Better understanding motion planning: A compared review of "Principles of Robot Motion: Theory, Algorithms, and Implementations", by H. Choset et al.

Pablo Jiménez

Institut de Robòtica i Informàtica Industrial (CSIC - UPC)
Llorens i Artigas 4-6, E-08028 Barcelona, Spain

Abstract

The textbook on Motion Planning "Principles of Robot Motion: Theory, Algorithms, and Implementations", by H. Choset et al., MIT Press, appeared on June 2005, is reviewed and compared to other two textbooks on the same subject, from 1991 and 2006 respectively. The ground-breaking developments over the last decade justify the necessity of the newer textbooks, that appear to be complementary, despite some overlap in the contents.

Key words: Robot Motion Planning
PACS: 45.40.Ln

1 Overview

1.1 Presenting the book

The book under review, *Principles of Robot Motion: Theory, Algorithms, and Implementations*, by H. Choset et al. [1] (from now on, we will refer to it as the *Principles*), appeared on June 2005. It is a textbook on **Robot Motion Planning**, thus covering not only the geometrical aspects of **Path Planning**, but also **Control** related issues. It begins with the very simple *bug algorithms*, as elemental but in many settings quite effective motion strategies, integrating these two main aspects of motion planning: geometry and sensor-based control.

Email address: pjimenez@iri.upc.edu (Pablo Jiménez).

Preprint submitted to Elsevier 17 May 2007

This approach inspires the structure of the book, with two clearly distinguishable parts. The first part is devoted to the geometry of path planning (a path is a linear geometric object connecting the initial with the final configuration of the robot). The basics of the *Configuration Space* are explained here, as well as the main families of planning algorithms (potential functions, complete roadmap methods, cell decompositions, and sampling-based methods). The second part of the book goes beyond geometry and covers all control topics related to robot motion. The subjects range from probabilistic analysis for localization and mapping to trajectory planning (in a trajectory, velocities along the path are considered), taking the robot's dynamics into account, as well as kinematic (in particular, nonholonomic) constraints for full Motion Planning.

The *Principles* is a textbook by virtue of its pedagogical values ("aimed at the advanced undergraduate or new graduate student interested in robot motion, as stated in the *Preface*): clearly structured, with a presentation that progresses from the basics to the elaborated. It is self-contained, and with proposed exercises at the end of each chapter (solutions are not provided in the text). The discourse proceeds smoothly, without cumbersome digressions: all the necessary mathematical background is provided in a number of Appendices at the end of the book. Special emphasis is put on an integrated view of the motion planning problem, not only by capturing the different involved disciplines -as mentioned above- but also by intertwining both theory and practice. Already the chapter on bug algorithms -the first after the Introduction- devotes some space to the implementation of these basic strategies, and the discussion about the problems encountered when setting the theoretical models into practice is quite enlightening.

The *Principles* has seven authors, which is justified by the broad scope covered by this book. This, together with the aforementioned pedagogical virtues, makes the reading of this book also very attractive for scholars and practitioners who want to get consistent notions about neighboring fields related to their expertise.

1.2 The historical precedent: Latombe's reference text

Robot Motion Planning is a relatively young field, whose bibliography consists mainly of journal papers and congress proceedings. Furthermore, as it covers many different issues, all the relevant information is quite spread out in diverse publications. Unifying or even compiling works are rare. However, there are some titles that are a must in Robot Motion Planning literature and they have to be considered here for situating the *Principles* in context.

Fifteen years ago, Jean-Claude Latombe published “Robot Motion Planning” [2], which is generally accepted to be the first textbook that appeared on the subject. Previous publications related to this problem were edited theses, like Canny’s Doctoral Dissertation “The Complexity of Robot Motion Planning” [3] or compilations like Hopcroft, Schwartz and Sharir’s “Planning, Geometry and Complexity of Robot Motion” [4]. Latombe’s book provided for the first time a common language, a notation that has become a standard, a founding ground for the formulation of the basic problem, and a characterization of the algorithms, classifying them into Roadmap Methods, Exact and Approximate Cell Decomposition, and Potential Field Methods. The book further contains chapters devoted to extensions of the basic problem (see Figure 1, chapters 8-11). The inclusion of proposed exercises at the end of every chapter underlines the pedagogical flavor of the book. This book has been the main reference over these past fifteen years, cited hundreds of times, and its illustrations appear repeatedly at presentation archives (powerpoint and pdf) and other didactic material on the web.

