

Neutrino mass in supersymmetry

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Abstract.

After summarizing neutrino oscillation results I discuss high and low-scale seesaw mechanisms, with or without supersymmetry, as well as recent attempts to understand the pattern of neutrino mixing from flavor symmetries. I also mention the possibility of intrinsic supersymmetric neutrino masses in the context of broken R parity models, showing how this leads to clear tests at the LHC.

NEUTRINO OSCILLATIONS

We now have uncontroversial evidence for neutrino flavor conversion coming from “celestial” (solar and atmospheric) as well as “laboratory” studies with reactor and accelerator neutrinos [1, 2]. Oscillations constitute the only viable explanation of the data and provide the first sign of physics beyond the Standard Model (SM). The basic concept in terms of which to describe them is the lepton mixing matrix, the leptonic analogue of the quark mixing matrix. In its simplest 3×3 unitary form it is given as [3]

$$K = \omega_{23}\omega_{13}\omega_{12} \quad (1)$$

where each ω is characterized by an angle and a corresponding CP phase. Present experiments are insensitive to CP violation, hence we set all three phases to zero. In this approximation oscillations depend on the three mixing angles $\theta_{12}, \theta_{23}, \theta_{13}$ and on the two squared-mass splittings $\Delta m_{21}^2 \equiv m_2^2 - m_1^2$ and $\Delta m_{31}^2 \equiv m_3^2 - m_1^2$ characterizing solar and atmospheric transitions. To a good approximation, one can set $\Delta m_{21}^2 = 0$ in the analysis of atmospheric and accelerator data, and Δm_{31}^2 to infinity in the analysis of solar and reactor data. The neutrino oscillation parameters obtained from a global analysis of the world’s neutrino oscillation data are summarized in Figs. 1 and 2. The left and right panels in Fig. 1 give the “atmospheric” and “solar” oscillation parameters, θ_{23} & Δm_{31}^2 , and θ_{12} & Δm_{21}^2 , respectively. The dot, star and diamond indicate the best fit points of atmospheric MINOS and global data, respectively. We minimize with respect to Δm_{21}^2 , θ_{12} and θ_{13} , including always all the relevant data. Similarly the “solar” oscillation parameters are obtained by combining solar and reactor neutrino data. The dot, star and diamond indicate the best fit points of solar, KamLAND and global data, respectively. We minimize with respect to Δm_{31}^2 , θ_{23} and θ_{13} , including always all relevant data. One sees that data from artificial and natural neutrino sources are

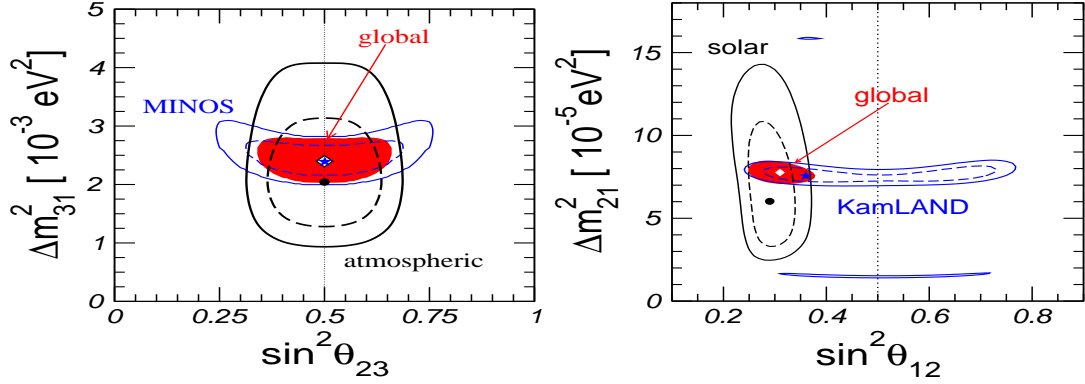


FIGURE 1. Current neutrino oscillation parameters, from Ref. [2].

clearly complementary: reactor and accelerators give the best determination of squared-mass-splittings, while solar and atmospheric data mainly determine mixings. Fig. 2

