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New Formulations of $D = 10$ Supersymmetry and D8–O8 Domain Walls

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ABSTRACT

We discuss a generalized form of IIA/IIB supergravity depending on *all* R-R potentials $C^{(p)}$ $(p = 0, 1, \ldots 9)$ as the effective field theory of Type IIA/IIB superstring theory. For the IIA case we explicitly break this R-R democracy to either $p \leq 3$ or $p \geq 5$ which allows us to write a new bulk action that can be coupled to $N = 1$ supersymmetric brane actions.

The case of 8-branes is studied in detail using the new bulk & brane action. The supersymmetric negative tension branes without matter excitations can be viewed as orientifolds in the effective action. These D8-branes and O8-planes are fundamental in Type I′ string theory. A BPS 8-brane solution is given which satisfies the jump conditions on the wall. It implies a quantization of the mass parameter in string units. Also we find a maximal distance between the two walls, depending on the string coupling and the mass parameter. We derive the same results via supersymmetric flow equations.

1 Introduction

The initial purpose of this work was to construct supersymmetric domain walls of string theory in $D = 10$ which may shed some light on the stringy origin of the brane world scenarios. In the process of pursuing this goal we have realized that all descriptions of the effective field theory of Type IIA/B string theory available in the literature are inefficient for our purpose. This has led us to introduce new versions of the effective supergravities corresponding to Type IIA/B string theory.

The standard IIA massless supergravity includes the $C^{(1)}$ and $C^{(3)}$ R-R potentials and the corresponding $G^{(2)}$ and $G^{(4)}$ gauge-invariant R-R forms. Type IIB supergravity includes the $C^{(0)}$, $C^{(2)}$ and $C^{(4)}$ R-R potentials and the corresponding $G^{(1)}$, $G^{(3)}$ and (self-dual) $G^{(5)}$ gauge-invariant R-R forms. On the other hand, string theory has all Dp-branes, odd and even, including the exotic ones, like 8-branes in IIA and 7-branes in IIB theory. These branes, of co-dimension 1 and 2, are special objects which are different in many respects from the other BPS-extended objects like the p-branes with $0 \leq p \leq 6$, which have co-dimension greater than or equal to 3. The basic difference is in the behavior of the form fields at large distance, $G^{(p+2)} \sim r^{p-8}$. For example the $G^{(10)}$ R-R form of the 8-brane does not fall off at infinity but takes a constant value there. It is believed that such extended objects can not exist independently but only in connection with orientifold planes [1]. However, the realization of the total system in supergravity is rather obscure.

It has been realized a while ago [2] that massive IIA supergravity, discovered by Romans [3], was the key to understand the spacetime picture of the 8-branes, which are domain walls in $D = 10$. A significant progress towards the understanding of the 8-brane solutions was made in [4, 5], where the bulk supergravity solution was found. Also, in [5], the description of the cosmological constant via a 9-form potential, based upon the work of [6, 7], was discussed. In [8] a standard 8-brane action coupling to this 9-form potential has been shown to be the appropriate source for the second Randall–Sundrum scenario [9]. Solutions for the coupled bulk & brane action system automatically satisfy the jump conditions and so they are consistent, at least from this point of view. A major unsolved problem was to find an explicitly supersymmetric description of coupled bulk & brane systems like it was done in [10]. Such a description should allow us to find out some important properties of the domain walls like the distance between the planes, the status of unbroken supersymmetry in the bulk and on the brane etc. We expect that realizing such a bulk & brane construction will lead to a better insight into the fundamental nature of extended objects of string theory.

The string backgrounds that we want to describe using an explicitly supersymmetric bulk & brane action are one-dimensional orbifolds obtained by modding out the circle S^1 by a reflection $\mathbb{Z}_2$. The orbifold direction is the transverse direction of the branes that fill the rest of the spacetime. Now, the orbifold $S^1/\mathbb{Z}_2$ being a compact space, we cannot place a single charged object (a D8-brane, say) in it, but we have to have at least two oppositely charged objects. However, this kind of system cannot be in supersymmetric equilibrium unless their tensions also have opposite signs. We are going to identify these negative-tension objects with O8-planes and we will propose an O8-plane action to be coupled to the bulk supergravity action. O8-planes can only sit at orbifold points because they require the spacetime to be mirror symmetric in their transverse direction and, thus, they can sit in any of the two endpoints of the segment $S^1/\mathbb{Z}_2$. We are going to place the other (positive tension, opposite R-R charge) brane at the other endpoint. Clearly we can, from the effective action point of view, identify the positive tension brane as a combination of O8-planes and D8-branes with positive total tension and the negative tension brane as a combination of O8-planes and D8-branes with negative total tension.

Our strategy will be to generalize the 5-dimensional construction of the supersymmetric bulk & brane action, proposed in [10]. The construction of [10] allowed to find a supersymmetric realization of the brane-world scenario of Randall and Sundrum [9]. We will repeat the construction of [10] in $D = 10$ with the aim to get a better understanding of branes and planes in string theory.

To solve the discrepancy between the bulk actions with limited field content (lower-rank R-R forms) and the wide range of brane actions that involve all the possible R-R forms, we have constructed a new formulation of IIA/IIB supergravity up to quartic order in fermions. In particular, the new formulation gives an easy control over the exotic $G^{(0)}$ and $G^{(10)}$ R-R forms associated with the mass and cosmological constant[1] of the $D = 10$ supergravity. This in turn allows a clear study of the D8–O8 system describing a pair of supersymmetric domain walls which are fundamental objects of the Type I$'$ string theory. The quantization of the mass parameter and cosmological constant in stringy units are simple consequences of the theory. Apart from being a tool to understand the supersymmetric domain walls we were interested in, it can be expected that the new effective theories of $D = 10$ supersymmetry will have more general applications in the future.

This paper is organized as follows. First, in section 2 we discuss the new formulations of $D = 10$ supersymmetry in the bulk. In subsection 2.1 we give a democratic formulation based upon a pseudo-action along the lines of [11]. This formulation is the one that treats all R-R potentials in a unified way, leading to the same equations of motion as have been encountered basically in the coupling to D-branes in [12]. For the supersymmetric case, this field content and equations of motion follow also from the superspace formulation of [13, 14]. However, only a pseudo-action is available, whose equations of motion are supplemented by duality constraints. It allows for a unified IIA/IIB treatment and has no Chern–Simons terms. The constraints do not follow from the action and have to be imposed by hand. These generalize the self-duality condition that relates the components of the five-form field strength $G^{(5)}$ of type IIB theory to a set of relations between all Hodge dual field strengths. In IIB this self-duality prevents the construction of a proper action. The same is true in this formulation with a pseudo-action and generalized self-duality equations, for both type IIA and IIB. In the IIA case, different field strengths are related by the constraints and hence the R-R democracy can be broken without paying a price. In the IIB theory, things are more complicated. The self-duality of $G^{(5)}$ prevents a similar construction. To allow for a proper action one has to resort to a non-covariant formulation [15] or the auxiliary fields of Pasti, Sorokin and Tonin [16].

[1] In this paper we will mean by cosmological constant the square of the mass parameter of the Type IIA supergravity. Strictly speaking it is not a cosmological constant because in the Einstein frame it carries a dilaton factor.

In subsection 2.2 we break the self-duality for type IIA, and in this way are able to obtain a supersymmetric proper action with potentials $C^{(p)}$, $p = 5, 7, 9$. The duality relations of the first formulation also do not allow to obtain the generalization of the mechanism of [10], involving the replacement of the mass parameter $G^{(0)}$ by a field $G^{(0)}(x)$. Indeed, to do so we need an independent nine-form auxiliary field. In the democratic formulation, its field strength $G^{(10)}$ is related to $G^{(0)}$ itself. In the formulation of section 2.2, the breaking of the self-duality, at the same time of allowing an action, also has the setting to allow for a varying $G^{(0)}(x)$. We will also see that the democratic formulation does not preserve a suitable $\mathbb{Z}_2$ symmetry that played an important role in the mechanism of [10], in case that the mass parameter is non-zero. Therefore, it is the formulation of section 2.2 that we will use for the bulk & 8-brane construction. A first version of this formulation was given in [5]. In subsection 2.3 we obtain the string-frame version of the standard Romans' massive IIA supergravity in which the self-duality is also broken but potentials $C^{(p)}$ with $p = 1, 3$ are kept.

Next, in section 3 we discuss the supersymmetry on the brane. The mechanism is the same for all branes, but only the action of one brane is added, such that this breaks the democracy. The supersymmetric bulk & brane construction for the D8–O8 system is discussed in section 4. We propose a supersymmetric action for the O8-planes, which together with 32 D8-branes allows us to reinterpret the set of positive–negative tension branes as a combination of O8-planes with D8-branes. In section 5 we give the 8-brane solution and calculate the corresponding Killing spinors. As an application of our results, we discuss in section 6 the quantization of the mass and cosmological constant parameter corresponding to this system. Also critical distances are discussed. In section 7 we give the BPS action for the domain wall and investigate the supersymmetric flow equations. Finally, in section 8 we give a summary of our results and a discussion. There are two appendices. In appendix A we give our conventions. In appendix B we discuss the world-sheet T-duality between the different brane systems.

2 Supersymmetry in the Bulk

The standard formulation of $D = 10$ IIA (massless [17, 18, 19] and massive [3]) and IIB [20, 21] supergravity has the following field content

$$\begin{aligned} \text{IIA} \quad &: \qquad \left\{g_{\mu\nu}, B_{\mu\nu}, \phi, C^{(1)}_{\mu}, C^{(3)}_{\mu\nu\rho}, \psi_\mu, \lambda\right\} \,, \\ \text{IIB} \quad &: \qquad \left\{g_{\mu\nu}, B_{\mu\nu}, \phi, C^{(0)}, C^{(2)}_{\mu\nu}, C^{(4)}_{\mu\nu\rho\sigma}, \psi_\mu, \lambda\right\} \,. \end{aligned} \tag{2.1}$$

In the IIA case, the massive theory contains an additional mass parameter $G^{(0)} = m$. In the IIB case, an extra self-duality condition is imposed on the field strength of the four-form. It turns out that one can realize the N=2 supersymmetry on the R-R gauge fields of higher rank as well. These are usually incorporated via duality relations. To treat the R-R potentials democratically we propose in the next subsection a new formulation based upon a pseudo-action. This democratic formulation describes the dynamics of the bulk supergravity in the most elegant way. However, it turns out that this formulation is not well suited for our purposes. For the IIA case, we therefore give a different formulation in subsection 2.2 where the constant mass parameter has been replaced by a field. The relation of this formulation with the standard IIA supergravity (with the above field content) will be discussed in subsection 2.3.

2.1 The Democratic Formulation

To explicitly introduce the democracy among the R-R potentials we propose a pseudo-action containing all potentials. Of course this enlarges the number of degrees of freedom. Since a p- and an $(8-p)$-form potential carry the same number of degrees of freedom, the introduction of the dual potentials doubles the R-R sector. Including the highest potential $C^{(9)}$ in IIA does not alter this, since it carries no degrees of freedom. This 9-form potential can be seen as the potential dual to the constant mass parameter $G^{(0)} = m$. The doubling of number of degrees of freedom will be taken care of by a constraint, relating the lower- and higher-rank potentials. This new formulation of supersymmetry is inspired by the bosonic construction of [22], and, in the case of IIB supergravity, is related to the pseudo-action construction of [11].

A pseudo-action [11] can be used as a mnemonic to derive the equations of motion. It differs from a usual action in the sense that not all equations of motion follow from varying the fields in the pseudo-action. To obtain the complete set of equations of motion, an additional constraint has to be substituted by hand into the set of equations of motion that follow from the pseudo-action. The constraint itself does not follow from the pseudo-action. The construction we present here generalizes the pseudo-action construction of [22, 11] in the sense that our construction (i) treats the IIA and IIB case in a unified way, introducing all R-R potentials in the pseudo-action, and (ii) describes also the massive IIA case via a 9-form potential $C^{(9)}$ and a constant mass parameter $G^{(0)} = m$.

