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Determination of M_b and α_s from the Upsilon System

M. Jamin^a and A. Pich^b

^a Institut für Theoretische Physik, Universität Heidelberg,
Philosophenweg 16, D-69120 Heidelberg, Germany

^b Departament de Física Teòrica, IFIC, CSIC — Universitat de València,
Dr. Moliner 50, E-46100 Burjassot, València, Spain*

The mass of the bottom quark (both the pole mass M_b and the \overline{MS} mass m_b) and the strong coupling constant α_s have been determined [1] from QCD moment sum rules for the Υ system. In the pole-mass scheme large perturbative corrections resulting from coulombic contributions have been resummed. The results of this analysis are: $M_b = 4.60 \pm 0.02$ GeV, $m_b(m_b) = 4.13 \pm 0.06$ GeV and $\alpha_s(M_Z) = 0.119 \pm 0.008$.

1. INTRODUCTION

A short-distance description in terms of quarks and gluons is well suited for inclusive quantities, where no reference to a particular hadronic state is needed. The vacuum polarization

$$(q_\mu q_\nu - g_{\mu\nu} q^2) \Pi(q^2) = i \int dx e^{iqx} \langle T \{ j_\mu(x) j_\nu(0) \} \rangle$$

induced by the vector current $j_\mu \equiv \bar{b} \gamma_\mu b$ can be calculated theoretically within the Operator Product Expansion (OPE), whereas its imaginary part can be experimentally determined from the $e^+ e^- \rightarrow b\bar{b}$ cross-section:

$$R(s) \equiv \frac{1}{Q_b^2} \frac{\sigma(e^+ e^- \rightarrow b\bar{b})}{\sigma(e^+ e^- \rightarrow \mu^+ \mu^-)} = 12\pi \text{Im} \Pi(s+i\epsilon).$$

Using a dispersion relation the n th derivative of $\Pi(s)$ at $s = 0$ can be expressed in terms of the n th integral moment of $R(s)$:

$$\begin{aligned} \mathcal{M}_n &\equiv \frac{12\pi^2}{n!} \left(4M_b^2 \frac{d}{ds} \right)^n \Pi(s) \Big|_{s=0} \\ &= (4M_b^2)^n \int_0^\infty ds \frac{R(s)}{s^{n+1}} \\ &= 2 \int_0^1 dv v (1-v^2)^{n-1} R(v), \end{aligned} \quad (1)$$

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where $v \equiv \sqrt{1 - 4M_b^2/s}$. M_b corresponds to the pole of the perturbatively renormalized propagator, whereas the running quark mass in the \overline{MS} scheme renormalized at a scale μ will be denoted by $m_b(\mu)$.

Under the assumption of quark-hadron duality, the moments \mathcal{M}_n can either be calculated theoretically in renormalization group improved perturbation theory, including non-perturbative condensate contributions, or can be obtained from experiment. In this way, hadronic quantities like masses and decay widths get related to the QCD parameters α_s , m_b and condensates.

For large values of n , the moments become dominated by the threshold region. Therefore, they are very sensitive to the heavy quark mass. Owing to the large size of the $(\alpha_s \sqrt{n})^k$ Coulombic corrections, the large- n moments can also be used to get a determination of α_s from the existing data on Υ resonances [2].

2. EXPERIMENTAL INPUT

The first six Upsilon resonances have been observed. Their masses are known rather accurately, and their electronic widths have been measured with an accuracy which ranges from 3% for the $\Upsilon(1S)$ to 23% for the $\Upsilon(6S)$. For our purposes, the narrow-width approximation provides a very good description of these states, because the full widths of the first three Υ resonances

are roughly a factor 10^{-5} smaller than the corresponding masses, and the higher-resonance contributions to the moments are suppressed:

$$\frac{\mathcal{M}_n}{(4M_b^2)^n} = \frac{9\pi}{\bar{\alpha}^2 Q_b^2} \sum_{k=1}^6 \frac{\Gamma_{kS}}{M_{kS}^{2n+1}} + \int_{s_0}^{\infty} ds \frac{R(s)}{s^{n+1}}, \quad (2)$$

where $\Gamma_{kS} \equiv \Gamma[\Upsilon(kS) \rightarrow e^+e^-]$ and [3] $\bar{\alpha}^2 = 1.07 \alpha^2$.

