Multirobot C-SLAM: Simultaneous Localization, Control and Mapping

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Abstract

This paper is about closing the low level control loop during Multirobot Simultaneous Localization and Map Building from an estimation-control theoretic viewpoint. We present a multi-vehicle control strategy that uses the state estimates generated from the SLAM algorithm as input to a multi-vehicle controller. Given the separability between optimal state estimation and regulation, we show that the tracking error does not influence the estimation performance of a fully observable EKF based multirobot SLAM implementation, and viceversa, that estimation errors do not undermine controller performance. Furthermore, both the controller and estimator are shown to be asymptotically stable. The feasibility of using this technique to close the perception-action loop during multirobot SLAM is validated with simulation results.

Key words: Multirobot SLAM, EKF, Feedback Linearization

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1 Introduction

This paper is about closing the control loop during Multirobot Simultaneous Localization and Map Building from an estimation-control theoretic viewpoint. Linear estimation theory has extensively being used for solving the SLAM problem [13,17], with the widely accepted use of the Extended Kalman Filter as the workhorse [8], as well as extensions to deal with nonlinearities, such as the Unscented Kalman Filter [3,10], or with the aid of non-deterministic particle filters [14]. Furthermore, the case of cooperative mapping and localization within the same estimation paradigm has also received significant attention recently [9,15,18,21].

The issue of combining control and estimation together during SLAM has in general been addressed with the idea of online trajectory generation. For example, by studying which vehicle maneuvers would most effectively reduce localization uncertainty [7,16], or what maneuvers would provide the greatest reward in terms of exploration gain [8]; by incorporating visual servoing techniques [6], or by implementing a PD controller over an A* searched trajectory [19].

We are not aware however, of any substantial contribution that guarantees that such planned trajectories will be followed accurately, in a systems theoretical sense, spite the duality between regulation and linear estimation. That is precisely the focus of this article: a unified approach to Multirobot Simultaneous Localization, Control and Mapping, from an estimation-control theoretic perspective, that would generate the necessary control commands to accurately follow a higher level planned trajectory, and that would guarantee that both the controller and the estimator are asymptotically stable. Given that observability is a requisite for stable SLAM [1,2,11], it is of uttermost importance to guarantee stability of the closed loop system as well. That is, not only during estimation, but also during vehicle control. The acronym C-SLAM has a twofold meaning: 'C' for closing the control loop, and 'See' for stressing that full observability is a requisite to stability of both state estimation and control as well.

More specifically, by using a nonlinear control technique called Feedback Linearization over the EKF state estimates, we are capable of accurately following any multirobot trajectory parameterized in time, while at the same time building an optimally estimated map. Such trajectory could be generated on line, for example, to reduce estimation error, or to maximize exploration gain. Furthermore, extending the Separability principle for the LQG regulator and the Kalman estimator to the feedback linearization scheme, we are able to decouple control error from estimation error, thus guaranteeing stability both for the controller as well as the estimator.

The paper is divided as follows. In Section 2 we briefly review the multirobot Gaussian SLAM case, and extend the notation in Section 3 to present a time-parameterized multirobot trajectory following scheme using a feedback linearization law for the control of vehicle states. To show the feasibility of the approach Section 4 presents simulations with a pair of nonlinear vehicles over a realistic scenario. Conclusions are presented in Section 5.

2 The EKF for Multirobot SLAM

The motion of the robots and the map measurements are governed by the discrete time stochastic state transition model

$$\mathbf{x}(k+1) = \mathbf{f}\left(\mathbf{x}(k), \mathbf{u}(k), \mathbf{v}(k)\right) \tag{1}$$

$$\mathbf{z}(k+1) = \mathbf{h}\left(\mathbf{x}(k+1)\right) + \mathbf{w}(k) \tag{2}$$

