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3–TYPE CURVES IN THE EUCLIDEAN SPACE E^6

BY

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Dedicated to the sixtieth birthday of Professor Bang-Yen Chen

Abstract. In [1] D. Blair gave a complete classification of 3–type curves in the space E^3. In two recent papers [8] and [9] we gave a complete classification of 3–type curves in the spaces E^4 and E^5. In this paper we give a complete classification of 3–type curves in the space E^6. This result completes the describing of 3–type curves in Euclidean spaces.

1. Introduction

The notion of curves of finite type was introduced by B. Y. Chen around 1980. A closed curve γ in a Euclidean space E^n is of finite type (k–type, $k \in N$) if its Fourier series expansion with respect to an arclength parameter is finite (has exactly k nonzero terms).

It is proved in [3] that a closed curve $\gamma\colon [0, 2\pi r] \mapsto E^n$ is of k–type ($k \in N$) if and only if there is a vector $A_0 \in E^n$, natural numbers $p_1 < p_2 < \cdots < p_k$ (frequency numbers of the curve), and vectors $A_1, \ldots, A_k, B_1, \ldots, B_k \in E^n$ such that $\|A_i\|^2 + \|B_i\|^2 \neq 0$ $(i = 1, \ldots, k)$ and

$$\gamma(s) = A_0 + \sum_{i=1}^{k}(A_i \cos\frac{p_i s}{r} + B_i \sin\frac{p_i s}{r}).$$

It is shown in [5] that every curve of k–type in the space E^n lies in an affine subspace of the dimension $2k$. Hence, the only interesting case is $n \leq 2k$.

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Consequently, considering the 3–type curves in the Euclidean space E^n, we can always assume that $n \leq 6$.

3–type curves in the space E^3, have been investigated several times in the literature (see e.g. [1, 8, 9]). So in [1] D. Blair gave a complete classification of all closed 3–type curves in the space E^3. Further in [8], we have classified all closed 3–type curves in the Euclidean space E^4, and in [9], all closed 3–type curves in the Euclidean space E^5. Doing a final step, in this paper we give a complete classification of all closed 3–type curves in the Euclidean space E^6. This obviously completes the classification of all closed 3–type curves in Euclidean spaces.

The methods which are used in this paper are similar to the corresponding methods from the papers [1], [8] and [9]. In view of these similarities, we very often mention only results omiting the proofs. But, comparing the spaces E^k and E^{k+1}, we often meet the cases which are really possible in the space E^{k+1}, but impossible in the space E^k. Hence, the investigations in the space E^{k+1} are not the complete analogies of the cases in the smaller space. Very often, some new cases there appear.

We also mention that by usual lifting $(x, y, z, t, u) \mapsto (x, y, z, t, u, 0)$ of the space E^5 into the space E^6, every 3–type curve in the space E^5 becomes a 3–type curve in the space E^6. Hence, if some of the following cases is possible in the space E^5, it is also possible in the space E^6.

By the general statement, we have that a closed curve $\gamma \subseteq E^6$ is of 3–type if there are natural numbers $p_1 < p_2 < p_3$ (frequency numbers of the curve) such that $\gamma\colon [0, 2\pi r] \mapsto E^6$ has the form

$$\gamma(s) = A_0 + \sum_{i=1}^{3}(A_i \cos\frac{p_i s}{r} + B_i \sin\frac{p_i s}{r}),$$

where $A_0 \in E^6$ and $A_1, A_2, A_3, B_1, B_2, B_3 \in E^6$ are such that $\|A_i\|^2 + \|B_i\|^2 \neq 0$ for each $i = 1, 2, 3$.

It is proved in [3] that the last condition is equivalent to the following system of equations:

$$(O) \qquad \sum_{i=1}^{3} p_i^2 D_{ii} = 2r^2,$$

$$I(l) \qquad \sum_{\substack{i=1\\2p_i=l}}^{3} p_i^2 A_{ii} + 2 \sum_{\substack{i,j=1\\i>j\\p_i+p_j=l}}^{3} p_i p_j A_{ij} - \sum_{\substack{i,j=1\\i>j\\p_i-p_j=l}}^{3} p_i p_j D_{ij} = 0,$$

$$\overline{I}(l) \qquad \sum_{\substack{i=1\\2p_i=l}}^{3} p_i^2 \overline{A}_{ii} + 2 \sum_{\substack{i,j=1\\i>j\\p_i+p_j=l}}^{3} p_i p_j \overline{A}_{ij} - \sum_{\substack{i,j=1\\i>j\\p_i-p_j=l}}^{3} p_i p_j \overline{D}_{ij} = 0,$$

where

$$A_{ij} = \langle A_i, A_j\rangle - \langle B_i, B_j\rangle, \quad \overline{A}_{ij} = \langle A_i, B_j\rangle + \langle A_j, B_i\rangle,$$
$$D_{ij} = \langle A_i, A_j\rangle + \langle B_i, B_j\rangle, \quad \overline{D}_{ij} = \langle A_i, B_j\rangle - \langle A_j, B_i\rangle,$$