The $e^+e^- \rightarrow b\bar{b}$ cross-section above threshold is unfortunately very badly measured [4]. The second term in Eq. (2) accounts for the contributions to R_b above the sixth resonance and is approximated by the perturbative QCD continuum. The continuum threshold $\sqrt{s_0}$ should lie around the mass of the next resonance, which has been estimated in potential models. We have used $\sqrt{s_0} = 11.2 \pm 0.2 \text{ GeV}$; the lower value includes the mass of the sixth resonance and should be a conservative estimate. The contribution from open B production above the $B\bar{B}$ threshold and below s_0 is very small [4] and has been included in the variation of s_0 .

The numerical weight of the heavier resonances in (2) decreases strongly for increasing values of n . The contribution of the $\Upsilon(5S)$ [$\Upsilon(6S)$] is 9.5% [4%] at $n = 0$; 1% [0.3%] at $n = 10$; and a tiny 0.08% [0.02%] at $n = 20$. Therefore, taking $n \gtrsim 10$, the uncertainties associated with the contributions of higher-mass states are very small.

3. THEORETICAL CALCULATION

3.1. Perturbation Theory

The vacuum polarization $\Pi(s)$ can be expanded in powers of the strong coupling constant:

$$\Pi(s) = \Pi^{(0)}(s) + a \Pi^{(1)}(s) + a^2 \Pi^{(2)}(s) + \dots,$$

with $a \equiv \alpha_s/\pi$. Analogous expansions for $R(v)$ and \mathcal{M}_n can be written. For the first two terms, analytic expressions are available [5,6].

$\Pi^{(2)}(s)$ is still not fully known analytically. However, the method of Padé approximants has been recently exploited to calculate $\Pi^{(2)}$ numerically, using available results at high energies, analytical results for the first seven moments $\mathcal{M}_i^{(2)}$ ($i = 1, \dots, 7$) and the known threshold behaviour

of $R^{(2)}(v)$ [7,8]. This information is good enough to obtain an accurate numerical evaluation of $\mathcal{M}_n^{(2)}$ for values of n not too large.

The contributions from diagrams with internal quark loops to the spectral function are fully known. This allows to check the accuracy of the Padé approximation, which for $n = 20$ is found to be better than 10^{-6} .

The numerical stability of the results can be analyzed, either using different Padé approximants with the full set of information, or constructing Padé approximants with one order less by removing one datum. For $n \leq 20$, the resulting numerical uncertainty is below 0.02%, being completely negligible for our application.

The reliability of the Padé approximation has been further corroborated in Ref. [9], where the first seven terms of the expansion of $\Pi^{(2)}(s)$ in powers of M_b^2/s have been computed.

3.2. Coulomb Resummation

For large n the higher-order perturbative corrections to the moments grow with respect to the leading order. At $n = 8$ ($n = 20$) the first order correction is roughly 120% (200%) of the leading term whereas the second order contribution is 140% (340%). This behaviour of the perturbation series originates from the fact that the relevant parameter in the Coulomb system is α_s/v which leads to a $\alpha_s\sqrt{n}$ dependence of the moments [10,11]. Thus, for higher n the perturbative corrections become increasingly more important and have to be summed up explicitly.