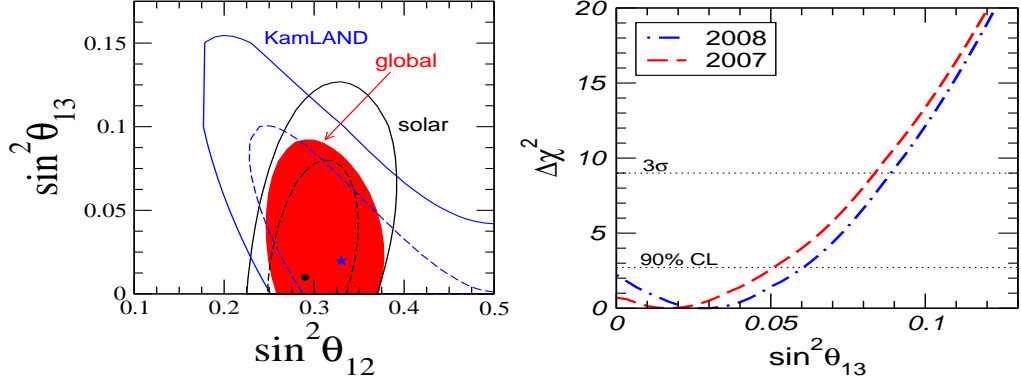


FIGURE 2. Constraints on $\sin^2 \theta_{13}$ from different parts of the global data given in Ref. [2].

summarizes the information on the remaining angle θ_{13} , the right panel shows how current data slightly prefer a nonzero value for θ_{13} . Since this is currently not significant, we prefer to interpret this as a weaker bound on θ_{13} ¹:

$$\sin^2 \theta_{13} \leq \begin{cases} 0.060 (0.089) & (\text{solar+KamLAND}) \\ 0.027 (0.058) & (\text{CHOOZ+atm+K2K+MINOS}) \\ 0.035 (0.056) & (\text{global data}) \end{cases} \quad (2)$$

A possible experimental confirmation of a non-zero θ_{13} would encourage the search for CP violation in upcoming neutrino oscillation experiments [4, 5]. Note that all CP violating observables, such as CP asymmetries, are proportional to the small parameter $\alpha \equiv \frac{\Delta m_{21}^2}{|\Delta m_{31}^2|}$ well-determined experimentally as $\alpha = 0.032$, $0.027 \leq \alpha \leq 0.038$ (3σ).

¹ Note: the bounds in Eq. (2) are given for 1 dof, while the regions in Fig. 2 (left) are 90% CL for 2 dof

ON THE ORIGIN OF NEUTRINO MASS

In spite of the great experimental progress summarized above, pinning-down the ultimate origin of neutrino mass remains a challenge for the next decades. To understand the pathways to neutrino masses it is important to note that, being electrically neutral, neutrino masses should, on general grounds, be of Majorana-type [3]. Indeed, specific neutrino mass generation mechanisms also differ from those of charged fermions in the SM. The latter come in two chiral species and get mass linearly in the electroweak symmetry breaking vacuum expectation value (vev) $\langle\Phi\rangle$ of the Higgs scalar doublet. In contrast, as shown in the left panel in Fig. 3, neutrinos acquire mass from an effective lepton number violating dimension-five operator $\lambda L\Phi L\Phi$ (where L denotes a lepton doublet) [6].

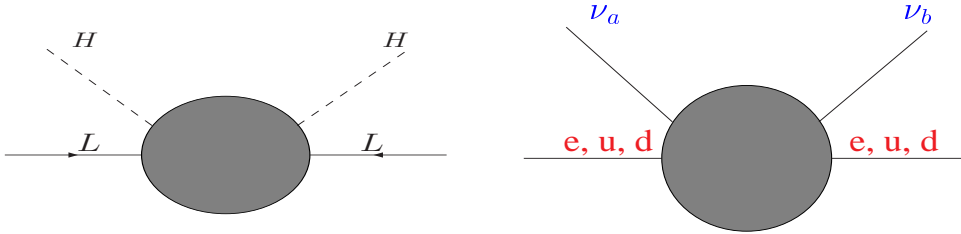


FIGURE 3. Operators characterizing neutrino masses and non-standard neutrino interactions (NSI) arising, say, from the non-trivial structure of lepton mixing in seesaw-type schemes [3].

