathcal{I}}^*_\infty\tilde{q})\Im\mathrm{m}\,(\vec{\nabla}\cdot\vec{C}) + |\tilde{q}|^2\Im\mathrm{m}\left(\frac{1}{\tilde{r}^*}\vec{\nabla}\cdot\vec{C}\right) = 0\,, \tag{4.33}$$

where we have defined

$$\vec{C} \equiv \frac{(x, y, z - i\alpha)}{[x^2 + y^2 + (z - i\alpha)^2]^{3/2}} \,. \tag{4.34}$$

The curl and divergence of $\vec{C}$ have been carefully computed in Ref. [30] in a distributional sense, i.e. as integrals of their products with test functions. For us it is enough to known that

$$\Re\mathrm{e}\,(\vec{\nabla} \times \vec{C}) = \Im\mathrm{m}\,(\vec{\nabla} \cdot \vec{C}) = 0\,, \tag{4.35}$$

and that $\Im\mathrm{m}\,(\vec{\nabla} \times \vec{C})$ vanishes for vanishing α. We are left with

$$\left[\Re\mathrm{e}\,(\tilde{\mathcal{I}}^*_\infty \tilde{q}) + |\tilde{q}|^2 \Re\mathrm{e}\,\frac{1}{\tilde{r}}\right] \Im\mathrm{m}\,(\vec{\nabla} \times \vec{C}) \;=\; 0\,, \tag{4.36}$$

$$\left[\Im\mathrm{m}\,(\tilde{\mathcal{I}}^*_\infty \tilde{q}) - |\tilde{q}|^2 \Im\mathrm{m}\,\frac{1}{\tilde{r}}\right] \Re\mathrm{e}\,(\vec{\nabla} \cdot \vec{C}) \;=\; 0\,. \tag{4.37}$$

The only way to satisfy the first condition is to have $\Im\mathrm{m}\,(\vec{\nabla} \times \vec{C}) = 0$, which requires $\alpha = 0$ (no sources of angular momentum). Since $\Re\mathrm{e}\,(\vec{\nabla} \cdot \vec{C}) \neq 0$ always, the only way to satisfy the second condition is to have $\Im\mathrm{m}\,(\tilde{\mathcal{I}}^*_\infty \tilde{q}) = 0$ as before (no sources of NUT charge) and $\Im\mathrm{m}\,\frac{1}{\tilde{r}} = 0$ which also requires $\alpha = 0$.

Thus, imposing supersymmetry everywhere is equivalent, yet again, to requiring absence of sources of NUT charge and angular momentum. In the supersymmetric Kerr-Newman solution all the angular momentum originates in that source[14] and, thus, that solution and its naked singularities can be excluded from the class of everywhere supersymmetric solutions of $N = 2, d = 4$ supergravity. Again, supersymmetry acts as a cosmic censor and, most importantly, there is agreement between the macroscopic description of black holes provided by Supergravity and the microscopic models provided by String Theory in which there seems to be no way of having angular momentum without breaking supersymmetry.

Therefore, we must only consider distributions of point-like charges, which correspond to complex harmonic functions of the form

$$\tilde{\mathcal{I}} = \tilde{\mathcal{I}}_\infty + \sum_A \frac{\tilde{q}_A}{|\vec{x} - \vec{x}_A|}\,, \tag{4.38}$$

from which dipole (and higher) momenta arise only in asymptotic expansions:

$$\tilde{\mathcal{I}} \sim \tilde{\mathcal{I}}_\infty + \frac{\sum_A \tilde{q}_A}{|\vec{x}|} + \frac{(\sum_A \tilde{q}_A \vec{x}_A) \cdot \vec{x}}{|\vec{x}|^3} + \cdots\,, \tag{4.39}$$

[14]We are going to see that there are solutions with angular momentum and no elementary sources of angular momentum.

and may give rise to non-vanishing angular momentum

$$\vec{J} = \Im\mathrm{m}\,(\tilde{\mathcal{I}}^*_\infty \tilde{\vec{m}}) = \langle\, \mathcal{I}_\infty \mid \vec{m}\,\rangle\,, \qquad \vec{m} = -\sum_A q_A \vec{x}_A\,, \tag{4.40}$$

but not to non-vanishing NUT charge.

$$N = \Im\mathrm{m}\,(\tilde{\mathcal{I}}^*_\infty \tilde{q}) = \langle\, \mathcal{I}_\infty \mid q\,\rangle = 0\,, \qquad q = \sum_A q_A\,. \tag{4.41}$$

We are going to look for this kind of solutions in pure $N = 2, d = 4$ supergravity next, recovering the (negative) Hartle and Hawking result [18]. We will have to look for them in more general $N = 2, d = 4$ theories.

