IFIC/03-43, FTUV/03-0930

Phenomenology of the ⟨VVP⟩ Green's function within the Resonance Chiral Theory *

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We analyse the odd-intrinsic-parity effective Lagrangian of QCD valid for processes involving one pseudoscalar with two vector mesons described in terms of antisymmetric tensor fields. Substantial information on the odd-intrinsic-parity couplings is obtained by constructing the vector-vector-pseudoscalar Green's three-point function, at leading order in $1/N_C$, and demanding that its short-distance behaviour matches the corresponding OPE result. The QCD constraints thus enforced allow us to predict the decay amplitude $\omega \to \pi\gamma$, the $\mathcal{O}(p^6)$ corrections to $\pi \to \gamma\gamma$ and the slope parameter in $\pi \to \gamma\gamma^*$.

1. Introduction

Effective field theories of QCD have provided efficient ways to explore hadron dynamics in those regimes where we are not able to solve the full theory. In the very low-energy domain, chiral perturbation theory (χPT) [1,2] has achieved a remarkable success in describing the strong interactions among pseudoscalar mesons. Moving up to the 1 GeV region the effects of vector resonances become dominant and must be accommodated in the theory. Several works [3,4] have provided a sound procedure to include resonance states within the chiral framework, namely the Resonance Chiral Theory (RχT). As the couplings entering the effective Lagrangian are not fixed by the symmetry alone, one should rely on the phenomenology or, alternatively, construct theoretical tools that could provide a meaningful way to compare the results of the effective theory with those of QCD. The pioneering work of Ref. [4] indicated that the analysis of Green's functions and form factors of QCD currents yields valuable information on the resonance sector.

Recently, several authors have pushed forward this direction, either by using a Lagrangian with explicit resonance degrees of freedom or within the framework of the lowest meson dominance (LMD) approximation to the large number of colours (N_C) limit of QCD [5–9]. In particular, the authors of Ref. [5] undertook a systematic study of several QCD three-point functions which are free of perturbative contributions from QCD at short distances. Therefore, their OPE expansion should be more reliable when descending to energies close to the resonance region. Under this hypothesis, it was shown [5] that while the ansatz derived from the LMD approach automatically incorporates the right short-distance behaviour of QCD by construction, the same Green's functions as calculated with a resonance Lagrangian, in the vector-field representation, are incompatible with the OPE outcome. Moreover the authors put forward that these discrepancies cannot be repaired just by introducing local counterterms from the chiral Lagrangian $\mathcal{L}_\chi^{(6)}$, as it was done at $\mathcal{O}(p^4)$ [4]. This result severely questions the usefulness of the resonance effective theory beyond the initial work of Ref. [4], and deserves further investigation.

With this aim, we have reanalysed the vector-vector-pseudoscalar three-point function, this time with the vector mesons described in terms of antisymmetric tensor fields. This requires the introduction of an odd-intrinsic-parity effective Lagrangian in the formulation of Ref. [3] containing all allowed interactions between two vector objects (currents or resonances) and one pseu-

*Talk given by P. D. Ruiz-Femenía at the High-Energy Physics International Conference in Quantum Chromodynamics (QCD 03), Montpellier, France, 2–8 July 2003.

doscalar meson. The details of the calculation can be found in Ref. [10].

2. RχT and the odd-intrinsic-parity sector

The low-energy behaviour of QCD for the light quark sector is ruled by the spontaneous breaking of chiral symmetry. The corresponding effective realization of QCD describing the interaction between the Goldstone fields is χPT given, at $\mathcal{O}(p^2)$, by

$$\mathcal{L}_\chi^{(2)} = (F^2/4)\,\langle u_\mu u^\mu + \chi^+\rangle\,. \qquad (1)$$

The inclusion of resonances as explicit degrees of freedom in the chiral framework was carried out in Ref. [3] for the even–intrinsic–parity sector ($\mathcal{L}_\mathrm{V}$). For the odd–intrinsic–parity sector, three different sources might contribute to the $\langle$VVP$\rangle$ Green's function :

(i) the Wess-Zumino action $Z_\mathrm{WZ}[v,a]$ [11], which is of $\mathcal{O}(p^4)$ and fulfills the chiral anomaly,