Our pseudo-action has the extended field content

$$\begin{array}{lll} \text{IIA} & : & \big\{g_{\mu\nu}, B_{\mu\nu}, \phi, C^{(1)}_{\mu}, C^{(3)}_{\mu\nu\rho}, C^{(5)}_{\mu\cdots\rho}, C^{(7)}_{\mu\cdots\rho}, C^{(9)}_{\mu\cdots\rho}, \psi_\mu, \lambda\big\}\ , \\ \text{IIB} & : & \big\{g_{\mu\nu}, B_{\mu\nu}, \phi, C^{(0)}, C^{(2)}_{\mu\nu}, C^{(4)}_{\mu\cdots\rho}, C^{(6)}_{\mu\cdots\rho}, C^{(8)}_{\mu\cdots\rho}, \psi_\mu, \lambda\big\}\ . \end{array} \tag{2.2}$$

It is understood that in the IIA case the fermions contain both chiralities, while in the IIB case they satisfy

$$\Gamma_{11}\psi_\mu = \psi_\mu\,, \qquad \Gamma_{11}\lambda = -\lambda\,, \qquad \text{(IIB)}. \tag{2.3}$$

In that case they are doublets, and we suppress the corresponding index. The explicit form of the pseudo-action is given by[2]

$$\begin{aligned} S_{\rm Pseudo} = -\frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}\Big\{e^{-2\phi}\Big[&R\big(\omega(e)\big) - 4\big(\partial\phi\big)^2 + \tfrac{1}{2}H\cdot H + \\ &- 2\partial^\mu\phi\chi^{(1)}_\mu + H\cdot\chi^{(3)} + 2\bar\psi_\mu\Gamma^{\mu\nu\rho}\nabla_\nu\psi_\rho - 2\bar\lambda\Gamma^\mu\nabla_\mu\lambda + 4\bar\lambda\Gamma^{\mu\nu}\nabla_\mu\psi_\nu\Big] + \end{aligned}$$

[2]We partly use form notation. For our conventions, see appendix A.

$$+\sum_{n=0,1/2}^{5,9/2} \tfrac{1}{4}G^{(2n)}\cdot G^{(2n)} + \tfrac{1}{2}G^{(2n)}\cdot \Psi^{(2n)}\Big\} + \text{ quartic fermionic terms}\,. \quad (2.4)$$

It is understood that the summation in the above pseudo-action is over integers ($n = 0, 1, \ldots, 5$) in the IIA case and over half-integers ($n = 1/2, 3/2, \ldots, 9/2$) in the IIB case. In the summation range we will always first indicate the lowest value for the IIA case, before the one for the IIB case. Furthermore,

$$\frac{1}{2\kappa_{10}^2} = \frac{g^2}{2\kappa^2} = \frac{2\pi}{(2\pi\ell_s)^8}\,, \quad (2.5)$$

where κ^2 is the physical gravitational coupling, g is the string coupling constant and $\ell_s = \sqrt{\alpha'}$ is the string length. For notational convenience we group all potentials and field strengths in the formal sums

$$\mathbf{G} = \sum_{n=0,1/2}^{5,9/2} G^{(2n)}\,, \qquad \mathbf{C} = \sum_{n=1,1/2}^{5,9/2} C^{(2n-1)}\,. \quad (2.6)$$

The bosonic field strengths are given by[3]

$$H = dB\,, \qquad \mathbf{G} = d\mathbf{C} - dB{\scriptstyle\wedge}\mathbf{C} + G^{(0)}\mathbf{e}^{B}\,, \quad (2.10)$$

where it is understood that each equation involves only one term from the formal sums (2.6) (only the relevant combinations are extracted). The corresponding Bianchi identities then read

$$dH = 0\,, \qquad d\mathbf{G} - H{\scriptstyle\wedge}\mathbf{G} = 0\,. \quad (2.11)$$

In this subsection $G^{(0)} = m$ indicates the constant mass parameter of IIA supergravity. In the IIB theory all equations should be read with vanishing $G^{(0)}$. The spin connection in the

[3] There is an alternative basis for the R-R potentials that we can call "**A**-basis" and can be useful in certain contexts. This basis is related to the "**C**-basis" just defined by

$$\mathbf{A} = \mathbf{C}{\scriptstyle\wedge}\mathbf{e}^{-B}\,, \quad (2.7)$$

with the following gauge transformations

$$\delta\mathbf{A} = d\mathbf{\Lambda} - G^{(0)}\Lambda - \mathbf{A}{\scriptstyle\wedge}d\Lambda\,, \quad (2.8)$$

and in it, the above R-R field strengths are written as follows:

$$\mathbf{G} = \left(d\mathbf{A} + G^{(0)}\right){\scriptstyle\wedge}\mathbf{e}^{B}\,. \quad (2.9)$$

The two main properties of this basis are that in it, the R-R potentials only appear in the field strengths through their derivatives (field strengths) and the standard Wess-Zumino term of the Dp-brane actions does not contain the NS-NS 2-form B. In the Type IIB action written in this basis, the invariance under constant shifts of the R-R scalar (axion) is manifest. It is the existence of this basis that makes the Scherk-Schwarz generalized dimensional reduction of Ref. [5].

covariant derivative ∇_μ is given by its zehnbein part: ${\omega_\mu}^{ab} = {\omega_\mu}^{ab}(e)$. The bosonic fields couple to the fermions via the bilinears $\chi^{(1,3)}$ and $\Psi^{(2n)}$, which read

$$
\begin{aligned}
\chi^{(1)}_\mu =& -2\bar\psi_\nu \Gamma^\nu \psi_\mu - 2\bar\lambda\Gamma^\nu\Gamma_\mu\psi_\nu\,,\\
\chi^{(3)}_{\mu\nu\rho} =& \tfrac{1}{2}\bar\psi_\alpha \Gamma^{[\alpha}\Gamma_{\mu\nu\rho}\Gamma^{\beta]}\mathcal{P}\psi_\beta + \bar\lambda\Gamma_{\mu\nu\rho}{}^\beta\mathcal{P}\psi_\beta - \tfrac{1}{2}\bar\lambda\mathcal{P}\Gamma_{\mu\nu\rho}\lambda\,,\\
\Psi^{(2n)}_{\mu_1\cdots\mu_{2n}} =& \tfrac{1}{2}e^{-\phi}\bar\psi_\alpha\Gamma^{[\alpha}\Gamma_{\mu_1\cdots\mu_{2n}}\Gamma^{\beta]}\mathcal{P}_n\psi_\beta + \tfrac{1}{2}e^{-\phi}\bar\lambda\Gamma_{\mu_1\cdots\mu_{2n}}\Gamma^\beta\mathcal{P}_n\psi_\beta +\\
& - \tfrac{1}{4}e^{-\phi}\bar\lambda\Gamma_{[\mu_1\cdots\mu_{2n-1}}\mathcal{P}_n\Gamma_{\mu_{2n}]}\lambda\,.
\end{aligned}
\tag{2.12}
$$

We have used the following definitions:

$$
\begin{aligned}
\mathcal{P} &= \Gamma_{11} \quad (\text{IIA}) \qquad \text{or} \qquad -\sigma^3 \ (\text{IIB})\,,\\
\mathcal{P}_n &= (\Gamma_{11})^n \ (\text{IIA}) \qquad \text{or} \qquad \sigma^1 \ (\text{n}+1/2 \text{ even}),\ \mathrm{i}\sigma^2 \ (\text{n}+1/2 \text{ odd}) \ (\text{IIB})\,.
\end{aligned}
\tag{2.13}
$$

Note that the fermions satisfy

$$
\Psi^{(2n)} = (-)^{\text{Int}[n]+1} \star \Psi^{(10-2n)}\,,
\tag{2.14}
$$

due to the Γ-matrices identity (A.7).

Due to the appearance of all R-R potentials, the number of degrees of freedom in the R-R sector has been doubled. Each R-R potential leads to a corresponding equation of motion:

$$
d\star(G^{(2n)} + \Psi^{(2n)}) + H_\wedge\star(G^{(2n+2)} + \Psi^{(2n+2)}) = 0\,.
\tag{2.15}
$$

Now, one must relate the different potentials to get the correct number of degrees of freedom. We therefore by hand impose the following duality relations

$$
G^{(2n)} + \Psi^{(2n)} = (-)^{\text{Int}[n]} \star G^{(10-2n)}\,,
\tag{2.16}
$$

in the equations of motion that follow from the pseudo-action (2.4). It is in this sense that the action (2.4) cannot be considered as a true action. Instead, it should be considered as a mnemonic to obtain the full equations of motion of the theory. As usual, the Bianchi identities and equations of motions of the dual potentials correspond to each other when employing the duality relation. For the above reason the democratic formulation can be viewed as self-dual, since (2.16) places constraints relating the field content (2.2).

The pseudo-action (2.4) is invariant under supersymmetry provided we impose the duality relations (2.16) after varying the action. The supersymmetry rules read (here given modulo cubic fermion terms):

$$
\begin{aligned}
\delta_\epsilon {e_\mu}^a =& \bar\epsilon\Gamma^a\psi_\mu\,,\\
\delta_\epsilon\psi_\mu =& \Big(\partial_\mu + \tfrac{1}{4}\not\omega_\mu + \tfrac{1}{8}\mathcal{P}\not H_\mu\Big)\epsilon + \tfrac{1}{16}e^\phi \sum_{n=0,1/2}^{5,9/2}\frac{1}{(2n)!}\not G^{(2n)}\Gamma_\mu\mathcal{P}_n\epsilon\,,\\
\delta_\epsilon B_{\mu\nu} =& -2\,\bar\epsilon\Gamma_{[\mu}\mathcal{P}\psi_{\nu]}\,,\\
\delta_\epsilon C^{(2n-1)}_{\mu_1\cdots\mu_{2n-1}} =& -e^{-\phi}\,\bar\epsilon\,\Gamma_{[\mu_1\cdots\mu_{2n-2}}\,\mathcal{P}_n\Big((2n-1)\psi_{\mu_{2n-1}]} - \tfrac{1}{2}\Gamma_{\mu_{2n-1}]}\lambda\Big) +\\
& + (n-1)(2n-1)\,C^{(2n-3)}_{[\mu_1\cdots\mu_{2n-3}}\,\delta_\epsilon B_{\mu_{2n-2}\mu_{2n-1}]}\,,
\end{aligned}
$$

$$\delta_\epsilon\lambda = \Big(\partial\!\!\!/\phi + \tfrac{1}{12}\, H\!\!\!\!/\mathcal{P}\Big)\epsilon + \tfrac{1}{8}e^\phi \sum_{n=0,1/2}^{5,9/2} (-)^{2n}\frac{5-2n}{(2n)!}\; G\!\!\!\!/^{(2n)}\mathcal{P}_n\epsilon\,,$$

$$\delta_\epsilon\phi = \tfrac{1}{2}\,\bar\epsilon\lambda\,, \tag{2.17}$$

where ϵ is a spinor similar to ψ_μ, i.e. in IIB: $\Gamma_{11}\epsilon = \epsilon$. Note that for n half-integer (the IIB case) these supersymmetry rules exactly reproduce the rules given in eq. (1.1) of [23].