The resummed spectral function resulting from the imaginary part of the Green function for the static Coulomb potential is well known [5,6]. Subtracting the zero-order contribution, already included in $R^{(0)} = \frac{3}{2}v(3-v^2)$, it has the form:

$$R_C = \frac{9}{2} \left[\frac{x_V}{(1 - e^{-x_V/v})} - v \right], \quad (3)$$

where $x_V \equiv \pi^2 C_F a_V$ and a_V is the effective coupling in the QCD potential; a_V is independent of the renormalization scale μ_a , but it does depend on the three-momentum transfer between the heavy quark and antiquark, $\vec{q}^2 = 4v^2 M_b^2/(1-v^2)$.

The expansion of a_V in terms of the \overline{MS} coupling,

$$a_V(\bar{q}^2) = a(\mu_a) \left[1 + a(\mu_a) r_V^{(1)}(\bar{q}^2/\mu_a^2) + \dots \right],$$

is known to $\mathcal{O}(a^3)$ [12–14].

R_C resums the leading $(a/v)^n$ and some of the sub-leading corrections [15]. The corresponding terms have to be subtracted from $R^{(1)}$ and $R^{(2)}$. Thus, we can rewrite the perturbative expansion of the spectral function in the form

$$R(v) = S(a) \left\{ R^{(0)} + R_C + \tilde{R}^{(1)}a + \tilde{R}^{(2)}a^2 \right\}, \quad (4)$$

with

$$\begin{aligned} S(a) &= \left(1 - 4C_F a + 16C_F^2 a^2 \right), \\ \tilde{R}^{(1)} &= R^{(1)} + 4C_F R^{(0)} - \frac{9}{4} \pi^2 C_F, \\ \tilde{R}^{(2)} &= R^{(2)} + 4C_F R^{(1)} - \frac{3\pi^4 C_F^2}{8v} - \frac{9}{4} \pi^2 C_F r_V^{(1)}. \end{aligned} \quad (5)$$

We have factored out in $S(a)$ the correction to the vector current “ $-4C_F a$ ”, which originates from transversal, hard gluons; we have added a term $16C_F^2 a^2$ in order not to generate additional corrections of $\mathcal{O}(a^2)$ proportional to $R^{(0)}$.

After performing the Coulomb resummation, the large-moment behaviour of the remaining terms is much weaker. For instance, although $\mathcal{M}_n^{(1)}/\mathcal{M}_n^{(0)}$ increases as \sqrt{n} , now $\tilde{\mathcal{M}}_n^{(1)}/\mathcal{M}_n^{(0)} \sim 1/\sqrt{n}$. One can easily check that, if $a_V(\bar{q}^2)$ is evaluated with $n_l = 4$ light quark flavours, all contributions to $R^{(2)}$ of $\mathcal{O}(1/v)$, $\mathcal{O}(\ln v^2)$ and $\mathcal{O}(1)$ are canceled in (5), such that $\tilde{R}^{(2)}$ vanishes in the limit $v \rightarrow 0$. Thus, to $\mathcal{O}(a^2)$, all threshold singularities are properly taken into account through the Coulomb factor R_C .

The contributions of $\mathcal{O}(v)$ which determine the constant terms in $\tilde{\mathcal{M}}_n^{(2)}/\mathcal{M}_n^{(0)}$ are not known analytically. Those terms correspond to the current correction from transversal, hard gluons. Thus, the splitting of the $\mathcal{O}(a^2 v)$ correction between $\tilde{R}^{(2)}$ and the global factor $S(a)$ is ambiguous.

3.3. Power Corrections

The leading non-perturbative correction is the gluon-condensate contribution to the massive vector correlator; it is known at the next-to-leading order [16].

The relative growth of $\mathcal{M}_{n,G^2}^{(0)}/\mathcal{M}_n^{(0)}$ is proportional to n^3 . Therefore, the non-perturbative contribution grows much faster than the perturbative moments. In addition, in the pole-mass scheme, the next-to-leading order correction to \mathcal{M}_{n,G^2} is of the same size or larger as the leading term. Because the perturbative expansion for the gluon condensate cannot be trusted, we shall restrict our analysis to the range $n \leq 20$ where its contribution to the $b\bar{b}$ moments is below 3%.