Since this fundamental work and until the appearance of the *Principles*, no attempts on a new textbook on this matter had been undertaken, despite the huge amount of original contributions that during the decade following Latombe’s book had populated the challenging fields of robot motion planning. In particular, sampling-based techniques were just newcomers when Latombe’s book was issued: randomized planning was treated there in the context of potential field methods only as a strategy for escaping local minima (the author cited his own work, later published in [5]). However, only a few years later the Probabilistic Roadmap Methods (described first in [6], in the following years numerous variants on this basic procedure appeared) had become hugely popular, and nowadays constitute the first option to tackle with high-dimensional complex problems like motion planning for deformable objects [7]. In the same stream of sampling-based strategies, Ariadne’s Clew Algorithm [8], Expansive Space Trees [9,10] and the Rapidly-exploring Random Trees [11,12] (the latter designed to cope with the broader category of kynodimanic planning problems), as well as combinations of these approaches like the sampling-based roadmap of trees [13], have also contributed to radically change, or, better said, widen the panorama of robot motion planning.

During this time some books appeared, but again they were compilations of different authors’ works, with a specific orientation in the field. Special mention deserve two books appeared in 1998. The first one is Laumond’s (Ed.) “Robot Motion Planning and Control” [14], which collects the work done in the context of the European Project PROMotion (Planning Robot Motion), mainly centered on nonholonomic motion planning (although other areas like probabilistic planners or collision detection are also covered). The second book appeared that year is Gupta and Del Pobil’s (Eds.) “Practical Motion Planning in Robotics” [15]. This book is fruit of a workshop held in the context of the 1996 IEEE International Conference on Robotics and Automation) which emphasizes -as hinted by the title- the practical aspects of motion planning. Thus, it not only collects the most efficient and easily implementable algorithms at that moment, but also a whole part of the book is devoted to the basic issue of collision detection, another one to industrial applications of planning algorithms, and -last but not least- another part to the more control-level related sensor-based approaches. The latter deserve again some space in the *Principles*, like the chapter on bug algorithms.

1.3 A third book in discussion: complementarity of LaValle's textbook

Very recently another textbook on planning has entered the arena, Steven LaValle's "Planning Algorithms" [16] (in fact, most parts of it were already available on the web as a work in progress at least one year ago). It is structured in three main parts: Motion Planning, Decision-Theoretic Planning, and Planning Under Differential Constraints. The scope of the first and third parts is coincident to some extent with the corresponding chapters of the *Principles*, whereas the second part is devoted to planning in information spaces, responding to the aim of giving a unified view on the topic of planning (ranging from the lowest to the highest abstraction level). Although planning is focused mainly on Robotics in this book, the described techniques are general enough to be applied to other contexts as well, as shown by multiple examples along the chapters. Like Latombe's book and the *Principles*, it is a pedagogically appropriate textbook, which means that it meets the aforementioned virtues, including the proposed exercises and implementations for the reader to rework the contents of each chapter.

1.4 Three complementary textbooks with common subjects

From the preceding brief survey, it becomes clear that a comparative study of the *Principles* with Latombe's and LaValle's books (from now on, Latombe and LaValle) may shed some light on the specific treatment of the subject in the reviewed book. However, it should be also kept in mind that this comparison is by no means a competition, as the different nature of the books -mainly of the two newer ones- precludes any rivalry. Latombe is the classical reference, the only textbook during a decade and a half. LaValle is the encyclopedic reference, where planning is treated in its most general, abstract and formal aspects (although many particular algorithms and examples are displayed along the book, mainly but not only in the context of Robotics). And the *Principles* is the textbook for roboticists interested in an integrated vision of the motion planning problem, from geometry to control, from theory to implementation, from theorems to algorithms and devices.

Having stressed the complementary nature of the books, we can proceed to examine the contents of the *Principles* considering also the treatment that on the common parts is given by the other two books. This analysis will be done in the main Section 2, according to the sequence of chapters (in boldface) of the *Principles* -as this is actually the book under review. After a short description of its contents, the relative covering of each particular issue by the other two books will be considered (some of the chapters may not be addressed by one or the other reference). An overview of the relationships between the three books is displayed in Figure 1.