(ii) chiral invariant $\epsilon_{\mu\nu\rho\sigma}$ terms involving vector mesons. Within the antisymmetric formalism, the basis of odd-intrinsic-parity operators which comprise all possible vertices involving two vector resonances and one pseudoscalar (VVP), and vertices with one vector resonance and one external vector source plus one pseudoscalar (VJP) reads:

$$\begin{aligned}
\mathcal{O}^1_\mathrm{VJP} &= \epsilon_{\mu\nu\rho\sigma}\,\langle\,\{V^{\mu\nu}, f_+^{\rho\alpha}\}\nabla_\alpha u^\sigma\,\rangle\ ,\\
\mathcal{O}^2_\mathrm{VJP} &= \epsilon_{\mu\nu\rho\sigma}\,\langle\,\{V^{\mu\alpha}, f_+^{\rho\sigma}\}\nabla_\alpha u^\nu\,\rangle\ ,\\
\mathcal{O}^3_\mathrm{VJP} &= i\,\epsilon_{\mu\nu\rho\sigma}\,\langle\,\{V^{\mu\nu}, f_+^{\rho\sigma}\}\,\chi_-\,\rangle\ ,\\
\mathcal{O}^4_\mathrm{VJP} &= i\,\epsilon_{\mu\nu\rho\sigma}\,\langle\,V^{\mu\nu}\,[\,f_-^{\rho\sigma}, \chi_+]\,\rangle\ ,\\
\mathcal{O}^5_\mathrm{VJP} &= \epsilon_{\mu\nu\rho\sigma}\,\langle\,\{\nabla_\alpha V^{\mu\nu}, f_+^{\rho\alpha}\}u^\sigma\,\rangle\ ,\\
\mathcal{O}^6_\mathrm{VJP} &= \epsilon_{\mu\nu\rho\sigma}\,\langle\,\{\nabla_\alpha V^{\mu\alpha}, f_+^{\rho\sigma}\}u^\nu\,\rangle\ ,\\
\mathcal{O}^7_\mathrm{VJP} &= \epsilon_{\mu\nu\rho\sigma}\,\langle\,\{\nabla^\sigma V^{\mu\nu}, f_+^{\rho\alpha}\}u_\alpha\,\rangle\ , \qquad (2)
\end{aligned}$$

$$\begin{aligned}
\mathcal{O}^1_\mathrm{VVP} &= \epsilon_{\mu\nu\rho\sigma}\,\langle\,\{V^{\mu\nu}, V^{\rho\alpha}\}\nabla_\alpha u^\sigma\,\rangle\ ,\\
\mathcal{O}^2_\mathrm{VVP} &= i\,\epsilon_{\mu\nu\rho\sigma}\,\langle\,\{V^{\mu\nu}, V^{\rho\sigma}\}\,\chi_-\,\rangle\ ,\\
\mathcal{O}^3_\mathrm{VVP} &= \epsilon_{\mu\nu\rho\sigma}\,\langle\,\{\nabla_\alpha V^{\mu\nu}, V^{\rho\alpha}\}u^\sigma\,\rangle\ ,\\
\mathcal{O}^4_\mathrm{VVP} &= \epsilon_{\mu\nu\rho\sigma}\,\langle\,\{\nabla^\sigma V^{\mu\nu}, V^{\rho\alpha}\}u_\alpha\,\rangle\ . \qquad (3)
\end{aligned}$$

The corresponding resonance Lagrangian will thus be defined as

$$\mathcal{L}_V^\mathrm{odd} = \sum_{a=1}^{7}\frac{c_a}{M_V}\,\mathcal{O}^a_\mathrm{VJP} + \sum_{a=1}^{4} d_a\,\mathcal{O}^a_\mathrm{VVP}\ . \qquad (4)$$

(iii) the relevant operators in the $\mathcal{O}(p^6)$ Goldstone chiral Lagrangian [12]. The successful resonance saturation of the chiral Lagrangian couplings at $\mathcal{O}(p^4)$ [3] might translate naturally to $\mathcal{O}(p^6)$ couplings too, implying that they are generated completely through integration of vector resonances. Accordingly we do not include $\mathcal{L}^{(6)}_\mathrm{odd}$ in our evaluation.

In summary we will proceed in the following by considering the relevant effective resonance theory (ERT) given by :

$$Z_\mathrm{ERT}[v,a,s,p] = Z_\mathrm{WZ}[v,a] \,+\, Z^\mathrm{odd}_{\mathrm{V}\chi}[v,a,s,p]\,, \qquad (5)$$

where $Z^\mathrm{odd}_{\mathrm{V}\chi}$ is generated by $\mathcal{L}_\chi^{(2)}$, $\mathcal{L}_\mathrm{V}$ and $\mathcal{L}_V^\mathrm{odd}$.

3. Short-distance information on the odd-intrinsic-parity couplings

The vector-vector-pseudoscalar QCD three-point function $\langle$VVP$\rangle$ is built from the octet vector current and the octet pseudoscalar density,

$$\begin{aligned}
(\Pi_\mathrm{VVP})^{(abc)}_{\mu\nu}(p,q) &= \int d^4x \int d^4y\, e^{i(p\cdot x + q\cdot y)}\\
&\quad\times \langle 0|\,T\,[\,V^a_\mu(x)\,V^b_\nu(y)\,P^c(0)\,]\,|0\rangle\\
&= \epsilon_{\mu\nu\alpha\beta}\,p^\alpha q^\beta d^{abc}\,\Pi_\mathrm{VVP}(p^2,q^2,r^2)\,, \qquad (6)
\end{aligned}$$

with the four-vector $r = -(p+q)$.