4.1.4 Solutions with two black holes

Let us consider, to start with, just two poles

$$\tilde{\mathcal{I}} = \tilde{\mathcal{I}}_\infty + \frac{\tilde{q}_1}{|\vec{x}-\vec{x}_1|} + \frac{\tilde{q}_2}{|\vec{x}-\vec{x}_2|}\,. \tag{4.42}$$

Asymptotic flatness requires $|\tilde{\mathcal{I}}_\infty| = 1$. The condition Eq. (3.14) is automatically satisfied and (3.15) takes the form

$$\left[\langle\, \mathcal{I}_\infty \mid q_1\,\rangle + \frac{\langle\, q_2 \mid q_1\,\rangle}{|\vec{x}_1-\vec{x}_2|}\right]\delta^{(3)}(\vec{x}-\vec{x}_1) + \left[\langle\, \mathcal{I}_\infty \mid q_2\,\rangle + \frac{\langle\, q_1 \mid q_2\,\rangle}{|\vec{x}_2-\vec{x}_1|}\right]\delta^{(3)}(\vec{x}-\vec{x}_2) = 0\,, \tag{4.43}$$

which leads to the two equations

$$\begin{aligned} \langle\, \mathcal{I}_\infty \mid q_1\,\rangle + \frac{\langle\, q_2 \mid q_1\,\rangle}{|\vec{x}_1-\vec{x}_2|} &= 0\,, \\ \langle\, \mathcal{I}_\infty \mid q_2\,\rangle + \frac{\langle\, q_1 \mid q_2\,\rangle}{|\vec{x}_2-\vec{x}_1|} &= 0\,, \end{aligned} \tag{4.44}$$

each of which expresses the absence of sources of NUT charge at $\vec{x}_1$ and $\vec{x}_2$. The antisymmetry of the symplectic product implies the consistency condition

$$\langle\, \mathcal{I}_\infty \mid q_1 + q_2\,\rangle = 0\,, \tag{4.45}$$

which means that the total charge of the two objects satisfies the same condition (no global NUT charge) as the charge of just one.

Expanding asymptotically $\mathcal{I}$ and using the above constraints we find that this two-body system has a total mass and angular momentum given by

$$
\begin{aligned}
M &= \sum_A \langle\, \mathcal{R}_\infty \mid q_A \,\rangle \equiv \sum_A M_A\,, && (4.46)\\
\vec{J} &= \langle\, \mathcal{I}_\infty \mid \vec{m}\,\rangle = \langle\, q_1 \mid q_2\,\rangle \frac{(\vec{x}_2-\vec{x}_1)}{|\vec{x}_2-\vec{x}_1|}\,. && (4.47)
\end{aligned}
$$

Observe that there is total angular momentum even though there are no sources of angular momentum.

There are two types of solutions to these equations required by condition I:

1. Each object's charge satisfies the condition for single independent objects $\langle\, \mathcal{I}_\infty \mid q_A\,\rangle = 0$ which requires $\langle\, q_2 \mid q_1\,\rangle = 0$. In this theory this means that the phases of $\tilde{\mathcal{I}}_\infty, \tilde{q}_1$ and $\tilde{q}_2$ are such that

   $$\tilde{I} = e^{i\beta}\left(1+\sum_A \frac{s_A|\tilde{q}_A|}{|\vec{x}-\vec{x}_A|}\right)\,, \tag{4.48}$$

   where $s_A = \pm 1$. The total mass is given by the formula Eq. (4.11)

   $$M = \Re\mathrm{e}(\tilde{\mathcal{I}}^*_\infty \sum_A \tilde{q}_A) = \langle\, \mathcal{R}_\infty \mid \sum_A q_A\,\rangle = \sum_A s_A|\tilde{q}_A|\,, \tag{4.49}$$

   and the angular momentum vanishes (ω vanishes).

   These are the Majumdar-Papapetrou solutions [21, 22]. Only the solutions with all $s_A = +1$ are regular, but one could argue that only those correspond to objects that would have positive masses $M_A = |\tilde{q}_A|$ if they were isolated [25]. This is the meaning of condition II.