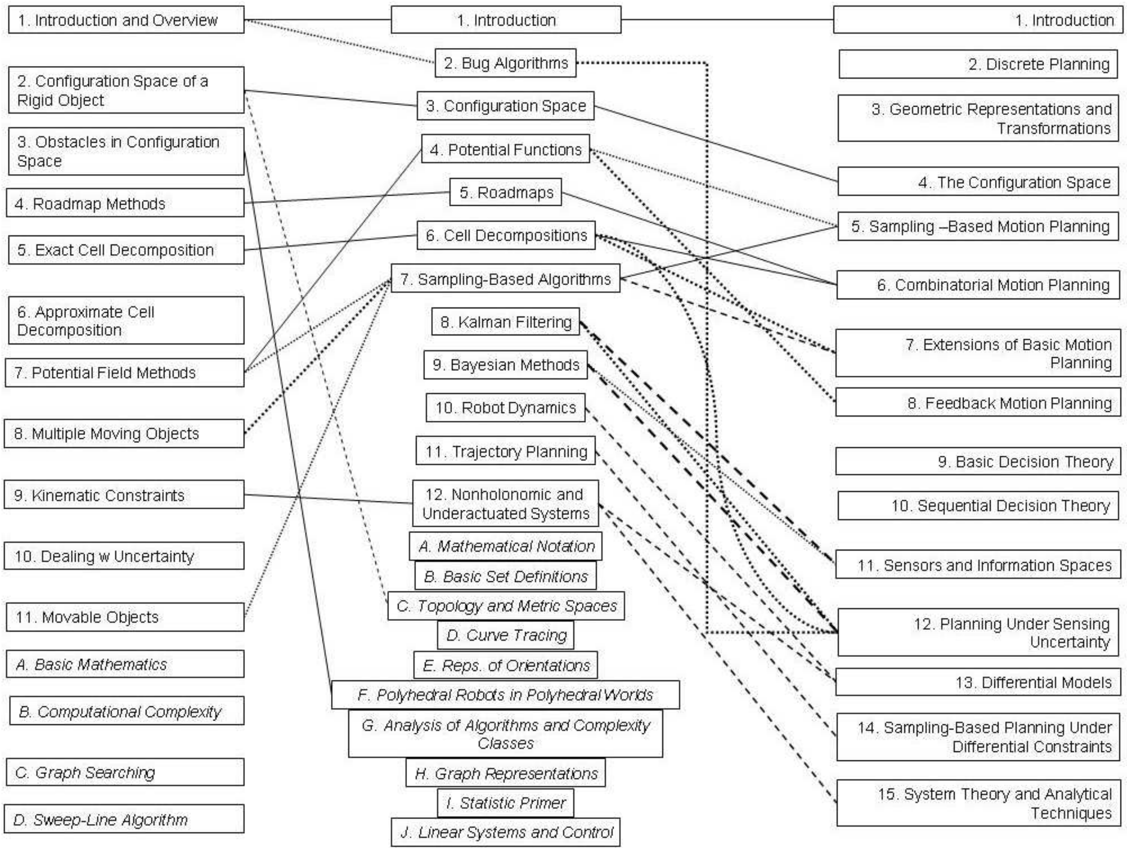

Fig. 1. Chapters and appendices (slanted) of the three mentioned books are shown together with their relationships, whether of strong coincidence in contents (solid lines), partial coincidence (dashed lines), or mention of specific methods (dotted lines). The *Principles* is in the middle, whereas Latombe and LaValle are on the left and right sides respectively. Some relations have not been depicted for the sake of clarity. These include a brief explanation of the methods of Latombe's Chapter 10 in LaValle's Chapter 12, as well as the following partial coincidences between appendices of the *Principles* and chapters of LaValle: B-4, C-4, E-3, E-4, F-4, G-6, H-9, H-12, I-9, and J-13.

After reviewing and comparing the contents of the books, Section 3 makes some considerations about formal aspects. Concluding remarks are drawn in

Section 4.