$(i, j = 1, 2, 3)$, and l runs the set

$$\mathcal{A} = \{2p_1,\, 2p_2,\, 2p_3,\, p_1+p_2,\, p_1+p_3,\, p_2+p_3,\, p_2-p_1,\, p_3-p_1,\, p_3-p_2\}.$$

2. Main Result

The main theorem of this paper is the following.

Theorem. *If $\gamma(s)$ is a 3–type curve in the Euclidean space E^6, then $\gamma(s)$ belongs to a p–parameter family of curves where p is one of the numbers* 3, 6, 8, 9, 13, 15, 17, *and which families of curves are described further on.*

The proof of this theorem follows from the next series of propositions.

First, we shall differ the cases when all indices in the set $\mathcal{A}$ are distinct, or some of them coincide.

The complete classification of all these cases is as follows.

(1^0) $\quad p_2 \neq 3p_1,\ p_3 \neq 3p_1, 3p_2, p_2+2p_1, 2p_2 \pm p_1,$

(2^0) $\quad p_2 = 3p_1,\ p_3 \neq 5p_1, 7p_1, 9p_1,$

(3^0) $\quad p_2 \neq 2p_1,\ p_3 = 3p_1,$

(4^0) $\quad p_2 \neq 3p_1,\ p_3 = 3p_2,$

(5^0) $\quad p_2 \neq 3p_1,\ p_3 = p_2 + 2p_1,$

(6^0) $\quad p_2 = 3p_1,\ p_3 = 5p_1,$

(7^0) $p_2 \neq 3p_1,\ p_3 = p_1 + 2p_2,$

(8^0) $p_2 = 3p_1,\ p_3 = 7p_1,$

(9^0) $p_2 \neq 2p_1,\ 3p_1, \quad p_3 = 2p_2 - p_1,$

(10^0) $p_2 = 2p_1,\ p_3 = 3p_1,$

(11^0) $p_2 = 3p_1,\ p_3 = 9p_1.$

We shall discuss all these cases separately. Introduce the following notations:

$$\begin{aligned} A_1 &= (a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}), \quad B_1 = (b_{11}, b_{12}, b_{13}, b_{14}, b_{15}, b_{16}),\\ A_2 &= (a_{21}, a_{22}, a_{23}, a_{24}, a_{25}, a_{26}), \quad B_2 = (b_{21}, b_{22}, b_{23}, b_{24}, b_{25}, b_{26}),\\ A_3 &= (a_{31}, a_{32}, a_{33}, a_{34}, a_{35}, a_{36}), \quad B_3 = (b_{31}, b_{32}, b_{33}, b_{34}, b_{35}, b_{36}). \end{aligned}$$

If some index in the set $\mathcal{A}$ differs of all other indices in this set, we shall call it “single”. The set $\mathcal{A}$ obviously has at least two single indices, namely $2p_3$ and $p_2 + p_3$. These indices are evidently the greatest in $\mathcal{A}$.

Lemma1. *By a suitable change of the coordinate system, we can assume that*

$$A_3 = (\mu, 0, 0, 0, 0, 0), \quad B_3 = (0, \mu, 0, 0, 0, 0), \qquad (\mu \neq 0).$$

In this system we have $b_{21} = -a_{22}, b_{22} = a_{21}$*, thus* $B_2 = (-a_{22}, a_{21}, b_{23},\ b_{24}, b_{25}, b_{26})$.

We omit the proof since it is quite similar to the corresponding proof of Lemma 1 in [8] and [9]. The similar is true in the next lemma.

Lemma 2. *If* $2p_2$ *and* $p_3 - p_2$ *are single parameters, then by a suitable change of coordinate system we can assume that*

$$A_2 = (0, 0, \nu, 0, 0, 0), \qquad B_2 = (0, 0, 0, \nu, 0, 0),$$

for some $\nu \neq 0$.

