arXiv:hep-ph/9810523v1 29 Oct 1998

Short– and long–distance contributions to the rare decay $K_L \to \mu^+\mu^-$

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The interplay between short– and long–distance contributions to the $K_L \to \mu^+\mu^-$ decay amplitude is analyzed. The long–distance piece is estimated using chiral perturbation theory techniques and large–N_C considerations, leading to a consistent description of the $\pi^0 \to e^+e^-$, $\eta \to \mu^+\mu^-$ and $K_L \to \mu^+\mu^-$ branching ratios [1].

1. INTRODUCTION

The rare decay $K_L \to \mu^+\mu^-$ has deserved a significant theoretical interest during the last three decades. It represents a potentially important channel to study the weak interaction within the Standard Model (SM), as well as possible effects of new physics, mainly in connection with flavour–changing neutral currents and CP violation.

This decay proceeds through two distinct mechanisms: a long–distance contribution from the 2γ intermediate state and a short–distance part, which in the SM arises from one–loop diagrams (*W boxes, Z penguins*) involving the weak gauge bosons. Since the short–distance amplitude is sensitive to the presence of a virtual top quark, it could be used to improve our present knowledge on the quark–mixing factor V_{td}; moreover, it offers a window into new–physics phenomena.

The short–distance SM amplitude is well–known [2]. Including QCD corrections at the next-to-leading logarithm order, it implies [3]:

$$
\begin{aligned}
\mathrm{Br}\,(K_L \to \mu^+\mu^-)_{\mathrm{SD}} &= 0.9 \times 10^{-9}\,(\rho_0 - \bar\rho)^2 \\
&\times \left(\frac{\overline{m}_t(m_t)}{170\ \mathrm{GeV}}\right)^{3.1} \left(\frac{|V_{cb}|}{0.040}\right)^4, \quad (1)
\end{aligned}
$$

where $\rho_0 \approx 1.2$ and $\bar\rho \equiv \rho\,(1 - \lambda^2/2)$, with ρ and λ the usual quark–mixing parameters, in the Wolfenstein parametrization. The deviation of ρ_0 from 1 is due to the charm contri-

bution. Using the presently allowed ranges for m_t and the quark–mixing factors, one gets [3] $\mathrm{Br}\,(K_L \to \mu^+\mu^-)_{\mathrm{SD}} = (1.2 \pm 0.6) \times 10^{-9}$. If this number is compared with the measured rate [4]

$$
\mathrm{Br}\,(K_L \to \mu^+\mu^-) = (7.2 \pm 0.5) \times 10^{-9}, \quad (2)
$$

it is seen that the decay process is strongly dominated by the long–distance amplitude.

Clearly, in order to extract useful information about the short–distance dynamics it is first necessary to have an accurate (and reliable) determination of the $K_L \to \gamma^*\gamma^* \to \mu^+\mu^-$ contribution.

Let us consider the normalized ratios

$$
\begin{aligned}
R(P \to l^+l^-) &\equiv \frac{\mathrm{Br}\,(P \to l^+l^-)}{\mathrm{Br}\,(P \to \gamma\gamma)} \\
&= 2\beta \left(\frac{\alpha}{\pi}\frac{m_l}{M_P}\right)^2 \left|F(P \to l^+l^-)\right|^2, \quad (3)
\end{aligned}
$$

where $\beta \equiv \sqrt{1 - 4m_l^2/M_P^2}$. The on–shell 2γ intermediate state generates the absorptive contribution [5]

$$
\mathrm{Im}\,[F(P \to l^+l^-)] = \frac{\pi}{2\beta}\ln\left(\frac{1-\beta}{1+\beta}\right), \quad (4)
$$

which, taking into account the measured $K_L \to \gamma\gamma$ branching ratio, leads to the so-called *unitarity bound*:

$$
\begin{aligned}
\mathrm{Br}\,(K_L \to \mu^+\mu^-) &\geq \mathrm{Br}\,(K_L \to \mu^+\mu^-)_{\mathrm{Abs}} \\
&= (7.07 \pm 0.18) \times 10^{-9}. \quad (5)
\end{aligned}
$$

Comparing this result with the experimental value in Eq. (2), we see that $\mathrm{Br}\,(K_L \to \mu^+\mu^-)$ is almost saturated by this absorptive piece.