2 Chapter by chapter

The **Introduction** is devoted to the description of some motivating examples and to terminology issues (types of motion planning tasks, properties of the robots and of the algorithms). An overview of the book, a declaration of intentions about the use of mathematics, and the introduction of the concepts of workspace, configuration space, path and trajectory planning complete this chapter. This structure is quite similar to the *Introduction* of LaValle's book, whereas Latombe's *Introduction and Overview* is basically the latter, a presentation of the concepts and methods that are developed along the chapters (plus some considerations about complexity and the relationship of Motion Planning to the control and the task planning levels).

Variants on the simple sensor-based **Bug Algorithms** are presented in Chapter 2, providing intuitive descriptions of the *modus operandi* as well as pseudocode. They are smartly used to illustrate the concepts of optimal vs. greedy in the output, exhaustive vs. opportunistic in the search. The more sophisticated "Tangent Bug" serves as theoretical basis of the sensor-based motion planning strategy presented at the end of this chapter. Implementational issues are addressed at carefully, so as to provide the tools to carry out theory into practice, a distinctive feature of this textbook. This kind of algorithms is only briefly mentioned in the Introduction of Latombe, whereas LaValle devotes six pages to the same algorithms in Chapter 12 ("Planning Under Sensing Uncertainty").

The formal description of the **Configuration Space** (Chapter 3) and related items is a must in any Robot Motion Planning textbook. Although the concepts and their definitions are standard ones, the way they are presented to the reader differ noticeably in the three textbooks. Latombe's treatment of the subject is quite formal (e.g., some relevant facts are stated as "Propositions" grounded on previous evidence or mathematical proofs). A detailed view on all related aspects is provided, even about items that are not dealt with further in the book, so as to give the reader a complete solid theoretical basis. A whole Chapter is devoted to the construction of configuration space obstacles (this subject is treated in the *Principles* in Appendix F, see Figure 1). LaValle provides a strong topological basis before introducing the configuration space. C-obstacles and their construction are dealt with into the same chapter, and a whole section is devoted to analyzing the case of closed kinematic chains. The *Principles* bets on a more intuitive approach. Concepts are defined precisely but not extensively, examples are quickly brought into the scene to help understanding. The focus lies on the most relevant items that are strictly necessary for the comprehension of further chapters (some concepts are introduced later, as needed, like some topological concepts in Chapter 5 when explaining the Generalized Voronoi Diagrams).

In Chapter 4, motion planning algorithms based on **Potential Functions** are described: how to compute the repulsive potential due to the obstacles and the attractive potential of the goal and combine them in a gradient descent algorithm, how to compute the necessary distances to the obstacles from range sensors or on a grid-representation of free space by the Brushfire Algorithm, how to overcome the problem of the presence of local minima (Randomized Path Planner, Wave-Front Planner, Navigation Potential Functions). Also, potential functions for non-Euclidean spaces are computed in workspace on some control points of the robot, and the resulting forces are related to the forces in the configuration space through the Jacobian matrix (examples include a planar robot allowed to rotate and articulated robots). The treatment of navigation potential functions is more extensive than in Latombe's chapter on potential field methods (it also cites more recent references), while Latombe devotes more space and detail to the Randomized Path Planner, and explains also the elliptical potentials that are not mentioned in the *Principles*. LaValle splits the issue between a section on randomized potential fields in Chapter 5 (Sampling-Based Motion Planning) and navigation functions for discrete and continuous environments (Chapter 8, Feedback Motion Planning).

After some definitions on maps and related items, the chapter on **Roadmaps** describes the main families: Visibility Graphs, Generalized Voronoi Diagrams (both in the planar and 3D Euclidean spaces, as well as in the non-Euclidean space of a rod allowed to translate and rotate in the plane), and the Silhouette Methods. Definitions are given for all three families, and construction methods are described. Particular mention deserve the Hierarchical Generalized Voronoi Diagrams, and the Opportunistic Path Planner (which is a generalization of the original Silhouette Method) which do not appear in Latombe as they are posterior to its publication (whereas this book describes also the "Freeway Method" in great detail, which is not even mentioned in the *Principles*). LaValle includes these methods in the chapter "Combinatorial Motion Planning" together with the cell decomposition methods. This allows the author to expose Canny's silhouette algorithm in the same context of computational algebraic geometry based methods as the cylindrical algebraic decomposition.

