Hard m_t Corrections as a Probe of the Symmetry Breaking Sector

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Nondecoupling effects related to a large m_t affecting nonoblique radiative corrections in vertices ($Z\bar{b}b$) and boxes (B-$\bar{B}$ mixing and ϵ_K) are sensitive to the mechanism of spontaneous symmetry breaking. In the framework of the effective chiral electroweak standard model there is only one $O(p^4)$ operator which modifies the longitudinal part of the W^+ boson without touching the oblique corrections. This operator affects the $Z\bar{b}b$ vertex, the B-$\bar{B}$ mixing, and the CP-violating parameter ϵ_K, generating interesting correlations among the hard $m_t^4 \ln m_t^2$ corrections to these observables. [S0031-9007(97)02873-1]

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One of the basic ingredients of the standard model (SM) is the spontaneous breaking of the electroweak gauge symmetry. In the SM it is implemented through the Higgs mechanism in which the would-be Goldstone excitations are absorbed into the longitudinal degrees of freedom of the gauge bosons. The spontaneous symmetry breaking (SSB) is realized linearly, that means, by the use of a scalar field which acquires a nonzero vacuum expectation value. The spectrum of physical particles contains then not only the massive vector bosons but also a neutral scalar Higgs field which must be relatively light.

In a more general scenario, the SSB can be parametrized in terms of a nonrenormalizable Lagrangian which contains the SM gauge symmetry realized nonlinearly [1,2]. This nonlinearly realized SM is also called the chiral realization of the SM (χSM) due to its similarity with low-energy quantum chromodynamics (QCD) chiral Lagrangians. It includes, with a particular choice of the parameters of the Lagrangian, the SM, as long as the energies involved are small compared with the Higgs mass which is not present in the effective Lagrangian. In addition, it can also accommodate any model that reduces to the SM at low energies as happens in many technicolor scenarios. The price to be paid for this general parametrization is the loss of renormalizability and, therefore, the appearance of many couplings which must be determined from experiment or computed in a more fundamental theory.

Since the SSB is related to the bosonic sector, one would expect that any deviation from the SM SSB mechanism would affect especially the gauge-boson propagation properties, the so-called oblique corrections, which are parametrized in terms of the S,T,U parameters [3] (or the ϵ_1, ϵ_2 and ϵ_3 parameters [4]). In fact, these corrections have been studied extensively in the framework of the χSM [5]. In particular, one would think that one should look into quantities which are M_H dependent in the SM to test the SSB sector. However, it is interesting to realize that the only M_H-dependent radiative correction, $\Delta\rho$, has an agreement with the SM prediction at the per-mil level. Vertex corrections, whose M_H dependence appears only at the two-loop level, are not so well known. [And, in fact, in the past there has been a big controversy about the $R_b = \Gamma(Z \to b\bar{b})/\Gamma(Z \to \text{hadrons})$ value.]

On the other hand, the would-be Goldstone bosons coming from SSB also couple to fermions. In fact, all nondecoupling effects of the SM related to a large top-quark mass, m_t, come from the coupling of the would-be Goldstone bosons to the top quark. Therefore, we can expect any nondecoupling quantity related to a heavy top quark to be sensitive to the would-be Goldstone boson propagation properties and couplings, that is, to the specific mechanism of SSB.

In the SM, large m_t^2 effects appear, in addition to the oblique corrections, in the vertex $Zb\bar{b}$, that is in $R_b = \Gamma_b/\Gamma_h$, and in B-$\bar{B}$ and K-$\bar{K}$ mixing. [Of course, nondecoupling effects appear in other observables, but present experiments are sensitive enough to see the effects only in the quantities we just mentioned.] Then, we will use these quantities to explore possible deviations of the SM spontaneous symmetry breaking mechanism. To do so, we will use a χSM only for the bosonic sector of the theory and leave fermion couplings as in the linear SM. [Possible modifications of the fermionic couplings of the gauge bosons have been investigated in Ref. [6]. However, these couplings affect the oblique corrections as well.]

It turns out that there is only one operator in the effective Lagrangian that affects the $Zb\bar{b}$ vertex without touching the oblique corrections (which, as mentioned before, agrees with the SM at the per-mil level). This operator modifies the propagation properties of the charged would-be Goldstone bosons, that is, the longitudinal component of the W^+ boson. Therefore, it will also affect any observable in which the nondecoupling effects of a large m_t are important, in particular, B-$\bar{B}$ mixing and ϵ_K.

