IIB seven–branes revisited

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Abstract. We re–investigate the construction of half–supersymmetric seven–brane solutions of IIB supergavity. In contrast to previous approaches we allow the occurrence of objects with monodromy Tr $\Lambda \neq 2$. We obtain non-trivial information from the requirement of a globally well–defined Killing spinor and by including $SL(2,\mathbb{Z})$–invariant source terms.

1. Introduction

The construction of half–supersymmetric seven–brane solutions goes back to the classic work of [1] where they were presented as cosmic string solutions of a D=4 gravity dilaton/axion system. Later, after the invention of D–branes [2], these solutions were oxidized to D=10 dimensions and re–interpreted as D7–brane solutions [3]. Since then D7–branes, in particular in the form of D3–D7 brane systems, have found important applications in model building, see e.g. [4–6], and cosmology [7, 8].

The original motivation of [1] was not the construction of cosmic string solutions as such, but the investigation of supersymmetric String Theory backgrounds more general than the direct product of 4-dimensional Minkowski spacetime and a Calabi-Yau 3-fold. In this context the gravity plus dilaton/axion system of [1] arises from compactification on a torus T^2, the complex axion-dilaton field τ being its modular parameter. Within the context of D=10 IIB supergravity one must for this purpose rely on a 12-dimensional F-theory. It is the purpose of this work to re–analyze half–supersymmetric seven–brane solutions of IIB supergravity without invoking the compactification of a higher–dimensional theory. Instead, we obtain non-trivial information by the requirement that there exists a globally well-defined Killing spinor.

In general, D7–branes do not come alone since this leads to singularities at a finite distance of the D7–brane. To obtain a globally well–defined solution one must add other seven–brane objects whose monodromy is not necessarily related to the monodromy of a D7–brane by a $SL(2,\mathbb{Z})$–transformation. In the case they are not related, we will call these objects, for reasons that will become clear soon, det $Q > 0$ branes. In the multiple D7–brane solutions of [1] these det $Q > 0$ branes occur in multiples such that their masses and monodromies cancel amongst each other and one is left with multiple D7–branes only. We will present new half–supersymmetric configurations where these cancellations do not occur and we will discuss the properties of these solutions. In particular we will show that they can be used as the basic building blocks for constructing general seven–brane solutions, including the multiple D7–branes of [1]

Another distinguishing feature of our analysis is that we add $SL(2,\mathbb{Z})$–invariant source terms to the equations of motion. Strictly speaking they are $SL(2,\mathbb{Z})$–invariant provided we also transform the constants that occur in these source terms, see eq. (1). The source terms both represent the embedding of a probe D7–brane as well as det $Q > 0$ objects into the IIB supergravity background. They enable us to derive an expression for both the seven–brane solution close to the position of the brane as well as the monodromy of the brane in terms of the charges describing the source term.

2. Seven-brane Source Terms

We begin by analyzing the source terms. The addition of probe 7-branes to a gravity-axion-dilaton background is described by the following action:

$$S = \int d^{10}x\sqrt{-g}\left(R - \frac{\partial_\mu\tau\partial^\mu\bar{\tau}}{2\,(\mathrm{Im}\tau)^2} - \int_\Sigma d^8\sigma\sqrt{-g_{(8)}}\frac{\delta(x - X(\sigma))}{\sqrt{-g}}\frac{1}{\mathrm{Im}\tau}\left(p + q|\tau|^2 + r\frac{\tau+\bar{\tau}}{2}\right)\right)\,, \tag{1}$$

where p, q, r are real numbers and $\tau = \chi + ie^{-\phi}$ with χ the axion and ϕ the dilaton. The world-volume, Σ, is parameterized by $\{\sigma^i, i = 0, 1, \ldots, 7\}$. The metric on the world-volume is $g_{(8)ij}$ which is the pull-back of the target-space Einstein frame metric $g_{\mu\nu}$. The embedding coordinates of the brane are denoted by $X^\mu(\sigma)$, and so the pull-back is given by

$$g_{(8)ij}(\sigma) = \frac{\partial X^\mu}{\partial\sigma^i}\frac{\partial X^\nu}{\partial\sigma^j}g_{\mu\nu}(X)\,. \tag{2}$$

We will only consider objects for which in the static gauge $X^8 = X^9 = 0$, i.e. we do not consider fluctuations of the world-volume. The source term in (1) should be interpreted as adding a purely static object to the theory. Note that the source term is linear in p, q and r. This is related to the fact that, unlike strings, all seven–branes have the same half–supersymmetry projection operator which is $SL(2,\mathbb{Z})$–invariant.