The state $\mathbf{x}(k) = [\mathbf{r}_1(k)^\top, \ldots, \mathbf{r}_m(k)^\top, \mathbf{m}_1^\top, \ldots, \mathbf{m}_n^\top]^\top$ contains the pose of the robots $\mathbf{r}_1, \ldots, \mathbf{r}_m$ at time step k, and a vector of stationary map features $\mathbf{m}_1, \ldots, \mathbf{m}_n$. The input vector $\mathbf{u}(k) = [\mathbf{u}_1(k)^\top, \ldots, \mathbf{u}_m(k)^\top]^\top$ is a multi-vehicle control command, $\mathbf{v}(k) = [\mathbf{v}_1(k)^\top, \ldots, \mathbf{v}_m(k)^\top]^\top$ is the plant noise, $\mathbf{w}(k) = [\mathbf{w}_1(k)^\top, \ldots, \mathbf{w}_n(k)^\top]^\top$ is the sensor noise, and both are Gaussian random vectors with zero mean and block diagonal covariance matrices $\mathbf{Q}$ and $\mathbf{R}$, respectively. An optimal estimate of (1), in a least squares sense, is given by the expression

$$\hat{\mathbf{x}}(k+1) = \mathbf{f}(\hat{\mathbf{x}}(k), \mathbf{u}(k), \mathbf{0}) + \mathbf{K}(\mathbf{z}(k+1) - \mathbf{h}(\mathbf{f}(\hat{\mathbf{x}}(k), \mathbf{u}(k), \mathbf{0}))) \tag{3}$$

with covariance

$$\mathbf{P}(k+1) = \mathbf{F}\mathbf{P}(k)\mathbf{F}^\top + \mathbf{G}\mathbf{Q}\mathbf{G}^\top - \mathbf{K}[\mathbf{H}[\mathbf{F}\mathbf{P}(k)\mathbf{F}^\top + \mathbf{G}\mathbf{Q}\mathbf{G}^\top]\mathbf{H}^\top + \mathbf{R}]\mathbf{K}^\top \tag{4}$$

The Jacobians $\mathbf{F}$ and $\mathbf{G}$ represent first order linearizations of the multi-vehicle model with respect to the state and the plant noise. Similarly, the Jacobian $\mathbf{H}$ contains first order linearizations of the measurement model with respect to the entire state. The details on how to compute the Kalman gain $\mathbf{K}$ can be found, for example, in [4]. The use of the Extended Kalman Filter for solving SLAM has a long history within the robotics community. Unfortunately, the strong assumption of unimodal Gaussian noise produces an estimation that ends up accumulating consistent errors and the effects of nonlinearities; preventing the approach from being able to map very large areas without the need for additional heuristics, such as nonlinear state estimation, or by merging multiple local submaps. The technique nevertheless is still widely accepted, given its simplicity, and the ability to prove stability of the filter [20], and convergence of the state covariance in the Riccati equation (4) [8,12].

3 Feedback Linearization

In this Section, we design a controller using feedback linearization for multi-robot trajectory tracking. The feedback linearization approach is commonly used to control nonlinear systems by algebraically transforming the system dynamics into a linear one, so that linear control techniques can be applied. It differs from conventional (Jacobian) linearization in that linearization is achieved by exact state transformations and feedback, rather than by linear approximations of the dynamics.

To apply feedback linearization to the multirobot position part of the state, the system dynamics (1) must be described in controllability canonical form. That is, linear with respect to the input $\mathbf{u}(k)$.

$$\mathbf{y}(k+1) = \mathbf{y}(k) + \mathbf{B}(\mathbf{u}(k) + \mathbf{v}(k)) \tag{5}$$

with $\mathbf{y}(k) = [x_1(k), y_1(k), \ldots, x_m(k), y_m(k)]^\top$ only the multi-vehicle location part of the state vector.

The nonlinear matrix $\mathbf{B}$ is a function of the multirobot part of the state (see Appendix A). Feedback linearization of the entire multirobot subset of the state vector, that is, including the orientation states, is not possible because in that case, $\mathbf{B}$ would be not invertible, and the resulting pseudo-inverse turns out to be rank-deficient (see [5]).

