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Crossing of Phantom Divide in $F(R)$ Gravity

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An explicit model of $F(R)$ gravity with realizing a crossing of the phantom divide is reconstructed. In particular, it is shown that the Big Rip singularity may appear in the reconstructed model of $F(R)$ gravity. Such a Big Rip singularity could be avoided by adding R^2 term or non-singular viable $F(R)$ theory¹ to the model because phantom behavior becomes transient.

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1. Introduction

It is observationally supported that the current expansion of the universe is accelerating.^{2,3,4,5,6,7,8,9} A number of scenarios to account for the current accelerated expansion of the universe have been proposed (for reviews, see Refs. 10, 11, 12, 13, 14, 15, 16, 17, 18, 19).

Approaches to explain the current accelerated expansion of the universe fall into two broad categories. One is the introduction of some unknown matter, which

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is called “dark energy” in the framework of general relativity. The other is the modification of the gravitational theory, e.g., “ $F(R)$ gravity”, where $F(R)$ is an arbitrary function of the scalar curvature R (for reviews, see Refs. 14, 15, 16, 17, 18, 19).

Recent various observational data^{20,21,22,23} imply that the effective equation of state (EoS), which is the ratio of the effective pressure of the universe to the effective energy density of it, may evolve from larger than -1 (non-phantom phase) to less than -1 (phantom one), namely, cross -1 (the phantom divide).

Various investigations to realize the crossing of the phantom divide have been executed in the framework of general relativity: Scalar-tensor theories with the non-minimal gravitational coupling between a scalar field and the scalar curvature or that between a scalar field and the Gauss-Bonnet term, one scalar field model with non-linear kinetic terms or a non-linear higher-derivative one, phantom coupled to dark matter with an appropriate coupling, the thermodynamical inhomogeneous dark energy model, multiple kinetic k-essence, multi-field models (two scalar fields model, “quintom” consisting of phantom and canonical scalar fields), and the description of those models through the Parameterized Post-Friedmann approach, or a classical Dirac field or string-inspired models, non-local gravity, a model in loop quantum cosmology and a general consideration of the crossing of the phantom divide (for a detailed review, see Ref. 13). However, explicit models of modified gravity realizing the crossing of the phantom divide have hardly been examined, although there were suggestive and interesting related works.^{14,24,25}

In the present paper, we review our results in Ref. 26 and reconstruct an explicit model of $F(R)$ gravity in which a crossing of the phantom divide can be realized by using the reconstruction method proposed in Refs. 27, 28 (for more detailed references, see references in Refs. 26, 29, 30, 31, 32). It is demonstrated that the Big Rip singularity may appear in the reconstructed model of $F(R)$ gravity.

2. Reconstruction of a $F(R)$ gravity model with realizing a crossing of the phantom divide

2.1. Reconstruction method

To begin with, we briefly review the reconstruction method of modified gravity.^{27,28}

The action of $F(R)$ gravity with general matter is given by

$$S = \int d^4x \sqrt{-g} \left[\frac{F(R)}{2\kappa^2} + \mathcal{L}_{\text{matter}} \right], \quad (1)$$

where g is the determinant of the metric tensor $g_{\mu\nu}$ and $\mathcal{L}_{\text{matter}}$ is the matter Lagrangian.

By using proper functions $P(\phi)$ and $Q(\phi)$ of a scalar field ϕ , the action in Eq. (1) can be rewritten to the following form:

$$S = \int d^4x \sqrt{-g} \left\{ \frac{1}{2\kappa^2} [P(\phi)R + Q(\phi)] + \mathcal{L}_{\text{matter}} \right\}. \quad (2)$$

The scalar field ϕ may be regarded as an auxiliary scalar field because ϕ has no kinetic term. From the action in Eq. (1), the equation of motion of ϕ is derived as

$$0 = \frac{dP(\phi)}{d\phi}R + \frac{dQ(\phi)}{d\phi}, \quad (3)$$

which may be solved with respect to ϕ as $\phi = \phi(R)$. Substituting $\phi = \phi(R)$ into the action (2), we find that the expression of $F(R)$ in the action of $F(R)$ gravity in Eq. (1) is given by