   These solutions describe two charged, static black holes in equilibrium with their event horizons placed at $\vec{x}_1$ and $\vec{x}_2$ which are really 2-spheres of finite areas equal to $4\pi|\tilde{q}_1|^2$ and $4\pi|\tilde{q}_2|^2$. They are, as argued by Hartle and Hawking, and as we are going to see, the only regular black-hole-type solutions in the whole IWP family [18]

2. $\langle\, \mathcal{I}_\infty \mid q_A\,\rangle \neq 0$ and we have two objects that cannot exist independently in the vacuum $\mathcal{I}_\infty$ (i.e. we have a bound state). The distance between them is fixed by the condition of absence of sources of NUT charge to be

   $$|\vec{x}_2-\vec{x}_1| = \frac{\langle\, q_1 \mid q_2\,\rangle}{\langle\, \mathcal{I}_\infty \mid q_1\,\rangle}\,. \tag{4.50}$$

   The sign of the r.h.s. can always be made positive by flipping the sign of $\mathcal{I}_\infty$, which is irrelevant for the moduli and for solving Eq. (4.45). Thus, this equation always has a solution. However, when all the above conditions have been satisfied, the total mass of the solution is negative. The simplest way to see this is by first making $\tilde{\mathcal{I}}_\infty = 1$ by a duality rotation that does not change the metric. After the duality rotation one finds $\tilde{q}'_A = M_A + iN_A$, meaning that they are complex combinations of the masses and NUT charges of each object. Using $N_2 = -N_1$, the above condition takes the form

$$N_1 + \frac{N_1 M_2 - N_2 M_1}{|\vec{x}_2 - \vec{x}_1|} = N_1 \left(1 + \frac{M_1 + M_2}{|\vec{x}_2 - \vec{x}_1|}\right) = 0\,, \tag{4.51}$$

which has solution only for vanishing NUT charges or for negative total mass $M_1 + M_2$ which violates condition II and produces naked singularities. Thus, we cannot simultaneously satisfy conditions I and II for bound states with $\langle\, q_1 \mid q_2 \,\rangle \neq 0$.

This result can be generalized to solutions with more poles: let us consider first the 3-pole harmonic function

$$\tilde{\mathcal{I}} = \tilde{\mathcal{I}}_\infty + \frac{\tilde{q}_1}{|\vec{x} - \vec{x}_1|} + \frac{\tilde{q}_2}{|\vec{x} - \vec{x}_2|} + \frac{\tilde{q}_3}{|\vec{x} - \vec{x}_3|}\,. \tag{4.52}$$

The ω integrability condition leads to three equations (one to cancel the NUT charge at each pole) which can be written as a linear system for the N_As:

$$\begin{pmatrix} \left(1 + \frac{M_2}{r_{12}} + \frac{M_3}{r_{14}}\right) & -\frac{M_1}{r_{12}} & -\frac{M_1}{r_{13}} \\ -\frac{M_2}{r_{12}} & \left(1 + \frac{M_1}{r_{12}} + \frac{M_3}{r_{23}}\right) & -\frac{M_2}{r_{23}} \\ -\frac{M_3}{r_{13}} & -\frac{M_3}{r_{23}} & \left(1 + \frac{M_1}{r_{13}} + \frac{M_2}{r_{23}}\right) \end{pmatrix} \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix} = 0\,. \tag{4.53}$$

It is easy to see that the determinant of the matrix is +1 plus terms linear and quadratic in the masses, all with positive sign. It will never vanish if all the masses are positive. This argument can be easily generalized to a higher number of poles and, therefore we conclude that the only solutions satisfying conditions I and II are the Majumdar-Papapetrou solutions. This result should be read in a positive sense: no singular solutions are allowed by the conditions proposed in the introduction, even if only static solutions are allowed in this simple theory. To find solutions with angular momentum satisfying conditions I-III we need to consider theories with scalars.