Seven-branes couple electrically to 8-forms. It is special to 7-branes that the scalar degrees of freedom which describe the tension of the brane and the 8-form degrees of freedom which describe the electric coupling are dual degrees of freedom. In the action (1) the bulk part only consists of scalars and gravity. Note that the Wess-Zumino term of the world-volume theory is absent in the coupling. This is related to the following. In order to properly derive (1) one needs to be able to describe scalars and 8-forms as independent fields of one and the same action. This can be realized using the Pasti-Sorokin-Tonin (PST) method [9, 10]. The PST method allows one to work with potentials and their duals as independent fields in one action. For the particular case of scalars and 8-forms such a bulk action has been constructed [11].

The details of adding the 7-brane world-volume action to the bulk action of [11] and how one arrives at the action (1) will be reported elsewhere.

$SL(2,\mathbb{Z})$–invariant 7-brane world-volume actions were considered in [12] and were shown to preserve $N=1$ supersymmetry for all possible values of p,q,r. The world-volume action describing a single D7 brane or any $SL(2,\mathbb{Z})$ transform thereof has values p,q,r which satisfy the condition $-r^2/4+pq=0$, or

$$\det Q = 0\,, \qquad\qquad Q = \begin{pmatrix} \frac{r}{2} & p \\ -q & -\frac{r}{2} \end{pmatrix}. \tag{3}$$

For this set of seven-brane actions one can introduce a single Born-Infeld vector in a target–space gauge–invariant and $SL(2,\mathbb{Z})$ invariant manner [13]. This confirms the identification of these objects as Dirichlet branes.

We will allow for objects which are not related to the D7 brane by some $SL(2,\mathbb{Z})$ transformation. Such objects have

$$\det Q \neq 0\,. \tag{4}$$

It turns out that in making solutions with a D7 brane globally well-defined, objects with $\det Q > 0$ play a crucial role. These objects cannot be described by brane actions containing a single Born-Infeld vector. They are therefore not Dirichlet branes.

3. The Equations of Motion

3.1. Supersymmetry and holonomy of the Killing spinor

We require that the following Killing spinor equations are satisfied:

$$\delta_\epsilon \lambda = \frac{i}{\tau-\bar\tau}\,(\gamma^\mu \partial_\mu \bar\tau)\,\epsilon_C = 0\,, \tag{5}$$

$$\delta_\epsilon \psi_\mu = \left(\partial_\mu + \frac{1}{4}\omega_\mu{}^{ab}\gamma_{ab} + \frac{1}{4}\frac{1}{\tau-\bar\tau}\partial_\mu(\tau+\bar\tau)\right)\epsilon = 0\,. \tag{6}$$

The equations (5) and (6) are invariant under the following $SL(2,\mathbb{Z})$ transformations

$$\tau \to \frac{a\tau+b}{c\tau+d}\,,\quad \lambda \to e^{3i\varphi}\lambda\,,\quad \psi_\mu \to e^{i\varphi}\psi_\mu\,,\quad \epsilon \to e^{i\varphi}\epsilon\,, \tag{7}$$

where φ is given by

$$\varphi = \frac{1}{2}\mathrm{arg}(\mathrm{c}\tau + \mathrm{d})\,, \qquad\qquad -\pi < \varphi \leq \pi\,. \tag{8}$$

One can organize the numbers a,b,c,d in an $SL(2,\mathbb{Z})$ matrix, Λ say, and write the τ transformation as

$$\tau \to \Lambda\tau \quad \text{where} \quad \Lambda = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{PSL}(2,\mathbb{Z}) = \mathrm{SL}(2,\mathbb{Z})/\{\pm \mathrm{I}\}\,. \tag{9}$$

The Killing spinor ϵ does transform under $\Lambda = -\mathbb{I}_2$ as $\epsilon \to i\epsilon$.