$$F(R) = P(\phi(R))R + Q(\phi(R)). \quad (4)$$

From the action in Eq. (2), the gravitational field equation is given by

$$\frac{1}{2}g_{\mu\nu} [P(\phi)R + Q(\phi)] - R_{\mu\nu}P(\phi) - g_{\mu\nu}\square P(\phi) + \nabla_\mu\nabla_\nu P(\phi) + \kappa^2 T_{\mu\nu}^{(\text{matter})} = 0, \quad (5)$$

where ∇_μ is the covariant derivative operator associated with $g_{\mu\nu}$, $\square \equiv g^{\mu\nu}\nabla_\mu\nabla_\nu$ is the covariant d'Alembertian for a scalar field, and $T_{\mu\nu}^{(\text{matter})}$ is the contribution to the matter energy-momentum tensor.

We assume the flat Friedmann-Robertson-Walker (FRW) space-time with the metric,

$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2, \quad (6)$$

where $a(t)$ is the scale factor.

In the flat FRW background (6), the $(\mu, \nu) = (0, 0)$ component and the trace part of the $(\mu, \nu) = (i, j)$ component of Eq. (5), where i and j run from 1 to 3, become

$$-6H^2P(\phi(t)) - Q(\phi(t)) - 6H\frac{dP(\phi(t))}{dt} + 2\kappa^2\rho = 0, \quad (7)$$

and

$$2\frac{d^2P(\phi(t))}{dt^2} + 4H\frac{dP(\phi(t))}{dt} + (4\dot{H} + 6H^2)P(\phi(t)) + Q(\phi(t)) + 2\kappa^2p = 0, \quad (8)$$

respectively, where $H = \dot{a}/a$ is the Hubble parameter and a dot denotes a time derivative, $\dot{} = \partial/\partial t$. Here, ρ and p are the sum of the energy density and pressure of matters with a constant EoS parameter w_i , respectively, where i denotes some component of the matters.

By eliminating $Q(\phi)$ from Eqs. (7) and (8), we obtain

$$\frac{d^2P(\phi(t))}{dt^2} - H\frac{dP(\phi(t))}{dt} + 2\dot{H}P(\phi(t)) + \kappa^2(\rho + p) = 0. \quad (9)$$

We note that the scalar field ϕ may be taken as $\phi = t$ because ϕ can be redefined properly.

We consider that $a(t)$ is expressed as

$$a(t) = \bar{a} \exp(\bar{g}(t)), \quad (10)$$

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where \bar{a} is a constant and $\tilde{g}(t)$ is a proper function. In this case, Eq. (9) is reduced to

$$\frac{d^2 P(\phi)}{d\phi^2} - \frac{d\tilde{g}(\phi)}{d\phi} \frac{dP(\phi)}{d\phi} + 2 \frac{d^2 \tilde{g}(\phi)}{d\phi^2} P(\phi) + \kappa^2 \sum_i (1 + w_i) \bar{\rho}_i \bar{a}^{-3(1+w_i)} \exp[-3(1+w_i)\tilde{g}(\phi)] = 0, \quad (11)$$

where $\bar{\rho}_i$ is a constant and we have used $H = d\tilde{g}(\phi)/(d\phi)$. Moreover, it follows from Eq. (7) that $Q(\phi)$ is given by

$$Q(\phi) = -6 \left[\frac{d\tilde{g}(\phi)}{d\phi} \right]^2 P(\phi) - 6 \frac{d\tilde{g}(\phi)}{d\phi} \frac{dP(\phi)}{d\phi} + 2\kappa^2 \sum_i \bar{\rho}_i \bar{a}^{-3(1+w_i)} \exp[-3(1+w_i)\tilde{g}(\phi)]. \quad (12)$$

Hence, if the solution of Eq. (11) with respect to $P(\phi)$ is obtained, we can find $Q(\phi)$.

2.2. *Explicit $F(R)$ gravity model with realizing a crossing of the phantom divide*

Next, we reconstruct an explicit model of $F(R)$ gravity in which a crossing of the phantom divide can be realized by using the reconstruction method explained in the preceding subsection.

