The lowest-lying spin-1/2 and spin-3/2 baryon magnetic moments in chiral perturbation theory

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Abstract   We review some recent progress in our understanding of the lowest-lying spin-1/2 and spin-3/2 baryon magnetic moments (MMs) in terms of Chiral Perturbation Theory (ChPT). In particular, we show that at next-to-leading-order ChPT can describe the MMs of the octet baryons quite well. We also make predictions for the decuplet MMs at the same chiral order. Among them, the MMs of the Δ^{++} and Δ^+ are found to agree well with data within the experimental uncertainties.

Key words   magnetic moments, octet baryons, decuplet baryons, chiral perturbation theory

PACS   13.40.Em, 14.20.-c, 12.39.Fe

arXiv:1001.0465v1 [hep-ph] 4 Jan 2010

1   Introduction

The magnetic moments (MMs) of the octet baryons have long been related to those of the proton and neutron, i.e., the celebrated Coleman-Glashow (CG) relations [1]. These relations are a result of (approximate) global SU(3) flavor symmetry. Of course, we know that SU(3) flavor symmetry is broken, as can also be clearly seen by comparing the predicted MMs with the corresponding experimental values (see Table 1). How to implement SU(3) breaking in a model-independent and systematic way has been pursued since then.

Chiral symmetry and its breaking pattern, in combination with the concept of effective field theory first systematically put forward by Weinberg [2], has led to a low-energy effective theory of QCD–Chiral Perturbation Theory (ChPT) [3–10]. It has long been realized that ChPT may be employed to study SU(3) breaking effects on the MMs of the baryon octet. The first effort was undertaken by Caldi and Pagels in 1974 [11], even before ChPT as we know today was formulated. It was found that at next-to-leading-order (NLO), SU(3) breaking effects are so large that the description of the octet baryon MMs by the CG rela-

tions tends to deteriorate, which was later confirmed by the calculations performed in Heavy Baryon (HB) ChPT [12–15] and Infrared (IR) ChPT [16]. This apparent failure has often been used to question the validity of SU(3) ChPT in the one-baryon sector. In order to solve this problem, different approaches have been suggested, including reordering the chiral series [17] or using a cutoff to reduce the loop contributions, i.e., the so-called long-range regularization [18].

We will show in this talk that the above-mentioned apparent failure of baryon SU(3) ChPT is caused by the power-counting-restoration (PCR) procedure used in removing the power-counting-breaking (PCB) terms, which are due to the large non-zero baryon masses in the chiral limit [5]. The HB, the IR, and the extended-on-mass-shell (EOMS)* approaches all remove the PCB terms, but the HB and IR approaches achieve this by also removing nominally higher-order terms in such a way that relativity and analyticity of the loop results are lost. These questions have been discussed quite extensively in the literature, e.g., see Refs. [10, 19, 20]. Once the relativity and analyticity of the loop results are properly conserved, e.g., in the EOMS regularization scheme, it was found that baryon SU(3) ChPT at NLO improves

*It is necessary to stress that the EOMS regularization scheme is nothing but a $\overline{MS}$ procedure with an additional subtraction removing the finite power-counting-breaking pieces, if they exist.

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the CG relations [20], contrary to the conclusions of most previous ChPT studies performed at this order.
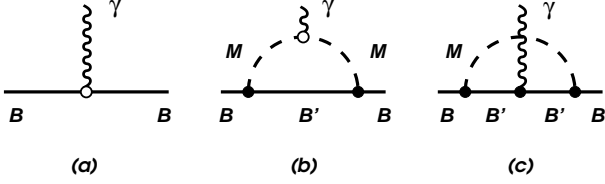

Fig. 1.   Feynman diagrams contributing to the octet baryon magnetic moments up to NLO.

It has often been argued that different regularization schemes should yield the same results since the difference between them is of nominal higher order. One must notice, however, that in order for this to be true, the regularization procedure should not break the analyticity of the loop results, which is certainly not true in certain cases for the HB and IR schemes, as demonstrated in Refs. [10, 19]. At a certain order, one scheme can converge faster than the other schemes. From a practical point of view, one should better choose the one that conserves the analyticity of the loop results, is covariant, and, meanwhile, converges faster. Among the HB, IR, and EOMS regularization schemes, the EOMS scheme has been found to satisfy the above criteria. Therefore, we have chosen the EOMS regularization scheme in all the calculations presented in this work.