The $SL(2,\mathbb{Z})$–invariant supersymmetry projection operator on the 7-brane is given by

$$P\epsilon = \frac{1}{2}\left(1 - i\gamma_{\underline{0}\ldots\underline{7}}\right)\epsilon = \frac{1}{2}\left(1 + i\gamma_{\underline{8}}\gamma_{\underline{9}}\right)\epsilon = 0\,. \tag{10}$$

Equation (5) tells us that $(\partial_8 - i\partial_9)\,\bar\tau = 0$. In deriving this result we have gauge fixed the transverse part of the metric to be manifestly conformally flat and assumed that τ only depends on the transverse space coordinates. We define the complex coordinate z to be $z = x^8 + ix^9$ so that we now have $\partial_z\bar\tau = 0$, that is, τ is a holomorphic function. Given the supersymmetry projection operator (10) the most general 7-brane solution to equations (5) and (6) is given by [3, 14]

$$ds^2 = -dt^2 + d\vec{x}_7^{\,2} + (\mathrm{Im}\tau)|\mathrm{f}|^2\mathrm{dzd\bar z}\,, \tag{11}$$

$$\tau = \tau(z)\,,\quad f = f(z)\,, \tag{12}$$

$$\epsilon = \left(\frac{f}{\bar f}\right)^{1/4}\epsilon_0\,, \tag{13}$$

where ϵ_0 is a constant spinor which satisfies $\gamma_{\underline{z}^*}\epsilon_0 = 0$. The functions τ and f are defined on the Riemann sphere. The form of the solution, equations (11) to (13), is therefore fixed up to $SL(2,\mathbb{C})$ transformations

$$z \to \frac{az+b}{cz+d}\,, \tag{14}$$

where a, b, c, d are arbitrary complex numbers. These are the global general coordinate transformations that do not change the structure of the branch cuts in the complex z-plane.

By parallel transporting the Killing spinor around a closed loop in the transverse space using the connection in (6), evaluated on the metric (11), it can be shown that the holonomy of ϵ is given by

$$\Lambda_b\,\epsilon(z=b) = \exp\left(\frac{i}{2}\,\mathrm{Im}\oint_{\gamma_\mathrm{b}}(\log \mathrm{f})'\mathrm{dz}\right)\epsilon(z=b)\,, \tag{15}$$

where the closed path γ_b has base point b. The prime denotes differentiation with respect to z. The holonomy phase factor, Λ_b, will depend on the base point b. We require the holonomy phase factor in (15) to coincide with an $SL(2,\mathbb{Z})$ transformation as given in equation (7). In this way the Lorentz rotation characterized by the holonomy phase factor Λ_b cancels the $SL(2,\mathbb{Z})$–transformation characterized by the monodromy phase factor $e^{\frac{i}{2}\mathrm{arg}(\mathrm{c}\tau+\mathrm{d})}$. This is in agreement with the fact that the Killing spinor is covariantly constant with respect to the generalized connection given in (6). We therefore require the following

$$\exp\left(\frac{i}{2}\,\mathrm{Im}\oint_{\gamma_\mathrm{b}}(\log \mathrm{f})'\mathrm{dz}\right) = e^{i\varphi}\quad\text{where}\quad -\pi < \varphi = \frac{1}{2}\mathrm{arg}(\mathrm{c}\tau+\mathrm{d}) \le \pi\,. \tag{16}$$