A solution of Eq. (11) without matter is given by

$$P(\phi) = e^{\tilde{g}(\phi)/2} \tilde{p}(\phi), \quad (13)$$

$$\tilde{g}(\phi) = -10 \ln \left[\left(\frac{\phi}{t_0} \right)^{-\gamma} - C \left(\frac{\phi}{t_0} \right)^{\gamma+1} \right], \quad (14)$$

$$\tilde{p}(\phi) = \tilde{p}_+ \phi^{\beta_+} + \tilde{p}_- \phi^{\beta_-}, \quad (15)$$

$$\beta_{\pm} = \frac{1 \pm \sqrt{1 + 100\gamma(\gamma+1)}}{2}, \quad (16)$$

where γ and C are positive constants, t_0 is the present time, and \tilde{p}_{\pm} are arbitrary constants.

From Eq. (14), we see that $\tilde{g}(\phi)$ diverges at finite ϕ when

$$\phi = t_s \equiv t_0 C^{-1/(2\gamma+1)}, \quad (17)$$

which implies that there could be the Big Rip singularity at $t = t_s$.

We consider only the period $0 < t < t_s$ because $\tilde{g}(\phi)$ should be real number. From Eq. (14), we obtain the following Hubble rate $H(t)$:

$$H(t) = \frac{d\tilde{g}(\phi)}{d\phi} = \left(\frac{10}{t_0} \right) \left[\frac{\gamma \left(\frac{\phi}{t_0} \right)^{-\gamma-1} + (\gamma+1) C \left(\frac{\phi}{t_0} \right)^{\gamma}}{\left(\frac{\phi}{t_0} \right)^{-\gamma} - C \left(\frac{\phi}{t_0} \right)^{\gamma+1}} \right], \quad (18)$$

where it is taken $\phi = t$.

In the flat FRW background (6), even for $F(R)$ gravity described by the action in Eq. (1), the effective energy-density and pressure of the universe are given by $\rho_{\text{eff}} = 3H^2/\kappa^2$ and $p_{\text{eff}} = -(2\dot{H} + 3H^2)/\kappa^2$, respectively. The effective EoS $w_{\text{eff}} = p_{\text{eff}}/\rho_{\text{eff}}$ is defined as¹⁴

$$w_{\text{eff}} \equiv -1 - \frac{2\dot{H}}{3H^2}. \quad (19)$$

For the case of $H(t)$ in Eq. (18), from Eq. (19) we find that w_{eff} is expressed as

$$w_{\text{eff}} = -1 + U(t), \quad (20)$$

where

$$U(t) \equiv -\frac{2\dot{H}}{3H^2} = -\frac{-\gamma + 4\gamma(\gamma + 1)\left(\frac{t}{t_s}\right)^{2\gamma+1} + (\gamma + 1)\left(\frac{t}{t_s}\right)^{2(2\gamma+1)}}{15\left[\gamma + (\gamma + 1)\left(\frac{t}{t_s}\right)^{2\gamma+1}\right]^2}. \quad (21)$$

Furthermore, the scalar curvature is given by $R = 6(\dot{H} + 2H^2)$. For the case of Eq. (18), R is described as

$$R = \frac{60\left[\gamma(20\gamma - 1) + 44\gamma(\gamma + 1)\left(\frac{t}{t_s}\right)^{2\gamma+1} + (\gamma + 1)(20\gamma + 21)\left(\frac{t}{t_s}\right)^{2(2\gamma+1)}\right]}{t^2\left[1 - \left(\frac{t}{t_s}\right)^{2\gamma+1}\right]^2}. \quad (22)$$

In deriving Eqs. (21) and (22), we have used Eq. (17).

When $t \rightarrow 0$, i.e., $t \ll t_s$, $H(t)$ behaves as

$$H(t) \sim \frac{10\gamma}{t}. \quad (23)$$

In this limit, it follows from Eq. (19) that the effective EoS parameter is given by

$$w_{\text{eff}} = -1 + \frac{1}{15\gamma}. \quad (24)$$

This behavior is identical with that in the Einstein gravity with matter whose EoS is greater than -1 .

On the other hand, when $t \rightarrow t_s$, we find

$$H(t) \sim \frac{10}{t_s - t}. \quad (25)$$

In this case, the scale factor is given by $a(t) \sim \bar{a}(t_s - t)^{-10}$. When $t \rightarrow t_s$, therefore, $a \rightarrow \infty$, namely, the Big Rip singularity appears. In this limit, the effective EoS parameter is given by

$$w_{\text{eff}} = -1 - \frac{1}{15} = -\frac{16}{15}. \quad (26)$$

This behavior is identical with the case in which there is a phantom matter with its EoS being smaller than -1 . As a consequence, we have reconstructed an explicit model with realizing a crossing of the phantom divide.