A natural way to account for the smallness of neutrino masses, irrespective of their specific origin, is that L-number is restored, in the absence of L-violating operator(s). Such may be naturally suppressed either by a high-scale M_X in the denominator or, alternatively, it may involve a low-mass-scale in the numerator. The big question is to identify which *mechanism* gives rise to this operator, its associated mass *scale* and its *flavor structure*. Gravity is often argued to break global symmetries [7, 8], and could induce the dimension-five operator, with M_X identified to the Planck scale. The resulting Majorana neutrino masses are too small, hence the need for physics beyond the SM [9].

The coefficient λ could vanish due to symmetry, so that the effective operator responsible for neutrino mass is of higher dimension [10, 11]. Alternatively it may be suppressed by small scales, Yukawa couplings and/or loop-factors [12]. To arrange our brief discussion I consider three options: (i) tree level, (ii) radiative, and (iii) hybrid mechanisms, all of which may have high- or low-scale realizations. If lepton-number symmetry is broken spontaneously there is either an extra neutral gauge boson or a Nambu-Goldstone boson coupled to neutrinos, depending on whether it is gauged or not. It is easy to construct models based on either high- or low-scale symmetry breaking, the former are more popular among theorists, because they are closer to the idea of unification.

However the most basic and general description of the seesaw is in terms of the SM $SU(3) \otimes SU(2) \otimes U(1)$ gauge structure [3]. In such a framework it is clear that the relevant scale can be large or small, depending on model details. Hence there is a fair chance that the origin of neutrino mass may be probed at accelerators like the LHC.

(i) Minimal seesaw schemes

The classic way to generate Weinberg’s dimension-5 operator [6] is the exchange of heavy fermion states with masses close to the “unification” scale.

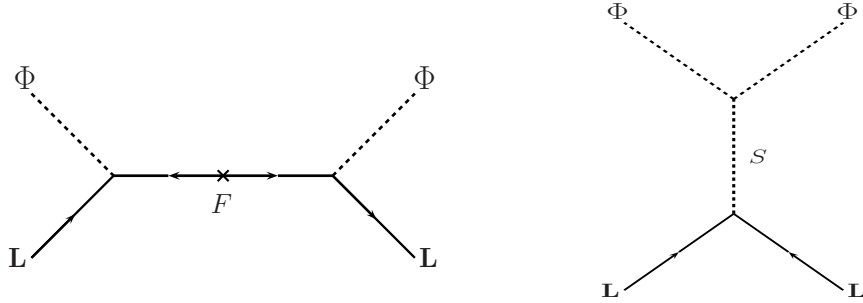


FIGURE 4. Type-I and III (left) and Type-II (right) realizations of the seesaw mechanism.

Depending on whether these are $SU(3) \otimes SU(2) \otimes U(1)$ singlets or triplets the mechanism is called **type-I** [3] [13, 14, 15, 16], or **type-III** seesaw [17], respectively. As seen in the right panel in Fig. 4, the seesaw may also be induced by the exchange of heavy triplet scalars, now called **type-II** seesaw [3] [18, 19], a convention opposite to the one used originally in [3]. The “complete seesaw” was thoroughly studied in [3] and its perturbative diagonalization was given Ref. [18] in a general form that may be adapted to different models. The hierarchy of vevs required to account the small neutrino masses $v_3 \ll v_2 \ll v_1$ in such seesaw was studied in detail in Ref. [18].