2   The octet baryon magnetic moments

2.1   Dynamical octet baryon contributions

In the following, we discuss the results for the octet baryon magnetic moments at NLO without considering the contributions of dynamical decuplet baryons, which will be studied in the next sub-section.

We will not show the detailed formalism here, which can be found in Ref. [20]. Up to NLO, one has the diagrams shown in Fig. 1. The tree-level coupling **(a)** gives the leading-order (LO) result

$$\kappa_B^{(2)} = \alpha_B b_6^D + \beta_B b_6^F, \tag{1}$$

where the coefficients α_B and β_B for each of the octet baryons are listed in Table I of Ref. [20]. This lowest-order contribution is nothing but the SU(3)-symmetric prediction leading to the CG relations [1, 12].

The $\mathcal{O}(p^3)$ diagrams **(b)** and **(c)** account for the leading SU(3)-breaking corrections that are induced by the corresponding degeneracy breaking in the masses of the pseudoscalar meson octet. Their contributions to the anomalous magnetic moment of a given member of the octet B can be written as

$$
\begin{aligned}
\kappa_B^{(3)} &= \frac{1}{8\pi^2 F_\phi^2}\left(\sum_{M=\pi,K}\xi_{BM}^{(b)}H^{(b)}(m_M)\right.\\
&\quad\left.+\sum_{M=\pi,K,\eta}\xi_{BM}^{(c)}H^{(c)}(m_M)\right)
\end{aligned}
\tag{2}
$$

with the coefficients $\xi_{BM}^{(b,c)}$ listed in Table I of Ref. [20]. The loop-functions, which are convergent, read

$$
\begin{aligned}
H^{(b)}(m) &= -M_B^2 + 2m^2 + \frac{m^2}{M_B^2}(2M_B^2 - m^2)\log\left(\frac{m^2}{M_B^2}\right)\\
&\quad + \frac{2m(m^4 - 4m^2 M_B^2 + 2M_B^4)}{M_B^2\sqrt{4M_B^2 - m^2}}\arccos\left(\frac{m}{2M_B}\right),\\
H^{(c)}(m) &= M_B^2 + 2m^2 + \frac{m^2}{M_B^2}(M_B^2 - m^2)\log\left(\frac{m^2}{M_B^2}\right)\\
&\quad + \frac{2m^3(m^2 - 3M_B^2)}{M_B^2\sqrt{4M_B^2 - m^2}}\arccos\left(\frac{m}{2M_B}\right).
\end{aligned}
\tag{3}
$$

One immediately notices that they contain pieces $\sim M_B^2$ that contribute at $\mathcal{O}(p^2)$ to the MMs, which break the naive PC.

Different regularization schemes differ in how they remove the PCB terms: HB performs a dual expansion while IR subtracts from the full result the regular part. The underlying reason that one can perform a regularization on the results shown in Eq. (3) lies in the fact that ChPT includes all symmetry allowed terms such that the PCB terms can be absorbed by the corresponding low-energy-constants (LECs). One can easily see that the PCB terms ($\sim M_B^2$) can be absorbed by redefining b_6^D and b_6^F. This is how one performs the regularization in the EOMS scheme. In the HB and IR schemes, one also removes higher order analytic terms while those LECs corresponding to these nominally higher-order terms are not explicitly taken into account in the NLO calculation. One should also notice that in order for the EOMS argument to be totally true, one has to use a common decay constant F_ϕ for pions, kaons, and etas, because one has only two LECs at his disposal at this order, which can not take care of higher-order effects leading to different values for F_ϕ.