Exact **Cell Decompositions** are reviewed in Chapter 6: trapezoidal decompositions of polygonal environments for path planning, Morse decompositions for coverage planning, and visibility-based decompositions for the pursuit/evasion problem. The latter two go beyond the basic motion planning formulation and are not treated in Latombe's textbook. On the other hand, Latombe describes an exact cell decomposition method for a translating and rotating rod in a plane, and includes also the cylindrical algebraic decomposition (with the Collins decomposition algorithm) of environments described as semi-algebraic sets. Furthermore, Latombe devotes a full chapter to approximate cell decomposition methods, that are not treated in the *Principles*. LaValle explains the same exact cell decomposition algorithms as in Latombe, but elaborates more on the vertical cell decomposition in 3D (only hinted at in Latombe). Coverage planning is a subject of Chapter 7 ("Extensions of Basic Motion Planning") and the visibility-based pursuit/evasion problem appears in Chapter 12 ("Planning Under Sensing Uncertainty).

Sampling-Based Algorithms include the basic formulation of Probabilistic Roadmaps (PRM, together with pseudo-code for the construction and query phases), a bunch of different sampling strategies, simple-query sampling-based planners (Expansive-Spaces Trees and Rapidly-exploring Random Trees) and their integration with PRMs. Further on, an analysis (on the probabilistic completeness and other issues) of basic PRMs is carried out. Finally some extensions and applications of sampling-based planners beyond basic path planning are described, including control-based planning, multiple robots, manipulation planning, assembly planning, flexible objects, and biological applications. Latombe explains only one sampling-based planner, the RPP mentioned above in the context of potential methods, as the textbook was written previously to the advent of PRMs and other sampling-based planners. LaValle devotes a whole chapter to these methods, where basically the same ones are reviewed, although the order of presentation is different (single-query models like RPPs, Ariadne's Clew algorithm or RRTs are presented before multiple-query models like PRMs) and basic issues like sampling theory or collision detection are treated more formally and extensively. As for extensions of the basic problem like multiple robots or manipulation planning, they are addressed in a specific chapter.

The next chapters of the *Principles* can be viewed as a second part of the book, where topics related to the inputs to the planning system (i.e., sensors that provide real noisy measurements) and to its output (the control module, that provides specific motion commands) are treated. The (purely geometric) *path planning* problem of the first part is transcended and becomes the complete *motion planning* problem. First, a whole chapter is devoted to **Kalman Filtering**, as a technique that can be used in the context of probabilistic position estimation. Linear Kalman Filtering and the Extended Kalman Filter are explained, as well as their use in Simultaneous Localization and Mapping (SLAM). Less restrictive assumptions than those required by Kalman filtering methods are allowed by the **Bayesian Methods** described in the next chapter: sensor and robot models can be nonlinear, and the distributions of the estimations can be non Gaussian. Discrete (grid-based) approximations and particle filters are discussed in the context of localization, and different sensor models are reviewed. Also the problems of mapping with known locations of the robot, as well as SLAM are treated within this general probabilistic formulation. In particular, the occupancy grid method and the Rao-Blackwellized particle filtering are explained in detail for these two problems respectively. A huge specific literature exists for probabilistic estimation, be it in general [17] or addressed to robot localization and mapping [18], where many other methods are described. As for LaValle, these topics are discussed first at a higher abstraction level, and afterwards dealt with in a more general context involving uncertainty and how to cope with: Kalman filtering and the sampling-based approaches have a short section ("Computing Probabilistic Information States") in Chapter 11, whereas the localization and mapping problems are dealt with, together with many others, in Chapter 12.