(ii) “Non-minimal” seesaw schemes

The seesaw may be implemented in the type I, type II or type III manner, with different gauge groups and multiplet contents, with gauged or ungauged B-L, broken explicitly or spontaneously, at a high or at a low energy scale, with or without supersymmetry. There are so many ways to seesaw, that a full taxonomy describing all variants will probably never be written, as nature may be more imaginative than physicists. Since any extended symmetry model must ultimately break to the SM, what is phenomenologically relevant is the seesaw description at the $SU(3) \otimes SU(2) \otimes U(1)$ level [3]. Such low-energy description is specially relevant in accurately describing low-scale variants of the seesaw mechanism, whose interest has now been revived with the coming of the LHC. An attractive class of such schemes employs, in addition to the left-handed SM neutrinos ν_L , two $SU(3) \otimes SU(2) \otimes U(1)$ singlets ν^c, S [20] (for other extended seesaw schemes see, e.g. [21, 22, 23, 24]). The basic lepton-number-violating parameter is small [25, 26] and may be calculable due to supersymmetric renormalization group evolution effects [27]. One may implement such schemes in the $SO(10)$ framework [28, 29], leaving, in addition a light Z' to be probed at the LHC [30].

(iii) Radiative schemes

Neutrino masses may be absent at tree level and calculable [32, 33], with no need for a large scale. In this case the coefficient λ is suppressed by small loop-factors, by Yukawa couplings and possibly by a small scale parameter characterizing the breaking of lepton number, leading to naturally small neutrino masses. Like low-scale seesaw schemes (see above) radiative models open the door to phenomenology associated with the new states required to provide the neutrino mass and which could be searched for, e. g., at the LHC.

(iv) R parity violation

It could well be that the origin of neutrino masses is intrinsically supersymmetric. This is the case in models with R parity violation [34, 35, 36], in which lepton number is broken together with the so-called R parity. This may happen spontaneously, driven by a nonzero vev of an $SU(3) \otimes SU(2) \otimes U(1)$ singlet sneutrino [37, 38, 39], leading to an effective model with bilinear violation of R parity [40, 41]. This provides the minimal way to break R parity and add neutrino masses to the MSSM [41]. The neutrino spectrum is hybrid, with one scale (typically the atmospheric) generated at tree level by neutralino-exchange *weak-scale seesaw*, and the other scale (solar) induced by *calculable* one-loop corrections [42].

Unprotected by any symmetry, the lightest supersymmetric particle (LSP) will decay. Given the scale of neutrino mass indicated by experiment these decays will happen inside typical detectors at the Tevatron or the LHC [42, 43] but with a decay path that can be experimentally resolved, leading to a so-called displaced vertex [44, 45] (left panel in Fig. 5). More strikingly, its decay properties correlate with the neutrino mixing angles. Indeed, as seen in the right panel in Fig. 5 the LSP decay pattern is predicted by

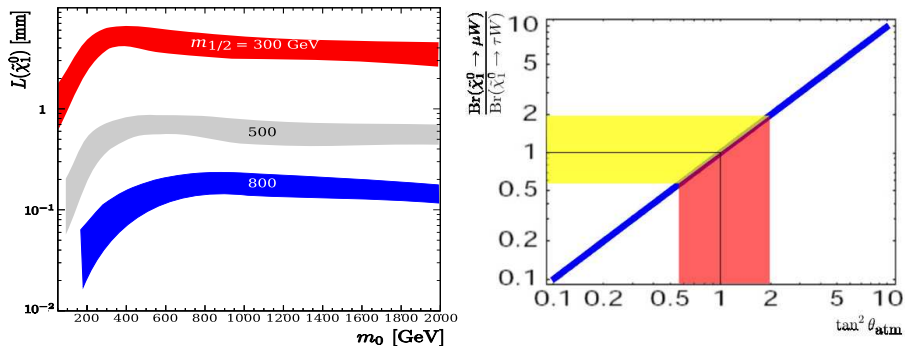


FIGURE 5. Left: $\tilde{\chi}_1^0$ decay length versus m_0 for $A_0 = -100$ GeV, $\tan\beta = 10$, $\mu > 0$, and several values of $m_{1/2}$. The widths of the three shaded bands around $m_{1/2} = 300, 500, 800$ GeV correspond to the variation of the BRpV parameters in such a way that the neutrino masses and mixing angles fit the required values within 3σ . Right: Ratio of branching ratios, $\text{Br}(\tilde{\chi}_1^0 \rightarrow \mu q' \bar{q})$ over $\text{Br}(\tilde{\chi}_1^0 \rightarrow \tau q' \bar{q})$ as a function of the atmospheric angle in bilinear R parity violation [31].

the low-energy measurement of the atmospheric angle [31, 46, 47]. Such a prediction will be tested at the LHC, and will potentially allow a high-energy redetermination of θ_{23} . Similar correlations hold in variant models which have other supersymmetric states as LSP [48].