Proposition 1. *In the case* (1^0) *a curve* $\gamma(s)$ *is a 3–type curve if and only if, in a coordinate system, we have*

$$A_1 = (0, 0, 0, 0, \rho, 0), \quad B_1 = (0, 0, 0, 0, 0, \rho), \qquad (\rho \neq 0),$$

$$A_2 = (0,0,\nu,0,0,0), \quad B_2 = (0,0,0,\nu,0,0), \qquad (\nu \neq 0),$$
$$A_3 = (\mu,0,0,0,0,0), \quad B_3 = (0,\mu,0,0,0,0), \qquad (\mu \neq 0).$$

We again omit the proof. Note that this case is impossible in the space E^5. But here, it obviously defines a 3–parameter family of curves.

Proposition 2. *In the case* (2^0), $\gamma(s)$ *is a* 3*–type curve if and only if, in a coordinate system, we have*

$$A_1 = (0,0,a_{13},a_{14},a_{15},a_{16}), \quad B_1 = (0,0,-a_{14},a_{13},b_{15},b_{16}),$$
$$A_2 = (0,0,\nu,0,0,0), \quad B_2 = (0,0,0,\nu,0,0), \qquad (\nu \neq 0),$$
$$A_3 = (\mu,0,0,0,0,0), \quad B_3 = (0,\mu,0,0,0,0), \qquad (\mu \neq 0),$$

where

$$a_{13} = \frac{a_{15}^2 + a_{16}^2 - b_{15}^2 - b_{16}^2}{6\nu}, \qquad a_{14} = -\frac{a_{15}b_{15} + a_{16}b_{16}}{3\nu}.$$

Hence in this case we get a 6–parameter family of curves.

Proposition 3. *In the case* (3^0), $\gamma(s)$ *is a* 3*–type curve if and only if, in a coordinate system, we have*

$$A_1 = (a_{11},a_{12},0,0,a_{15},a_{16}), \quad B_1 = (-a_{12},a_{11},0,0,b_{15},b_{16}),$$
$$A_2 = (0,0,\nu,0,0,0), \quad B_2 = (0,0,0,\nu,0,0), \qquad (\nu \neq 0),$$
$$A_3 = (\mu,0,0,0,0,0), \quad B_3 = (0,\mu,0,0,0,0), \qquad (\mu \neq 0),$$

where

$$a_{11} = \frac{a_{15}^2 + a_{16}^2 - b_{15}^2 - b_{16}^2}{6\mu}, \qquad a_{12} = -\frac{a_{15}b_{15} + a_{16}b_{16}}{3\mu}.$$

Therefore, in this case we also get a 6–parameter family of curves.

Proposition 4. *In the case* (4^0), $\gamma(s)$ *is a* 3*–type curve if and only if, in a coordinate system, we have*

$$A_1 = (0,0,\nu,0,0,0), \quad B_1 = (0,0,0,\nu,0,0), \qquad (\nu \neq 0),$$
$$A_2 = (a_{21},a_{22},0,0,a_{25},a_{26}), \quad B_2 = (-a_{22},a_{21},0,0,b_{25},b_{26}),$$

$$A_3 = (\mu, 0, 0, 0, 0, 0), \quad B_3 = (0, \mu, 0, 0, 0, 0), \qquad (\mu \neq 0),$$

where

$$a_{21} = \frac{a_{25}^2 + a_{26}^2 - b_{25}^2 - b_{26}^2}{6\mu}, \qquad a_{22} = -\frac{a_{25}b_{25} + a_{26}b_{26}}{3\mu},$$

and $a_{21}^2 + a_{22}^2 \neq 0$.

Hence, in this case the above system again defines a 6–parameter family of curves.

Proposition 5. (Case (5^0)) $(p_2 \neq 3p_1, p_3 = p_2 + 2p_1)$. *In this case,* $\gamma(s)$ *is a 3–type curve if and only if in a coordinate system we have*

$$\begin{aligned} A_1 &= (a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}), \quad B_1 = (-a_{12}, a_{11}, b_{13}, b_{14}, b_{15}, b_{16}), \\ A_2 &= (a_{21}, a_{22}, \nu, 0, 0, 0), \quad B_2 = (-a_{22}, a_{21}, 0, \nu, 0, 0), \\ A_3 &= (\mu, 0, 0, 0, 0, 0), \quad B_3 = (0, \mu, 0, 0, 0, 0), \end{aligned}$$

where $\mu \neq 0$, $a_{21}^2 + a_{22}^2 + \nu^2 \neq 0$ *and*

(1) $\quad a_{11}a_{21} + a_{12}a_{22} = -\dfrac{\nu}{2}(a_{13} + b_{14}),$

(2) $\quad a_{11}a_{22} - a_{12}a_{21} = \dfrac{\nu}{2}(a_{14} - b_{13}),$

(3) $\quad \displaystyle\sum_{i=3}^{6}(a_{1i}^2 - b_{1i}^2) = \frac{2p_2p_3}{p_1^2}\mu a_{21},$