Let the closed contour γ_b be parameterized by λ which runs from 0 to 1. The holonomy phase Λ_b is equal to

$$\exp\left(\frac{i}{2}\,\mathrm{Im}\oint_{\gamma_\mathrm{b}}(\log \mathrm{f})'\mathrm{dz}\right) = \left(\frac{f(\lambda=1)}{|f(\lambda=1)|)}\right)^{1/2}\left(\frac{|f(\lambda=0)|}{f(\lambda=0))}\right)^{1/2}\,. \tag{17}$$

Combining this with the requirement (16) leads to the following monodromy condition for the function f

$$f(z) \to (c\tau+d)f(z)\,. \tag{18}$$

3.2. The scalar equations of motion

We perform a variation of the action (1) with respect to $\bar{\tau}$ and use the metric, given in (11). This leads to the following equation of motion for τ:

$$\partial\bar{\partial}\tau - 2\frac{\partial\tau\bar{\partial}\tau}{\tau - \bar{\tau}} = -\frac{i}{4}\delta(z - z_0, \bar{z} - \bar{z}_0)\left(p + q\tau^2 + r\tau\right) . \tag{19}$$

Equation (19) can be integrated to give the following equation:

$$\lim_{\delta\to 0}\oint_{\gamma_\delta}\left(2\pi i\tau' - p\frac{1}{z - z_0} - q\frac{\tau^2}{z - z_0} - r\frac{\tau}{z - z_0}\right)dz = 0\,. \tag{20}$$

This implies that the integrand of (20) is an analytic function which does not have any poles in the interior of γ_δ, so that we may deform the contour γ_δ to any contour which is homotopic to γ_δ. Hence we can write

$$2\pi i\tau' - p\frac{1}{z - z_0} - q\frac{\tau^2}{z - z_0} - r\frac{\tau}{z - z_0} = \sum_{n=0}^{\infty} a_n(z - z_0)^n\,, \tag{21}$$

for arbitrary coefficients a_n. In the limit $|z - z_0| \to 0$ the poles on the left hand-side will dominate all the terms on the right hand-side. In this approximation the right hand-side of (21) can be put to zero, and we are left with the homogeneous version of equation (21), i.e.

$$2\pi i\tau' - p\frac{1}{z - z_0} - q\frac{\tau^2}{z - z_0} - r\frac{\tau}{z - z_0} = 0\,. \tag{22}$$

The solutions to (22) are

$$\begin{aligned} e^{2\pi i\tau/p} &= z - z_0 && \text{for } \det \mathrm{Q} = 0 \text{ and } \mathrm{q} = \mathrm{r} = 0\,, \\ c\left(\frac{\tau - \tau_0}{\tau - \bar{\tau}_0}\right)^{\frac{\pi}{\sqrt{\det \mathrm{Q}}}} &= z - z_0 && \text{for } \det \mathrm{Q} > 0 \text{ and } \mathrm{q} \neq 0\,, \end{aligned} \tag{23}$$

where Q is the matrix defined in (3), $\tau_0 \equiv \tau(z = z_0) = -\frac{r}{2q} + \frac{i}{q}\sqrt{\det \mathrm{Q}}$ is a fixed point of e^Q and $c \neq 0$ is a constant. Any $PSL(2,\mathbb{Z})$ transformation of these solutions is again a solution, leading to solutions for all values of p, q and r subject to $\det \mathrm{Q} \geq 0$. The monodromy is always given by

$$\tau \to e^Q\tau \quad \text{where} \quad \mathrm{e}^{\mathrm{Q}} = \cos(\sqrt{\det \mathrm{Q}})\mathrm{I} + \frac{\sin(\sqrt{\det \mathrm{Q}})}{\sqrt{\det \mathrm{Q}}}\mathrm{Q}\,. \tag{24}$$

This identifies e^Q as the monodromy matrix. These solutions were first found in [14].

The left hand-side of expressions (23) can be recognized as expansions of modular functions around fixed points of some modular group of transformations. In the next section we will discuss the full solutions to the scalar field equation (19) in terms of such modular functions that incorporate the above solutions as approximations around certain fixed points.