Secondly, the pseudo-action (2.4) is also invariant under the usual bosonic NS-NS and R-R gauge symmetries with parameters Λ and $\Lambda^{(2n)}$ respectively:

$$\delta_\Lambda B = d\Lambda\,, \qquad \delta_\Lambda \mathbf{C} = (d\boldsymbol{\Lambda} - G^{(0)}\Lambda)_\wedge \mathbf{e}^B\,, \qquad \text{with } \boldsymbol{\Lambda} = \sum_{n=0,1/2}^{4,7/2} \Lambda^{(2n)}\,. \tag{2.18}$$

Finally, there is a number of $\mathbb{Z}_2$-symmetries. However, in the IIA case these $\mathbb{Z}_2$-symmetries are *only valid for* $G^{(0)} = m = 0$. The world-sheet form of these symmetries is given in appendix B. Below we show how these symmetries of the action act on supergravity fields. For both massless IIA and IIB there is a fermion number symmetry $(-)^{F_L}$ given by

$$\begin{aligned}
\big\{\phi, g_{\mu\nu}, B_{\mu\nu}\big\} &\to \big\{\phi, g_{\mu\nu}, B_{\mu\nu}\big\}\,,\\
\big\{C^{(2n-1)}_{\mu_1\cdots\mu_{2n-1}}\big\} &\to -\big\{C^{(2n-1)}_{\mu_1\cdots\mu_{2n-1}}\big\}\,,\\
\big\{\psi_\mu, \lambda, \epsilon\big\} &\to +\mathcal{P}\big\{\psi_\mu, -\lambda, \epsilon\big\}\,, \quad \text{(IIA)},\\
\big\{\psi_\mu, \lambda, \epsilon\big\} &\to +\mathcal{P}\big\{\psi_\mu, \lambda, \epsilon\big\}\,, \quad \text{(IIB)}.
\end{aligned} \tag{2.19}$$

In the IIB case there is an additional worldsheet parity symmetry Ω given by

$$\begin{aligned}
\big\{\phi, g_{\mu\nu}, B_{\mu\nu}\big\} &\to \big\{\phi, g_{\mu\nu}, -B_{\mu\nu}\big\}\,,\\
\big\{C^{(2n-1)}_{\mu_1\cdots\mu_{2n-1}}\big\} &\to (-)^{n+1/2}\big\{C^{(2n-1)}_{\mu_1\cdots\mu_{2n-1}}\big\}\,,\\
\big\{\psi_\mu, \lambda, \epsilon\big\} &\to \sigma^1\big\{\psi_\mu, \lambda, \epsilon\big\}\,,
\end{aligned} \tag{2.20}$$

In the massless IIA case there is a similar $I_9\Omega$-symmetry involving an additional parity transformation in the 9-direction. Writing $\mu = (\underline{\mu}, \dot 9)$, the rules are given by

$$\begin{aligned}
x^{\dot 9} &\to -x^{\dot 9}\,,\\
\big\{\phi, g_{\underline{\mu\nu}}, B_{\underline{\mu\nu}}\big\} &\to \big\{\phi, g_{\underline{\mu\nu}}, -B_{\underline{\mu\nu}}\big\}\,,\\
\big\{C^{(2n-1)}_{\underline{\mu_1}\cdots\underline{\mu_{2n-1}}}\big\} &\to (-)^{n+1}\big\{C^{(2n-1)}_{\underline{\mu_1}\cdots\underline{\mu_{2n-1}}}\big\}\,,\\
\big\{\psi_{\underline{\mu}}, \lambda, \epsilon\big\} &\to +\Gamma^9\big\{\psi_{\underline{\mu}}, -\lambda, \epsilon\big\}\,.
\end{aligned} \tag{2.21}$$

The parity of the fields with one or more indices in the $\dot 9$-direction is given by the rule that every index in the $\dot 9$-direction gives an extra minus sign compared to the above rules.

In both IIA and IIB there is also the obvious symmetry of interchanging all fermions by minus the fermions, leaving the bosons invariant.

The $\mathbb{Z}_2$-symmetries are used for the construction of superstring theories with sixteen supercharges, see [24]. $(-)^{F_L}$ gives a projection to the $E_8 \times E_8$ heterotic superstring (IIA) or the SO(32) heterotic superstring theory (IIB). Ω is used to reduce the IIB theory to the SO(32) Type I superstring, while the $I_9\Omega$-symmetry reduces the IIA theory to the Type I′ $SO(16) \times SO(16)$ superstring theory.

One might wonder at the advantages of the generalized pseudo-action (2.4) above the standard supergravity formulation. At the cost of an extra duality relation we were able to realize the R-R democracy in the action. Note that only kinetic terms are present; by allowing for a larger field content the Chern–Simons term is eliminated. Under T-duality all kinetic terms are easily seen to transform into each other [25]. The same goes for the duality constraints. This formulation is elegant and comprises all potentials. However, it is impossible to construct a proper action in this formulation due to the doubling of the degrees of freedom. Therefore, to add brane actions to the bulk system, the democratic formulation is not suitable. This is due to two reasons. First, the $I_9\Omega$ symmetry is only valid for $G^{(0)} = 0$, but we will need this symmetry in our construction of the bulk & 8-brane system. Secondly, to describe a charged domain wall, we would like to have opposite values for $G^{(0)}$ at the two sides of the domain wall, i.e. we want to allow for a mass parameter that is only piecewise constant. The R-R democracy has to be broken to accommodate for an action and this will be discussed in the following two subsections.

2.2 The Dual Formulation of IIA

We will present here the new dual formulation with action, available for the IIA case only. A proper action will be constructed in this formulation. It is this formulation that we will apply in our construction of the bulk & brane system. We will call this the dual formulation and explain why in the next subsection.

The independent fields in this formulation are

$$\big\{e_\mu^a, B_{\mu\nu}, \phi, G^{(0)}, G^{(2)}_{\mu\nu}, G^{(4)}_{\mu_1\cdots\mu_4}, A^{(5)}_{\mu_1\cdots\mu_5}, A^{(7)}_{\mu_1\cdots\mu_7}, A^{(9)}_{\mu_1\cdots\mu_9}, \psi_\mu, \lambda\big\}\,. \tag{2.22}$$

The bulk action reads

$$\begin{aligned} S_{\rm bulk} = &-\frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}\Big\{e^{-2\phi}\big[R\big(\omega(e)\big) - 4\big(\partial\phi\big)^2 + \tfrac{1}{2}H\cdot H - 2\partial^\mu\phi\chi^{(1)}_\mu + H\cdot\chi^{(3)} + \\ &+ 2\bar\psi_\mu\Gamma^{\mu\nu\rho}\nabla_\nu\psi_\rho - 2\bar\lambda\Gamma^\mu\nabla_\mu\lambda + 4\bar\lambda\Gamma^{\mu\nu}\nabla_\mu\psi_\nu\big] + \sum_{n=0,1,2}\tfrac{1}{2}G^{(2n)}\cdot G^{(2n)} + G^{(2n)}\cdot\Psi^{(2n)} + \\ &- \star\big[\tfrac{1}{2}\,G^{(4)}G^{(4)}B - \tfrac{1}{2}\,G^{(2)}G^{(4)}B^2 + \tfrac{1}{6}\,G^{(2)2}B^3 + \tfrac{1}{6}\,G^{(0)}G^{(4)}B^3 - \tfrac{1}{8}\,G^{(0)}G^{(2)}B^4 + \\ &+ \tfrac{1}{40}\,G^{(0)2}B^5 + \mathbf{e}^{-B}\mathbf{G}d(A^{(5)} - A^{(7)} + A^{(9)})\big]\Big\} + \text{ quartic fermionic terms}\,, \end{aligned} \tag{2.23}$$

where all $\wedge$'s have been omitted in the last two lines. In the last term a projection on the 10-form is understood. Here $\mathbf{G}$ is defined as in (2.6) but where $G^{(0)}$, $G^{(2)}$ and $G^{(4)}$ are now independent fields (which we will call black boxes) and are no longer given by (2.10). Note that their Bianchi identities are imposed by the Lagrange multipliers $A^{(9)}$, $A^{(7)}$ and $A^{(5)}$. The NS-NS three-form field strength is given by (2.10).

The symmetries of the action are similar to those of the democratic formulation with some small changes. In the supersymmetry transformations of gravitino and gaugino, the sums now extend only over $n = 0, 1, 2$:

$$
\begin{aligned}
\delta_\epsilon e_\mu{}^a =& \bar\epsilon\Gamma^a\psi_\mu\,,\\
\delta_\epsilon\psi_\mu =& \Big(\partial_\mu + \tfrac{1}{4}\,\not{\omega}_\mu + \tfrac{1}{8}\Gamma_{11}\,\not{H}_\mu\Big)\epsilon + \tfrac{1}{8}e^\phi \sum_{n=0,1,2}\frac{1}{(2n)!}\,\not{G}^{(2n)}\Gamma_\mu(\Gamma_{11})^n\epsilon\,,\\
\delta_\epsilon B_{\mu\nu} =& -2\,\bar\epsilon\Gamma_{[\mu}\Gamma_{11}\psi_{\nu]}\,,\\
\delta_\epsilon\lambda =& \Big(\not{\partial}\phi - \tfrac{1}{12}\Gamma_{11}\,\not{H}\Big)\epsilon + \tfrac{1}{4}e^\phi\sum_{n=0,1,2}\frac{5-2n}{(2n)!}\,\not{G}^{(2n)}(\Gamma_{11})^n\epsilon\,,\\
\delta_\epsilon\phi =& \tfrac{1}{2}\,\bar\epsilon\lambda\,,\\
\delta_\epsilon\mathbf{A} =& \mathbf{e}^{-B}{}_\wedge\mathbf{E}\,,\\
\delta_\epsilon\mathbf{G} =& d\mathbf{E} + \mathbf{G}{}_\wedge\delta_\epsilon B - H{}_\wedge\mathbf{E}\,,\\
\text{with}\quad & E^{(2n-1)}_{\mu_1\cdots\mu_{2n-1}} \equiv -e^{-\phi}\,\bar\epsilon\,\Gamma_{[\mu_1\cdots\mu_{2n-2}}(\Gamma_{11})^n\Big((2n-1)\psi_{\mu_{2n-1}]} - \tfrac{1}{2}\Gamma_{\mu_{2n-1}]}\lambda\Big)\,.
\end{aligned}
\tag{2.24}
$$

The transformation of the black boxes $\mathbf{G}$ follow from the requirement that $\mathbf{e}^{-B}\mathbf{G}$ transforms in a total derivative. Here the formal sums

$$
\mathbf{A} = \sum_{n=1}^{5} A^{(2n-1)}\,,\qquad \mathbf{E} = \sum_{n=1}^{5} E^{(2n-1)}\,,\qquad \mathbf{G} = \sum_{n=0}^{5} G^{(2n)}\,,
\tag{2.25}
$$

have been used. Note that the first formal sum in (2.25) contains fields, $A^{(1)}$ and $A^{(3)}$, that do not occur in the action. The same applies to $\mathbf{G}$, which contains the extra fields $G^{(6)}, G^{(8)}$ and $G^{(10)}$. Although these fields do not occur in the action, one can nevertheless show that the supersymmetry algebra is realized on them. To do so one must use the supersymmetry rules of (2.24) and the equations of motion that follow from the action (2.23).

The gauge symmetries with parameters Λ and $\Lambda^{(2n)}$ are

$$
\begin{aligned}
\delta_\Lambda B &= d\Lambda\,,\qquad \delta_\Lambda\mathbf{A} = d\boldsymbol{\Lambda} - G^{(0)}\Lambda - d\Lambda{}_\wedge\mathbf{A}\,,\\
\delta_\Lambda\mathbf{G} &= d\Lambda{}_\wedge\big(\mathbf{G} - \mathbf{e}^B{}_\wedge(d\mathbf{A} + G^{(0)})\big) + \mathbf{e}^B{}_\wedge\Lambda{}_\wedge dG^{(0)}\,.
\end{aligned}
\tag{2.26}
$$

Note that, with respect to the R-R gauge symmetry, the $\mathbf{A}$ potentials transform as a total derivative while the black boxes are invariant.

Finally, there are $\mathbb{Z}_2$-symmetries, $(-)^{F_L}$ and $I_9\Omega$, which leave the action invariant. In contrast to the democratic formulation these two $\mathbb{Z}_2$-symmetries are valid symmetries even for $G^{(0)} \neq 0$. The $(-)^{F_L}$-symmetry is given by

$$
\begin{aligned}
\big\{\phi, g_{\mu\nu}, B_{\mu\nu}\big\} &\to \big\{\phi, g_{\mu\nu}, B_{\mu\nu}\big\}\,,\\
\big\{G^{(2n)}_{\mu_1\cdots\mu_{2n}}, A^{(2n-1)}_{\mu_1\cdots\mu_{2n-1}}\big\} &\to -\big\{G^{(2n)}_{\mu_1\cdots\mu_{2n}}, A^{(2n-1)}_{\mu_1\cdots\mu_{2n-1}}\big\}\,,\\
\big\{\psi_\mu, \lambda, \epsilon\big\} &\to +\Gamma_{11}\big\{\psi_\mu, -\lambda, \epsilon\big\}\,,
\end{aligned}
\tag{2.27}
$$

while the second $I_9\Omega$-symmetry reads

$$\begin{aligned}
x^9 &\to -x^9\,, \\
\{\phi, g_{\underline{\mu\nu}}, B_{\underline{\mu\nu}}\} &\to \{\phi, g_{\underline{\mu\nu}}, -B_{\underline{\mu\nu}}\}\,, \\
\{G^{(2n)}_{\underline{\mu_1}\cdots\underline{\mu_{2n}}}, A^{(2n-1)}_{\underline{\mu_1}\cdots\underline{\mu_{2n-1}}}\} &\to (-)^{n+1}\{G^{(2n)}_{\underline{\mu_1}\cdots\underline{\mu_{2n}}}, A^{(2n-1)}_{\underline{\mu_1}\cdots\underline{\mu_{2n-1}}}\}\,, \\
\{\psi_{\underline{\mu}}, \lambda, \epsilon\} &\to +\Gamma^9\{\psi_{\underline{\mu}}, -\lambda, \epsilon\}\,.
\end{aligned} \tag{2.28}$$

2.3 The Standard Formulation of IIA

Our dual action (2.23) can be reduced to the string-frame version of the standard formulation of massive IIA supergravity (originally written in Einstein frame in [3]). Here we will show how to go from one formulation to the other.

