

PPN limit and cosmological gravitational waves as tools to constrain $f(R)$ -gravity

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We discuss the PPN Solar-System constraints and the GW stochastic background considering some recently proposed $f(R)$ gravity models which satisfy both cosmological and stability conditions. Using the definition of PPN-parameters γ and β in terms of $f(R)$ -models and the definition of scalar GWs, we compare and discuss if it is possible to search for parameter ranges of $f(R)$ -models working at Solar System and GW stochastic background scale.

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1 Field equations and viable $f(R)$ -model

Let us start from the following action (see [1])

$$S = S_g + S_m = \frac{1}{k^2} \int d^4x \sqrt{-g} [R + f(R) + \mathcal{L}_m], \quad (1)$$

where we have considered the gravitational and matter contributions and $k^2 \equiv 16\pi G$. The non-linear $f(R)$ term has been put in evidence with respect to the standard Hilbert-Einstein term R and \mathcal{L}_m is the perfect-fluid matter Lagrangian. The field equations are

$$\frac{1}{2}g_{\mu\nu}F(R) - R_{\mu\nu}F'(R) - g_{\mu\nu}\square F'(R) + \nabla_\mu \nabla_\nu F'(R) = -\frac{k^2}{2}T_{\mu\nu}^{(m)}. \quad (2)$$

Here $F(R) = R + f(R)$ and $T_{\mu\nu}^{(m)}$ is the matter energy - momentum tensor. Action (1) can be recast in a scalar-tensor form. By using the conformal scale transformation $g_{\mu\nu} \rightarrow e^\sigma g_{\mu\nu}$ with $\sigma = -\ln(1 + f'(R))$, the action can be written in the Einstein frame as follows [2]:

$$S_E = \frac{1}{k^2} \int d^4x \sqrt{-g} \left(R - \frac{3}{2}g^{\rho\sigma} \partial_\rho \sigma \partial_\sigma \sigma - V(\sigma) \right), \quad (3)$$

where

$$V(\sigma) = e^\sigma g(e^{-\sigma}) - e^{2\sigma} f(g(e^{-\sigma})) = \frac{R}{F'(R)} - \frac{F(R)}{F'(R)^2}. \quad (4)$$

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The form of $g(e^{-\sigma})$ is given by solving $\sigma = -\ln(1 + f'(R)) = \ln F'(R)$ as $R = g(e^{-\sigma})$. The transformation $g_{\mu\nu} \rightarrow e^\sigma g_{\mu\nu}$ induces a coupling of the scalar field σ with matter.

Let us consider now a class of $f(R)$ models which do not contain cosmological constant and are explicitly designed to satisfy cosmological and Solar-System constraints in given limits of the parameter space. In practice, we choose a class of functional forms of $f(R)$ capable of matching, in principle, observational data. Firstly, the cosmological model should reproduce the CMBR constraints in the high-redshift regime (which agree with the presence of an effective cosmological constant). Secondly, it should give rise to an accelerated expansion, at low redshift, according to the Λ CDM model. Thirdly, there should be sufficient degrees of freedom in the parameterization to encompass low redshift phenomena (e.g. the large scale structure) according to the observations. Finally, small deviations from GR should be consistent with Solar System tests. All these requirements suggest that we can assume the limits

$$\lim_{R \rightarrow \infty} f(R) = \text{constant}, \quad \lim_{R \rightarrow 0} f(R) = 0, \quad (5)$$

which are satisfied by a general class of broken power law models, proposed in [3], which are

$$F(R) = R - \lambda R_c \frac{\left(\frac{R}{R_c}\right)^{2n}}{\left(\frac{R}{R_c}\right)^{2n} + 1} \quad (6)$$

where parameters $\{n, \lambda, R_c\}$ are constants which should be determined by experimental bounds.

2 Constraining $f(R)$ -models by PPN parameters

The above model can be constrained at Solar System level by considering the PPN formalism. This approach is extremely important in order to test gravitational theories and to compare them with GR. As it is shown in [4, 5], one can derive the PPN-parameters γ and β in terms of a generic analytic function $F(R)$ and its derivative

$$\gamma - 1 = -\frac{F''(R)^2}{F'(R) + 2F''(R)^2}, \quad \beta - 1 = \frac{1}{4} \left[\frac{F'(R) \cdot F''(R)}{2F'(R) + 3F''(R)^2} \right] \frac{d\gamma}{dR}. \quad (7)$$

These quantities have to fulfill the constraints coming from the Solar System experimental tests summarized in Table I. They are the perihelion shift of Mercury, the Lunar Laser Ranging, the upper limits coming from the Very Long Baseline Interferometry (VLBI) and the results obtained from the Cassini spacecraft mission in the delay of the radio waves transmission near the Solar conjunction.

Mercury perihelion Shift	$ 2\gamma - \beta - 1 < 3 \times 10^{-3}$
Lunar Laser Ranging	$4\beta - \gamma - 3 = (0.7 \pm 1) \times 10^{-3}$
Very Long Baseline Interferometer	$ \gamma - 1 < 4 \times 10^{-4}$
Cassini Spacecraft	$\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$

Table 1 Solar System experimental constraints on the PPN parameters.