In the nonlinear realization of the SM the Goldstone bosons π^a associated with the SSB of $SU(2)_L \times SU(2)_R \to SU(2)_{L+R}$ are collected in a matrix field $U(x) = \exp(i\pi^a\tau^a/v)$. The operators in the effective

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chiral Lagrangian are classified according to the number of covariant derivatives acting on $U(x)$.

The lowest-order operators just fix the values of the Z and W mass at tree level and do not carry any information on the underlying physics. Therefore, in order to extract some information on new physics, we must start studying the effects coming from higher-order operators. Departure of those coefficients from the SM predictions can be a hint for the existence of new physics.

The lowest-order effective chiral Lagrangian can be written in the following way:

$$L = L_B + L_\psi + L_Y \, , \tag{1}$$

where

$$L_B = -\frac{1}{2}\,\mathrm{Tr}(\hat{W}_{\mu\nu}\hat{W}^{\mu\nu} + \hat{B}_{\mu\nu}\hat{B}^{\mu\nu}) + \frac{v^2}{4}\,\mathrm{Tr}(D_\mu U^+ D^\mu U)\, , \tag{2}$$

with $\hat{W}_{\mu\nu} = W^a_{\mu\nu}\tau^a/2$, $\hat{B}^{\mu\nu} = B^{\mu\nu}\tau^3/2$, and $D^\mu U = \partial^\mu U + i\,\frac{g}{2}\,W^\mu_a\tau^a U - i\,\frac{g'}{2}\,B^\mu U\tau^3$. L_ψ is the usual fermionic kinetic Lagrangian and

$$L_Y = -\bar{Q}_L U M_q Q_R + \mathrm{H.c.}\, , \tag{3}$$

where M_q is a 2×2 block-diagonal matrix containing the 3×3 mass matrices of the up and down quarks, and Q_L and Q_R are doublets containing the up and down quarks for the three families in the weak basis.

At the next order that contains, at most, four derivatives, the CP and $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ invariant effective chiral Lagrangian with only gauge bosons and Goldstone fields is described by the 15 operators reported in Ref. [2]: $L = \sum_{i=0}^{14} a_i O_i$.

The usual oblique corrections are sensitive to a_0 $(a_8 + a_{13})$ and $(a_1 + a_{13})$; the present data bound these couplings below the 1% level. On the other hand, the operators proportional to a_2, a_3, a_9, and a_{14} parametrize the effective non-Abelian gauge couplings that are tested by LEP2. The operators contributing to three- and four-point Green functions $(a_2, a_3, a_4, a_5, a_6, a_7, a_9, a_{10}, a_{14})$ modify the oblique corrections only at the one-loop level; thus, the present bounds on those couplings are rather weak ($\sim$10%). The other couplings (a_{11}, a_{12}) remain untested because, although quadratic in the Goldstone fields, they do not contribute to the oblique corrections even at one loop. For instance, the operator proportional to a_{11},

$$O_{11} = \mathrm{Tr}[(D_\mu V^\mu)^2]\, , \tag{4}$$

with $V_\mu = (D_\mu U)U^+$ and $D^\mu V_\mu = \partial^\mu V_\mu + ig[\hat{W}_\mu, V^\mu]$ generates corrections to the two-point Green function of the W^+, Z, and would-be Goldstone bosons:

$$O_{11} = g^2 W^+_\mu \partial^\mu\partial^\nu W^-_\nu + \frac{g_Z^2}{2}\,Z_\mu\partial^\mu\partial^\nu Z_\nu - 4\pi^+\,\frac{\partial^4}{v^2}\,\pi^- - 2\pi_3\,\frac{\partial^4}{v^2}\,\pi_3 + \frac{2g}{v}\,W^+_\mu\partial^\mu\partial^2\pi^- + \frac{2g}{v}\,W^-_\mu\partial^\mu\partial^2\pi^+ + \frac{2g_Z}{v}\,Z^+_\mu\partial^\mu\partial^2\pi_3 + O(\pi^3)\, . \tag{5}$$

However, all these interactions always involve the longitudinal components of the gauge bosons and so do not enter directly into the ϵ_i parameters. The same happens to the operators O_{12} and O_{13}, which affect only the longitudinal part of the neutral Z boson.