Consider the field equations for the $A^{(5)}$, $A^{(7)}$ and $A^{(9)}$ potentials given by

$$d(\mathbf{e}^{-B}{\scriptstyle\wedge}\mathbf{G}) = 0\,. \tag{2.29}$$

The most general solutions can be taken in the form

$$\mathbf{e}^{-B}{\scriptstyle\wedge}\mathbf{G} = d\mathbf{A} + \mathbf{G}_{\text{flux}}\,, \tag{2.30}$$

or, explicitly,

$$\begin{aligned}
G^{(0)} &= G^{(0)}_{\text{flux}}\,, \qquad G^{(2)} = dA^{(1)} + G^{(0)}B + G^{(2)}_{\text{flux}}\,, \\
G^{(4)} &= dA^{(3)} + G^{(2)}{\scriptstyle\wedge}B - \tfrac{1}{2}G^{(0)}B{\scriptstyle\wedge}B + G^{(4)}_{\text{flux}}\,.
\end{aligned} \tag{2.31}$$

The $\mathbf{G}_{\text{flux}}$ are cohomological solutions. If there is full 10-dimensional Lorentz symmetry, then only $G^{(0)}_{\text{flux}}$ can be non-zero and is a constant, which is the mass parameter m in the theory of Romans. We will mostly consider this situation. However, before proceeding, we can remark that we could consider that constant fluxes $G^{(2)}_{\text{flux}}$, $G^{(4)}_{\text{flux}}$ are present in our configuration in addition to m and only 4-dimensional Lorentz symmetry is preserved (e.g. [26, 27]). See section 8 for more comments.

From now on we restrict ourselves to

$$G^{(0)}_{\text{flux}} = m\,, \qquad G^{(2)}_{\text{flux}} = G^{(4)}_{\text{flux}} = 0\,. \tag{2.32}$$

Substituting these solutions in the bulk action (2.23), we obtain a theory without black boxes but with a mass parameter and a one- and a three- form. Note that this is the same field content (2.1) as the massive theory of Romans. A close inspection of the action reveals that in fact, we are dealing with the standard Romans' theory written in the **A**-basis introduced in (2.7).

Note that, as we have already remarked, $\mathbf{A} = \mathbf{C}{\scriptstyle\wedge}\mathbf{e}^{-B}$ is exactly the combination that occurs in the Wess–Zumino terms of D-brane actions, with the world-volume fields put to zero.

In the **C**-basis, the standard formulation has the action

$$S_{\text{bulk}} = -\frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}\Big\{e^{-2\phi}\big[R\big(\omega(e)\big) - 4\big(\partial\phi\big)^2 + \tfrac{1}{2}H\cdot H - 2\partial^\mu\phi\chi^{(1)}_\mu + H\cdot\chi^{(3)} + \\ + 2\bar\psi_\mu\Gamma^{\mu\nu\rho}\nabla_\nu\psi_\rho - 2\bar\lambda\Gamma^\mu\nabla_\mu\lambda + 4\bar\lambda\Gamma^{\mu\nu}\nabla_\mu\psi_\nu\big] + \sum_{n=0,1,2}\tfrac{1}{2}G^{(2n)}\cdot G^{(2n)} + G^{(2n)}\cdot\Psi^{(2n)} + \\ -\star\big[\tfrac{1}{2}\,dC^{(3)}\,dC^{(3)}\,B + \tfrac{1}{6}\,G^{(0)}\,dC^{(3)}\,B^3 + \tfrac{1}{40}\,G^{(0)2}\,B^5\big]\Big\} + \text{ quartic fermionic terms}\,, \tag{2.33}$$

where all $\wedge$'s have been omitted in the last line. Here all field strengths $G^{(2n)}$ are given by (2.10) and $G^{(0)}$ is constant. Note that the six Chern-Simons terms in (2.23) can be written in only three terms when the $G^{(2n)}$'s are field strengths. The standard IIA action is invariant under the $N = 2$ supersymmetry rules

$$\begin{aligned}
\delta_\epsilon e_\mu{}^a =& \bar\epsilon\Gamma^a\psi_\mu\,,\\
\delta_\epsilon\psi_\mu =& \Big(\partial_\mu + \tfrac{1}{4}\not\omega_\mu + \tfrac{1}{8}\Gamma_{11}\not H_\mu\Big)\epsilon + \tfrac{1}{8}e^\phi\sum_{n=0,1,2}\frac{1}{(2n)!}\not G^{(2n)}\Gamma_\mu(\Gamma_{11})^n\,\epsilon\,,\\
\delta_\epsilon B_{\mu\nu} =& -2\,\bar\epsilon\Gamma_{[\mu}\Gamma_{11}\psi_{\nu]}\,,\\
\delta_\epsilon C^{(2n-1)}_{\mu_1\cdots\mu_{2n-1}} =& -e^{-\phi}\,\bar\epsilon\,\Gamma_{[\mu_1\cdots\mu_{2n-2}}\,(\Gamma_{11})^n\,\Big((2n-1)\psi_{\mu_{2n-1}]} - \tfrac{1}{2}\Gamma_{\mu_{2n-1}]}\lambda\Big) + \\
& + (n-1)(2n-1)\,C^{(2n-3)}_{[\mu_1\cdots\mu_{2n-3}}\,\delta_\epsilon B_{\mu_{2n-2}\mu_{2n-1}]}\,,\\
\delta_\epsilon\lambda =& \Big(\not\partial\phi + \tfrac{1}{12}\not H\Gamma_{11}\Big)\epsilon + \tfrac{1}{4}e^\phi\sum_{n=0,1,2}\frac{5-2n}{(2n)!}\not G^{(2n)}(\Gamma_{11})^n\epsilon\,,\\
\delta_\epsilon\phi =& \tfrac{1}{2}\,\bar\epsilon\lambda\,,
\end{aligned} \tag{2.34}$$

and the gauge transformations (2.18). Also the $\mathbb{Z}_2$-symmetries (2.19) and (2.21) are valid but *only for vanishing mass.*

The dual formulation of section 2.2 thus can be converted to the standard formulation. But it is also possible to revert this: Romans' theory can be used to derive (2.23). To do so, one must perform the first step in the dualization of the one- and three-form. That is, the field strengths are promoted to black boxes and their Bianchi identities are imposed by Lagrange multipliers, considering $G^{(0)}$ as a zero-form field strength. The full dualization then implies the elimination of the black boxes by solving their equations of motion. Performing only the first step one obtains the action (2.23). It is for this reason that we call it the dual formulation.

Thus we have three different formulations of one theory at our disposal. The democratic formulation comprises all potentials but can not be used to add brane actions since it has no proper action. The standard formulation does have a proper action, containing $C^{(1)}$ and $C^{(3)}$, and thus is suitable for the 0- and 2-branes. The dual action has the dual potentials $A^{(5)}$, $A^{(7)}$ and $A^{(9)}$, accommodating for the 4-, 6- and 8-branes. With the latter two formulations we have set the stage for the addition of brane actions of any (even) dimension as long as we do not add simultaneously higher- and lower-dimensional branes.

3 Supersymmetry on the Brane

Having established supersymmetry in the bulk, we now turn to supersymmetry on the brane. As mentioned in the introduction, our main interest is in one-dimensional orbifold constructions with 8-branes at the orbifold points. Using the techniques of the three-brane on the orbifold in five dimensions [10], we want to construct an orientifold using a $\mathbb{Z}_2$-symmetry of the bulk action. On the fixed points we insert brane actions, which will turn out to be invariant under the reduced ($N = 1$) supersymmetry. For the moment we will not restrict to domain walls (in this case eight-branes) since our brane analysis is similar for orientifolds of lower dimension. In the previous section we have seen that our bulk action possesses a number of symmetries, among which a parity operation. To construct an orientifold, the relevant $\mathbb{Z}_2$-symmetry must contain parity operations in the transverse directions. Furthermore, in order to construct a charged domain wall, we want for a p-brane the $(p+1)$-form R-R potential to be even. For the 8-brane the $I_9\Omega$ symmetry satisfies the desired properties. For the other p-branes, it would seem natural to use the $\mathbb{Z}_2$-symmetry

$$I_{9,8,\ldots,p+1}\Omega \equiv (I_9\Omega)(I_8\Omega)\cdots(I_{p+1}\Omega)\,, \tag{3.1}$$

where $I_q\Omega$ is the transformation (2.21) with 9 replaced by q, and I_q and Ω commute. However, for some p-branes ($p = 2, 3, 6, 7$) the corresponding $C^{(p+1)}$ R-R-potential is odd under this $\mathbb{Z}_2$-symmetry. To obtain the correct parity one must include an extra $(-)^{F_L}$ transformation in these cases, which also follows from T-duality [28], see appendix B. This leads for each p-brane to the $\mathbb{Z}_2$-symmetry indicated in Table 1.

p	IIB	IIA
9	Ω	-
8	-	$I_9\Omega$
7	$(-)^{F_L}I_{9,8}\Omega$	-
6	-	$(-)^{F_L}I_{9,8,7}\Omega$
5	$I_{9,8,\ldots,6}\Omega$	-
4	-	$I_{9,8,\ldots,5}\Omega$
3	$(-)^{F_L}I_{9,8,\ldots,4}\Omega$	-
2	-	$(-)^{F_L}I_{9,8,\ldots,3}\Omega$
1	$I_{9,8,\ldots,2}\Omega$	-
0	-	$I_{9,8,\ldots,1}\Omega$

Table 1: The $\mathbb{Z}_2$-symmetries used in the orientifold construction of an Op-plane. The T-duality transformation from IIA to IIB in the lower dimension induces each time a $(-)^{F_L}$.

Thus the correct $\mathbb{Z}_2$-symmetry for a general IIA Op-plane is given by

$$((-)^{F_L})^{p/2}I_{9,8,\ldots,p+1}\Omega\,. \tag{3.2}$$

The effect of this $\mathbb{Z}_2$-symmetry on the bulk fields reads (the underlined indices refer to the worldvolume directions, i.e. $\mu = (\underline{\mu}, p+1, \ldots, 9)$

$$
\begin{aligned}
\{x^{p+1}, \ldots, x^9\} &\to -\{x^{p+1}, \ldots, x^9\}\,, \\
\{\phi, g_{\underline{\mu\nu}}, B_{\underline{\mu\nu}}\} &\to \{\phi, g_{\underline{\mu\nu}}, -B_{\underline{\mu\nu}}\}\,, \\
\{A^{(5)}_{\underline{\mu_1\cdots\mu_5}}, A^{(9)}_{\underline{\mu_1\cdots\mu_9}}, G^{(2)}_{\underline{\mu\nu}}\} &\to (-)^{\frac{p}{2}}\{A^{(5)}_{\underline{\mu_1\cdots\mu_5}}, A^{(9)}_{\underline{\mu_1\cdots\mu_9}}, G^{(2)}_{\underline{\mu\nu}}\}\,, \\
\{A^{(7)}_{\underline{\mu_1\cdots\mu_7}}, G^{(0)}, G^{(4)}_{\underline{\mu_1\cdots\mu_4}}\} &\to (-)^{\frac{p}{2}+1}\{A^{(7)}_{\underline{\mu_1\cdots\mu_7}}, G^{(0)}, G^{(4)}_{\underline{\mu_1\cdots\mu_4}}\}\,, \\
\{\psi_{\underline{\mu}}, \epsilon\} &\to -\alpha\Gamma^{p+1\cdots 9}(-\Gamma_{11})^{\frac{p}{2}}\{\psi_{\underline{\mu}}, \epsilon\}\,, \\
\{\lambda\} &\to +\alpha\Gamma^{p+1\cdots 9}(+\Gamma_{11})^{\frac{p}{2}}\{\lambda\}\,,
\end{aligned} \tag{3.3}
$$

and for fields with other indices there is an extra minus sign for each replacement of a worldvolume index $\underline{\mu}$ by an index in a transverse direction. We have left open the possibility of combining the symmetry with the sign change of all fermions. This possibility introduces a number $\alpha = \pm 1$ in the above rules. This symmetry will be used for the orientifold construction.