By choosing a control input of the form

$$\mathbf{u}(k) = \mathbf{B}^{-1}(\mathbf{u}'(k) - \mathbf{y}(k)) \tag{6}$$

we can cancel the nonlinearities in that subset of the state, $\mathbf{y}(k)$, obtaining a single input-state linear relation

$$\mathbf{y}(k+1) = \mathbf{u}'(k) + \mathbf{B}\mathbf{v}(k) \tag{7}$$

The term $\mathbf{u}'(k)$ in (7) is a new input to be determined, that can be chosen using traditional linear control techniques. In this case, we have opted for a control law to track a higher level planned multi-robot trajectory parameterized in time $\mathbf{y}^*(k)$, guaranteeing at the same time exponential vehicle location dynamics. That is, by defining the trajectory tracking error as

$$\mathbf{e}(k) = \mathbf{y}(k) - \mathbf{y}^*(k) \tag{8}$$

the desired error control dynamics is designed such that

$$\mathbf{e}(k+1) + \mathbf{Q}_1\mathbf{e}(k) = \mathbf{0} \tag{9}$$

where $\mathbf{Q}_1$ is constant and positive definite, and as will be seen later, with λ's in $\det(\lambda\mathbf{I} + \mathbf{Q}_1) = 0$ within the unitary circle.

Solving for $\mathbf{y}(k+1)$ in (9), substituting in (7), and assuming that the expectations for the estimation error $E[\tilde{\mathbf{y}}(k)] = \mathbf{0}$, and the plant noise $E[\mathbf{v}(k)] = \mathbf{0}$ hold, we get the control law

$$\mathbf{u}(k) = \mathbf{B}^{-1}(\mathbf{y}^*(k+1) - \mathbf{Q}_1\mathbf{y}^*(k) - (\mathbf{Q}_1 + \mathbf{I})\hat{\mathbf{y}}(k)) \tag{10}$$

The control law $\mathbf{u}(k)$ is written as a function of available data. That is, it is a function of the time parameterized multi-vehicle trajectories $\mathbf{y}^*(k)$, and of the current multi-vehicle state estimates $\hat{\mathbf{y}}(k)$.

Notice that in order to have zero mean estimation error of the vehicle states, SLAM must be fully observable [1]. Filter stability turns out to be a prerequisite for this or any other low level control strategy to be asymptotically stable as well. The intuition is straightforward, to accurately control a troupe of vehicles through a predefined trajectory, one must have means to accurately measure their location at all times.

So the control law (10), will stabilize the system about the time parameterized trajectory $\mathbf{y}^*(k)$.

In order to validate our feedback control scheme, we write the closed loop equations for the multi-vehicle state and multi-vehicle state estimate error, using the fact that $\mathbf{y}(k) = \hat{\mathbf{y}}(k) + \tilde{\mathbf{y}}(k)$.

$$\mathbf{y}(k+1) = -\mathbf{Q}_1\mathbf{y}(k) + (\mathbf{I} + \mathbf{Q}_1)\tilde{\mathbf{y}}(k) + \mathbf{B}\mathbf{v}(k) + \mathbf{y}^*(k+1) + \mathbf{Q}_1\mathbf{y}^*(k) \tag{11}$$

$$\tilde{\mathbf{x}}(k+1) = (\mathbf{F} - \mathbf{KHF})\tilde{\mathbf{x}}(k) + (\mathbf{G} - \mathbf{KHG})\mathbf{v}(k) - \mathbf{K}\mathbf{w}(k) \tag{12}$$

The separation of the problem in two parts, the optimal state estimation, and the controller, gives a Kalman filter independent of the matrix $\mathbf{Q}_1$, which specifies the control strategy. In the same way that the controller does not depend on the statistics $\mathbf{P}$, $\mathbf{Q}$, and $\mathbf{R}$ of the random noises.

The eigenvalues of the closed-loop system are given by those of the linearized-feedback dynamics $-\mathbf{Q}_1$, together with those of the state estimator dynamics $\mathbf{F} - \mathbf{KHF}$. Only when both matrices are stable, so is the closed-loop. We have designed $\mathbf{Q}_1$ for a stable controller, and for a fully observable estimation problem, it is straightforward to verify $\mathbf{F} - \mathbf{KHF}$ always stable [4,20].