Then, one immediate question is whether the small room left for the dispersive contribution, $\mathrm{Br}\,(K_L \to \mu^+\mu^-)_{\mathrm{Dis}} = (0.1 \pm 0.5) \times 10^{-9}$, can be understood dynamically.

2. $K_L \to \gamma\gamma$

The obvious theoretical framework to perform a well-defined analysis of the long–distance $K_L \to \mu^+\mu^-$ amplitude is chiral perturbation theory (ChPT). Unfortunately, the chiral symmetry constraints are not powerful enough to make an accurate determination of the dispersive part [6–8].

The problem can be easily understood by looking at the $K_L \to \gamma\gamma$ amplitude,

$$A(K_L \to \gamma\gamma) = c(q_1^2, q_2^2)\, \varepsilon^{\mu\nu\rho\sigma}\, \epsilon_{1\mu}\epsilon_{2\nu}q_{1\rho}q_{2\sigma}\,, \quad (6)$$

which, at lowest order in momenta, proceeds through the chain $K_L \to \pi^0, \eta, \eta' \to 2\gamma$. The lowest–order —$O(p^4)$— chiral prediction can only generate a constant form factor $c(q_1^2, q_2^2)$, corresponding to the decay into on-shell photons [9]:

$$c(0,0) = \frac{2G_8 \alpha f_\pi}{\pi}\, (c_\pi + c_\eta + c_{\eta'})\,, \quad (7)$$

where $c_{\pi,\eta,\eta'}$ stand for the π, η and η' pole contributions respectively, and the global parameter $G_8 \equiv 2^{-1/2} G_F V_{ud} V_{us}^* g_8$ characterizes [10] the strength of the weak $\Delta S = 1$ transition.

In the standard $SU(3)_L \otimes SU(3)_R$ ChPT, the η' contribution is absent, and $(c_\pi + c_\eta) \propto (3M_\eta^2 + M_\pi^2 - 4M_K^2)$, which vanishes owing to the Gell-Mann–Okubo mass relation. The physical $K_L \to \gamma\gamma$ amplitude is then a higher–order —$O(p^6)$— effect in the chiral counting, which makes difficult to perform a reliable calculation.

The situation is very different if one considers the large–N_C limit, in which the symmetry of the effective theory is enlarged to $U(3)_L \otimes U(3)_R$ [11], and the singlet η_1 field is included. The large mass of the η' originates in the $U(1)_A$ anomaly which, although formally of $O(1/N_C)$, is numerically important. Thus, it makes sense to perform a combined chiral expansion [12] in powers of momenta and $1/N_C$, around the nonet–symmetry limit, but keeping the anomaly contribution (i.e. the η' mass) together with the lowest–order term. In fact, the usual successful description of the $\eta/\eta' \to 2\gamma$ decays corresponds to the lowest–order contribution within this framework, plus some amount of symmetry breaking through $f_\eta \neq f_{\eta'} \neq f_\pi$. The mixing between the η_8 and η_1 states provides a large enhancement of the $\eta \to 2\gamma$ amplitude, which is clearly needed to understand the data.

Although the resulting numerical prediction for $c(0,0)$ contains several theoretical uncertainties (values of f_η and $f_{\eta'}$, deviations from the nonet symmetry limit, accuracy in θ_P and $|G_8|$), it is seen that the actual value of the $K_L \to \gamma\gamma$ rate can be easily fitted within the $U(3)_L \otimes U(3)_R$ framework for a reasonable choice of the parameters. The amplitude is found to be dominated by the pion pole, owing to a destructive interference between the c_η and $c_{\eta'}$ contributions.

3. DISPERSIVE $K_L \to l^+l^-$ AMPLITUDE

The description of the $K_L \to \gamma\gamma$ transition with off-shell photons is a priori more complicated because the $q_{1,2}^2$ dependence of the form factor originates from higher–order terms in the chiral lagrangian. This is the reason why only model–dependent estimates of the dispersive $K_L \to l^+l^-$ transition amplitude have been obtained so far. At lowest–order in momenta, $c(q_1^2, q_2^2) = c(0,0)$; thus, the (divergent) photon loop can be explicitly calculated up to a global normalization, which is determined by the known absorptive piece (i.e. by the experimental value of $c(0,0)$). The model–dependence appears in the local contributions from direct $K_L l^+ l^-$ terms in the chiral lagrangian [6,8] (allowed by symmetry considerations), which reabsorb the loop divergence.