From Eq. (19), we see that the effective EoS w_{eff} becomes -1 when $\dot{H} = 0$. By solving $w_{\text{eff}} = -1$ with respect to t by using Eq. (20), namely, $U(t) = 0$, we find that the effective EoS parameter crosses the phantom divide at $t = t_c$, given by

$$t_c = t_s \left(-2\gamma + \sqrt{4\gamma^2 + \frac{\gamma}{\gamma+1}} \right)^{1/(2\gamma+1)}. \quad (27)$$

It follows from Eq. (21) that when $t < t_c$, $U(t) > 0$ because $\gamma > 0$. Moreover, the time derivative of $U(t)$ is given by

$$\frac{dU(t)}{dt} = -\frac{2\gamma(\gamma+1)(2\gamma+1)^2}{15 \left[\gamma + (\gamma+1) \left(\frac{t}{t_s} \right)^{2\gamma+1} \right]^3} \left(\frac{1}{t_s} \right) \left(\frac{t}{t_s} \right)^{2\gamma} \left[1 - \left(\frac{t}{t_s} \right)^{2\gamma+1} \right]. \quad (28)$$

Eq. (28) implies that the relation $dU(t)/(dt) < 0$ is always satisfied because we consider only the period $0 < t < t_s$ as mentioned above. This means that $U(t)$ decreases monotonously. Thus, the value of $U(t)$ evolves from positive to negative. From Eq. (20), we see that the value of w_{eff} crosses -1 . Once the universe enters the phantom phase, it stays in this phase, namely, the value of w_{eff} remains less than -1 , and finally the Big Rip singularity appears because $U(t)$ decreases monotonically. We note that there could be other types of the finite-time future singularities in modified gravity as shown in Refs. 33, 34.

By using Eqs. (13), (14), (15) and (17), $P(t)$ is obtained as

$$P(t) = \left[\frac{\left(\frac{t}{t_0} \right)^\gamma}{1 - \left(\frac{t}{t_s} \right)^{2\gamma+1}} \right]^5 \sum_{j=\pm} \tilde{p}_j t^{\beta_j}. \quad (29)$$

It follows from Eqs. (12) and (29) that $Q(t)$ is given by

$$Q(t) = -6H \left[\frac{\left(\frac{t}{t_0} \right)^\gamma}{1 - \left(\frac{t}{t_s} \right)^{2\gamma+1}} \right]^5 \sum_{j=\pm} \left(\frac{3}{2}H + \frac{\beta_j}{t} \right) \tilde{p}_j t^{\beta_j}. \quad (30)$$

If Eq. (22) can be solved with respect to t as $t = t(R)$, in principle we can find the form of $F(R)$ by using this solution and Eqs. (4), (29) and (30). However, for the general case it is difficult to solve Eq. (22) as $t = t(R)$. Therefore, as an solvable example, we illustrate the behavior of $t_s^2 F(\tilde{R})$ as a function of $\tilde{R} \equiv t_s^2 R$ in Fig. 1 for $\gamma = 1/2$, $\tilde{p}_+ = -1/t_s^{\beta_+}$, $\tilde{p}_- = 0$, $\beta_+ = (1 + 2\sqrt{19})/2$ and $t_s = 2t_0$. The quantities in Fig. 1 are described in dimensionless quantities. The horizontal and vertical axes show \tilde{R} and $t_s^2 F$, respectively. (Here, $\tilde{R} = t_s^2 R = 4R/R_0$, where R_0 is the current curvature. In deriving this relation, we have used $t_s = 2t_0$, $t_0 \approx H_0^{-1}$, where H_0 is the present Hubble parameter.) From Fig. 1, we see that the value of $F(R)$ increases as that of R becomes larger.