For this purpose we choose spacetime to be $\mathcal{M}^{p+1} \times T^{9-p}$ with radii $R^{\overline{\mu}}$ of the torus that may depend on the world-volume coordinates. All fields satisfy

$$
\Phi(x^{\overline{\mu}}) = \Phi(x^{\overline{\mu}} + 2\pi R^{\overline{\mu}})\,, \tag{3.4}
$$

with $\overline{\mu} = (p+1, \ldots, 9)$. We only keep fields that are even under the appropriate parity symmetry (3.2). In the bulk this relates fields at $x^{\overline{\mu}}$ and $-x^{\overline{\mu}}$. At the fixed point of the orientifolds, however, this relation is local and projects out half the fields. This means that we are left with only $N = 1$ supersymmetry on the fixed points, where the branes will be inserted. Consider for example a nine-dimensional orientifold. The projection truncates our bulk $N = 2$ supersymmetry to $N = 1$ on the brane; only half of the 32 components of ϵ are even under (3.3). The original field content, a $D = 10$, $(128 + 128)$, $N = 2$ supergravity multiplet, gets truncated on the brane to a reducible $D = 9$, $(64 + 64)$, $N = 1$ theory consisting of a supergravity plus a vector multiplet. One may further restrict to a constant torus. This particular choice of spacetime then projects out a $N = 1$ $(8+8)$ vector multiplet (containing $e_{\dot{9}}{}^{9}$), leaving us with the irreducible $D = 9$, $(56 + 56)$, $N = 1$ supergravity multiplet. Similar truncations are possible in lower dimensional orientifolds, on which the $(64 + 64)$ $N = 1$ theory also consists of a number of multiplets.

We propose the p-brane action ($p = 0, 2, 4, 6, 8$) to be proportional to

$$
\mathcal{L}_p = -e^{-\phi}\sqrt{-g_{(p+1)}} - \alpha\tfrac{1}{(p+1)!}\varepsilon^{(p+1)}C^{(p+1)}\,, \quad \text{with } \varepsilon^{(p+1)}C^{(p+1)} \equiv \varepsilon^{(p+1)}_{\underline{\mu_0\cdots\mu_p}}\, C^{(p+1)\underline{\mu_0\cdots\mu_p}}\,, \tag{3.5}
$$

with $\varepsilon^{(p+1)\,\underline{\mu_0\cdots\mu_p}} = \varepsilon^{(10)\,\underline{\mu_0\cdots\mu_p}\dot{p}+\dot{1}\cdots\dot{9}}$, which follows from $e_{\underline{\mu}}{}^{\overline{a}} = 0$ (being odd). Here the underlined indices are $(p+1)$-dimensional and refer to the world-volume. The parameter α is the same that appears in (3.3) and takes the values $\alpha = +1$ for *branes*, which are defined to have tension and charge with the same sign in our conventions, and $\alpha = -1$ for *anti-branes*, which are defined to have tension and charge of opposite signs. Note that due to the vanishing of B on the brane the potentials $C^{(p+1)}$ and $A^{(p+1)}$ are equal. The p-brane action can easily be shown to be invariant under the appropriate $N=1$ supersymmetry:

$$\delta_\epsilon \mathcal{L}_p = -e^{-\phi}\sqrt{-g_{(p+1)}}\,\bar\epsilon\big(1-\alpha\Gamma^{p+1\cdots 9}(\Gamma_{11})^{\frac{p}{2}}\big)\,\Gamma^{\underline{\mu}}\left(\psi_{\underline{\mu}}-\tfrac{1}{18}\Gamma_{\underline{\mu}}\lambda\right). \tag{3.6}$$

The above variation vanishes due to the projection under (3.3) that selects branes or anti-branes depending on the sign of α ($+1$ or -1 respectively). In the following discussions we will assume $\alpha=1$ but the other case just amounts to replacing branes by anti-branes.

By truncating our theory we are able to construct a brane action that only consists of bosons and yet is separately supersymmetric. Having these at our disposal, we can introduce source terms for the various potentials. In general there are 2^{9-p} fixed points. The compactness of the transverse space implies that the total charge must vanish. Thus the total action will read

$$\begin{aligned} \mathcal{L} =& \mathcal{L}_{\text{bulk}} + k_p\mathcal{L}_p\Delta_p\,, \\ \text{with}\quad \Delta_p \equiv& \big(\delta(x^{p+1})-\delta(x^{p+1}-\pi R^{p+1})\big)\cdots\big(\delta(x^9)-\delta(x^9-\pi R^9)\big) \end{aligned} \tag{3.7}$$

where the branes at all fixed points have a tension and a charge proportional to $\pm k_p$, a parameter of dimension $1/[\text{length}]^{p+1}$. Since anti-branes do not satisfy the supersymmetry condition (3.6), we need both positive and negative tension branes to accomplish vanishing total charge. As explained in the introduction we are going to interpret the negative tension branes as O-planes.

The equations of motion following from (3.7) induce a δ-function in the Bianchi identity of the $8-p$-form field strength. In general, an elegant solution is difficult to find, but in one special case the situation simplifies. This is the eight-brane case and will be discussed in the next section.

But first let us notice another possibility. With the above choice of spacetime, $\mathcal{M}^{p+1}\times(T^{9-p}/\mathbb{Z}_2)$, one can place the branes only at the fixed points without breaking all supersymmetry. However, making the identification

$$x^{\overline{\mu}} \sim -x^{\overline{\mu}}\,, \tag{3.8}$$

the projection under (3.3) would be local everywhere. The odd fields would not only vanish on the fixed points but in all spacetime. Thus the action (3.5) would be invariant everywhere and the branes would be allowed in between the fixed points. Note that not only on the fixed points but also in the bulk, only $N=1$ supersymmetry would survive the modding out of the $\mathbb{Z}_2$-symmetry. We will not consider this choice of spacetime.

4 Supersymmetry of the D8–O8 System

Mimicking the set-up of the five-dimensional case, we have now set the stage to add eight-brane actions. Replacing the mass parameter by a field $G^{(0)}$ at the cost of a nine-form $A^{(9)}$, our bulk action has two $\mathbb{Z}_2$-symmetries. The one involving parity will be used to truncate our $N=2$ theory to $N=1$ on the fixed points. The addition of brane actions will modify the equation of motion of the nine-form: $G^{(0)}$ only has to be constant between the branes.

First we choose our spacetime to be $\mathcal{M}^9 \times \mathcal{S}^1$. All fields satisfy $\Phi(x^9) = \Phi(x^9 + 2\pi R)$ with $R = R(x^{\underline{\mu}})$ the radius of $\mathcal{S}^1$. Furthermore, the fields can be split up in even and odd under $I_9\Omega$. Modding out this $\mathbb{Z}_2$-symmetry the odd fields vanish on the fixed points $x^9 = 0$ and $x^9 = \pi R \equiv -\pi R$ of the orientifold, where we will put the branes. Using the parity symmetry, and taking a constant radius for the circle, the $D = 10$, $(128+128)$, $N = 2$ supergravity multiplet gets truncated on the brane to a reducible $D = 9$, $(56 + 56)$, $N = 1$ supergravity (see the previous section).

We start with the nine-dimensional 8-brane action placed at $x^9 = 0$

$$(S_{D8})_{x^9=0} = \mu_8 \int d^{10}x \mathcal{L}_8 \, \delta(x^9) = -\tau_8 g_s \int d^{10}x \{e^{-\phi}\sqrt{-g_{(9)}} + \alpha \tfrac{1}{9!}\varepsilon^{(9)} C^{(9)}\}\delta(x^9) \,, \tag{4.1}$$

with $\varepsilon^{(9)\,\underline{\mu_0\cdots\mu_8}} \equiv \varepsilon^{(10)\,\underline{\mu_0\cdots\mu_8}\dot{9}}$ and $\sqrt{-g_{(9)}} = \sqrt{-g_{(10)}}$ where we use that $e_{\underline{\mu}}{}^9 = 0$ (being odd). The overall constant μ_8 is related to the 'physical' tension τ_8 as follows:

$$\mu_8 = \tau_8 g_s = \frac{2\pi}{(2\pi\ell_s)^9} \,. \tag{4.2}$$

The underlined indices are nine-dimensional in this section.

In this paper we assume that there is no matter on the branes. Thus, we are describing the vacuum solution of the D-brane system, switching off the excitations on the branes. As discussed in the Introduction, for the total charge to vanish while maintaining supersymmetric equilibrium, one needs negative tension branes rather than anti-branes. We associate the negative-tension branes to Orientifold(O)-planes. We suggest the following action of the supersymmetric O8-plane at $x^9 = 0$:

$$(S_{O8})_{x^9=0} = 16\tau_8 g_s \int d^{10}x \{e^{-\phi}\sqrt{-g_{(9)}} + \alpha \tfrac{1}{9!}\epsilon^{(9)} C^{(9)}\}\delta(x^9) \,. \tag{4.3}$$

Each plane has a charge -16 and thus we may associate this object with 16 negative tension D8-branes without matter. Both brane & plane action satisfy the supersymmetry condition (3.6). We would like to stress that supersymmetry of the brane action tells us to use always the same value of α, the relative sign between the kinetic and the WZ term independently of the sign in front of the total brane action. Thus we are left with only D-branes and O-planes.

There are two ways to interpret *our complete effective action.* In the first picture it consists of the bulk & branes with positive and negative tensions. In the alternative picture we have two O-planes as well as a number of D-branes at each plane.

i) *First picture: positive and negative tension branes.* In analogy with the supersymmetric RS construction in $D = 5$ [10] we may first consider the complete action for the IIA theory in the bulk with $2k$ coincident positive tension branes placed at $x^9 = 0$ and $2k$ coincident negative tension branes placed at $x^9 = \pi R$:

$$S_1 = S_{\text{bulk}} + 2k(S_{D8})_{x^9=0} - 2k(S_{D8})_{x^9=\pi R} \,. \tag{4.4}$$

Here k is an arbitrary integer. We take $2k$ branes with positive tension and $2k$ with negative tension to have a simple relation of this picture with the orientifold construction where one counts branes and their images, so that the total number is even.

ii) *Alternative interpretation: planes & branes.* Following [4], we consider O8-planes at the fixed points $x^9 = 0$ and $x^9 = \pi R = -\pi R$ and they carry the 8-brane charge -16 each. The theory of Type IIA supergravity under orientifold truncation would be inconsistent unless an $SO(32)$ gauge multiplet appears in the theory. This means that between these O-planes we have to place 32 D8-branes. This is the effective action of type I′ string theory. It is T-dual to Type I string theory, which is obtained by modding the IIB theory with the $\mathbb{Z}_2$-symmetry Ω (2.20). This also explains the origin of the 32 D8-branes: the Type I gauge group $SO(32)$ can be seen to come from 32 unoriented (spacetime filling) D9-branes and performing T-duality yields the 32 D8-branes [29].