It would be useful to have a reliable determination in some symmetry limit. The large–N_C description of $K_L \to \gamma^*\gamma^*$ provides such a possibility [1]. At leading order, this process occurs through the π^0, η, η' poles, as represented in Fig. 1. Therefore, the problematic electromagnetic loop in Fig. 1(a) is actually the same governing the decays $\pi^0 \to e^+e^-$ and $\eta \to \mu^+\mu^-$, and the unknown local contribution (Fig. 1(b)) can be fixed from the measured rates for these transitions [13,14]. It can be seen [13] that the same combination of local chiral couplings shows up in both decays, leading to a relation that is

well satisfied by the data. Moreover, this combination is also the relevant one for the $\eta' \to l^+l^-$ transition in the large N_C limit [1].
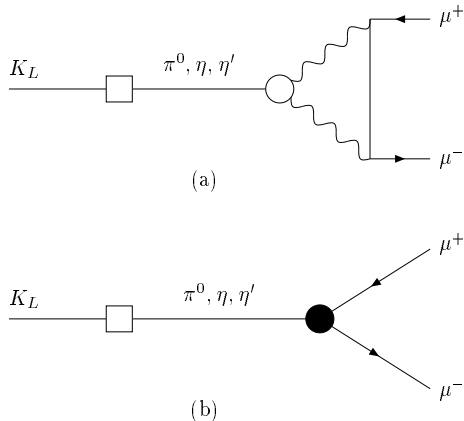

(a)

(b)

Figure 1. (a) Photon loop and (b) associated counterterm contributions to the $K_L \to \mu^+\mu^-$ process.

Nonet symmetry should provide a good estimate of the ratio $R(K_L \to l^+l^-)$. Since $K_L \to \gamma\gamma$ is dominated by the pion pole, we can expect that symmetry–breaking corrections would play a rather small role. In this limit, the dispersive amplitude for all $R(P \to l^+l^-)$ is given by

$$\mathrm{Re}\,[F(P \to l^+l^-)] = \frac{1}{4\beta}\ln^2\left(\frac{1-\beta}{1+\beta}\right)$$

$$+\frac{1}{\beta}Li_2\left(\frac{\beta-1}{\beta+1}\right) + \frac{\pi^2}{12\beta} + 3\ln\left(\frac{m_l}{\mu}\right) + \chi(\mu), \quad (8)$$

where $\chi(\mu)$ is the relevant local contribution, renormalized in the $\overline{\mathrm{MS}}$ scheme. The μ dependence of the $\chi(\mu)$ and $\ln(m_l/\mu)$ terms compensate each other, so that the total amplitude is μ–independent.

4. RESULTS

Table 1 shows the fitted values of $\chi(M_\rho)$ from the three measured ratios $R(\pi^0 \to e^+e^-)$, $R(\eta \to \mu^+\mu^-)$ and $R(K_L \to \mu^+\mu^-)$. Subtracting the known absorptive contribution, the experimental data provide two possible solutions

Table 1
Fitted values of $\chi(M_\rho)$ from different $R(P \to l^+l^-)$ ratios. The numbers quoted for $K_L \to \mu^+\mu^-$ refer to the difference $\chi(M_\rho) - \delta\chi_{\mathrm{SD}}$.

	[Solution 1]	[Solution 2]
$\pi^0 \to e^+e^-$	$4\,^{+4}_{-6}$	-24 ± 5
$\eta \to \mu^+\mu^-$	$5.5\,^{+0.8}_{-1.0}$	$-0.8\,^{+1.0}_{-0.8}$
$K_L \to \mu^+\mu^-$	$3.3\,^{+0.9}_{-0.7}$	$1.9\,^{+0.7}_{-0.9}$

for each ratio; they correspond to a total positive (solution 1) or negative (solution 2) dispersive amplitude. We see from the Table that the second solution from the decay $\pi^0 \to e^+e^-$ is clearly ruled out; owing to the smallness of the electron mass, the logarithmic loop contribution dominates the dispersive amplitude, which has then a definite positive sign (an unnaturally large and negative value of $\chi(M_\rho)$ is needed to make it negative). The large experimental errors do not allow to discard at this point any of the other solutions: the remaining value from $\pi^0 \to e^+e^-$ is consistent with the results from the $\eta \to \mu^+\mu^-$ and $K_L \to \mu^+\mu^-$ decays, and these are also in agreement with each other if the same solution (either the first or the second) is taken for both. We see that, in any case, the three experimental ratios are well described by a common value of $\chi(M_\rho)$. In this way, the experimentally observed small dispersive contribution to the $K_L \to \mu^+\mu^-$ decay rate fits perfectly well within the large–N_C description of this process.