In Table 1, we show the LO and NLO results obtained in the EOMS scheme [20]. For the sake of comparison, we also show the NLO results obtained by using the HB and IR schemes. To compare with the results of earlier studies, we define

$$\tilde{\chi}^2 = \sum(\mu_{\text{th}} - \mu_{\text{exp}})^2, \tag{4}$$

while μ_{th} and μ_{exp} are theoretical and experimental MMs of the octet baryons. The results shown in Table 1 are obtained by minimizing $\tilde{\chi}^2$ with respect to the two LECs $\tilde{b}_6^D$ and $\tilde{b}_6^F$, renormalized b_6^D and b_6^F. It

is clear that the HB and IR results spoil the CG relations, as found in previous studies, while the EOMS results improve them.

Table 1.   The baryon-octet magnetic moments (in nuclear magnetons) up to $\mathcal{O}(p^3)$ obtained in different χPT approaches in comparison with data.

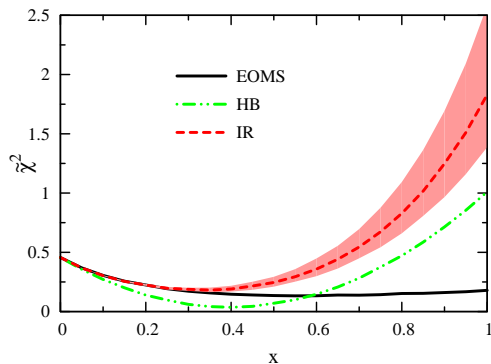|	p	n	Λ	Σ^-	Σ^+	Σ^0	Ξ^-	Ξ^0	$\Lambda\Sigma^0$	$\tilde{\chi}^2$
					$\mathcal{O}(p^2)$					
Tree level	2.56	-1.60	-0.80	-0.97	2.56	0.80	-1.60	-0.97	1.38	0.46
					$\mathcal{O}(p^3)$					
HB	3.01	-2.62	-0.42	-1.35	2.18	0.42	-0.70	-0.52	1.68	1.01
IR	2.08	-2.74	-0.64	-1.13	2.41	0.64	-1.17	-1.45	1.89	1.86
EOMS	2.58	-2.10	-0.66	-1.10	2.43	0.66	-0.95	-1.27	1.58	0.18
Exp.	2.793(0)	-1.913(0)	-0.613(4)	-1.160(25)	2.458(10)	—	-0.651(3)	-1.250(14)	$\pm$ 1.61(8)	

Fig. 2.   SU(3)-breaking evolution of the minimal $\tilde{\chi}^2$ in the $\mathcal{O}(p^3)$ χPT approaches under study. The shaded bands are produced by varying M_B from 0.8 to 1.1 GeV.

The difference between the EOMS, HB, and IR approaches can also be seen from Fig. 2, where we show the evolution of the minimal $\tilde{\chi}^2$ as a function of $x = M_M/M_{M,phys}$, while M_M, $M_{M,phys}$ are the masses of the pion, kaon, eta used in the calculation and their physical values. It is clear that at $x = 0$, the chiral limit, all the results are identical to the CG relations. As x approaches 1, where the meson masses equal to the physical values, only the EOMS results show a proper behavior, while both the HB and IR results rise sharply. This shows clearly that relativity and analyticity of the loop results play an important role in the present case.

2.2   Dynamical decuplet baryon contributions

Chiral perturbation theory relies on the assumption that there exists a natural cutoff such that high-energy degrees of freedom can be integrated out, with their effects approximated by the LECs. In the case of baryon SU(3) ChPT, the average mass gap between the baryon octet and the baryon decuplet is

only 0.231 GeV, similar to the pion mass and even smaller than the kaon mass. Therefore, in baryon SU(3) ChPT, one has to be careful about the contributions of the decuplet baryons.

It must be pointed out that description of spin-3/2 baryons in a fully consistent quantum field theory framework is an unsolved problem, see e.g. Refs. [21, 22] and references therein.

Using the "consistent coupling" scheme to describe the self-interaction of spin-3/2 baryons and their interaction with spin-1/2 baryons, we have shown in Ref. [22] that the inclusion of dynamical spin-3/2 baryons has only a small effect on our description of the octet baryon MMs as described above. It is also shown that this conclusion is stable with respect to all of the model parameters within their uncertainties [22].