In general these D8-branes can move between the O8-planes. However, we will only place them at the fixed points. At the point $x^9 = 0$ we have an O8-plane which contributes -16 to $2k$ in (4.4) and we have there also a stack of $2n$ D8-branes, thus $2k = 2(n-8)$. At the second fixed point of the orientifold we have -16 from the O8-plane and $2(16-n)$ from the stack of D8-branes so that $-2k = -2[8-(16-n)] = -2(n-8)$. This means that at $x^9 = 0$ for $n > 8$ there will be an effective action of the positive tension branes. At πR for $n > 8$ there will be an effective action of the negative tension branes. The total action is

$$S_2 = S_{\text{bulk}} + (S_{O8} + 2nS_{D8})_{x^9=0} + (S_{O8} + 2(16-n)S_{D8})_{x^9=\pi R}\,. \tag{4.5}$$

The two actions are equal for the special choice $k = n - 8$:

$$S_1 = S_2 = S_{\text{bulk}+O8+D8} \quad : \qquad k = n - 8\,. \tag{4.6}$$

It also follows that if at $x^9 = 0$ we want to have the total tension from branes & planes positive, i.e. $k > 0$, there is a restriction $n \geq 8$ so that

$$8 \leq n \leq 16\,. \tag{4.7}$$

5 8-Brane Solution and Killing Spinors

The total effective action (4.5) is given by the bulk action and an O8-plane and $2n$ D8-branes at $x^9 = 0$ and an O8-plane and $32-2n$ D8-branes at πR. To analyze its equations of motion, we will only keep the fields participating in D8–O8 dynamics: the metric, the dilaton, and the 0-form field strength and the 9-form potential. Thus, our starting point will be the following bulk, brane & plane supersymmetric action:

$$\begin{aligned} S_{\text{bulk}+O8+D8} \;=\; & \frac{2\pi}{(2\pi\ell_s)^8}\Bigg[\int d^{10}x\, -\sqrt{|g|}\left\{e^{-2\phi}\left[R - 4\left(\partial\phi\right)^2\right] + \tfrac{1}{2}\left(G^{(0)}\right)^2 - G^{(0)}\star\left(dC^{(9)}\right)\right\} \\ & - \frac{2(n-8)}{(2\pi\ell_s)}\{e^{-\phi}\sqrt{|g_{(9)}|} + \alpha\tfrac{1}{9!}\varepsilon^{(9)}C^{(9)}\}\left(\delta(x^9) - \delta(x^9 - \pi R)\right)\Bigg]\,. \end{aligned} \tag{5.1}$$

The D8-brane solution is given by

$$
\begin{aligned}
\mathrm{d}s_s^2 &= H_{D8}^{-1/2}[-\mathrm{d}t^2 + (\mathrm{d}x^{\underline{\mu}})^2] + H_{D8}^{1/2}(\mathrm{d}x^9)^2\,, \\
e^{\phi} &= e^{\phi_0} H_{D8}^{-5/4}\,, \\
G^{(0)} &= \alpha e^{-\phi_0}\partial_{\dot{9}} H_{D8} = \alpha\,\frac{n-8}{2\pi\ell_s}\varepsilon(x^9) \\
C^{(9)}_{\dot{0}\cdots\dot{8}} &= \alpha\, e^{-\phi_0}\left(H_{D8}^{-1}-1\right)\,, \qquad \text{with } \; H_{D8} = 1 - h_{D8}|x^9|\,, \qquad h_{D8} = \frac{(n-8)g_s}{2\pi\ell_s}\,,
\end{aligned}
\tag{5.2}
$$

where the first term of H_{D8} is fixed by requiring $e^{\phi} = e^{\phi_0}$ at $x^9 = 0$. This constant can be identified with the string coupling constant g_s. This is a natural identification in the non-asymptotically flat spacetimes associated to the higher branes and is also consistent, via T-duality, with the standard identification of $g_s = e^{\phi_0}$ where now this is the value of the dilaton at infinity in the asymptotically flat spacetimes associated to lower-dimensional branes.

For the non-vanishing fields of the D8-brane solution the Killing spinor equations take the form

$$
\left(\partial_\mu + \tfrac{1}{4}\,\not{\omega}_\mu + \tfrac{1}{8}e^{\phi}G^{(0)}\Gamma_\mu\right)\epsilon = 0\,, \qquad \left(\not{\partial}\phi + \tfrac{5}{4}e^{\phi}G^{(0)}\right)\epsilon = 0\,. \tag{5.3}
$$

The ingredients of these equations are the non-vanishing zehnbeins and spin connection components, the dilaton and $G^{(0)}$. These can be read off from (5.2) while the spin connection takes the form

$$
\not{\omega}_{\underline{\mu}} = 2\Gamma^9\Gamma_{\underline{\mu}}\partial_{\dot{9}}H_{D8}^{-1/4}\,, \qquad \not{\omega}_{\dot{9}} = 0\,. \tag{5.4}
$$

For these fields the Killing spinor equations are solved by

$$
\epsilon = H^{-1/8}\epsilon_0\,, \quad \text{with } (1+\alpha\Gamma^9)\,\epsilon_0 = 0\,, \tag{5.5}
$$

where ϵ_0 is a constant spinor that satisfies the above linear constraint. Thus it has 1/2 of unbroken supersymmetry of Type IIA theory, i.e. 16 unbroken supersymmetries.

6 Critical Distances and Quantization of Mass

From the 8-brane solution (5.2) one can read off that H_{D8} is zero for $|x^9| = 1/h_{D8}$, implying singularities. Thus the distance between the branes must be less than $1/h_{D8}$ so that the harmonic function does not vanish. The radius of the circle and distance between the O-planes is therefore restricted to

$$
R < \frac{2\pi\ell_s}{(n-8)g_s}\,. \tag{6.1}
$$

$1/h_{D8}$ is called the critical distance. Thus it seems that type I′ supergravity is consistent only on $\mathcal{M}^9 \times (\mathcal{S}^1/\mathbb{Z}_2)$ with a circle of restricted radius. Of course we have only considered a special case of the type I′ theory with all D-branes on one of the fixed points. However, also with D-branes in between the O-planes we expect the vacuum solution to imply a critical distance. The same phenomenon of Type I′ was found in [4] in the context of the duality between the Heterotic and Type I theories. Note that the maximal distance depends on the distribution of the D-branes. In the most asymmetric case ($n = 16$) it is smallest while in the most symmetric case ($n = 8$) there is no restriction on R.

The 8-brane solution (7.8) has other consequences as well. The equation of motion of the nine-form is modified by the brane & plane actions and leads to

$$G^{(0)} = \alpha \, \frac{n-8}{2\pi\ell_s} \varepsilon(x^9) \, . \tag{6.2}$$

Thus we may identify the mass parameter of Type IIA supergravity as follows:

$$m = \begin{cases} \alpha \dfrac{n-8}{2\pi\ell_s} \, , & x^9 > 0 \, , \\[2ex] -\alpha \dfrac{n-8}{2\pi\ell_s} \, , & x^9 < 0 \, . \end{cases} \tag{6.3}$$

The mass is quantized in string units and it is proportional to $n-8$ where there are $2n$ and $2(16-n)$ D8-branes at each O8-plane. The mass vanishes only in the special case $n = 8$ when the contribution from the D8-branes cancels exactly the contribution from the O8-planes. In general, due to restriction (4.7) the mass takes only the restricted values

$$2\pi\ell_s|m| = 0, \, 1, \, 2, \, 3, \, 4, \, 5, \, 6, \, 7, \, 8. \tag{6.4}$$

This is a quantization of our mass parameter, and for the cosmological constant it follows that

$$m^2 = (G^{(0)})^2 = \left(\frac{n-8}{2\pi\ell_s}\right)^2 \, . \tag{6.5}$$

Thus the mass parameter and the cosmological constant are quantized in the units of the string length in terms of the integers $n - 8$.

The quantization of the mass and of the cosmological constant in $D = 10$ was discussed before in [2, 4, 26] as well as in [5, 12]. In the latter two references, two independent derivations of the quantization condition were given. In [5], the T-duality between a 7-brane & 8-brane solution was investigated. Here it was pointed out that, in the presence of a cosmological constant, the relation between the $D = 10$ IIB R-R scalar $C^{(0)}$ and the one reduced to $D = 9$, $c^{(0)}$, is given via a generalized Scherk–Schwarz prescription:

$$C^{(0)} = c^{(0)}(x^9) + mx^8 \, . \tag{6.6}$$

Here (x^8, x^9) parametrize the 2-dimensional space transverse to the 7-brane. x^9 is a radial coordinate whereas x^8 is periodically identified (it corresponds to a U(1) Killing vector field):

$$x^8 \sim x^8 + 1 \, . \tag{6.7}$$

Furthermore, due to the $SL(2,\mathbb{Z})$ U-duality, the R-R scalar $C^{(0)}$ is also periodically identified:

$$C^{(0)} \sim C^{(0)} + 1\,. \tag{6.8}$$

Combining the two identifications with the reduction rule for $C^{(0)}$ leads to a quantization condition for m of the form

$$m \sim \frac{n}{\ell_s}\,, \qquad\qquad n \;\text{ integer}\,. \tag{6.9}$$

The same result was obtained by a different method in [12].

We are able to give a new, and independent, derivation of the quantization condition for the mass and cosmological constant. The conditions given in (6.3), (6.5) follow straightforwardly from our construction of the bulk & brane & plane action (5.1).

Note that the Scherk–Schwarz reduction in (6.6) and the quantization of $SL(2,\mathbb{R})$ were essential in deriving the quantization of m. In the new dual formulation we can derive a similar T-duality relation between the 7-brane and the 8-brane, including the source terms. However, in this case the T-duality relation does not imply a quantization condition for m since we do not know how to realize the $SL(2,\mathbb{R})$ symmetry in the dual formulation. Another noteworthy feature is that the derivation of the T-duality rules in the dual formulation does not require a Scherk-Schwarz reduction. This is possible due to the fact that the R-R scalar only appears after solving the equations of motion.

7 BPS Action and Supersymmetric Flow Equations

Following our work in $D = 5$, we consider the supersymmetric flow equations corresponding to a domain wall solution. Originally, supersymmetric flow equations were introduced in the context of black holes in [30]. They follow from the BPS-type energy functional, which has a form of a sum of perfect squares, up to total derivatives. Supersymmetric flow equations are simply the requirement that each expression in the perfect square vanishes. Typically it is the same condition that may be derived from the Killing spinor equations or from the field equations.

For the domain walls, the corresponding BPS-type energy functional was derived in [31, 32, 33]. It consists usually of two perfect squares, one with a positive and one with a negative sign and some total derivative terms. The requirement that each expression in the perfect square vanishes, leads to the supersymmetric flow equations. However, for the domain walls the presence of kinks requires a more careful treatment of the jump conditions at the wall, as shown in [10] for the 3-branes in $D = 5$. With supersymmetric bulk & brane & plane actions this will be taken care of automatically as we will show below.

We start with the action (5.1) and look for configuration which depends only on x^9. We choose the metric in the form suitable for Dp-branes in general.

$$\mathrm{d}s^2 = f^2(x^9)\,(-\mathrm{d}t^2 + (\mathrm{d}x^{\underline{\mu}})^2) + f^{-2}(x^9)\,(\mathrm{d}x^9)^2\,. \tag{7.1}$$

The scalar curvature is (with prime indicating a derivative with respect to x^9)

$$R = 18\left[5(f')^2 + ff''\right]. \tag{7.2}$$

The expression for the energy functional consists of the contribution from the bulk, and from the planes & branes. The branes are at the positions of the O-planes.

For our ansatz for the time-independent configuration, the energy is minus the Lagrangian $E = -\mathcal{L}$. We find the following expression for the energy functional of the bulk & brane-plane actions with $k = n - 8 > 0$

$$\begin{aligned}\frac{(2\pi\ell_s)^8}{2\pi}E_{\text{total}} =& \tfrac{1}{2}f^8 e^{-2\phi}(f\phi' - \tfrac{5}{4}\alpha G^{(0)}e^{\phi})^2 - 18 f^8 e^{-2\phi}(2f' - \tfrac{1}{2}f\phi' + \tfrac{1}{8}\alpha G^{(0)}e^{\phi})^2 + \\ &+ 18(f^9 f' e^{-2\phi})' + (\alpha G^{(0)} f^9 e^{-\phi})' \\ &+ \left(\frac{2k}{2\pi\ell_s}(\delta(x^9) - \delta(x^9 - \pi R) - \alpha G^{(0)\prime}\right)(f^9 e^{-\phi} + \alpha\tfrac{1}{9!}\varepsilon^{(9)}C^{(9)}). \end{aligned} \tag{7.3}$$

The third line in this expression can be removed by solving equations of motion for the 9-form field which leads to

$$\frac{2k}{2\pi\ell_s}\big((\delta(x^9) - \delta(x^9 - \pi R)\big) - \alpha G^{(0)\prime} = 0 \qquad \Longrightarrow \qquad G^{(0)} = \alpha\frac{k}{2\pi\ell_s}\varepsilon(x^9), \tag{7.4}$$

which is the same result as before. The second line of (7.3) consists of total derivatives, which vanish in our space with fields satisfying (3.4). Thus the final expression for the total energy is given by

$$\frac{(2\pi\ell_s)^8}{2\pi}\oint E_{total} = \oint \tfrac{1}{2}f^8 e^{-2\phi}\big[(f\phi' - \tfrac{5}{4}\alpha G^{(0)}e^{\phi})^2 - 36(2f' - \tfrac{1}{2}f\phi' + \tfrac{1}{8}\alpha G^{(0)}e^{\phi})^2\big]. \tag{7.5}$$

The difference with the structure of the BPS action in previous cases [31, 32, 33] is due to the presence of the mixed $f'\phi'$ terms, which originate from the second derivative of the metric converted into the first derivative of the metric and the dilaton. In Einstein frame the procedure of getting rid of the second derivative of the metric does not involve the dilaton derivative.