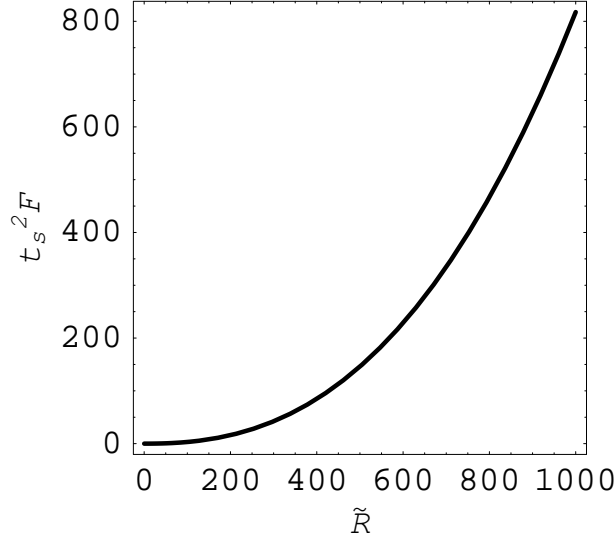


Fig. 1. Behavior of $t_s^2 F(\tilde{R})$ as a function of \tilde{R} for $\gamma = 1/2$, $\tilde{p}_+ = -1/t_s^{\beta_+}$, $\tilde{p}_- = 0$, $\beta_+ = (1 + 2\sqrt{19})/2$ and $t_s = 2t_0$.

To explore the analytic form of $F(R)$ for the general case, we examine the behavior of $F(R)$ in the limits $t \rightarrow 0$ and $t \rightarrow t_s$. When $t \rightarrow 0$, from Eq. (23) we get

$$t \sim \sqrt{\frac{60\gamma(20\gamma-1)}{R}}. \quad (31)$$

In this limit, it follows from Eqs. (4), (23), (29), (30) and (31) that the form of $F(R)$ is given by

$$F(R) \sim \left\{ \frac{\left[\frac{1}{t_0} \sqrt{60\gamma(20\gamma-1)} R^{-1/2} \right]^\gamma}{1 - \left[\frac{1}{t_s} \sqrt{60\gamma(20\gamma-1)} R^{-1/2} \right]^{2\gamma+1}} \right\}^5 R \times \sum_{j=\pm} \left\{ \left(\frac{5\gamma-1-\beta_j}{20\gamma-1} \right) \tilde{p}_j [60\gamma(20\gamma-1)]^{\beta_j/2} R^{-\beta_j/2} \right\}. \quad (32)$$

Note that such action belongs to general class of actions with positive and negative powers of curvature introduced in Ref. 35.

On the other hand, when $t \rightarrow t_s$, from Eq. (25) we obtain

$$t \sim t_s - 3\sqrt{\frac{140}{R}}. \quad (33)$$

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In this limit, it follows from Eqs. (4), (25), (29), (30) and (33) that the form of $F(R)$ is given by

$$F(R) \sim \left(\frac{\left\{ \frac{1}{t_0} [t_s - 3\sqrt{140}R^{-1/2}] \right\}^\gamma}{1 - \left[1 - \frac{3\sqrt{140}}{t_s} R^{-1/2} \right]^{2\gamma+1}} \right)^5 R \sum_{j=\pm} \tilde{p}_j [t_s - 3\sqrt{140}R^{-1/2}]^{\beta_j} \times \left\{ 1 - \sqrt{\frac{20}{7}} \left[\sqrt{\frac{15}{84}} t_s + (\beta_j - 15) R^{-1/2} \right] \frac{1}{t_s - 3\sqrt{140}R^{-1/2}} \right\}. \quad (34)$$

For large R , namely, $t_s^2 R \gg 1$, the expression of $F(R)$ in (34) can be approximately written as

$$F(R) \approx \frac{2}{7} \left[\frac{1}{3\sqrt{140}(2\gamma+1)} \left(\frac{t_s}{t_0} \right)^\gamma \right]^5 \left(\sum_{j=\pm} \tilde{p}_j t_s^{\beta_j} \right) t_s^5 R^{7/2}. \quad (35)$$

3. Summary

We have studied a crossing of the phantom divide in $F(R)$ gravity. We have reconstructed an explicit model of $F(R)$ gravity in which a crossing of the phantom divide can occur by using the reconstruction method.^{27,28} As a result, we have shown that the Big Rip singularity may appear in the reconstructed model of $F(R)$ gravity. We finally mention that by adding R^2 term (as it was first proposed in Ref. 24) to the model or by adding non-singular theory,¹ $R^2 (R^n + c_1) / (R^n + c_2)$, where n , c_1 and c_2 are constants, Big Rip singularities could be avoided because phantom behavior becomes transient.^{33,34}