As mentioned before, given the kinematics constraint of the vehicle model used, the entire vehicle pose including orientation cannot be stabilized, and we have decided to let $\mathbf{y}(k) = [x(k), y(k)]^\top$ be the Cartesian coordinates of the vehicle location only, at the expense of optimally controlling the vehicle orientation. Our experiments show however, that by controlling the vehicle position only, and letting the vehicle orientation be a free variable, after an initial transient interval, the predefined time parameterized trajectories can still be accurately followed with a troupe of vehicles.

4 Simulations

In order to show the feasibility of using Feedback Linearization during multi-robot SLAM, we simulated an environment with 16 landmarks over a $600m^2$ area. The vehicle model used in the simulations corresponds to the all-terrain planar vehicle from Figure 1, and is given in Appendix A. Note that the pose of the robot, and hence, the control point is located apart from the vehicle axis of rotation in order to avoid singularities in the computation of the Jacobians. The vehicle state and control point is chosen at the origin of a laser range scanner placed on the front of the vehicle, thus simplifying the measurement model.

Figure 2 presents an simulation for a pair of robots simultaneously tracking two time parameterized circular paths, while performing SLAM. The objective is to track the desired path as accurately as possible. The desired trajectory should come from a higher-level planning strategy. But since that is not the scope of this paper, but to guarantee concurrent tracking and estimation stabilities, simple circular paths are chosen instead.

Figure 3 shows plots for the vehicle state estimates, the state estimation error, and the history of control commands. Note in the last plot, that when the motion is initiated, the control law chooses a saturated translational velocity to reach the circular path, stabilizing then around 0.6 m/sec. Disregarding the uncontrollability of the angular orientation produces a drastic fluctuation of the angular velocity signal during this initial transient interval, then stabilizing to the desired angular velocity, set at 5 deg/sec.

Finally, Figure 4 shows the asymptotic landmark state estimate trace covari-

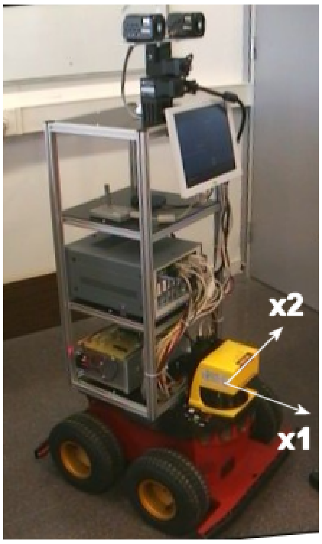


Fig. 1. The model for this nonlinear vehicle is used in the simulations, and is given in Appendix A. The Cartesian coordinates of the control point (x, y) are located on the projection center of the laser range scanner, and were porpousedly chosen not coincident with the vehicel axle center.

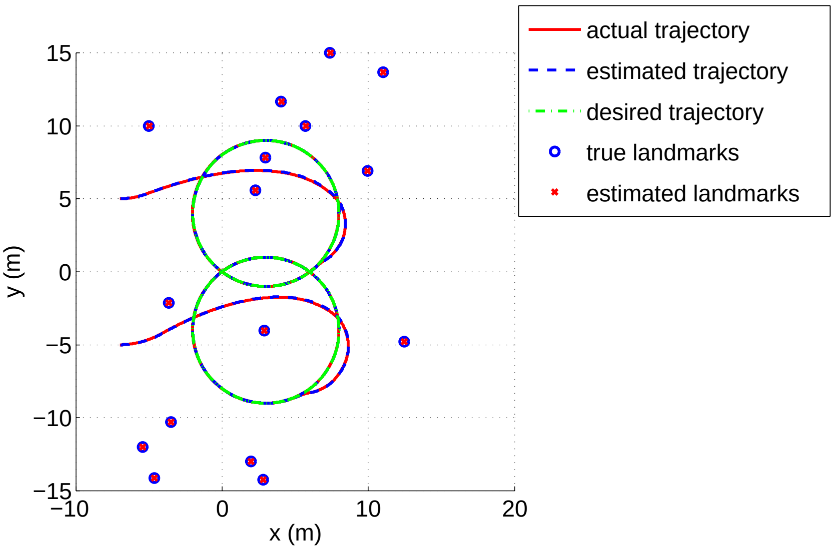
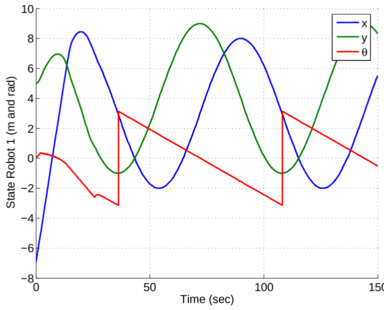
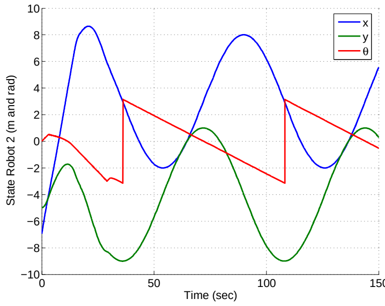