4. RESULTS

To suppress higher resonances as well as power corrections, we have restricted the analysis to the range $n = 8, \dots, 20$. Solving the moment sum rules for M_b , we can fit M_b to a constant by varying M_b and $\alpha_s(M_b)$. In Figure 1 the resulting values for M_b are displayed as a function of n . This illustrates that a constant M_b in the range $8 \leq n \leq 20$ really produces an excellent fit.

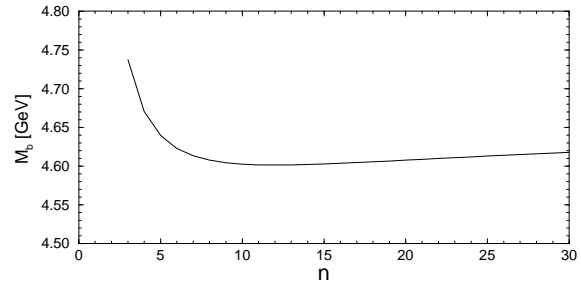


Figure 1. M_b as a function of n .

A compilation of all different contributions to the errors on M_b and $\alpha_s(M_b)$ is summarized in Table 1. The dominant uncertainty, due to the unknown higher-order perturbative corrections, has been estimated in three different ways: 1) the size of the known $\mathcal{O}(a^3)$ correction (through a_V [14]) to R_C —only the $\mathcal{O}(a^2)$ contribution has been included in the central values—; 2) the size of the $\mathcal{O}(a^2)$ term $\tilde{R}^{(2)}$; and 3) the variation under a change of the scale at which α_s is evaluated, in the range $M_b/2 \leq \mu_a \leq 2M_b$. Adding all

Table 1
Error estimates for M_b and $\alpha_s(M_b)$.

	ΔM_b [MeV]	$\Delta\alpha_s(M_b)$
statistical	± 9.1	± 0.0097
$\mathcal{O}(a^3)$ Coulomb	± 7.3	± 0.0222
$\mathcal{O}(a^2)$	± 0.9	± 0.0082
scale μ_a	$\pm_{1.3}^{2.1}$	$\pm_{0.0196}^{0.0287}$
continuum	$\pm_{2.0}^{2.6}$	$\pm_{0.0027}^{0.0037}$
$\langle aGG \rangle$	± 5.3	± 0.0036
Γ_{kS}	± 3.1	± 0.0067
total	± 13.5	± 0.0269

three would double count the error, because the uncertainty in an asymptotic series, such as the perturbative expansion, is bounded by the size of the last known term. For our final results, we have chosen to include the errors of varying the Coulomb and the $\mathcal{O}(a^2)$ terms.

The entry for the gluon condensate results from removing the non-perturbative contribution completely.

Adding all errors in quadrature, we finally get:

$$M_b = 4.604 \pm 0.014 \text{ GeV}, \quad (6)$$

$$\alpha_s(M_b) = 0.2197 \pm 0.0269, \quad (7)$$

which implies

$$\alpha_s(M_Z) = 0.1184 \pm_{0.0080}^{0.0070}. \quad (8)$$

We have also investigated the same sum rules using the \overline{MS} quark mass. The convergence of the perturbative series turns out to be better in the \overline{MS} scheme. However, we have refrained from performing a resummation of the Coulomb corrections because now the velocity v depends on the renormalization scale μ_m , used to define the running mass $m_b(\mu_m)$. To restrict the $\mathcal{O}(a^2)$ corrections to a reasonable size ($< 50\%$), μ_m should lie in the range $\mu_m = 3.2 \pm 0.5 \text{ GeV}$.

The separate contributions to the theoretical errors have been listed in Table 2. The uncertainty from higher-order corrections is now due to the variation of the scales $\mu_m = 3.2 \pm 0.5 \text{ GeV}$ and $2.6 \text{ GeV} \leq \mu_a \leq 2m_b$. The scale μ_a should

Table 2
Error estimates for $m_b(m_b)$ and $\alpha_s(m_b)$.