By integrating last equations (7), one obtains $f(R)$ solutions depending on γ and β which has to be confronted with γ_{exp} and β_{exp} plug into such equations the model and the experimental values of PPN parameters and then we will obtain algebraic constraints for the phenomenological parameters n and λ . Determining the value from the equation for de Sitter solutions according to the stability conditions $F'(R) > 0$ and $F''(R) > 0$. Finally we obtain a good sets of parameters for the model [2]:

- $\frac{R}{R_c} = 3.38$, $n = 1$, $\lambda = 2$
- $\frac{R}{R_c} = \sqrt{3}$, $n = 2$, $\lambda > \frac{8}{3\sqrt{3}}$

3 Constraining $f(R)$ -models by stochastic backgrounds of gravitational waves

Also the stochastic background of GWs can be taken into account in order to constrain models. This approach could reveal very interesting because production of primordial GWs could be a robust prediction for any model attempting to describe the cosmological evolution at primordial epochs. The main characteristics of the gravitational backgrounds produced by cosmological sources depend both on the emission properties of each single source and on the source rate evolution with redshift. It is therefore interesting to compare and contrast the probing power of these classes of $f(R)$ -models at high, intermediate and zero redshift [6,7]. To this purpose, let us take into account the primordial physical process which gave rise to a characteristic spectrum Ω_{sgw} for the early stochastic background of relic scalar GWs by which we can recast the further degrees of freedom coming from fourth-order gravity. This approach can greatly contribute to constrain viable cosmological models. The stochastic background of scalar GWs can be described in terms of a scalar field Φ and characterized by a dimensionless spectrum. We can write the energy density of scalar GWs in terms of the closure energy density of GWs per logarithmic frequency interval as ([8])

$$\Omega_{sgw}(f) = \frac{1}{\rho_c} \frac{d\rho_{sgw}}{d \ln f}, \quad \rho_c \equiv \frac{3H_0^2}{8\pi G} \quad (8)$$

is the critical energy density of the Universe, H_0 the today observed Hubble expansion rate, and $d\rho_{sgw}$ is the energy density of the gravitational radiation scalar part contained in the frequency range from f to $f + df$. We are considering now standard units.

The calculation for a simple inflationary model can be performed assuming that the early Universe is described by an inflationary de Sitter phase emerging in the radiation dominated era [9]. The conformal metric element is

$$ds^2 = a^2(\eta)[-d\eta^2 + d\vec{x}^2 + h_{\mu\nu}(\eta, \vec{x})dx^\mu dx^\nu], \quad (9)$$

and a GW with tensor and scalar modes in the $z+$ direction is given by [10]

$$\tilde{h}_{\mu\nu}(t-z) = A^+(t-z)e_{\mu\nu}^{(+)} + A^\times(t-z)e_{\mu\nu}^{(\times)} + \Phi(t-z)e_{\mu\nu}^{(s)}. \quad (10)$$

The pure scalar component is then

$$h_{\mu\nu} = \Phi e_{\mu\nu}^{(s)}, \quad (11)$$

where $e_{\mu\nu}^{(s)}$ is the polarization tensor. At lower frequencies, the spectrum is given by

$$\Omega_{sgw}(f) \propto f^{-2}. \quad (12)$$

It is interesting to calculate the corresponding strain, where interferometers like VIRGO, LIGO and LISA reach a maximum in sensitivity. The well known equation for the characteristic amplitude [8], adapted to the scalar component of GWs, can be used. It is

$$\Phi_c(f) \simeq 1.26 \times 10^{-18} \left(\frac{1Hz}{f} \right) \sqrt{h_{100}^2 \Omega_{sgw}(f)}, \quad (13)$$

and then we obtain the values in the Table 2.

At this point, using the upper bounds in Table 2, calculated for the characteristic amplitude of GW scalar component, let us test the $f(R)$ -gravity models, considered in the previous sections, to see whether they are compatible both with the Solar System and GW stochastic background.

Before starting with the analysis, taking into account the above discussion, we have that the GW scalar component is derived considering

$$\Phi = -\frac{\delta\sigma}{\sigma_0}, \quad \sigma = -\ln(1 + f'(R)) = \ln F'(R), \quad \delta\sigma = \frac{f''(R)}{1 + f'(R)} \delta R. \quad (14)$$

Then we obtain a good sets of parameters for the model [2]:

$\Phi_c(100Hz) < 2 \times 10^{-26}$	LIGO
$\Phi_c(100Hz) < 2 \times 10^{-25}$	VIRGO
$\Phi_c(100Hz) < 2 \times 10^{-21}$	LISA

Table 2 Upper limits on the expected amplitude for the GW scalar component by ground-based-interferometers LIGO-VIRGO and space-interferometer LISA.

- $\frac{R}{R_c} = 3.38$, $n = 1$, $\lambda = 2$
- $\frac{R}{R_c} = \sqrt{3}$, $n = 2$, $\lambda > \frac{8}{3\sqrt{3}}$

such sets of parameters are same as bounds coming from the PPN and some sets reproduce quite well both the PPN upper limits and the constraints on the scalar component amplitude of GWs. The results indicate that self-consistent models could be achieved comparing experimental data at very different scales without extrapolating results obtained only at a given scale [2].

4 Conclusions

The interesting feature, and the main result of this paper, is that such sets of parameters are not in conflict with bounds coming from the cosmological stochastic background of GWs. In particular, some sets of parameters reproduce quite well both the PPN upper limits and the constraints on the scalar component amplitude of GWs.

Far to be definitive, these preliminary results indicate that self-consistent models could be achieved comparing experimental data at very different scales without extrapolating results obtained only at a given scale.

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