4.2 General $N = 2, d = 4$ supergravity

The setup of our problem in general $N = 2, d = 4$ theories is similar to pure supergravity case. Let us first consider spherically-symmetric, static, single black-hole-type solutions with magnetic and electric charges p^Λ and q_Λ. They are determined by a symplectic vector of $2\bar{n}$ real harmonic functions

$$\mathcal{I} = \begin{pmatrix} \mathcal{I}^\Lambda \\ \mathcal{I}_\Lambda \end{pmatrix} = \mathcal{I}_\infty + \frac{q}{r}\,, \qquad q \equiv \begin{pmatrix} p^\Lambda \\ q_\Lambda \end{pmatrix}\,, \qquad \mathcal{I}_\infty \equiv \begin{pmatrix} \mathcal{I}^\Lambda_\infty \\ \mathcal{I}_{\Lambda\,\infty} \end{pmatrix}\,. \tag{4.54}$$

We assume that the stabilization equations have been solved and $\mathcal{R}(\mathcal{I})$ has been found in order to be able to construct the fields of the solutions.

The n complex scalars are constructed using the general formula Eq. (2.5). The moduli (the values of the n complex scalars Z^i at infinity, Z^i_∞) are complicated functions $Z^i_\infty(\mathcal{I}_\infty)$ of these $2n+2$ real constant components of $\mathcal{I}_\infty$. One of the components of $\mathcal{I}_\infty$ can be determined as a function of the remaining $2n+1$ by imposing asymptotic flatness of the metric, that is, $\langle\, \mathcal{R}_\infty \mid \mathcal{I}_\infty\,\rangle = 1$, and another one can be determined by imposing condition I, since Eq. (3.8) implies

$$N = \langle\, \mathcal{I}_\infty \mid q\,\rangle = 0\,. \tag{4.55}$$

It should always be possible to give the $2n$ real moduli any admissible value within their definition domain with the remaining $2n$ unconstrained real components of $\mathcal{I}_\infty$. This is difficult to prove explicitly due to the complicated and theory-dependent relations between $\mathcal{I}_\infty$ and the moduli Z^i_∞, but it is safe to assume that in general it is possible.

Let us turn to condition II. The positivity of the masses, which is given by the general expression Eq. (2.53) has to be imposed by hand and, although this can always be done, it is a non-trivial constraint on the charges and moduli. The positivity of the masses can be also understood as part of a stronger requirement that the scalar fields take values only within their definition domain for all values of r. Actually, this requirement should suffice to ensure the finiteness of $-g_{rr}$ for $r \neq 0$.

The finiteness of $-g_{rr}$ for $r \neq 0$ is not enough to have a black hole and condition III has to be imposed to find a finite horizon area at $r = 0$.

If we want to describe more than one black hole we have to use harmonic functions with two point-like singularities:

$$\mathcal{I} = \mathcal{I}_\infty + \frac{q_1}{|\vec{x}-\vec{x}_1|} + \frac{q_2}{|\vec{x}-\vec{x}_2|}\,. \tag{4.56}$$

Again, one of the components of $\mathcal{I}_\infty$ is determined by imposing asymptotic flatness. Condition I now leads to the two equations Eqs. (4.44) which should determine another component of $\mathcal{I}_\infty$ and the parameter $|\vec{x}_1 - \vec{x}_2|$ if $\langle q_2 \mid q_1\rangle \neq 0$. The question now is whether these solutions can be obtained while maintaining the positivity of the masses (condition II)

$$M_i \equiv \langle\, \mathcal{R}_\infty \mid q_i\,\rangle > 0\,, \tag{4.57}$$

and solving the attractor equations for each of the singularities of the harmonic functions. We have no general answer to these questions and, what we are going to do is to study how the three conditions can actually be imposed in a particularly simple example and suffice to ensure regularity of the solutions.

4.2.1 A toy model with a complex scalar field

We are going to consider the $\bar{n} = 2$ theory with prepotential

$$\mathcal{F} = -i\mathcal{X}^0\mathcal{X}^1 \,. \tag{4.58}$$

This theory has only one complex scalar

$$\tau \equiv i\mathcal{X}^1/\mathcal{X}^0 \,, \tag{4.59}$$

in terms of which the period matrix is given by

$$(\mathcal{N}_{\Lambda\Sigma}) = \begin{pmatrix} -\tau & 0 \\ 0 & 1/\tau \end{pmatrix} \tag{4.60}$$

and, in the $\mathcal{X}^0 = i/2$ gauge, the Kähler potential and metric are

$$\mathcal{K} = -\ln \Im\mathrm{m}\tau \,, \qquad \mathcal{G}_{\tau\tau^*} = (2\Im\mathrm{m}\tau)^{-2} \,. \tag{4.61}$$

The reality of the Kähler potential requires the positivity of $\Im\mathrm{m}\tau$. Therefore, τ parametrizes the coset $SL(2,\mathbb{R})/SO(2)$ and can be identified with the *axidilaton* and this theory is a truncation of the $SO(4)$ formulation of $N = 4, d = 4$ supergravity.