The supersymmetric flow equations are

$$f\phi' = \tfrac{5}{4}\alpha G^{(0)}e^{\phi}, \qquad 2f' - \tfrac{1}{2}f\phi' = -\tfrac{1}{8}\alpha G^{(0)}e^{\phi}. \tag{7.6}$$

We may eliminate $\alpha G^{(0)}$ from these equations so that $\phi' = 5\,f'/f$. If we choose $cf = e^{\frac{1}{5}\phi}$ where c is an arbitrary positive[4] constant, we get

$$\left(e^{-\frac{4}{5}\phi}\right)' = -c\alpha G^{(0)}, \qquad \left(e^{-\frac{4}{5}\phi}\right)'' = -c\alpha(G^{(0)})' = -c\frac{2k}{2\pi\ell_s}\big(\delta(x^9) - \delta(x^9 - \pi R)\big). \tag{7.7}$$

[4] We focus here on the special case when the energy of the brane at $x = 0$ is positive. Note that the energy is given by $\frac{2k}{2\pi\ell_s}f^9 e^{-\phi}$, thus $f > 0$.

We may choose two constants in our solution to be defined by the initial values at $x^9 = 0$ of the metric and of the dilaton so that $f(0) = 1$ and $e^{\phi}(0) = g_s = c^5$. For such a choice, the solution is given in terms of a harmonic function H_{D8}

$$H_{D8} \equiv f^{-4} = c^4 e^{-\frac{4}{5}\phi} = 1 - g_s \frac{k}{2\pi\ell_s}|x^9| = 1 - g_s|m||x^9| \,. \tag{7.8}$$

where $k = n - 8 > 0$. Note that the first term in the harmonic function has to be positive. At small $|x|$ the exponent has to be positive, and also we should be able to take a square root of it to have $f^2 = H^{-1/2}$. The second term comes out negative: this means that there is a singularity at $H = 0$. The string coupling e^{ϕ} blows up at $|x^9| = |x^9|_{\text{critical}} = 1/(g_s\,|m|)$ and the metric is singular. Thus we have to place the second wall at $\pi R < |x|_{\text{critical}}$.

In the solutions of the second order equations we have also found that the two constants in the harmonic function are of opposite sign. This supports the present picture that one needs two O-planes with branes on them at the finite distance from each other, to describe a physically meaningful configuration for supersymmetric domain walls (8-branes). It is plausible that by adding some more non-vanishing fluxes in our configurations that break more supersymmetries, one can find a solution where the runaway behaviour of the dilaton is replaced by the critical point with the fixed value of the dilaton. In such a solution one may expect that an infinite distance between orientifold planes will be possible. One natural candidate for such solution is the embedding of the FGPW type solution [34] into our $D = 10$ bulk & brane construction.

8 Summary of Results and Discussion

The main new results of this paper of general nature are the new formulations of Type II $D = 10$ supergravity (section 2). For both Type IIA and IIB theories, we constructed democratic bulk theories with a unified treatment of all R-R potentials. Due to the doubling of R-R degrees of freedom one had to impose extra duality constraints and thus a proper action was not possible. A so-called pseudo-action, containing kinetic terms for all R-R potentials but without Chern-Simons terms, was discussed. Furthermore, we have broken the self-duality explicitly in the IIA case, allowing for a proper action. Instead of all R-R potentials only half of the $C^{(p)}$'s occur in these theories. Both the standard ($p = 1, 3$) as well as the dual ($p = 5, 7, 9$) formulations were discussed. Using these actions all bulk & brane systems can be described.

In section 3 we have studied brane actions at the fixed points of orientifolds. It turned out that, on the appropriate orientifolds, all brane actions preserve half of the $N = 2$ supersymmetry. Either branes or anti-branes fulfilled this condition but not both. This can also be understood from the point of view of supersymmetric equilibrium of forces. Thus, in a compact space, to have vanishing charge, both positive and negative tension (anti-)branes must be used. The latter can be interpreted as orientifold planes while the first correspond to Dirichlet branes. One particular case of this was studied in detail: the eight-brane. Our supersymmetric bulk & brane action gave us a description of the D8–O8 system. These are the fundamental objects in Type I′ string theory. By studying this explicit construction we have found several interesting features.

We have carefully studied the issue of the supersymmetric D8–O8 sources and have found that the jump conditions on the walls are satisfied for our BPS solutions in presence of sources. This is a highly non-trivial issue in view of the recent studies of the difficulties to include the brane sources in the uplifted RS scenario in [35]. Our BPS 8-brane solution implied a maximal distance between the two walls in order to avoid singularities. At

$$|x^9|_{\text{critical}} = \frac{2\pi\ell_s}{(n-8)g_s}\,, \qquad n \neq 8\,, \tag{8.1}$$

the harmonic function vanishes and thus the second wall has to be placed before this. Note that the maximal distance depends on the positions of the D-branes. The $n = 8$ case is special since the D8-branes are symmetrically distributed and hence are cancelled by the O-planes contributions. Also the mass parameter is found to be quantized in string units:

$$|m| = \frac{n-8}{2\pi\ell_s}\,, \tag{8.2}$$

(again for $n \neq 8$). Note that it is proportional to the D-brane distribution factor $n - 8$ while the maximal distance was inversely proportional. In fact, we have the simple relation $|x^9|_{\text{critical}} = 1/(g_s\,|m|)$. This clearly shows the twofold effect of introducing source terms: they induce a non-zero value of the mass parameter but also imply a maximal distance between the walls. Of course this is all special to the exotic 8-brane since its corresponding field strength $G^{(10)}$ does not fall off at infinity.

The D8-brane configuration presented in this paper requires two domain walls at the finite distance from each other. This is necessary to cut off the singularity of the metric and to keep the dilaton from blowing up. On the other hand this solution also has too many unbroken supersymmetries, (1/2 of all supersymmetries of Type II string theories). However, solutions with more non-vanishing form fields might exist with a dilaton that does not blow up but tends to a fixed value and with a metric given by the product of the AdS_5 space and some Euclidean 5-dimensional manifold. Our D8-branes can be wrapped around this 5d manifold and produce the 3-branes in 5d space with Minkowski signature. These kind of 3-branes have been used in the bulk & brane construction of [10]. The distance between orientifold planes would not be limited in such a situation and in particular, the second wall at πR may be pushed to infinity. The number of unbroken supersymmetries in the bulk would be equal to 1/4 or 1/8 of original 32 and on the wall we would have the desirable $D = 4$, $N = 1$ supersymmetry. We expect the relevant bulk solution to be the uplifted FGPW solution [34] which has one IR fixed point at $|\tilde{x}^9| \to \infty$. The UV fixed point would be cut off by our O8-D8 plane at $x^9 = 0$. Due to the presence of the D8 branes and O8 planes such solution would realize the 5d RSII scenario in the framework of fundamental objects of string theory.

A notable difference of our scenario from the HW [36, 37] scenario is that the walls are the O8 and D8 objects which exist in string theory. They may be wrapped around some $D = 5$ manifold. The main goal of the HW theory was to present a scenario for appearance of chiral fermions starting with $D = 11$ supersymmetric theory with non-chiral fermions. Our O8-D8 construction may reach this precise goal in an interesting and controllable way due to stringy nature of this construction and due to the complete control over supersymmetries in the bulk & on the walls. We remark that the strong coupling limit of Type I′ string theory is equal to the HW theory. Using the results of this paper, it would be interesting to investigate whether and how in this limit the O8-D8 objects can be related to the HW branes.

It is clear that the D8-O8 system can be generalized much further. To start with, placing D-branes in any compact transverse space requires the presence of oppositely charged branes that will have to have opposite tensions in order to be in supersymmetric equilibrium. If all the negative-tension branes are identified with orientifold planes, as we are suggesting here that should be done, then the compact transverse spaces must be orbifolds with the orientifold planes placed in the orbifold points. The $\mathbb{Z}_2$ reflection symmetries associated to the orientifold planes can be part of more general orbifold groups ($\mathbb{Z}_n$ etc.). It would be interesting to realize these bulk & brane configurations explicitly.

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A Conventions

A.1 General

We use mostly plus signature $(- + \cdots +)$. Greek indices $\mu, \nu, \rho \ldots$ denote world coordinates and Latin indices $a, b, c \ldots$ represent tangent spacetime. They are related by the vielbeins $e_a{}^\mu$ and inverse vielbeins $e_\mu{}^a$. Explicit indices $0, \ldots, 9$ are dotted for world coordinates and undotted in the tangent spacetime case. The covariant derivative (with respect to general coordinate and local Lorentz transformations) is denoted by ∇_μ. Acting on tensors ξ and spinors χ it reads

$$\begin{aligned} \nabla_\mu \xi &= \partial_\mu \xi \,, \\ \nabla_\mu \xi^\nu &= \partial_\mu \xi^\nu + \Gamma_{\mu\rho}{}^\nu \xi^\rho \,, \\ \nabla_\mu \chi &= \partial_\mu \chi + \tfrac{1}{4} \omega_\mu{}^{ab} \Gamma_{ab} \chi \,, \\ \nabla_\mu \chi^\nu &= \partial_\mu \chi^\nu + \Gamma_{\mu\rho}{}^\nu \chi^\rho + \tfrac{1}{4} \omega_\mu{}^{ab} \Gamma_{ab} \chi \,. \end{aligned} \tag{A.1}$$

Here $\Gamma_{\mu\rho}{}^{\nu}$ and $\omega_{\mu}{}^{ab}$ are the affine and spin connection, respectively. Indices with an additional underlining indicate lower-dimensional brane indices. We symmetrize and antisymmetrize with weight one. Slashes are also used in the following sense: $\not{H} = H^{\mu\nu\rho}\Gamma_{\mu\nu\rho}$, and $\not{H}_{\mu} = H_{\mu\nu\rho}\Gamma^{\nu\rho}$.

Our conventions in form notation are as follows:

$$
\begin{aligned}
P^{(p)} &\equiv \frac{1}{p!} P^{(p)}_{\mu_1\cdots\mu_p} \mathrm{d}x^{\mu_1}{\scriptstyle\wedge}\cdots{\scriptstyle\wedge}\mathrm{d}x^{\mu_p}\,, \\
P^{(p)}\cdot Q^{(p)} &\equiv \frac{1}{p!} P^{(p)}_{\mu_1\cdots\mu_p} Q^{(p)\,\mu_1\cdots\mu_p}\,, \\
P^{(p)}{\scriptstyle\wedge}Q^{(q)} &\equiv \frac{1}{p!q!} P^{(p)}_{\mu_1\cdots\mu_p} Q^{(q)}_{\mu_{p+1}\cdots\mu_{p+q}} \mathrm{d}x^{\mu_1}{\scriptstyle\wedge}\cdots{\scriptstyle\wedge}\mathrm{d}x^{\mu_{p+q}}\,, \\
\star\, P^{(p)} &\equiv \frac{1}{(10-p)!p!}\sqrt{-g}\varepsilon^{(10)}_{\mu_1\cdots\mu_{10}} P^{(p)\,\mu_{11-p}\cdots\mu_{10}} \mathrm{d}x^{\mu_1}{\scriptstyle\wedge}\cdots{\scriptstyle\wedge}\mathrm{d}x^{\mu_{10-p}}\,, \\
&\quad \text{with} \quad \varepsilon^{(10)}_{0123\cdots9} = -\varepsilon^{0123\ldots9} = 1\,, \\
\star\star\; P^{(p)} &= (-)^{p+1} P^{(p)}\,, \\
d &\equiv \partial_\mu \mathrm{d}x^\mu\,,
\end{aligned}
\tag{A.2}
$$

where the last line is the exterior derivative, acting from the left. Also we will use the following abbreviation:

$$\mathbf{e}^{\pm B} \equiv \pm B + \tfrac{1}{2} B{\scriptstyle\wedge}B \pm \tfrac{1}{3!} B{\scriptstyle\wedge}B{\scriptstyle\wedge}B + \ldots \tag{A.3}$$

A.2 Spinors in Ten Dimensions

The ten-dimensional Γ-matrices are defined to satisfy the anticommutation relations

$$\left\{\Gamma^a, \Gamma^b\right\} = +2\eta^{ab}\,. \tag{A.4}$$

We can choose a Majorana representation where they are purely real, with the choice $\mathcal{C} = \Gamma_0$ for the charge conjugation matrix. Their Hermiticity properties are

$$\Gamma^{0\,\dagger} = \Gamma^{0\,T} = -\Gamma^0\,, \qquad \Gamma^{i\,\dagger} = \Gamma^{i\,T} = \Gamma^i\,, \quad i = 1,\ldots,9\,. \tag{A.5}$$