(4) $\quad \displaystyle\sum_{i=3}^{6} a_{1i}b_{1i} = -\frac{p_2p_3\,\mu\,a_{22}}{p_1^2},$

(5) $\quad \nu(a_{13} - b_{14}) = \dfrac{p_3\,\mu\,a_{11}}{p_2},$

(6) $\quad a_{11}a_{22} - a_{12}a_{21} + \nu\,b_{13} = -\dfrac{p_3\,\mu\,a_{12}}{p_2}.$

Since the case (5^0) generates a whole family of curves in the space E^5 (see [9]), the similar is true in the space E^6. So, it is not necessary to construct at least one solution of the above system of equations. It is easy to see that in this case, the above system defines a 8–parameter family of curves.

A similar situation is true in all cases (6^0)–(11^0), so in each of these cases there is at least one 3–type curve (and even more the whole family of such curves) in the space E^6.

Proposition 6. (Case (6^0)) ($p_1 : p_2 : p_3 = 1 : 3 : 5$). *In this case* $\gamma(s)$ *is a 3–type curve if and only if, in a coordinate system we have*

$$A_1 = (a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}), \quad B_1 = (b_{11}, b_{12}, b_{13}, b_{14}, b_{15}, b_{16}),$$
$$A_2 = (a_{21}, a_{22}, a_{23}, a_{24}, a_{25}, a_{26}), \quad B_2 = (-a_{22}, a_{21}, b_{23}, b_{24}, b_{25}, b_{26}),$$
$$A_3 = (\mu, 0, 0, 0, 0, 0), \quad B_3 = (0, \mu, 0, 0, 0, 0),$$

where $\mu \neq 0$ *and*

(1) $\quad \sum_{i=1}^{6}(a_{1i}^2 - b_{1i}^2) = 3[\sum_{i=1}^{6} a_{1i}a_{2i} - b_{11}a_{22} + b_{12}a_{21} + \sum_{i=3}^{6} b_{1i}b_{2i}] + 30\,\mu a_{21},$

(2) $\quad \sum_{i=1}^{6} a_{1i}b_{1i} = \frac{3}{2}\sum_{i=1}^{6}(a_{2i}b_{1i} - a_{1i}b_{2i}) - 15\mu\, a_{22},$

(3) $\quad \sum_{i=1}^{6} a_{1i}a_{2i} - (-a_{22}b_{11} + a_{21}b_{12} + \sum_{i=3}^{6} b_{1i}b_{2i}) = \frac{5\,\mu}{6}\,(a_{11} + b_{12}),$

(4) $\quad \sum_{i=1}^{6} b_{1i}a_{2i} - a_{11}a_{22} + a_{12}a_{21} + \sum_{i=3}^{6} a_{1i}b_{2i} = \frac{5\,\mu}{6}\,(b_{11} - a_{12}),$

(5) $\quad \sum_{i=3}^{6}(a_{2i}^2 - b_{2i}^2) = -\frac{10}{9}\,\mu\,(a_{11} - b_{12}),$

(6) $\quad \sum_{i=3}^{6} a_{2i}b_{2i} = -\frac{5}{9}\,\mu\,(a_{12} + b_{11}).$

In this case we obtain a 17–parameter family of curves.

Proposition 7. (Case (7^0)) ($p_2 \neq 3p_1, p_3 = p_1 + 2p_2$). *In this case* $\gamma(s)$ *is a 3–type curve if and only if, in a coordinate system we have*

$$A_1 = (a_{11}, a_{12}, \nu, 0, 0, 0), \quad B_1 = (-a_{12}, a_{11}, 0, \nu, 0, 0),$$
$$A_2 = (a_{21}, a_{22}, a_{23}, a_{24}, a_{25}, a_{26}), \quad B_2 = (-a_{22}, a_{21}, b_{23}, b_{24}, b_{25}, b_{26}),$$
$$A_3 = (\mu, 0, 0, 0, 0, 0), \quad B_3 = (0, \mu, 0, 0, 0, 0),$$