The effects of the operator O_{11} can be seen more easily once we use the following equation of motion involving the operators of the Lagrangian to lowest order: [This is allowed in the effective Lagrangian, even at the one-loop level, as long as we keep only the dominant pieces. The use of the equations of motion is equivalent to a redefinition of the fields which affects only higher-order operators in the effective Lagrangian.]

$$D_\mu V^\mu = \frac{i}{v^2}\,D_\mu(\bar{Q}_L\gamma^\mu\tau^a Q_L\tau^a)\, , \tag{6}$$

$$i\not{D}Q_L = UM_qQ_R\, , \qquad i\not{D}Q_R = M_q^+U^+Q_L\, . \tag{7}$$

Then the operator O_{11} can be rewritten as

$$O_{11} = \frac{g^4}{8M_W^4}\,[\bar{Q}(\tau^a UM_qP_R - M_q^+U^+\tau^aP_L)Q]^2, \tag{8}$$

where P_L and P_R are the left and right chirality projectors. By writing (8) in terms of the mass eigenstates, and keeping only the terms proportional to the top-quark mass, we obtain

$$O_{11} = \frac{g^4}{8M_W^4}\,m_t^2\left((\bar{t}\gamma_5 t)^2 - 4\sum_{f,f'}^{d,s,b}(\bar{f}'_L t_R)(\bar{t}_R f_L)V_{tf}V^*_{tf'}\right). \tag{9}$$

Therefore, the effect to the lowest order of the modification of the would-be Goldstone propagator can be written as a four-fermion interaction proportional to quark masses. This kind of operator also appears in the analysis of new physics with an effective Lagrangian with SSB realized linearly [7]. However, the explicit m_t^2/M_W^4 factor in Eq. (9) has its origin in the bosonic operator of Eq. (4).

Four-fermion interactions are much more convenient for explicit calculations and also to understand the effects of the new operator. For instance, it is clear that the four-fermion interaction can only contribute to the gauge-boson self-energies at two loops and therefore do not contribute to the ϵ_i parameters at one loop.

We now discuss some observables affected by the new interaction.

R_b.—We start with the evaluation of the corrections to the $Z\bar{b}b$ vertex. We parametrize the effective $Z\bar{b}b$ vertex as

$$\frac{g}{c_W} Z^\mu (g_L^b \bar{b}_L \gamma_\mu b_L + g_R^b \bar{b}_R \gamma_\mu b_R), \tag{10}$$

with the values of the tree level couplings, $g_L^b = -1/2 + s_W^2/3$ and $g_R^b = s_W^2/3$.

At one loop we parametrize the effect of new physics as a shift in the couplings:

$$g_{L,R}^b \to g_{L,R}^b + \delta g_{L,R}^b. \tag{11}$$

We calculate the one-loop contribution of the operator O_{11}, keeping only the divergent logarithmic piece. This means we neglect any possible local contribution from the chiral Lagrangian at order p^6. The relevant diagram is depicted in Fig. 1(a) and the result is

$$\delta g_L = -\frac{\alpha}{4\pi s_w^2} a_{11} \frac{g^2}{4} \frac{m_t^4}{M_W^4} \ln \frac{\Lambda^2}{m_t^2}. \tag{12}$$

[The same result is, of course, obtained using the original form (4) for O_{11}, where the effect of this operator appears as a modification of the longitudinal W propagator. However, one needs to consider a larger number of Feynman diagrams in this case.] A shift in the $Zb\bar{b}$ couplings gives a shift in R_b given by