Control issues begin by describing **Robot Dynamics**, expressed through the Lagrangian formulation for mechanical systems. The standard equations are rewritten in different ways and its components analyzed in detail, considering also the inclusion of linear constraints on the velocities. Several examples illustrate the use and the meaning of the formulas. These models allow to perform **Trajectory Planning**, i.e., to determine the forces or control inputs that are needed for an adequate timing of a collision-free path (path + time = trajectory) while optimizing some objective function like the total time needed or the consumed energy. The first (decoupled) approach consists in finding previously a collision-free path in the configuration space and then determining a time-optimal scaling of this path subject to actuator limits. The second approach performs a direct trajectory planning in the state space of the robot. Nonlinear optimization is used to approximately solve the problem (ensuring that the constraints are satisfied at a fixed number of points along the time interval, and using a finite-parameter representation of the state and the control histories). Another method for numerically solving the optimization problem, grid-based search, is also described in detail. The final chapter is devoted to **Nonholonomic and Underactuated Systems**, i.e., systems with non-integrable constraints on the velocities and systems with fewer controls than degrees of freedom. The necessary background on Lie algebra and notions on controllability is provided in order to determine whether given configurations are actually reachable. Different motion planning approaches including control-theoretic methods for particular structures in absence of obstacles and search-based algorithms for car-like (possibly with trailers) vehicles are given. This kind of systems and methods are treated in Chapter 9 of Latombe, obviously without reflecting the wealth of new algorithms developed during the last decade. As for LaValle, the whole third part (three chapters) of the book concerns "planning under differential constraints": Chapter 13 explains the different models for systems with kinematic constraints, Chapter 14 revisits sampling-based approaches now running in phase spaces, and Chapter 15 reviews system theory and analytical techniques for planning without obstacles (many of them to be used as local planners for the strategies explained in the previous chapter).

3 Formal aspects

From a strictly formal point of view, all three books accomplish only too well the standards on what should be expected from a good and useful textbook. Focusing on the *Principles*, despite being coauthored by seven researchers from different fields of expertise, the book exhibits a coherent and integrated discourse. The language is precise and the organization is clear, allowing the reader to be perfectly situated at every moment. Each chapter begins with a sort of introduction that states the motivation of its contents, its relationships to other parts of the book, and a brief overview (in some chapters more explicitly described than in others). Mathematical notation is used when necessary, without overwhelming the reader. Quite useful and distinctive is also the frequent inclusion of pseudo-code for the different algorithms described along the book. Figures have a strong presence along the book (the ratio between the number of figures and the number of pages is quite good) and comprise schematic representations to illustrate concepts as well as graphical output of implemented algorithms.

The Table shown below aims at illustrating the relative dimensions of the three textbooks. However, care should be taken before extracting conclusions: for example, the index terms of the *Principles* include only common words (with the exception of concepts or methods known by their discoverers), whereas the other two books include some names of researchers as well.

Concept	Latombe	Principles	LaValle
Year of Publication	1991	2005	2006
N of Pages	651	603	826
N of References	302	433	1005
N of Index Terms	716	469	1715
N of Figures	221	313	416

Some words should be said about the differences in the structure of the presentation in the three books: Latombe begins with theoretical foundations of Configuration Space and its description, goes through different categories of models to solve the basic problem, where potential functions are the last examined approach, and finally reviews extensions. The *Principles*, on the contrary, goes from the simplest sensory driven bug algorithms to model-based ones, traversing the necessary (but simplified) definition of configuration space and the potential functions formulation as an intermediate step due to the possibility of straightforward implementation on a robot with range-sensors. Then, it goes further through the genuine model-based approaches like roadmaps and cell decompositions, and it ends up with the wide category of sampling-based algorithms, which constitute the main -and necessary- extension with respect to Latombe. Issues related to the uncertainty of sensor measurements and to the control of robots deserve several chapters, thus providing an overall vision on the motion planning problem. Finally, LaValle's discourse has a similar structure, although conditioned by the broader scope of the book. For example, the subject on uncertainty related to sensorial feedback and its use in localization and mapping is included in Part II, "Decision-Theoretic Planning", which spans four chapters and covers issues ranging from game theory to manipulation planning with sensing uncertainty.

4 Conclusions

The textbook "Principles of Robot Motion" (2005) by H. Choset et al. has been reviewed, revising both its contents and presentation. It has been compared with two other textbooks on the subject, Latombe's "Robot Motion Planning" (1991) and LaValle's "Planning Algorithms" (2006). The structure of the *Principles* has guided the exposition of the contents, which has allowed to evaluate the relative covering of the different issues. The two newer books clearly reflect the evolution of the field in the last years, mainly by the relevance gained by sampling-based algorithms and their applications beyond the basic formulation of the motion planning problem.