3.3. The Einstein equations of motion

Varying the action (1) with respect to the metric and substituting equations (11) and (12) one finds that the $z\bar{z}$ component of the Einstein equations is given by

$$\partial\bar{\partial}\log|f|^2 = -\frac{1}{2}\delta(z-z_0,\bar{z}-\bar{z}_0)\frac{i}{\tau-\bar{\tau}}\left(p+q|\tau|^2+r\frac{\tau+\bar{\tau}}{2}\right), \tag{25}$$

where $\partial = \frac{\partial}{\partial z}$. All other components of the Einstein equations are identically zero.

Integrating equation (25) we obtain

$$\lim_{\delta\to 0}\,\mathrm{Im}\oint_{\gamma_\delta}(\log \mathrm{f})'\mathrm{dz} = -\,\mathrm{sign(q)}\,\sqrt{\det \mathrm{Q}}\,, \tag{26}$$

where e^Q is the monodromy matrix of τ measured when going around the contour γ_δ.

The orders of the zeros/poles of the function $f(z)$ at $z = z_0$ determines the deficit angle at the location of the source. These orders can be computed as follows. Let γ_δ be a closed circular contour of radius δ which encircles the point z_0. This leads to the following expression for the deficit angle:

$$\delta = i\lim_{\delta\to 0}\oint_{\gamma_\delta}(\log f)'dz = \mathrm{sign(q)}\sqrt{\det \mathrm{Q}}\,. \tag{27}$$

These deficit angles occur in the formula for the total mass of the seven-brane configuration:

$$m \;=\; \frac{1}{16\pi G_3}\left(\int\frac{i}{2}dz\wedge d\bar{z}\frac{\partial\tau\bar{\partial}\bar{\tau}}{(\mathrm{Im}\tau)^2}+2\sum_j\delta_j\right), \tag{28}$$

where

$$\delta_j = \mathrm{sign(q_j)}\sqrt{\det \mathrm{Q_j}} \tag{29}$$

is the deficit angle of the $\det Q > 0$ brane at the location z_j. Note that there is no deficit angle at the position of the $\det \mathrm{Q} = 0$ branes.

4. Constructing Solutions

A single D7 brane corresponds to $p = 1$ in equation (23). The monodromy of τ measured when going around a single D7 brane is $\tau \to T\tau \equiv \tau + 1$ where

$$T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}. \tag{30}$$

Further, from equation (18) it follows that $f \to f$. The element $T \in PSL(2,\mathbb{Z})$ is of infinite order. However, solutions containing only an object with this monodromy will have infinite mass per volume element [1]. This is related to the fact that after modding out the complex plane with T the fundamental domain one gets has infinite area (measured with respect to $\frac{i}{2}\frac{d\tau\wedge d\bar{\tau}}{(\mathrm{Im}\,\tau)^2}$), and this leads to an infinite mass per volume element. Thus, to obtain solutions of finite mass, we are forced to include objects with other monodromies. Here we employ the S-duality of the theory. We take S to have the following matrix representation

$$S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}. \tag{31}$$

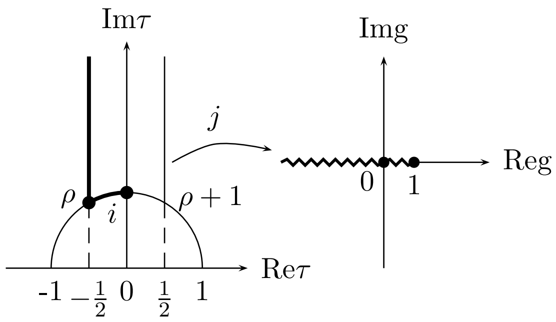

Figure 1. The fundamental domain of the group $PSL(2,\mathbb{Z})$. The point ρ denotes the point $-\frac{1}{2}+\frac{i}{2}\sqrt{3}$. The wavy lines in the g-plane represent branch cuts. The g-plane is mapped $1 \to N$ to the z-plane.

Note that $S^2 = -\mathbb{I}_2$ and that, when acting on the Killing spinor ϵ, $S^8 = \mathbb{I}_2$.