FLAVOR SYMMETRIES

As seen above current neutrino oscillation data indicate solar and atmospheric mixing angles which are unexpectedly large when compared with quark mixing angles. This challenges our attempts to explain the flavor problem in unified schemes where quarks and leptons are related. It has been noted that the neutrino mixing angles are approximately given by [49],

$$\begin{aligned}\tan^2 \theta_{\text{ATM}} &= \tan^2 \theta_{23}^0 = 1 \\ \sin^2 \theta_{\text{Chooz}} &= \sin^2 \theta_{13}^0 = 0 \\ \tan^2 \theta_{\text{SOL}} &= \tan^2 \theta_{12}^0 = 0.5.\end{aligned}\tag{3}$$

There have been many schemes suggested in the literature in order to reproduce the full tri-bi-maximal pattern, or at least to predict maximal atmospheric mixing using various discrete flavor symmetry groups containing mu-tau symmetry, e. g. [50, 51, 52, 53, 54, 55, 56, 57, 58, 59]. One expects the flavor symmetry to be valid at high energy scales. Deviations from tri-bi-maximal ansatz [60] may be calculable by renormalization group evolution [61, 62, 63].

A specially simple ansatz is that, as a result of a given flavor symmetry such as A4 [50, 51], neutrino masses unify at high energies M_X [64], the same way as gauge couplings unify at high energies due to supersymmetry [65]. Such quasi-degenerate neutrino scheme predicts maximal atmospheric angle and vanishing θ_{13} ,

$$\theta_{23} = \pi/4 \quad \text{and} \quad \theta_{13} = 0,$$

leaving the solar angle θ_{12} unpredicted, but Cabibbo-unsuppressed,

$$\theta_{12} = \mathcal{O}(1).$$

If CP is violated θ_{13} becomes arbitrary and the Dirac phase is maximal [53]. One can show that lepton and slepton mixings are related and that at least one slepton lies below 200 GeV, within reach of the LHC. The absolute Majorana neutrino mass scale $m_0 \gtrsim 0.3$ eV ensures that the model will be probed by future cosmological tests and $\beta\beta_{0\nu}$ searches. Rates for lepton flavour violating processes $l_j \rightarrow l_i + \gamma$ typically lie in the range of sensitivity of coming experiments, with $\text{BR}(\mu \rightarrow e\gamma) \gtrsim 10^{-15}$ and $\text{BR}(\tau \rightarrow \mu\gamma) > 10^{-9}$.

LEPTON FLAVOR VIOLATION (LFV)

Flavor is violated in neutrino propagation [1, 2]. It is therefore natural to expect that, at some level, it will also show up as transitions involving the charged leptons, since these sit in the same electroweak doublet. Two basic mechanisms are: (i) neutral heavy lepton exchange [66, 67, 68] and (ii) supersymmetry [69, 70, 71]. Both exist in supersymmetric seesaw-type schemes of neutrino mass, the interplay of both types of contributions depends on the seesaw scale and has been analysed in [72]. Barring fine-tunings, high-scale seesaw models require supersymmetry in order to have sizeable LFV rates. Moreover, supersymmetry brings in the possibility of direct lepton flavour violation in the production of supersymmetric particles. This will provide the most direct way to probe LFV at the LHC in high-scale seesaw models, as seen in Fig. 5, from Ref. [73].