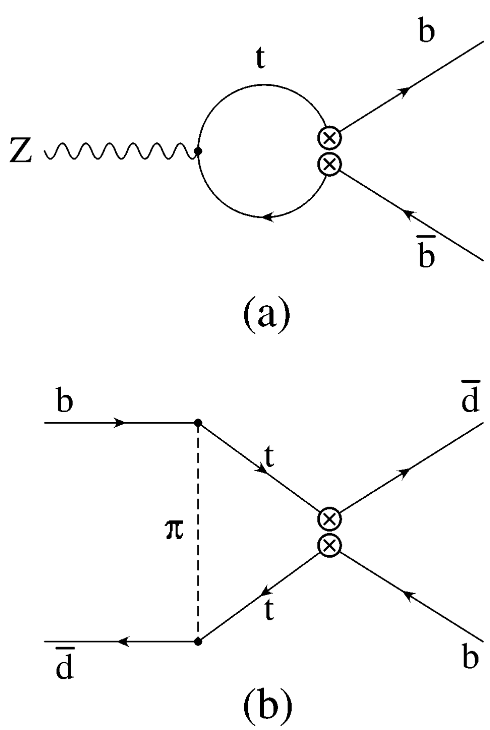

FIG. 1. (a) Contribution of the effective operator O_{11} to $Z \to b\bar{b}$. (b) Contribution of the effective operator O_{11} to B-$\bar{B}$ mixing

$$R_b = R_b^{\mathrm{SM}} \frac{1 + \delta_{bV}^{\mathrm{NP}}}{1 + R_b^{\mathrm{SM}} \delta_{bV}^{\mathrm{NP}}}, \tag{13}$$

with

$$\delta_{bV}^{\mathrm{NP}} = \frac{\delta \Gamma_b}{\Gamma_b^{\mathrm{SM}}} \approx 2 \frac{g_L^b}{(g_L^b)^2 + (g_R^b)^2} \delta g_L^b = -4.58\, \delta g_L^b. \tag{14}$$

The ALEPH collaboration has presented a new analysis of R_b data which leads to results which are compatible with the standard model predictions at the one-sigma level [8]. In fact, the new world average is [9] $R_b = 0.2178 \pm 0.0011$ to be compared with the SM expectation for $m_t = 175$ GeV, $R_b^{\mathrm{SM}} = 0.2157 \pm 0.0002$. Clearly, the new value of R_b is within two standard deviations of the standard model predictions [10].

Using these data on R_b, we get

$$\delta_{bV}^{\mathrm{NP}} = 0.012 \pm 0.007. \tag{15}$$

K-$\bar{K}$ and B-$\bar{B}$ mixing.—In the SM, the mixing between the B^0 meson and its antiparticle is completely dominated by the top contribution. The explicit m_t dependence of the corresponding box diagram is given by the loop function [11]

$$S(x_t)_{\mathrm{SM}} = \frac{x_t}{4}\left[1 + \frac{9}{1 - x_t} - \frac{6}{(1 - x_t)^2} - \frac{6x_t^2 \ln x_t}{(1 - x_t)^3}\right],$$
$$x_t \equiv \frac{\bar{m}_t^2}{M_W^2}, \tag{16}$$

which contains the hard m_t^2 term, $S(x_t) \sim x_t/4$, induced by the longitudinal W exchanges. The same function regulates the top-quark contribution to the K-$\bar{K}$ mixing parameter ε_K. The measured top-mass, $m_t = 175 \pm 6$ GeV [$\overline{m}_t \equiv \overline{m}_t(m_t) = 167 \pm 6$ GeV], implies $S(x_t)_{\mathrm{SM}} = 2.40 \pm 0.13$.

The correction induced by the new operator, O_{11}, can be parametrized as a shift on the function $S(x_t)$. The calculation of the diagrams in Fig. 1(b) leads to the following result:

$$S(x_t) = S(x_t)_{\mathrm{SM}} + \delta S(x_t),$$
$$\delta S(x_t) = -a_{11} \frac{g^2 m_t^4}{2M_W^4} \ln \frac{\Lambda^2}{m_t^2}. \tag{17}$$

Thus, the hard $m_t^4 \ln m_t^2$ contributions to $\delta_{bV}^{\mathrm{NP}}$ and $\delta S(x_t)$ are correlated:

$$\delta S(x_t) = \frac{32\pi^2}{|V_{tb}|^2 g^2} \delta g_L^b = -163 \delta_{bV}^{\mathrm{NP}}. \tag{18}$$

We can use the measured B_d^0-$\bar{B}_d^0$ mixing [12], $\Delta M_{B_d^0} = (0.464 \pm 0.018) \times 10^{12}\ \mathrm{s}^{-1}$, to infer the experimental value of $S(x_t)$ and, therefore, to set a limit on the δg_L^b contribution. The explicit dependence on the quark-mixing parameters can be resolved by putting together the constraints from $\Delta M_{B_d^0}$, ε_K, and $\Gamma(b \to u)/\Gamma(b \to c)$.