We will consider only solutions whose monodromy group is $PSL(2,\mathbb{Z})$. Having chosen a monodromy group we can specify the functions τ and f. From equation (23) it can be concluded that there must exist a function, $j(\tau)$, which is monodromy neutral, i.e. is an automorphic function $j(\Lambda\tau) = j(\tau)$. Here Λ is an element of the monodromy group. The set of τ values which are inequivalent under Λ form what is called the fundamental domain, see Figure 1. This fundamental domain is an orbifold containing three orbifold points with monodromies in distinct conjugacy classes:

$$\tau = i\infty : \underbrace{e^Q = T}_{\det Q = 0}, \qquad \tau = i : \underbrace{e^Q = S}_{\det Q > 0}, \qquad \tau = \rho : \underbrace{e^Q = T^{-1}S}_{\det Q > 0}.$$

We require that the function $j(\tau)$ maps the fundamental domain onto the Riemann sphere $\hat{\mathbb{C}}$ in a one-to-one fashion, so that the inverse function j^{-1} exists. Note that the inverse mapping j^{-1} has branch cuts connecting the points $z_{i\infty}$ to z_ρ and z_ρ to z_i, where the subscript indicates the value τ has at that point. In the case of $PSL(2,\mathbb{Z})$ the function $j(\tau)$ is given by Klein's modular j function. In general we will map the Riemann sphere via a mapping $g(z)$ to N Riemann spheres so that the function $\tau(z)$ is defined by

$$j(\tau(z)) = g(z)\,, \tag{32}$$

where $g(z)$ is a quotient of polynomials. The explicit form of $g(z)$ is determined by the fact that (i) we require that the modular j function maps the points $\{i\infty,\, \rho,\, i\}$ to $\{\infty,\, 0,\, 1\}$, respectively and (ii) we require a certain number of branes at each orbifold point.

orbifold point τ	order	monodromy	deficit angle δ
$i\infty$	∞	$\pm T$	0
i	2	$\pm S$	$\mp\pi/2$
ρ	3	$\pm(T^{-1}S)$	$-\,2\pi/3, \pi/3$

Table 1. The order of the orbifold points τ, the monodromy of τ and f and the deficit angles for $\tau = i\infty$, ρ, i.

Our next task is to find an explicit realization of the function $f(z)$. We first choose a function $F(\tau)$ which transforms as

$$F(\tau) \to \delta(a,b,c,d)(c\tau + d)F(\tau) \quad \text{whenever} \quad \tau \to \frac{\mathrm{a}\tau + \mathrm{b}}{\mathrm{c}\tau + \mathrm{d}}\,, \tag{33}$$

where $\delta(a,b,c,d)$ is a phase factor and then write

$$f(z) = F(\tau)h(z)\,, \tag{34}$$

where the function $h(z)$ cannot be written as a function of τ only and has monodromies which are such that they cancel the phases $\delta(a,b,c,d)$. The unique realization of $F(\tau)$ for $PSL(2,\mathbb{Z})$ is given by $F(\tau) = \eta^2(\tau)$ where $\eta(\tau)$ is the Dedekind eta function. This function transforms under the different $PSL(2,\mathbb{Z})$-transformations as follows:

$$\eta^2(\tau+1) = e^{\pi i/6}\eta^2(\tau)\,, \qquad \eta^2(-\frac{1}{\tau}) = e^{-\pi i/2}\tau\eta^2(\tau)\,, \qquad \eta^2(-\frac{\tau+1}{\tau}) = e^{-2\pi i/3}\tau\eta^2(\tau)\,.$$

Using these monodromies and the required monodromies of $f(z)$ it is not difficult to derive an explicit realization of the function h(z). This completes the construction of a solution.

For the convenience of the reader we have presented in table 1 the monodromies of τ and f and the deficit angles measured when going around the points $z_{i\infty}$, z_ρ, z_i in a counter clockwise direction.