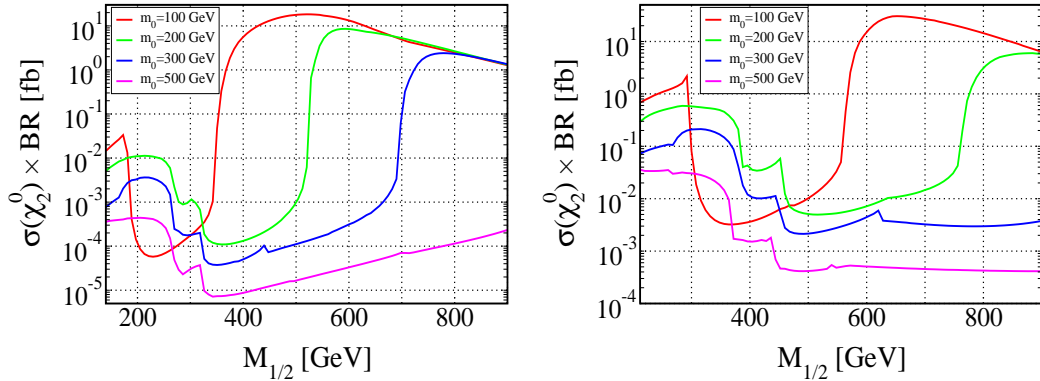


FIGURE 6. LFV rate for μ - τ lepton pair production from χ_2^0 decays versus $M_{1/2}$ for the indicated m_0 values, assuming minimal supergravity parameters: $\mu > 0$, $\tan\beta = 10$ and $A_0 = 0$ GeV, for type-I (left) and for type-II seesaw (right). Here $\lambda_1 = 0.02$ and $\lambda_2 = 0.5$ are Type-II seesaw parameters, and we imposed the constraint $\text{Br}(\mu \rightarrow e + \gamma) \leq 1.2 \cdot 10^{-11}$.

In contrast, the sizeable admixture of right-handed neutrinos in the charged current (rectangular nature of the lepton mixing matrix [3]) in low-scale seesaw schemes induces potentially large LFV rates even in the absence of supersymmetry [66]. Indeed, an important point to stress is that LFV [66, 67] and CP violation [74, 75] can occur in the massless neutrino limit, hence their attainable magnitude is unrestricted by the smallness of neutrino masses. In Fig. 7 we display $\text{Br}(\mu \rightarrow e\gamma)$ versus the small lepton number violating (LNV) parameters μ and v_L for two different low-scale seesaw models, the inverse and the linear seesaw, respectively. Clearly the LFV rates are sizeable in both cases, the different slopes with respect to μ and v_L follow from the fact that LNV occurs differently in the two models, $\Delta L = 2$ versus the $\Delta L = 1$, respectively. Similarly one can show [76] that in low-scale seesaw models the nuclear $\mu^- - e^-$ conversion rates lie within planned sensitivities of future experiments such as PRISM [77]. Note that models with specific flavor symmetries, such as those in [25, 26] relate different LFV rates. To conclude we mention that the some seesaw schemes, like type-III [17] or inverse type-III [26], may be directly probed at the LHC by directly producing the TeV RH neutrinos at accelerators.