Fig. 2. Simultaneous Multirobot Localization, Control, and Mapping

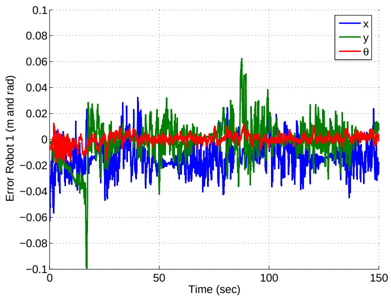
ances. The plot will look familiar to any experienced SLAM researcher, and specifically shows the decrease in all landmark localization uncertainties as the algorithm proceeds, showing asymptotic convergence of the estimation part of the problem.



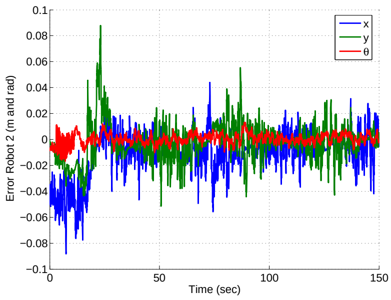
(a) Robot 1 state estimates



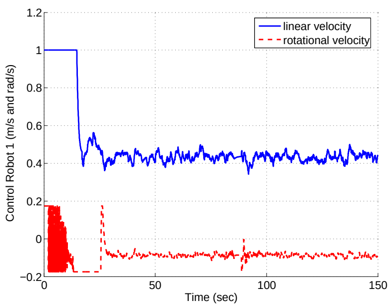
(b) Robot 2 state estimates



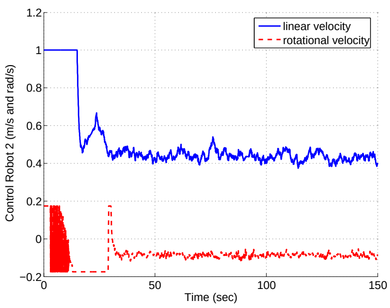
(c) Robot 1 estimation error



(d) Robot 2 estimation error



(e) Robot 1 control command



(f) Robot 2 control command

Fig. 3. State Estimation and Control using Feedback Linearization.

5 Conclusions

Given the separability between optimal state estimation and regulation, we have been able to present a multi-vehicle low-level control strategy that does not affect the estimation performance of a fully observable EKF based multirobot SLAM: a feedback linearization control strategy that is guaranteed