	Δm_b [MeV]	$\Delta\alpha_s(m_b)$
statistical	± 2	± 0.0044
scale μ_m	\pm_{36}^{33}	$\pm_{0.0058}^{0.0362}$
scale μ_a	\pm_{31}^{49}	$\pm_{0.0280}^{0.0251}$
continuum	± 1	$\pm_{0.0026}^{0.0035}$
$\langle aGG \rangle$	± 2	± 0.0023
Γ_{kS}	± 3	± 0.0060
total	\pm_{48}^{59}	$\pm_{0.0297}^{0.0449}$

not be taken lower than roughly 2.6 GeV because otherwise the $\mathcal{O}(a^2)$ correction $\overline{\mathcal{M}}_n^{(2)}$ becomes unacceptably large. Adding all errors in quadrature, we get:

$$m_b(m_b) = 4.13 \pm 0.06 \text{ GeV}, \quad (9)$$

$$\alpha_s(m_b) = 0.2325 \pm_{0.0297}^{0.0449}, \quad (10)$$

which implies

$$\alpha_s(M_Z) = 0.1196 \pm_{0.0080}^{0.0102}. \quad (11)$$

5. DISCUSSION

The resulting values for $\alpha_s(M_Z)$ from the pole and \overline{MS} mass schemes turn out to be in very good agreement. This is a further indication that the uncertainty from unknown higher-order corrections is under control. In addition, our results $m_b(m_b)$ and M_b for the b -quark mass satisfy the known perturbative relation between the pole and \overline{MS} masses [17,18], within the errors.

Combining both determinations of the strong coupling constant α_s , we find

$$\alpha_s(M_Z) = 0.119 \pm 0.008. \quad (12)$$

We have not averaged the errors of the two determinations because they are not independent. This result is surprisingly close to the current world average [19], $\alpha_s(M_Z) = 0.1186 \pm 0.0036$, although the error is certainly larger.

Our results differ from the ones originally quoted in Ref. [2], showing that the errors were

grossly underestimated. The main differences stem from our inclusion of $\mathcal{O}(a^2)$ corrections, which are large, and from our more complete numerical analysis. (In Ref. [2], the analysis was performed expanding in powers of $1/n$ and keeping only the leading term; this is a quite bad numerical approximation; moreover, the actual expansion parameter turns out to be $1/\sqrt{n}$).

The bottom quark mass values obtained by us are in good agreement to previous determinations from QCD sum rules [20–23] and a very recent calculation from lattice QCD [24]. Evolving $m_b(m_b)$ to the Z peak, it also agrees with the DELPHI $m_b(M_Z)$ determination [25]. Owing to the big sensitivity of the moment sum rules for the Υ system to the quark mass, and the good control over higher-order α_s corrections, our result is more precise. This result is lower than the one obtained in potential models [26].

The main uncertainty of our results originates in the perturbative series. We have estimated the effect of unknown higher-order corrections in a quite conservative way. However, it is known that the pole mass suffers from a renormalon ambiguity which has been argued to be of $\mathcal{O}(100\text{MeV})$ [27–29]. Throughout our analysis, the pole mass has been defined as the pole of the perturbatively renormalized quark propagator. Our determination (6) might therefore be subject to additional uncertainties which go beyond perturbation theory but which we cannot assess in a precise way.

It has been pointed out recently [30] that additional Coulombic singularities, not included in R_C , could show up at $\mathcal{O}(a^3)$. If true, that could generate sizeable corrections to the threshold region ($v \rightarrow 0$) and, therefore, to the very large- n moments. However, our results have been obtained at relatively low values of n , and they are very stable in the whole range $8 \leq n \leq 20$. Thus, this kind of higher-order corrections seems to be safely included in our quoted uncertainties.

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