The symplectic section $\mathcal{V}$ is

$$\mathcal{V} = \frac{1}{2(\Im\mathrm{m}\tau)^{1/2}} \begin{pmatrix} i \\ \tau \\ -i\tau \\ 1 \end{pmatrix} , \tag{4.62}$$

and the central charge is

$$\mathcal{Z}(\tau, \tau^*, q) = \langle \mathcal{V} \mid q \rangle = \frac{1}{2(\Im\mathrm{m}\tau)^{1/2}} [(p^1 - iq_0) - (q_1 + ip^0)\tau] \,. \tag{4.63}$$

The attractor equation is

$$\frac{d}{d\tau} \frac{1}{\Im\mathrm{m}\tau} [(p^1 - iq_0) - (q_1 + ip^0)\tau] \Big|_{\tau=\tau_{\text{fix}}} = 0 \,, \tag{4.64}$$

and has the general solution

$$\tau_{\text{fix}} = \frac{p^1 + iq_0}{q_1 - ip^0} \,, \tag{4.65}$$

which is admissible (belongs to the definition domain of τ) if

$$\Im\mathrm{m}\tau_{\text{fix}} = p^0 p^1 + q_0 q_1 > 0 \,. \tag{4.66}$$

The central charge at the fixed point of the scalar takes the value

$$\mathcal{Z}_{\text{fix}} = -i\frac{q_1 + ip^0}{|q_1 + ip^0|}\sqrt{p^0p^1 + q_0q_1}\,, \tag{4.67}$$

and it is always finite for $\tau_{\text{fix}} \neq 0$.

<u>*Solutions with a single black hole*</u>

Let us now consider solutions with

$$\mathcal{I} = \mathcal{I}_\infty + \frac{q}{r}\,. \tag{4.68}$$

In this theory the stabilization equations can be easily solved and they lead to

$$\mathcal{R} = \begin{pmatrix} 0 & -\sigma^1 \\ \sigma^1 & 0 \end{pmatrix}\mathcal{I}\,, \quad \Rightarrow \quad -g_{rr} = \langle\,\mathcal{R}\mid\mathcal{I}\,\rangle = 2(\mathcal{I}^0\mathcal{I}^1 + \mathcal{I}_0\mathcal{I}_1)\,, \tag{4.69}$$

which shows that the area of the horizon (if any) is related to $|\mathcal{Z}_{\text{fix}}|^2$ above according to the general formula Eq. (2.55).

We also have

$$\tau = i\frac{\mathcal{L}^1/X}{\mathcal{L}^0/X} = \frac{\mathcal{I}^1 + i\mathcal{I}_0}{\mathcal{I}_1 - i\mathcal{I}^0}\,, \tag{4.70}$$

which implies that the 4 harmonic functions are not entirely independent but have to satisfy

$$\Im\text{m}\tau = \mathcal{I}^0\mathcal{I}^1 + \mathcal{I}_0\mathcal{I}_1 > 0\,, \tag{4.71}$$

which ensures that, if there are no pathologies that make a black-hole interpretation of the solution impossible, the attractor equations will always have solutions and $\mathcal{Z}_{\text{fix}} \neq 0$. Thus, we will not have to worry about condition III but only about the positive definiteness of $\Im\text{m}\tau$.

The only possible pathologies (negative mass and presence of NUT charge) are clearly avoided by imposing conditions I and II, which is always possible and presents no difficulties.

<u>*Solutions with two black holes*</u>

Let us now consider solutions of the form

$$\mathcal{I} = \mathcal{I}_\infty + \frac{q_1}{r_1} + \frac{q_2}{r_2}\,, \qquad r_i \equiv |\vec{x} - \vec{x}_i|\,. \tag{4.72}$$