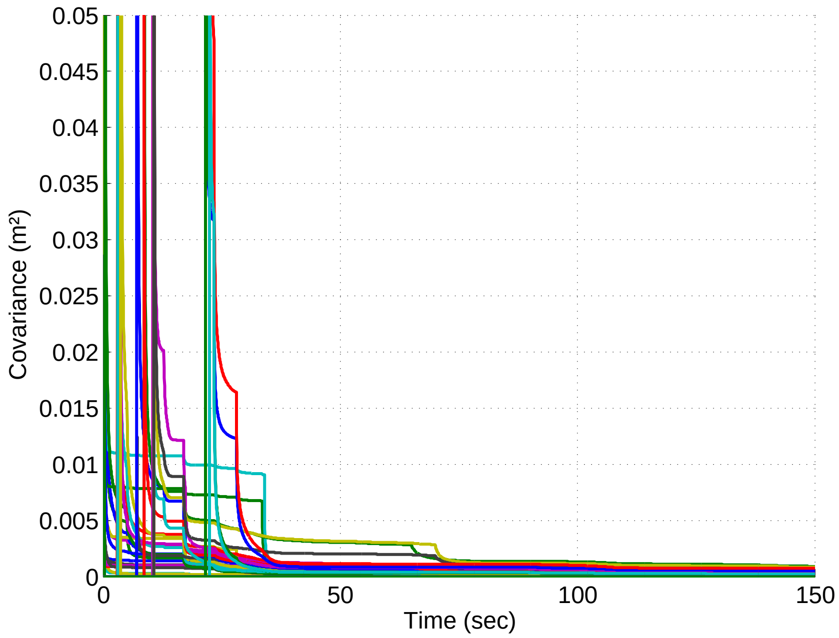


Fig. 4. Landmark trace covariances.

asymptotically stable for close tracking of any time-parameterized high-level computed trajectory. The feasibility of using the approach was validated with simulation results. In order to avoid the initial transient performance of the forward linearization control strategy, the effects of the kinematics constraints on the chosen vehicle model will be further investigated.

The beauty of this paper is precisely in that it points out the dependence on fully observable SLAM in order to be able to use the SLAM estimates as input to any type of controller. Then, both estimation and control can be decoupled and standard techniques such as the ones used here, Kalman filtering for estimation, and feedback linearization for control, are plausible for closing the perception-action-loop in multirobot SLAM.

A Nonlinear Vehicle and Measurement Models

The vehicle used in our simulations is a Pioneer robotics platform, controlled by a velocity v and a steering velocity ω. The process model used to predict the trajectory of the center of projection of the laser range scanner is given by

$$\begin{bmatrix} x(k+1) \\ y(k+1) \\ \theta(k+1) \end{bmatrix} = \begin{bmatrix} x(k) \\ y(k) \\ \theta(k) \end{bmatrix} + \tau \begin{bmatrix} \begin{bmatrix} \cos\theta(k) & -l\sin\theta(k) \\ \sin\theta(k) & l\cos\theta(k) \end{bmatrix} \begin{bmatrix} v(k)+v_v(k) \\ \omega(k)+v_\omega(k) \end{bmatrix} \\ \omega(k)+v_\omega(k) \end{bmatrix} \quad \text{(A.1)}$$

where l is the distance from the center of the wheel axle to the center of projection of the laser range scanner, τ is the time constant, and v_v, v_ω are zero mean Gaussian model noises.

The first two terms in (A.1) indicate the position of the vehicle, and are expressed in controllable canonical form, whereas the third term, the vehicle orientation is given as an incremental function of the input angular velocity. Thus, the nonlinear matrix $\mathbf{B}$ is in this case

$$\mathbf{B}(\theta(k)) = \begin{bmatrix} \cos\theta(k) & -l\sin\theta(k) \\ \sin\theta(k) & l\cos\theta(k) \end{bmatrix} \tag{A.2}$$

The observation model is

$$\begin{bmatrix} d_i(k) \\ \beta_i(k) \end{bmatrix} = \begin{bmatrix} \sqrt{(x_i - x(k))^2 + (y_i - y(k))^2} + w_d(k) \\ \tan^{-1}\left(\frac{(y_i - y(k))}{(x_i - x(k))}\right) - \theta(k) + \frac{\pi}{2} + w_\beta(k) \end{bmatrix} \tag{A.3}$$

with d_i and β_i the distance and bearing of an observed point landmark with respect to the laser center of projection. x_i and y_i are the absolute coordinates of such landmark, and i is used for the labeling of landmarks. $i = 0$ indicates an anchor feature not under estimation in order to guarantee full observability. w_d and w_β are zero mean Gaussian measurement noises.

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