where $\mu \neq 0$, $a_{11}^2 + a_{12}^2 + \nu^2 \neq 0$ *and we have*

(1) $\quad a_{11}a_{21} + a_{12}a_{22} = -\frac{\nu}{2}\,(a_{23} + b_{24}),$

(2) $\quad a_{11}a_{22} - a_{12}a_{21} = \frac{\nu}{2}\,(b_{23} - a_{24}),$

(3) $\sum_{i=3}^{6}(a_{2i}^2 - b_{2i}^2) = \frac{2p_1p_3\mu}{p_2^2}\, a_{11},$

(4) $\sum_{i=3}^{6} a_{2i}b_{2i} = -\frac{p_1p_3\mu}{p_2^2}\, a_{12},$

(5) $\nu\,(a_{23} - b_{24}) = \frac{p_3\,\mu}{p_1} a_{21},$

(6) $\nu\,(a_{24} + b_{23}) = -\frac{p_3\mu}{p_1} a_{22}.$

In that case we get a 8–parameter family of curves.

Proposition 8. (Case (8^0)) $(p_1 : p_2 : p_3 = 1 : 3 : 7)$. *In this case* $\gamma(s)$ *is a* 3*–type curve if and only if, in a coordinate system, we have*

$$A_1 = (a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}), \quad B_1 = (-a_{12}, a_{11}, b_{13}, b_{14}, b_{15}, b_{16}),$$
$$A_2 = (a_{21}, a_{22}, a_{23}, a_{24}, a_{25}, a_{26}), \quad B_2 = (-a_{22}, a_{21}, b_{23}, b_{24}, b_{25}, b_{26}),$$
$$A_3 = (\mu, 0, 0, 0, 0, 0), \quad B_3 = (0, \mu, 0, 0, 0, 0),$$

where $\mu \neq 0$ *and*

(1) $\sum_{i=3}^{6}(a_{2i}^2 - b_{2i}^2) = \frac{14}{9}\,\mu\, a_{11},$

(2) $\sum_{i=3}^{6} a_{2i}b_{2i} = -\frac{7}{9}\,\mu\, a_{12},$

(3) $\sum_{i=3}^{6}(a_{1i}a_{2i} - b_{1i}b_{2i}) = 7\,\mu\, a_{21},$

(4) $\sum_{i=3}^{6}(a_{1i}b_{2i} + b_{1i}a_{2i}) = -7\,\mu\, a_{22},$

(5) $\sum_{i=3}^{6}(a_{1i}^2 - b_{1i}^2) = 6a_{11}a_{21} + 6a_{12}a_{22} + 3\sum_{i=3}^{6}(a_{1i}a_{2i} + b_{1i}b_{2i}),$

(6) $\sum_{i=3}^{6} a_{1i}b_{1i} = 3a_{11}a_{22} - 3a_{12}a_{21} + \frac{3}{2}\sum_{i=3}^{6}(b_{1i}a_{2i} - a_{1i}b_{2i}).$

In this case we get a 15–parameter family of curves.

Proposition 9. (Case (9^0)) $(p_2 \neq 2p_1, 3p_1,\ p_3 = 2p_2 - p_1)$. *In this case* $\gamma(s)$

is a 3*–type curve if and only if in a coordinate system we have*

$$A_1 = (a_{11}, a_{12}, \nu, 0, 0, 0), \quad B_1 = (a_{12}, -a_{11}, 0, \nu, 0, 0),$$
$$A_2 = (a_{21}, a_{22}, a_{23}, a_{24}, a_{25}, a_{26}), \quad B_2 = (-a_{22}, a_{21}, b_{23}, b_{24}, b_{25}, b_{26}),$$
$$A_3 = (\mu, 0, 0, 0, 0, 0), \quad B_3 = (0, \mu, 0, 0, 0, 0),$$

where $\mu \neq 0$*,* $a_{11}^2 + a_{12}^2 + \nu^2 \neq 0$ *and*

(1) $$a_{11}a_{21} + a_{12}a_{22} = \frac{\nu}{2}(b_{24} - a_{23}),$$

(2) $$a_{11}a_{22} - a_{12}a_{21} = \frac{\nu}{2}(a_{24} + b_{23}),$$

(3) $$\sum_{i=3}^{6}(a_{2i}^2 - b_{2i}^2) = -\frac{4p_1p_3\,\mu}{p_2^2}\,a_{11},$$

(4) $$\sum_{i=3}^{6} a_{2i}b_{2i} = -\frac{2p_1p_3\,\mu}{p_2^2}\,a_{12},$$

(5) $$\nu\,(a_{23} + b_{24}) = -\frac{2p_3}{p_1}\,\mu\,a_{21},$$

(6) $$\nu\,(a_{24} - b_{23}) = \frac{2p_3}{p_1}\,\mu\,a_{22}.$$

In that case it can be proved that above system defines a 9–parameter family of curves.