Our goal is to find a configuration (i.e. a set of asymptotic values $\mathcal{I}_\infty$ and charges $q_{1,2}$) that satisfy conditions I-III. The previous discussions indicate how this has to be done and which formulas need to be applied. There is no systematic procedure to find such a configuration but it is not too difficult to find one:

$$
\begin{aligned}
\mathcal{I}^0 &= \frac{1}{\sqrt{2}} + \frac{q}{r_1} + \frac{q}{r_2}\,, \\
\mathcal{I}^1 &= \frac{1}{\sqrt{2}} + \frac{8q}{r_1} + \frac{8q}{r_2}\,, \\
\mathcal{I}_0 &= -\frac{4q}{r_2}\,, \\
\mathcal{I}_1 &= -\frac{1}{4\sqrt{2}} - \frac{q}{r_1} + \frac{q}{r_2}\,,
\end{aligned}
\tag{4.73}
$$

where $q > 0$ in order to guarantee Eq. (4.71). The metric component

$$
-g_{rr} = 1 + \frac{9\sqrt{2}q}{r_1} + \frac{10\sqrt{2}q}{r_2} + \frac{16q^2}{r_1^2} + \frac{8q^2}{r_2^2} + \frac{40q^2}{r_1 r_2}\,, \tag{4.74}
$$

is finite everywhere outside $r_{1,2} = 0$, and therefore, so is $\Im\mathrm{m}\tau$. In particular the "mass" of each of the two objects is positive

$$
M_1 = 9q/\sqrt{2}\,, \qquad M_2 = 5\sqrt{2}q\,, \qquad M = M_1 + M_2 = 19q/\sqrt{2}\,, \tag{4.75}
$$

and in the $r_{1,2} \to 0$ limits we find spheres of finite areas

$$
\frac{A_1}{4\pi} = 16q^2 = 2|\mathcal{Z}_{\mathrm{fix},1}|^2\,, \qquad \frac{A_2}{4\pi} = 8q^2 = 2|\mathcal{Z}_{\mathrm{fix},2}|^2\,. \tag{4.76}
$$

The total horizon area is

$$
\frac{A}{4\pi} = \frac{A_1}{4\pi} + \frac{A_2}{4\pi} = 24q^2 < 2|\mathcal{Z}_{\mathrm{fix,tot}}|^2 = 64q^2\,, \tag{4.77}
$$

which is the area of the horizon of a single black hole having the sum of the charges of the two black holes.

For this configuration

$$
\langle\, \mathcal{I}_\infty \mid q_1 \,\rangle = -\langle\, \mathcal{I}_\infty \mid q_2 \,\rangle = -q/\sqrt{2}\,, \qquad \langle\, q_2 \mid q_1 \,\rangle = 12q^2\,, \tag{4.78}
$$

so, choosing

$$
r_{12} = |\vec{x}_2 - \vec{x}_1| = 12\sqrt{2}q\,, \tag{4.79}
$$

we satisfy condition I (no NUT charges). The system has nevertheless angular momentum given by the general formula Eq. (4.47):

$$
|J| = |\langle\, q_2 \mid q_1 \,\rangle| = 12q^2\,. \tag{4.80}
$$

5 Conclusions

We have formulated three conditions that supersymmetric black-hole-type solutions have to satisfy in order to be supersymmetric everywhere, including at the sources. We have shown how these conditions constrain the possible sources by, basically, excluding those with NUT charge, angular momentum, negative energy and scalar hair, which seemingly cannot be modeled in String Theory. We arrived at a picture in which if an observer far away from one of the globally supersymmetric configurations we have considered, detects angular momentum and non-trivial scalar fields he/she will only find static electromagnetic sources in equilibrium when approaching the system.

These conditions and this picture should be improved by considering quantum corrections. Another interesting course of action would be to consider regularity of black-hole solutions in $N > 2$ theories, e.g. [33], and investigate the rôle played by the attractor [11].

It is also clear that the situation in $d = 5$ is completely different as there are regular rotating supersymmetric black holes for which microscopic String Theory models are known [34]. Work on these issues is already in progress [35].

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A Proofs of some identities

Let us consider the generalized stabilization equations derived from Eq. (2.3). Differentiating the imaginary part of that equation (i.e. Eq.(2.4), we get

$$d\mathcal{I}_\Lambda = d\Im\mathrm{m}\mathcal{F}_\Lambda = \tfrac{1}{2i}(d\mathcal{X}^\Lambda\mathcal{F}_{\Sigma\Lambda} - d\mathcal{X}^{*\,\Lambda}\mathcal{F}^*_{\Sigma\Lambda}) = d\mathcal{R}^\Sigma\Im\mathrm{m}\mathcal{F}_{\Sigma\Lambda} + d\mathcal{I}^\Sigma\Re\mathrm{e}\mathcal{F}_{\Sigma\Lambda}\,, \tag{A.1}$$

where we have used $\mathcal{X}^\Lambda = \mathcal{R}^\Lambda + i\mathcal{I}^\Lambda$. Using the invertibility of the imaginary part of $\mathcal{F}_{\Sigma\Lambda}$ we get

$$d\mathcal{R}^\Sigma = \Im\mathrm{m}\mathcal{F}^{\Sigma\Lambda}d\mathcal{I}_\Lambda - \Im\mathrm{m}\mathcal{F}^{\Sigma\Omega}\Re\mathrm{e}\mathcal{F}_{\Omega\Lambda}d\mathcal{I}^\Lambda\,. \tag{A.2}$$