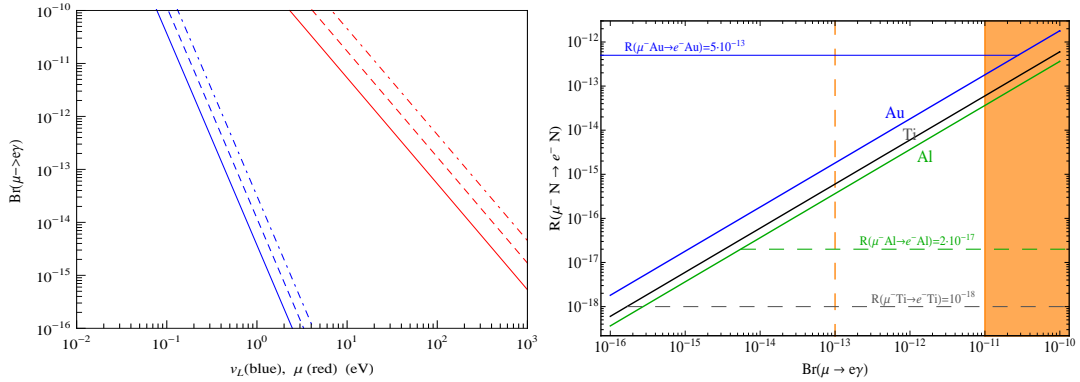


FIGURE 7. Left: $Br(\mu \rightarrow e\gamma)$ versus the lepton number violation scale for two low-scale seesaw models: the inverse seesaw in top (red color), the linear seesaw in bottom (blue color). In both cases, M is fixed as $M = 100 GeV$ (continuous line), $M = 200 GeV$ (dashed line) and $M = 1000 GeV$ (dot-dashed line). The right panel shows typical correlation between mu-e conversion in nuclei and $Br(\mu \rightarrow e\gamma)$.

LEPTON NUMBER VIOLATION

Neutrino oscillations can not distinguish Dirac from Majorana neutrinos. In contrast, LNV processes, such as $0\nu\beta\beta$ [78] hold the key to the issue. Indeed, in a gauge theory, *irrespective of the mechanism that induces $0\nu\beta\beta$* , it implies a Majorana mass for at least one neutrino [78], as illustrated in Fig. 8.

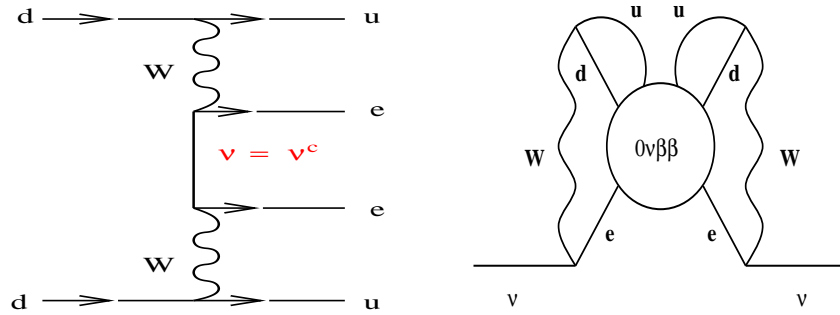


FIGURE 8. Neutrino mass mechanism for $0\nu\beta\beta$ (left), and black box theorem (right) [78].

Such “black-box” theorem [78] holds in any “natural” gauge theory, though quantitative implications are very model-dependent, for a recent discussion see [79]. The detection of neutrinoless double beta decay remains a major challenge [80].

The observation of neutrino oscillations suggests that the exchange of light Majorana neutrinos will induce $0\nu\beta\beta$ through the so-called *mass-mechanism*. The corresponding amplitude is sensitive both to the Majorana CP violation [3], and also to the absolute scale of neutrino mass, neither of which can be probed in oscillations. Together with high sensitivity beta decay studies [81], and with cosmic microwave background and large scale structure observations [82], neutrinoless double beta decay provides complementary information on the absolute scale of neutrino mass.

Taking into account current neutrino oscillation parameters [1, 2] and state-of-the-art nuclear matrix elements [83] one can determine the average mass parameter $\langle m_\nu \rangle$ characterizing the neutrino exchange contribution to $0\nu\beta\beta$, as in Fig. 42 of Ref. [5]. Quasi-degenerate neutrino models [50, 51] give the largest possible $0\nu\beta\beta$ signal. In normal hierarchy models there is in general no lower bound on $\langle m_\nu \rangle$ as there can be a destructive interference among the three neutrinos. In contrast, the inverted neutrino mass hierarchy implies a generic “lower” bound for the $0\nu\beta\beta$ amplitude. Specific flavor models may, however, imply a lower bound for $0\nu\beta\beta$ even with normal hierarchy, as discussed in [25] [84, 85, 86]. The best current limit on $\langle m_\nu \rangle$ comes from the Heidelberg-Moscow experiment, for the current experimental status and perspectives, see Ref. [80], which should be compared with nuclear theory [83].

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