Proposition 10. (Case (10^0)) $(p_1 : p_2 : p_3 = 1 : 2 : 3)$. *In this case* $\gamma(s)$ *is a* 3*–type curve if and only if, in a coordinate system, we have*

$$A_1 = (a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}), \quad B_1 = (b_{11}, b_{12}, b_{13}, b_{14}, b_{15}, b_{16}),$$
$$A_2 = (a_{21}, a_{22}, a_{23}, a_{24}, a_{25}, a_{26}), \quad B_2 = (-a_{22}, a_{21}, b_{23}, b_{24}, b_{25}, b_{26}),$$
$$A_3 = (\mu, 0, 0, 0, 0, 0), \quad B_3 = (0, \mu, 0, 0, 0, 0),$$

where $\mu \neq 0$ *and*

(1) $$\sum_{i=1}^{6} a_{1i}a_{2i} = -b_{11}a_{22} + a_{21}b_{12} + \sum_{i=3}^{6} b_{1i}b_{2i},$$

(2) $$\sum_{i=1}^{6} b_{1i}a_{2i} = a_{11}a_{22} - a_{12}a_{21} - \sum_{i=3}^{6} a_{1i}b_{2i},$$

(3) $$\sum_{i=1}^{6}(a_{1i}^2 - b_{1i}^2) = 3\,\mu\,(a_{11} + b_{12}),$$

(4) $\sum_{i=1}^{6} a_{1i}b_{1i} = \frac{3}{2}\,\mu\,(b_{11} - a_{12}),$

(5) $\sum_{i=1}^{6} a_{1i}a_{2i} - b_{11}a_{22} + b_{12}a_{21} + \sum_{i=3}^{6} b_{1i}b_{2i} = -6\,\mu a_{21},$

(6) $\sum_{i=1}^{6} a_{2i}b_{1i} + a_{11}a_{22} - a_{12}a_{21} - \sum_{i=3}^{6} a_{1i}b_{2i} = 6\mu\, a_{22},$

(7) $\sum_{i=3}^{6}(a_{2i}^2 - b_{2i}^2) = -\frac{3}{2}\,\mu\,(a_{11} - b_{12}),$

(8) $\sum_{i=3}^{6} a_{2i}b_{2i} = -\frac{3}{4}\,\mu\,(b_{11} + a_{12}).$

In this case we get a 15–parameter family of curves.

Proposition 11. (Case (11^0)) $(p_1 : p_2 : p_3 = 1 : 3 : 9)$. *In this case* $\gamma(s)$ *is a* 3*–type curve if and only if in a coordinate system we have*

$$A_1 = (0, 0, a_{13}, a_{14}, a_{15}, a_{16}), \quad B_1 = (0, 0, b_{13}, b_{14}, b_{15}, b_{16}),$$
$$A_2 = (a_{21}, a_{22}, a_{23}, a_{24}, a_{25}, a_{26}), \quad B_2 = (-a_{22}, a_{21}, b_{23}, b_{24}, b_{25}, b_{26}),$$
$$A_3 = (\mu, 0, 0, 0, 0, 0), \quad B_3 = (0, \mu, 0, 0, 0, 0),$$

where $\mu \neq 0$ *and*

(1) $\sum_{i=3}^{6}(a_{1i}a_{2i} - b_{1i}b_{2i}) = 0,$

(2) $\sum_{i=3}^{6}(a_{1i}b_{2i} + a_{2i}b_{1i}) = 0,$

(3) $\sum_{i=3}^{6}(a_{1i}^2 - b_{1i}^2) = 3\sum_{i=3}^{6}(a_{1i}a_{2i} + b_{1i}b_{2i}),$

(4) $\sum_{i=3}^{6} a_{1i}b_{1i} = 1,5\sum_{i=3}^{6}(b_{1i}a_{2i} - a_{1i}b_{2i}),$

(5) $\sum_{i=3}^{6}(a_{2i}^2 - b_{2i}^2) = 6\mu\, a_{21},$

(6) $\sum_{i=3}^{6} a_{2i}b_{2i} = -3\mu\, a_{22}.$

Finally, in that case, it can be proved that we obtain a 13–parameter family of curves.

Summarizing all Propositions 1–11, we get the theorem.

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