On the other hand, differentiating the real part of Eq. (2.3)

$$d\mathcal{R}_\Lambda = d\Re\mathrm{e}\mathcal{F}_\Lambda = \tfrac{1}{2}(d\mathcal{X}^\Lambda\mathcal{F}_{\Sigma\Lambda} + d\mathcal{X}^{*\,\Lambda}\mathcal{F}^*_{\Sigma\Lambda}) = d\mathcal{R}^\Sigma\Re\mathrm{e}\mathcal{F}_{\Sigma\Lambda} - d\mathcal{I}^\Sigma\Im\mathrm{m}\mathcal{F}_{\Sigma\Lambda}\,, \tag{A.3}$$

and, substituting our previous result for $d\mathcal{R}^\Lambda$

$$d\mathcal{R}_\Sigma = \Re\mathrm{e}\mathcal{F}_{\Sigma\Omega}\Im\mathrm{m}\mathcal{F}^{\Omega\Lambda}dH_\Lambda - (\Im\mathrm{m}\mathcal{F}_{\Sigma\Lambda} + \Re\mathrm{e}\mathcal{F}_{\Sigma\Omega}\Im\mathrm{m}\mathcal{F}^{\Omega\Delta}\Re\mathrm{e}\mathcal{F}_{\Delta\Lambda})d\mathcal{I}^\Lambda\,. \tag{A.4}$$

We can write all these results in the form

$$d\mathcal{R} = \begin{pmatrix} -\Im\mathrm{m}\mathcal{F}^{-1}\Re\mathrm{e}\mathcal{F} & \Im\mathrm{m}\mathcal{F}^{-1} \\ -(\Im\mathrm{m}\mathcal{F} + \Re\mathrm{e}\mathcal{F}\Im\mathrm{m}\mathcal{F}^{-1}\Re\mathrm{e}\mathcal{F}) & \Re\mathrm{e}\mathcal{F}\Im\mathrm{m}\mathcal{F}^{-1} \end{pmatrix} d\mathcal{I}\,, \tag{A.5}$$

$$d\mathcal{I} = \begin{pmatrix} \Im\mathrm{m}\mathcal{F}^{-1}\Re\mathrm{e}\mathcal{F} & -\Im\mathrm{m}\mathcal{F}^{-1} \\ \Im\mathrm{m}\mathcal{F} + \Re\mathrm{e}\mathcal{F}\Im\mathrm{m}\mathcal{F}^{-1}\Re\mathrm{e}\mathcal{F} & -\Re\mathrm{e}\mathcal{F}\Im\mathrm{m}\mathcal{F}^{-1} \end{pmatrix} d\mathcal{R}\,, \tag{A.6}$$

from which we can read identities such as

$$\begin{aligned} \frac{\partial\mathcal{R}^\Sigma}{\partial\mathcal{I}_\Lambda} &= \frac{\partial\mathcal{R}^\Lambda}{\partial\mathcal{I}_\Sigma} = \frac{\partial\mathcal{I}^\Lambda}{\partial\mathcal{R}_\Sigma}\,, & \frac{\partial\mathcal{R}_\Sigma}{\partial\mathcal{I}^\Lambda} &= \frac{\partial\mathcal{R}_\Lambda}{\partial\mathcal{I}^\Sigma} = -\frac{\partial\mathcal{I}_\Lambda}{\partial\mathcal{R}^\Sigma}\,, \\ \frac{\partial\mathcal{R}^\Sigma}{\partial\mathcal{I}^\Lambda} &= -\frac{\partial\mathcal{R}_\Lambda}{\partial\mathcal{I}_\Sigma} = \frac{\partial\mathcal{I}_\Lambda}{\partial\mathcal{R}_\Sigma}\,, & \frac{\partial\mathcal{R}_\Sigma}{\partial\mathcal{I}_\Lambda} &= -\frac{\partial\mathcal{R}^\Lambda}{\partial\mathcal{I}^\Sigma} = \frac{\partial\mathcal{I}^\Lambda}{\partial\mathcal{R}^\Sigma}\,. \end{aligned} \tag{A.7}$$