When both momenta p, q in Π_VVP become simultaneously large, the QCD calculation within the OPE framework gives, in the chiral limit and up to corrections of $\mathcal{O}(\alpha_s)$, [7]:

$$\begin{aligned}
&\lim_{\lambda\to\infty}\Pi_\mathrm{VVP}\left((\lambda p)^2,(\lambda q)^2,(\lambda r)^2\right)\\
&\qquad = -\frac{\langle\bar\psi\psi\rangle_0}{2\lambda^4}\,\frac{p^2+q^2+r^2}{p^2q^2r^2} + \mathcal{O}\left(\frac{1}{\lambda^6}\right)\,, \qquad (7)
\end{aligned}$$

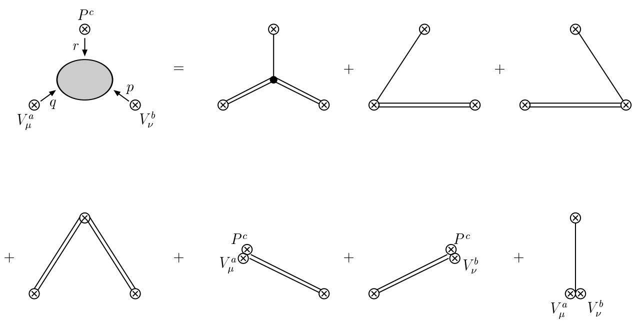

Figure 1. Diagrams entering the calculation of the VVP 3-point function with the ERT action. Double lines represent vector resonances, single lines are short for pseudoscalar mesons.

where $\langle\bar{\psi}\psi\rangle_0$ is the single flavour bilinear quark condensate. At leading order in the $1/N_C$ expansion of QCD, the three-point correlator in the effective resonance theory given by $Z_{\rm ERT}$ is evaluated from the tree-level diagrams shown in Fig. 1. The LMD approximation, which assumes that a single resonance in each channel saturates the requirements of QCD, is sufficient to satisfy the short–distance constraint (7) up to order $1/\lambda^4$, provided the following conditions among the $\mathcal{L}_V^{\rm odd}$ couplings hold:

$$\begin{aligned} 4\,c_3 + c_1 &= 0 \quad , \\ c_1 - c_2 + c_5 &= 0 \quad , \\ c_5 - c_6 &= \frac{N_C}{64\pi^2}\frac{M_V}{\sqrt{2}F_V} \quad , \\ d_1 + 8\,d_2 &= -\frac{N_C}{64\pi^2}\frac{M_V^2}{F_V^2} + \frac{F^2}{4F_V^2} \quad , \\ d_3 &= -\frac{N_C}{64\pi^2}\frac{M_V^2}{F_V^2} + \frac{F^2}{8F_V^2}\,. \end{aligned} \tag{8}$$

As the couplings of the Effective Lagrangian do not depend on the masses of the Goldstone fields the constraints above apply for non–zero pseudoscalar masses too.

Actually our $\langle$VVP$\rangle$ three-point function fully reproduces the LMD ansatz suggested in Ref. [7] :

$$\Pi_{\rm VVP}^{\rm res} = -\frac{\langle\bar{\psi}\psi\rangle_0}{2}\,\frac{(p^2+q^2+r^2) - \frac{N_C}{4\pi^2}\frac{M_V^4}{F^2}}{(p^2-M_V^2)(q^2-M_V^2)r^2}\,, \tag{9}$$

which has been successfully tested in previous works [7,9]. The authors of Ref. [5] found that the same agreement with the short-distance QCD behaviour could not be reached working with the resonance Lagrangian in the vector representation, not even at the expense of introducing local contributions from the $\mathcal{O}(p^6)$ chiral Lagrangian. They then suggested that the problem may be inherent to the effective Lagrangian approach and unlikely to be fixed just by using other representations for the resonance fields; our result, derived in the antisymmetric tensor-field formulation with an odd-intrinsic-parity sector, contradicts this assertion, at least in what concerns the $\langle$VVP$\rangle$ Green's function.