5. Example: the $N = 6$ solution with only D7–branes

As an application of the previous sections we now present a $N = 6$ solutions where all monodromies at the det $Q > 0$ orbifold points have been cancelled out. Therefore, the metric has no deficit angles in these points. It is convenient to use the following form of the mass formula

$$m = \frac{1}{16\pi G_3}\Big(N \times \text{area} \;+\; 2\sum_{\mathrm{j}} \delta_{\mathrm{j}}\Big)\,, \tag{35}$$

where δ_j are the different deficit angles and N is the number of times the fundamantal domain is covered by $\tau(z)$. We see that a cancelletion of monodromies can be achieved by taking a

S and a $-S$ object at $\tau = i$ and a $T^{-1}S$ object together with two $-T^{-1}S$–objects at $z = z_\rho$. This leads us to consider an $N = 6$ solution which is asymptotically a cone with deficit angle π. Similarly, the $N = 12$ and $N = 24$ cases lead to a cylinder and a sphere. These special cases were first given in [1]. Note that our methods allow a much wider class of solutions including $\det Q > 0$ objects. We can, for instance, make supersymmetric solutions with $N > 24$ by employing $\det Q > 0$ objects with negative deficit angles. For examples, see [15].

The $N = 6$ solution is best represented by giving a picture of its branch cuts in the z-plane: The explicit expressions for the functions $\tau(z)$ and $f(z)$ corresponding to this solution can

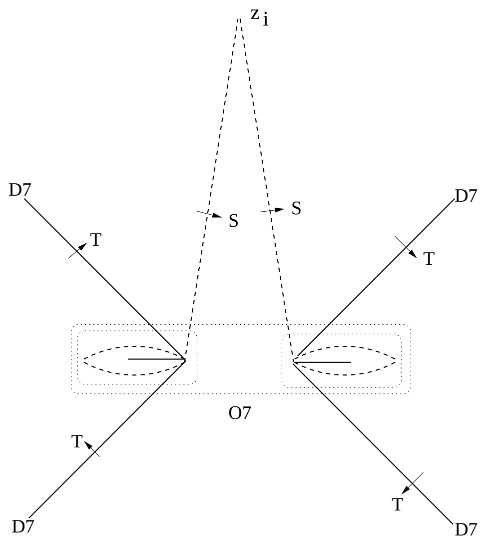

Figure 2. *The most general supersymmetric $N = 6$ solution with only non-trivial T-monodromies around the orbifold points $z_{i\infty}$, where the D7-branes are located. The filled (dashed) lines are T (S) branch cuts.*

be found in [15]. The $N = 6$ solution corresponds in the weak coupling limit $\mathrm{Im}\,\tau \to \infty$ to a system of 4 D7–branes and a O7–plane [17]. A more detailed discussion of this solution can be found in [15].

6. Remarks

We have presented a class of seven-brane solutions of IIB supergravity thereby generalizing the existing class of solutions by allowing additional objects with monodromies $\mathrm{Tr}\,\Lambda \neq 2$ at the orbifold points $z = \rho, i$. It would be interesting to better understand the explicit form of the source terms we used via the PST method.

It is not clear what the correct interpretation of the $\det Q > 0$ branes are within string theory. They are not Dirichlet branes like the D7–brane. This can, for instance, be seen from the fact that no worldvolume action containing a single Born–Infeld vector can be constructed that describes the dynamics close to these branes [13]. One could perhaps consider them as effective bound states of two D7–branes whose monodromies are related to each other by a $SL(2,\mathbb{Z})$–transformation. This is based on the observation that the monodromy of a $\det Q > 0$ brane can be written as the product of two such monodromies. A related interpretation was discussed in [16], where it was shown that the approximate solution (23) with $\det Q > 0$ can be written as a distribution of det $Q = 0$ branes. Alternatively, the work of [17] suggests that they have something to do with the non-perturbative description of O7–planes.

An interesting generalization of our results would be to add D3–branes and consider D3–D7 brane sytems. This could lead to applications both in the AdS/CFT correspondence and in cosmology. It would be interesting to see if, in addition to the solutions with only D7–branes of [1], the more general solutions with det $Q > 0$ objects can also play a role in these applications.

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