3   The decuplet baryon magnetic moments

In recent years, there have been increasing interest from both the experimental side and the lattice QCD community to study the magnetic moments of the lowest-lying decuplet baryons, particularly those of the $\Delta(1232)$'s. Encouraged by the success of the baryon ChPT in describing the octet baryon magnetic moments, we have extended the same framework to study the decuplet baryon magnetic moments. Details of this study can be found in Ref. [23].

In Table 2, we show our EOMS ChPT NLO results [23] in comparison with those of a number of theoretical models and available data. We have fitted the only LEC at this order by reproducing the MM of the Ω. It can be clearly seen that our results for the Δ^{++} and Δ^+ agree quite well with data within the experimental uncertainties.

Table 2.   Decuplet magnetic dipole moments (in nuclear magnetons) obtained in covariant ChPT up to $\mathcal{O}(p^3)$, in comparison with those obtained in other theoretical approaches and data.

	Δ^{++}	Δ^{+}	Δ^{0}	Δ^{-}	Σ^{*+}	Σ^{*0}	Σ^{*-}	Ξ^{*0}	Ξ^{*-}	Ω^{-}
SU(3)-symm.	4.04	2.02	0	-2.02	2.02	0	-2.02	0	-2.02	-2.02
NQM [24]	5.56	2.73	-0.09	-2.92	3.09	0.27	-2.56	0.63	-2.2	-1.84
RQM [25]	4.76	2.38	0	-2.38	1.82	-0.27	-2.36	-0.60	-2.41	-2.35
χQM [26]	6.93	3.47	0	-3.47	4.12	0.53	-3.06	1.10	-2.61	-2.13
χQSM [27]	4.85	2.35	-0.14	-2.63	2.47	-0.02	-2.52	0.09	-2.40	-2.29
QCD-SR [28]	4.1(1.3)	2.07(65)	0	-2.07(65)	2.13(82)	-0.32(15)	-1.66(73)	-0.69(29)	-1.51(52)	-1.49(45)
lQCD [29]	6.09(88)	3.05(44)	0	-3.05(44)	3.16(40)	0.329(67)	-2.50(29)	0.58(10)	-2.08(24)	-1.73(22)
lQCD [30]	5.24(18)	0.97(8)	-0.035(2)	-2.98(19)	1.27(6)	0.33(5)	-1.88(4)	0.16(4)	-0.62(1)	—
large N_c [31]	5.9(4)	2.9(2)	—	-2.9(2)	3.3(2)	0.3(1)	-2.8(3)	0.65(20)	-2.30(15)	-1.94
HBχPT [32]	4.0(4)	2.1(2)	-0.17(4)	-2.25(19)	2.0(2)	-0.07(2)	-2.2(2)	0.10(4)	-2.0(2)	-1.94
ChPT [23]	6.04(13)	2.84(2)	-0.36(9)	-3.56(20)	3.07(12)	0	-3.07(12)	0.36(9)	-2.56(6)	-2.02
Expt. [33]	5.6$\pm$1.9	$2.7^{+1.0}_{-1.3}\pm1.5\pm3$	—	—	—	—	—	—	—	-2.02$\pm$0.05

4 Summary and conclusions

EOMS SU(3) baryon ChPT provides a successful description of the SU(3) breaking effects on the octet baryon MMs. It has been found in this particular case that the relativity and analyticity of the loop results play an important role. We have also studied the dynamical decuplet contributions and found that their inclusion only has a small effect on the SU(3) breaking effects on the MMs of the octet baryons.

Encouraged by the success of the EOMS approach, we have studied the decuplet baryon MMs. Fitting our only LEC at this order to reproduce the MM of the Ω, we have been able to predict the MMs of the other members of the baryon decuplet. In particular, those of the Δ^{++} and Δ^{+} seem to agree well with data within the experimental uncertainties.

This approach has also been employed to study the SU(3) breaking corrections to the hyperon vector coupling $f_1(0)$ [34], which plays a decisive role in the extraction of V_{US} from hyperon semi-leptonic decay (HSD) data. It will also be interesting to apply the same approach to study the hyperon axial-vector couplings, which could provide us vital information about the spin structure of the baryons.

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