Furthermore we have the useful Γ-matrices identity

$$
\begin{aligned}
\Gamma_{11}\Gamma^{(n)}_{a_1\cdots a_n} &= \frac{(-1)^{\mathrm{Int}[(10-n)/2]+1}}{(10-n)!}\varepsilon^{(10)}_{a_1\cdots a_n b_1\cdots b_{10-n}}\Gamma^{(10-n)\;b_1\cdots b_{10-n}}\,, \\
\text{with} \quad \Gamma^{(n)}_{a_1\cdots a_n} &\equiv \Gamma_{[a_1}\cdots\Gamma_{a_n]} \quad \text{and} \quad \Gamma_{11} \equiv \Gamma^0\cdots\Gamma^9\,,
\end{aligned}
\tag{A.6}
$$

which combines with the star operation to

$$\Gamma_{11}\Gamma^{(n)} = (-)^{\mathrm{Int}\,\frac{n+1}{2}}\;\star\,\Gamma^{(10-n)}\,. \tag{A.7}$$

The Majorana condition is equivalent to requiring all components of a Majorana spinor to be real. We do not change order of the fermions in performing complex conjugation. Using the above properties and the definition of Majorana spinors one finds

$$\bar{\chi}\Gamma^{a_1\cdots a_n}\psi = (-1)^{n+\mathrm{Int}[n/2]}\;\bar{\psi}\Gamma^{a_1\cdots a_n}\chi\,. \tag{A.8}$$

B World-sheet T-duality

Although the discussion on bulk & brane supersymmetry has been in the context of supergravity, it is also useful to consider the string origin. Here we will derive a number of $\mathbb{Z}_2$-symmetries of Type IIA and IIB string theories, that are relevant for the orbifold[5] procedure. Since these are symmetries of the world-sheet, not only effective supergravity actions but also the full string theories must be invariant under the corresponding spacetime operations. It will turn out that they are related via simple world-sheet T-duality relations. We will discuss this in the Neveu–Schwarz–Ramond formalism. On the world-sheet one has the following field content:

$$\left\{ X_L{}^\mu, X_R{}^\mu, \psi_L{}^\mu, \psi_R{}^\mu \right\} \tag{B.1}$$

with left- and right-moving bosons and fermions. To construct symmetries of the two theories the following operations on these fields will be used:

$$\begin{array}{llll}
T_q : & X_L{}^q \to -X_L{}^q & \psi_L{}^q \to -\psi_L{}^q & \\[2ex]
I_q : & X_L{}^q \to -X_L{}^q & \psi_L{}^q \to -\psi_L{}^q & \\
 & X_R{}^q \to -X_R{}^q & \psi_R{}^q \to -\psi_R{}^q & \\[2ex]
\Omega : & X_L{}^\mu \leftrightarrow X_R{}^\mu & \psi_L{}^\mu \leftrightarrow \psi_R{}^\mu & (\sigma \to -\sigma) \\[2ex]
(-)^{F_L^c} : & & \psi_L{}^\mu \to -\psi_L{}^\mu &
\end{array}$$

with the other fields invariant. Note that all these operations square to one. Although the world-sheet action may be invariant under the above operations, in string theory boundary conditions are involved as well. Different choices of boundary conditions lead to different types of string theories. Therefore, for an operation to be a symmetry of a certain theory, it should also leave the boundary conditions invariant. Bearing this in mind, it is easy to see that Ω is a symmetry of Type IIB theory only. Using T-duality between Type IIA and IIB theory we can derive other symmetries. This chain of symmetries, connected via T-duality, reads

$$S_p \equiv ((-)^{F_L^c})^{\mathrm{Int}\left[\frac{p}{2}\right]} \cdot I_9 \cdots I_{p+1}\Omega = T_p S_{p+1} T_p \,, \tag{B.2}$$

with $S_{10} \equiv \Omega$. It is explicitly given in table 1. Thus, by applying T-duality to a $(p+1)$-dimensionally extended parity operation, we get a parity operation with one more direction involved. The S_p with p odd are symmetries of Type IIA theory; the S_p with p even of Type IIB theory. Using these Z_2-symmetries it is possible to construct orientifolds of any

[5] An *orbifold* corresponds to modding out a *spacetime* $\mathbb{Z}_2$-symmetry. From the supergravity point of view this is the correct terminology for section 2. Here we will discuss the string theory origin; a *world-sheet* operation is involved and the corresponding construction is called an *orientifold*. See [28] for a general introduction.

dimension that are charged under the corresponding R-R potential. Furthermore, under T-duality the actions of orientifold planes of all dimensions are related. This should not come as a surprise; an orientifold plane can be seen as a truncation of a D-brane and the latter are also related under T-duality.

References

[1] J. Polchinski, *String Theory.* Cambridge University Press, 1999.

[2] J. Polchinski, *Dirichlet-branes and Ramond–Ramond charges,* Phys. Rev. Lett. **75** (1995) 4724–4727, `hep-th/9510017`.

[3] L. J. Romans, *Massive $N = 2a$ Supergravity in Ten Dimensions,* Phys. Lett. **169B** (1986) 374.

[4] J. Polchinski and E. Witten, *Evidence for heterotic - type I string duality,* Nucl. Phys. **B460** (1996) 525–540, `hep-th/9510169`.

[5] E. Bergshoeff, M. de Roo, M. B. Green, G. Papadopoulos, and P. K. Townsend, *Duality of type II 7-branes and 8-branes,* Nucl. Phys. **B470** (1996) 113–135, `hep-th/9601150`.

[6] M. J. Duff and P. van Nieuwenhuizen, *Quantum inequivalence of different field representations,* Phys. Lett. **B94** (1980) 179.

[7] A. Aurilia, H. Nicolai, and P. K. Townsend, *Hidden constants: the theta parameter of QCD and the cosmological constant of $N = 8$ supergravity,* Nucl. Phys. **B176** (1980) 509.

[8] N. Alonso-Alberca, P. Meessen, and T. Ortín, *Supersymmetric brane-worlds,* Phys. Lett. **B482** (2000) 400–408, `hep-th/0003248`.

[9] L. Randall and R. Sundrum, *A large mass hierarchy from a small extra dimension,* Phys. Rev. Lett. **83** (1999) 3370–3373, `hep-ph/9905221`.
An alternative to compactification, Phys. Rev. Lett. **83**, 4690 (1999), `hep-th/9906064`.

[10] E. Bergshoeff, R. Kallosh, and A. Van Proeyen, *Supersymmetry in singular spaces,* JHEP **10** (2000) 033, `hep-th/0007044`.

[11] E. Bergshoeff, H. J. Boonstra, and T. Ortín, *S duality and dyonic p-brane solutions in type II string theory,* Phys. Rev. **D53** (1996) 7206–7212, `hep-th/9508091`.

[12] M. B. Green, C. M. Hull, and P. K. Townsend, *D-Brane Wess–Zumino actions, T-Duality and the cosmological constant,* Phys. Lett. **B382** (1996) 65–72, `hep-th/9604119`.

[13] M. Cederwall, A. von Gussich, B. E. W. Nilsson, P. Sundell, and A. Westerberg, *The Dirichlet super-p-branes in ten-dimensional type IIA and IIB supergravity,* Nucl. Phys. **B490** (1997) 179–201, `hep-th/9611159`.

[14] E. Bergshoeff and P. K. Townsend, *Super D-branes,* Nucl. Phys. **B490** (1997) 145–162, `hep-th/9611173`.

[15] M. Perry and J. H. Schwarz, *Interacting chiral gauge fields in six dimensions and Born–Infeld theory,* Nucl. Phys. **B489** (1997) 47–64, `hep-th/9611065`.

[16] P. Pasti, D. Sorokin, and M. Tonin, *Duality symmetric actions with manifest space-time symmetries,* Phys. Rev. **D52** (1995) 4277–4281, `hep-th/9506109`.
P. Pasti, D. Sorokin and M. Tonin, *On Lorentz invariant actions for chiral p-forms,* Phys. Rev. **D55** (1997) 6292-6298, `hep-th/9611100`;
P. Pasti, D. Sorokin and M. Tonin, *Covariant action for a $D = 11$ five-brane with the chiral field,* Phys. Lett. **B398** (1997) 41, `hep-th/9701037`.

[17] M. Huq and M. A. Namazie, *Kaluza–Klein supergravity in ten dimensions,* Class. Quant. Grav. **2** (1985) 293.

[18] F. Giani and M. Pernici, *$N = 2$ supergravity in ten dimensions,* Phys. Rev. **D30** (1984) 325–333.

[19] I. C. G. Campbell and P. C. West, *$N = 2$ $D = 10$ nonchiral supergravity and its spontaneous compactification,* Nucl. Phys. **B243** (1984) 112.

[20] J. H. Schwarz, *Covariant field equations of chiral $N = 2$ $D = 10$ supergravity,* Nucl. Phys. **B226** (1983) 269.

[21] P. S. Howe and P. C. West, *The complete $N = 2$, $d = 10$ supergravity,* Nucl. Phys. **B238** (1984) 181.

[22] M. Fukuma, T. Oota, and H. Tanaka, *Comments on T-dualities of Ramond–Ramond potentials on tori,* Prog. Theor. Phys. **103** (2000) 425–446, `hep-th/9907132`.

[23] E. Bergshoeff, M. de Roo, B. Janssen, and T. Ortín, *The super D9-brane and its truncations,* Nucl. Phys. **B550** (1999) 289–302, `hep-th/9901055`.

[24] E. Bergshoeff, E. Eyras, R. Halbersma, J. P. van der Schaar, C. M. Hull, and Y. Lozano, *Spacetime-filling branes and strings with sixteen supercharges,* Nucl. Phys. **B564** (2000) 29–59, `hep-th/9812224`.

[25] P. Meessen and T. Ortin, *An $Sl(2, Z)$ multiplet of nine-dimensional type II supergravity theories,* Nucl. Phys. **B541** (1999) 195, `hep-th/9806120`.

[26] J. Polchinski and A. Strominger, *New vacua for type II string theory,* Phys. Lett. **B388** (1996) 736–742, `hep-th/9510227`.

[27] S. Gukov, *Solitons, superpotentials and calibrations,* Nucl. Phys. **B574** (2000) 169–188, `hep-th/9911011`.

[28] A. Dabholkar, *Lectures on orientifolds and duality,* `hep-th/9804208`. in 'High energy physics and cosmology 1997', Proceedings Trieste 1997, The ICTP Series in Theoretical Physics, Vol. 14, eds. E. Gava et al., Singapore, World Scientific, 1998, p. 128.

[29] P. Horava and E. Witten, *Heterotic and type I string dynamics from eleven dimensions,* Nucl. Phys. **B460** (1996) 506–524, `hep-th/9510209`.

[30] S. Ferrara, G. W. Gibbons, and R. Kallosh, *Black holes and critical points in moduli space,* Nucl. Phys. **B500** (1997) 75–93, `hep-th/9702103`.

[31] K. Skenderis and P. K. Townsend, *Gravitational stability and renormalization-group flow,* Phys. Lett. **B468** (1999) 46–51, `hep-th/9909070`.

[32] O. DeWolfe, D. Z. Freedman, S. S. Gubser, and A. Karch, *Modeling the fifth dimension with scalars and gravity,* Phys. Rev. **D62** (2000) 046008, `hep-th/9909134`.

[33] R. Kallosh and A. Linde, *Supersymmetry and the brane world,* JHEP **02** (2000) 005, `hep-th/0001071`.

[34] D. Z. Freedman, S. S. Gubser, K. Pilch, and N. P. Warner, *Renormalization group flows from holography supersymmetry and a c-theorem,* Adv. Theor. Math. Phys. **3** (1999) 363–417, `hep-th/9904017`.

[35] M. Cvetič, M. Duff, J. T. Liu, H. Lü, C. Pope, and K. Stelle, *Randall–Sundrum brane tensions,* `hep-th/0011167`.

[36] P. Hořava and E. Witten, *Eleven-dimensional supergravity on a manifold with boundary,* Nucl. Phys. **B475** (1996) 94–114, `hep-th/9603142`.

[37] A. Lukas, B. A. Ovrut, K. S. Stelle, and D. Waldram, *The universe as a domain wall,* Phys. Rev. **D59** (1999) 086001, `hep-th/9803235`.
A. Lukas, B. A. Ovrut, K. S. Stelle and D. Waldram, *Heterotic M-theory in five dimensions,* Nucl. Phys. **B552** (1999) 246 `hep-th/9806051`;
K. S. Stelle, *Domain walls and the universe,* in 'Nonperturbative aspects of strings, branes and supersymmetry', proc. Trieste 1998, eds. M. Duff et al., World Scientific, 1999, p. 396, `hep-th/9812086`.