Using the Wolfenstein parametrization [13] of the quark-mixing matrix, one has

$$\left|\frac{V_{td}}{\lambda V_{cb}}\right| = \sqrt{(1-\rho)^2+\eta^2} = \frac{(1.21 \pm 0.09)}{\sqrt{S(x_t)}} \frac{185\text{ MeV}}{\sqrt{\eta_B}(\sqrt{2}f_B\sqrt{B_B})} = \frac{(1.21^{+0.50}_{-0.30})}{\sqrt{S(x_t)}}, \tag{19}$$

$$\eta[(1-\rho)A^2\eta_2 S(x_t) + P_0]A^2 B_K = 0.226, \tag{20}$$

$$\left|\frac{V_{ub}}{\lambda V_{cb}}\right| = \sqrt{\rho^2+\eta^2} = 0.36 \pm 0.09. \tag{21}$$

We have taken $\lambda \equiv |V_{us}| = 0.2205 \pm 0.0018$, $|V_{cb}| \equiv A\lambda^2 = 0.040 \pm 0.003$, and $|V_{ub}|/|V_{cb}| = 0.08 \pm 0.02$. The numerical factor on the right-hand side of Eq. (19) should be understood as an allowed range, because the error is dominated by the large theoretical uncertainties in the hadronic matrix element of the $\Delta B = 2$ operator; it corresponds to [14,15] $\sqrt{\eta_B}(\sqrt{2}f_B\sqrt{B_B}) = (185 \pm 45)$ MeV. In Eq. (20), $\eta_2 = 0.57 \pm 0.01$ is the short-distance QCD correction [16], while $P_0 = 0.31 \pm 0.02$ takes into account the charm contributions [14]. For the $\Delta S = 2$ hadronic matrix element we have chosen the range [15] $B_K = 0.6 \pm 0.2$.

Both the circle (19) and the hyperbola (20) depend on the value of $S(x_t)$. The intersection of the two circles (19) and (21) restricts $S(x_t)$ to be in the range $0.39 < |S(x_t)| < 9.7$. The request of simultaneous intersection with the hyperbola ϵ_K imposes a further constraint. Since a positive value of B_K is obtained by all present calculations and $S(x_t)_{\mathrm{SM}} > 0$, the SM implies a positive value for η. In our case, the constraint that the total $S(x_t) = S(x_t)_{\mathrm{SM}} + \delta S(x_t)$ is positive does not exist and this opens the possibility of solutions also with $\eta < 0$; however, this would imply a huge correction $\delta S(x_t)$. Taking $\eta > 0$, the three curves (bands) intersect if $S(x_t) > S(x_t)_{\min} = 1.0$.

The minimum value of $S(x_t)$ is reached for $V_{cd}^{\max}$, $B_K^{\max}$, and $|V_{ub}/V_{cb}|^{\max}$. Taking a more conservative ± 0.14 error in Eq. (21) (corresponding to $|V_{ub}/V_{cb}| = 0.08 \pm 0.03$) would result in $S(x_t)_{\min} = 0.8$.

The shift in g_L^b required by R_b [Eq. (15)] and relation (18) imply

$$\delta S = -2.0 \pm 1.1, \tag{22}$$

i.e., $-0.7 < S(x_t) < 1.5$. Thus, the present experimental measurements of R_b and the low-energy constraints from the usual *unitarity triangle fits* are compatible with the introduction of the operator O_{11}.

From Eqs. (18) and (15) and the constraint $S \geq S_{\min} = 1$, we can see that the maximum (positive) value of $\delta_{bV}^{\mathrm{NP}}$ allowed by low-energy physics is

$$\delta_{bV}^{\mathrm{NP}} < 0.01,$$

which is even stronger than the values obtained by the present direct measurements of R_b [Eq. (15)]. For $\Lambda \sim 1$ TeV, this translates into an O (10%) upper bound on a_{11}; this is comparable to the present limits for those couplings which contribute to the oblique corrections at the one-loop level.

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