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References

1. S. Nojiri and S. D. Odintsov, arXiv:0910.1464 [hep-th].
2. D. N. Spergel *et al.* [WMAP Collaboration], *Astrophys. J. Suppl.* **148**, 175 (2003).
3. H. V. Peiris *et al.* [WMAP Collaboration], *Astrophys. J. Suppl.* **148**, 213 (2003).
4. D. N. Spergel *et al.* [WMAP Collaboration], *Astrophys. J. Suppl.* **170**, 377 (2007).
5. E. Komatsu *et al.* [WMAP Collaboration], *Astrophys. J. Suppl.* **180**, 330 (2009).
6. S. Perlmutter *et al.* [Supernova Cosmology Project Collaboration], *Astrophys. J.* **517**, 565 (1999).

7. A. G. Riess *et al.* [Supernova Search Team Collaboration], *Astron. J.* **116**, 1009 (1998).
8. P. Astier *et al.* [The SNLS Collaboration], *Astron. Astrophys.* **447**, 31 (2006).
9. A. G. Riess *et al.*, *Astrophys. J.* **659**, 98 (2007).
10. P. J. E. Peebles and B. Ratra, *Rev. Mod. Phys.* **75**, 559 (2003).
11. V. Sahni, *AIP Conf. Proc.* **782**, 166 (2005) [*J. Phys. Conf. Ser.* **31**, 115 (2006)].
12. T. Padmanabhan, *Phys. Rept.* **380**, 235 (2003).
13. E. J. Copeland, M. Sami and S. Tsujikawa, *Int. J. Mod. Phys. D* **15**, 1753 (2006).
14. S. Nojiri and S. D. Odintsov, *Int. J. Geom. Meth. Mod. Phys.* **4**, 115 (2007) [arXiv:hep-th/0601213].
15. S. Nojiri and S. D. Odintsov, arXiv:0801.4843 [astro-ph].
16. S. Nojiri and S. D. Odintsov, arXiv:0807.0685 [hep-th].
17. T. P. Sotiriou and V. Faraoni, arXiv:0805.1726 [gr-qc].
18. F. S. N. Lobo, arXiv:0807.1640 [gr-qc].
19. S. Capozziello and M. Francaviglia, *Gen. Rel. Grav.* **40**, 357 (2008).
20. U. Alam, V. Sahni and A. A. Starobinsky, *JCAP* **0406**, 008 (2004).
21. S. Nesseris and L. Perivolaropoulos, *JCAP* **0701**, 018 (2007).
22. P. U. Wu and H. W. Yu, *Phys. Lett. B* **643**, 315 (2006).
23. H. K. Jassal, J. S. Bagla and T. Padmanabhan, arXiv:astro-ph/0601389.
24. M. C. B. Abdalla, S. Nojiri and S. D. Odintsov, *Class. Quant. Grav.* **22**, L35 (2005).
25. L. Amendola and S. Tsujikawa, *Phys. Lett. B* **660**, 125 (2008).
26. K. Bamba, C. Q. Geng, S. Nojiri and S. D. Odintsov, *Phys. Rev. D* **79**, 083014 (2009).
27. S. Nojiri and S. D. Odintsov, *Phys. Rev. D* **74**, 086005 (2006).
28. S. Nojiri and S. D. Odintsov, *J. Phys. Conf. Ser.* **66**, 012005 (2007) [arXiv:hep-th/0611071].
29. K. Bamba and C. Q. Geng, *Phys. Lett. B* **679**, 282 (2009).
30. K. Bamba and C. Q. Geng, *Prog. Theor. Phys.* **122**, 1267 (2009).
31. K. Bamba, arXiv:0904.2655 [gr-qc].
32. K. Bamba, arXiv:0909.2991 [astro-ph.CO].
33. K. Bamba, S. Nojiri and S. D. Odintsov, *JCAP* **0810**, 045 (2008).
34. S. Nojiri and S. D. Odintsov, *Phys. Rev. D* **78**, 046006 (2008).
35. S. Nojiri and S. D. Odintsov, *Phys. Rev. D* **68**, 123512 (2003).