4. Phenomenology of intrinsic-parity violating processes

4.1. $\omega \to \pi\gamma$

At tree-level, the intrinsic-parity violating transition $\omega \to \pi\gamma$ receives contributions from both the VJP and VVP terms of $\mathcal{L}_V^{\rm odd}$ (direct and ρ-mediated diagrams respectively). If we plug in the QCD constraints, Eq. (8), we find a full prediction for this process :

$$\Gamma(\omega\to\pi\gamma) = \frac{\alpha}{192}\,M_\omega\left(1-\frac{m_\pi^2}{M_\omega^2}\right)^3 \times\left[\,\frac{N_C}{4\pi^2}\,\frac{M_\omega^2}{F^2} - \frac{M_\omega^2}{M_V^2}\left(1+\frac{m_\pi^2}{M_\omega^2}\right)\,\right]^2 . \tag{10}$$

The direct and the ρ exchange diagrams almost contribute to similar extent to this process. This means that contrary to what we would expect from VMD, the $\omega\rho\pi$ coupling does not saturate the decay $\omega\to\pi\gamma$. This has immediate consequences to other channels where VMD alone was thought to be the relevant mechanism of decay, as in $\omega\to\pi^+\pi^-\pi^0$, where the direct amplitude competes in size with the intermediate meson exchange term [10].

Varying the parameter F from the bare value $F_0 \simeq 87$ MeV to the dressed one (i.e. the pion decay constant), $F_\pi \simeq 92.4$ MeV [13], we get that $\Gamma(\omega\to\pi\gamma)$ ranges from 0.703 MeV to 0.524 MeV, with $M_V = M_\rho = 771.1$ MeV and $M_\omega = 782.6$ MeV [13]. This 5–30% deviation from the experimental value, $\Gamma(\omega \to \pi\gamma)|_{\rm exp} = (0.734 \pm 0.035)\,\text{MeV}$, is in accordance with the expected size of next-to-leading $1/N_C$ corrections. Also the $\rho \to \pi\gamma$ decay widths, related with $\omega \to \pi\gamma$ by a $\text{SU}(3)_\text{V}$-symmetry factor, are extracted from the analysis [10].

4.2. $\pi \to \gamma\gamma$

In the chiral limit, the amplitude for the $\pi \to \gamma\gamma$ process is non-vanishing and exactly predicted by the ABJ anomaly. The odd-intrinsic-parity interactions among vector resonances introduced in Section 2 generate $\mathcal{O}(p^6)$ chiral corrections to this process. Only the two-resonance driven diagram survives after the short-distance conditions are applied. The correction induced into the $\pi \to \gamma\gamma$ width gives :

$$\Gamma(\pi \to \gamma\gamma) = \frac{\alpha^2}{64\,\pi^3\,F^2} m_\pi^3 \left[\,1\,-\,\Delta\,\right]^2 \quad , \tag{11}$$

where

$$\Delta \;=\; \frac{4\pi^2}{3}\,\frac{F^2}{M_V^2}\,\frac{m_\pi^2}{M_V^2} \;\simeq\; 0.006 \quad . \tag{12}$$

This result provides a tiny 1% correction to the width, and it is perfectly compatible with the experimental uncertainty, $\Gamma(\pi \to \gamma\gamma)|_{\rm exp} = (7.7 \pm 0.6)$ eV.

4.3. $\pi \to \gamma\gamma^*$

The $\pi \to \gamma\gamma^*$ amplitude is usually written as a slope parameter α which modifies the on-shell behaviour:

$$\mathcal{M}_{\pi\to\gamma\gamma^*} \;=\; \mathcal{M}_{\pi\to\gamma\gamma}\left(1 + \alpha\, k^{*2}\right)\,, \tag{13}$$

where k^* is the off-shell photon momentum. The interactions contained in $\mathcal{L}_V^{\rm odd}$ yield a contribution to the parameter α that amounts

$$\alpha^{\rm odd} \;=\; \frac{1}{M_V^2}\left[1 - \frac{4\pi^2F^2}{N_C M_V^2}\right] \simeq 1.36\ \text{GeV}^{-2}\,,$$

which is smaller than the VMD estimate, $\alpha^{\rm VMD} = 1/M_V^2 \simeq 1.68\ \text{GeV}^{-2}$. The chiral loops contributions to this slope, $\alpha^\chi \simeq 0.26\ \text{GeV}^{-2}$ were calculated in Ref. [14]. We can add both contributions to get $m_\pi^2\alpha \simeq 0.029$, to be compared with the averaged value $m_\pi^2\alpha|_{\rm exp} = 0.032 \pm 0.004$ in the PDG [13]. $\alpha^{\rm odd}$ has been extended beyond the LMD approximation by the inclusion of a second vector resonance into the $\langle\text{VVP}\rangle$ ansatz, Eq. (9), in Ref. [5]. The latter is in fact needed to have the right $1/k^{*2}$ behaviour for large k^* [15,16] in the form factor $\mathcal{F}_{\pi\gamma\gamma^*}(k^*)$.

Acknowledgements

We wish to thank S. Narison and his team for the organization of the QCD 03 conference. This work has been supported in part by TMR EURIDICE, EC Contract No. HPRN-CT-2002-00311, by MCYT (Spain) under grant FPA2001-3031, and by ERDF funds from the EU.

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