The three books should be owned by everyone interested in the field of robot motion planning. Latombe is the classical reference but this is not the only reason for having this book in the library: the chapter of approximate cell decomposition methods describes a family of algorithms that are practically uncovered in the other two books, although their relevance may have diminished after the emergence of probabilistic sampling-based methods. LaValle is a huge work with an ambitious scope, which does not prevent it from being a quite enjoyable reading. Due to the great variety of treated issues and the different fields of contextualization, it is a very useful reference book. As for the *Principles*, the discourse is more focused on robot motion planning along the whole book, which, together with its pedagogical virtues, converts it in a good guide for getting introduced in the field, as textbook in a course on motion planning, or for providing the expert in one of the subjects a comprehensive view on the other related fields. In sum, as for the newest two references, despite the circumstance of their almost simultaneous appearance after more than a decade of absence of textbooks on motion planning, far from being in competition they are rather complementary and both set the standard of quality on a very high level.

5 Acknowledgement

The author would like to thank the editors and the anonymous reviewers for their insightful comments.

References

[1] H. Choset, K. M. Lynch, S. Hutchinson, G. A. Kantor, W. Burgard, L. E. Kavraki, S. Thrun, Principles of Robot Motion: Theory, Algorithms, and Implementations, MIT Press, 2005.

[2] J.-C. Latombe, Robot Motion Planning, Vol. SECS 0124, Kluwer, Dordrecht, The Netherlands, 1991.

[3] J. Canny, The Complexity of Robot Motion Planning, MIT Press, Cambridge, MA, 1987.

[4] J. T. Schwartz, M. Sharir, J. Hopcroft, Planning, Geometry, and Complexity of Robot Motion, Ablex, Norwood, NJ, 1987.

[5] J. Barraquand, J.-C. Latombe, Robot motion planning: A distributed representation approach, International Journal of Robotics Research 10 (6) (1991) 628–649.

[6] L. E. Kavraki, P. Svestka, J.-C. Latombe, M. H. Overmars, Probabilistic roadmaps for path planning in high-dimensional configuration spaces, IEEE Transactions on Robotics & Automation 12 (4) (1996) 566–580.

[7] F. Lamiraux, L. Kavraki, Planning paths for elastic objects under manipulation constraints, International Journal of Robotics Research 20 (2001) 188–208.

[8] E. Mazer, J. M. Ahuactzin, P. Bessière, The Ariadne's clew algorithm, Journal of Artificial Intelligence Research 9 (1998) 295–316.

[9] D. Hsu, J.-C. Latombe, R. Motwani, Path planning in expansive configuration spaces, International Journal Computational Geometry & Applications 4 (1999) 495512.

[10] D. Hsu, R. Kindel, J.-C. Latombe, S. Rock, Randomized kynodinamic motion planning with moving obstacles, International Journal of Robotics Research 21 (3) (2002) 233–255.

[11] S. M. LaValle, Rapidly-exploring random trees: A new tool for path planning, Tech. Rep. 98-11, Computer Science Dept., Iowa State University (Oct. 1998).

[12] S. M. LaValle, J. J. Kuffner, Rapidly-exploring random trees: Progress and prospects, in: B. R. Donald, K. M. Lynch, D. Rus (Eds.), Algorithmic and Computational Robotics: New Directions, A K Peters, Wellesley, MA, 2001, pp. 293–308.

[13] E. Plaku, K. E. Bekris, B. Y. Chen, A. M. Ladd, L. E. Kavraki, Sampling-based roadmap of trees for parallel motion planning, IEEE Transactions on Robotics 21 (4) (2005) 597–608.

[14] J.-P. Laumond, Robot Motion Planning and Control, Springer-Verlag, Berlin, 1998, available online at http://www.laas.fr/∼jpl/book.html.

[15] K. Gupta, A. del Pobil (Eds.), Practical Motion Planning in Robotics: Current Approaches and Future Directions, John Wiley, 1998.

[16] S. M. LaValle, Planning Algorithms, Cambridge University Press, Cambridge, U.K., 2006, available at http://planning.cs.uiuc.edu/.

[17] Y. Bar-Shalom, X. R. Li, T. Kirubarajan, Estimation with Applications to Tracking and Navigation, John Wiley & Sons, New York, 2001.

[18] S. Thrun, W. Burgard, D. Fox, Probabilistic Robotics, MIT Press, Cambridge, 2005.