We have not considered up to now the short–distance contribution to the $K_L \to \mu^+\mu^-$ decay amplitude [3]. This can be done through a shift of the effective $\chi(M_\rho)$ value[2]:

$$\chi(M_\rho)_{\mathrm{eff}} = \chi(M_\rho) - \delta\chi_{\mathrm{SD}}, \quad (9)$$

$$\delta\chi_{\mathrm{SD}} \approx 1.7\,(\rho_0 - \bar\rho)\left(\frac{\overline{m}_t(m_t)}{170\,\mathrm{GeV}}\right)^{1.56}\left(\frac{|V_{cb}|}{0.040}\right)^2.$$

For the allowed range $|\bar\rho| \leq 0.3$, one has $\delta\chi_{\mathrm{SD}} \approx 1.8 \pm 0.6$, which allows to exclude the solution 2 for $\chi(M_\rho)$ obtained from $\eta \to \mu^+\mu^-$. The solution 1,

[2]The relative sign between the short– and long–distance dispersive amplitudes is fixed by the known positive sign of g_8 in the large–N_C limit [15].

4

on the contrary, is found to be compatible with the results from $K_L \to \mu^+\mu^-$, and can be used to get a constraint for $\delta\chi_{\rm SD}$. Indeed, taking as the best determination

$$\chi(M_\rho) = 5.5 \, ^{+0.8}_{-1.0} \, , \tag{10}$$

the first solution for $K_L \to \mu^+\mu^-$ leads to

$$\delta\chi_{\rm SD} = 2.2 \, ^{+1.1}_{-1.3} \, , \tag{11}$$

in agreement with the $\delta\chi_{\rm SD}$ value quoted above. The second solution for $K_L \to \mu^+\mu^-$ appears to be less favoured, yielding $\delta\chi_{\rm SD} = 3.6 \pm 1.2$; this shows a discrepancy of about $1.4\,\sigma$ with the short–distance estimate. Notice that the precision of the result in (11) is still relatively low. However, the errors could be reduced by improving the measurements of the $\eta \to \mu^+\mu^-$ and $K_L \to \mu^+\mu^-$ branching ratios.

Once the local contribution to the $P \to l^+l^-$ decay amplitude has been fixed, it is possible to obtain definite predictions for the decays into e^+e^- pairs:

$$\begin{aligned} {\rm Br}(\pi^0 \to e^+e^-) &= (8.3 \pm 0.4) \times 10^{-8} \, , \\ {\rm Br}(\eta \to e^+e^-) &= (5.8 \pm 0.2) \times 10^{-9} \, , \\ {\rm Br}(K_L \to e^+e^-) &= (9.0 \pm 0.4) \times 10^{-12} \, . \end{aligned} \tag{12}$$

The predicted $K_L \to e^+e^-$ decay rate has been confirmed by the recent BNL-E871 measurement [16], ${\rm Br}(K_L \to e^+e^-) = (8.7^{+5.7}_{-4.1}) \times 10^{-12}$.

In the same way, the amplitudes corresponding to the η' decays are found to be ${\rm Br}(\eta' \to e^+e^-) = (1.5 \pm 0.1) \times 10^{-10}$ and ${\rm Br}(\eta' \to \mu^+\mu^-) = (2.1 \pm 0.3) \times 10^{-7}$. However, in view of the large mass of the η', these predictions could receive important corrections from higher–order terms in the chiral lagrangian.

To summarize, we have shown that in the nonet symmetry limit it is possible to make a reliable determination of the ratios $R(P \to l^+l^-)$, at lowest non-trivial order in the chiral expansion. A consistent picture of all measured $P \to l^+l^-$ modes is obtained within the SM. In the case of the $K_L \to \mu^+\mu^-$ decay, the present data allow to get a constraint for the short–distance amplitude, which could be improved by more precise measurements of the $\eta \to \mu^+\mu^-$ and $K_L \to \mu^+\mu^-$

branching ratios. Although a more detailed investigation of the underlying theoretical uncertainties is still required, this analysis offers a new possibility for testing the flavour-mixing structure of the Standard Model.

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