We can now prove Eq. (2.46): taking the derivative of $\mathcal{R}$ as a function of $\mathcal{I}$ we have

$$\begin{aligned} \langle\,\nabla_\mu\mathcal{R}\mid\mathcal{I}\,\rangle &= \langle\,\frac{\partial\mathcal{R}}{\partial\mathcal{I}^\Lambda}\nabla_\mu\mathcal{I}^\Lambda + \frac{\partial\mathcal{R}}{\partial\mathcal{I}_\Lambda}\nabla_\mu\mathcal{I}_\Lambda \mid \mathcal{I}\,\rangle \\ &= \nabla_\mu\mathcal{I}^\Lambda\left(\mathcal{I}^\Sigma\frac{\partial\mathcal{R}_\Sigma}{\partial\mathcal{I}^\Lambda} - \mathcal{I}_\Sigma\frac{\partial\mathcal{R}^\Sigma}{\partial\mathcal{I}^\Lambda}\right) + \nabla_\mu\mathcal{I}_\Lambda\left(\mathcal{I}^\Sigma\frac{\partial\mathcal{R}_\Sigma}{\partial\mathcal{I}_\Lambda} - \mathcal{I}_\Sigma\frac{\partial\mathcal{R}^\Sigma}{\partial\mathcal{I}_\Lambda}\right)\,, \end{aligned} \tag{A.8}$$

and using now the above relations between partial derivatives

$$\langle\,\nabla_\mu\mathcal{R}\mid\mathcal{I}\,\rangle = \nabla_\mu\mathcal{I}^\Lambda\left(\mathcal{I}^\Sigma\frac{\partial\mathcal{R}_\Lambda}{\partial\mathcal{I}^\Sigma} + \mathcal{I}_\Sigma\frac{\partial\mathcal{R}_\Lambda}{\partial\mathcal{I}_\Sigma}\right) - \nabla_\mu\mathcal{I}_\Lambda\left(\mathcal{I}^\Sigma\frac{\partial\mathcal{R}^\Lambda}{\partial\mathcal{I}^\Sigma} + \mathcal{I}_\Sigma\frac{\partial\mathcal{R}^\Lambda}{\partial\mathcal{I}_\Sigma}\right)\,. \tag{A.9}$$

Given that the real section $\mathcal{R}$ is homogeneous of first order in the $\mathcal{I}$'s

$$\mathcal{I}^\Sigma\frac{\partial\mathcal{R}_\Lambda}{\partial\mathcal{I}^\Sigma} + \mathcal{I}_\Sigma\frac{\partial\mathcal{R}_\Lambda}{\partial\mathcal{I}_\Sigma} = \mathcal{R}_\Lambda\,, \qquad \mathcal{I}^\Sigma\frac{\partial\mathcal{R}^\Lambda}{\partial\mathcal{I}^\Sigma} + \mathcal{I}_\Sigma\frac{\partial\mathcal{R}^\Lambda}{\partial\mathcal{I}_\Sigma} = \mathcal{R}^\Lambda\,, \tag{A.10}$$

which proves the identity.

Similarly, expanding the r.h.s. of Eq. (2.45) we get

$$\langle\, \mathcal{R} \mid \nabla_\mu \mathcal{R}\, \rangle = \left(\frac{\partial \mathcal{R}^\Lambda}{\partial \mathcal{I}^\Sigma} R_\Lambda - \frac{\partial \mathcal{R}_\Lambda}{\partial \mathcal{I}^\Sigma} R^\Lambda \right) d\mathcal{I}^\Sigma + \left(\frac{\partial \mathcal{R}^\Lambda}{\partial \mathcal{I}_\Sigma} R_\Lambda - \frac{\partial \mathcal{R}_\Lambda}{\partial \mathcal{I}_\Sigma} R^\Lambda \right) d\mathcal{I}_\Sigma \,, \qquad \text{(A.11)}$$

and using the identities between partial derivatives and the fact that the real section $\mathcal{I}$ is homogeneous of first order in $\mathcal{R}$, we arrive at the result we wanted.

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