Hyperfine mixing in electromagnetic decay of doubly heavy bc baryons

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We investigate the role of hyperfine mixing in the electromagnetic decay of ground state doubly heavy bc baryons. As in the case of a previous calculation on $b \to c$ semileptonic decays of doubly heavy baryons, we find large corrections to the electromagnetic decay widths due to this mixing. Contrary to the weak case just mentioned, we find here that one can not use electromagnetic width relations obtained in the infinite heavy quark mass limit to experimentally extract information on the admixtures in a model independent way.

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Baryon	Quark content (l=u,d)	S_h^π	J^π	M [MeV]	Baryon	Quark content	S_h^π	J^π	M [MeV]
Ξ_{bc}^*	{b c} l	1^+	$3/2^+$	6996	Ω_{bc}^*	{b c} s	1^+	$3/2^+$	7075
Ξ_{bc}'	[b c] l	0^+	$1/2^+$	6958	Ω_{bc}'	[b c] s	0^+	$1/2^+$	7038
Ξ_{bc}	{b c} l	1^+	$1/2^+$	6928	Ω_{bc}	{b c} s	1^+	$1/2^+$	7013

TABLE I. Quantum numbers and quark content for unmixed ground state doubly heavy bc baryons with well defined S_h (spin of the heavy quark subsystem). The masses were obtained in Ref. [17]. S_h^π stands for the spin and parity of the heavy quark subsystem, while J^π stands for the total spin and parity of the baryon. For $J = 1/2$, actual physical states are mixtures of the Ξ_{bc}, Ξ_{bc}' (Ω_{bc}, Ω_{bc}') states, and they appear in Table II.

I. INTRODUCTION

In the infinite heavy quark mass limit, and according to heavy quark spin symmetry (HQSS) [1], one can select the heavy quark subsystem of a doubly heavy baryon to have a well defined total spin $S_h = 0, 1$. This has been the default assumption by most calculations of doubly heavy baryon spectroscopy [2–14]. However, due to the finite value of the heavy quark masses, the hyperfine interaction between the light quark and any of the heavy quarks can admix both $S_h = 0$ and $S_h = 1$ spin components into the wave function. For ground state (total orbital angular momentum $L = 0$) bc baryons, one should expect the actual physical Ξ particles (quark content bcu or bcd) to be mixtures of the $\Xi_{bc}\,(S_h = 1)$ and $\Xi_{bc}'\,(S_h = 0)$ states. Similarly in the strange sector, the physical Ω particles (quark content bcs) will be mixtures of $\Omega_{bc}\,(S_h = 1)$ and $\Omega_{bc}'\,(S_h = 0)$) states.

While mixing effects are negligible in the spectrum, it was pointed out in Ref. [15] that hyperfine mixing could greatly affect the decay widths of doubly heavy baryons. The calculation for $b \to c$ semileptonic decay of doubly heavy baryons was conducted by the same authors in Ref. [16], where they found that hyperfine mixing in the bc states had a tremendous impact on the decay widths. We qualitatively confirmed their results in Ref. [17], although our predictions for the decay widths were roughly a factor of two larger. There, we also showed how HQSS predictions for $b \to c$ semileptonic decay, could be used to experimentally obtain information on the admixtures of the bc baryons in a model independent manner. Unfortunately those ratios involved weak decays that have competing electromagnetic (e.m.) decays and thus, they will be difficult to observe experimentally. In this context, it was clear the possible relevance of hyperfine mixing effects in e.m. decays. In fact, the authors of Ref. [16] expected hyperfine mixing effects to play an important role also for e.m. transitions. As a result of these considerations we included in Ref. [17] predictions for ratios that involved e.m. decay widths evaluated in the infinite heavy quark mass limit. This limit implies that the spin of the heavy quark subsystem can not change in an e.m. transition.

In this letter we perform the full calculation using the same quark model as in Ref. [17]. To our knowledge there is only one prior calculation of e.m. decays of doubly heavy bc baryons [18]. There, the authors used the e.m. radiation as a means to investigate the diquark structure but no hyperfine mixing was considered. In this work we restrict ourselves to transitions involving ground state ($L = 0$) bc baryons and our emphasis is put on the relevance of hyperfine mixing for those transitions. In Table I we show the quantum numbers for the ground state of unmixed bc baryons classified so that S_h is well defined. The physical spin-1/2 bc states are mixtures of the Ξ_{bc}, Ξ_{bc}' (Ω_{bc}, Ω_{bc}') states shown in that table. Their quantum numbers and admixture coefficients appear in Table II. As for the weak $b \to c$ decays analyzed in Ref. [17], we find here that hyperfine mixing largely affects the e.m. decay widths. On the other hand we find that contributions that change the spin of the heavy quark subsystem are very important in the evaluation of the e.m. decay widths. We are thus far from the infinite heavy quark mass limit according to which the spin of the heavy quark subsystem can not change in an e.m. transition. Due to this fact the e.m. decay width ratios proposed in Ref. [17], and obtained within that assumption, are not valid for the actual heavy quark masses and can not be used to experimentally extract information on the admixtures in a model independent way.

The paper is organized as follows: in Sec. II we collect general formulas to evaluate the e.m. decay width. We also give an appropriate form factor decomposition of the electromagnetic current matrix elements as well as showing how the form factors can be obtained in terms of those matrix elements. In Sec. III we present our nonrelativistic states and the way the matrix elements are evaluated in our model. Finally in Sec. IV we present the results and the conclusions of our work.

Baryon		J^π	M [MeV]	Baryon		J^π	M [MeV]
$\Xi_{bc}^{(1)}$ =	$0.902\ \Xi'_{bc} + 0.431\ \Xi_{bc}$	$1/2^+$	6967	$\Omega_{bc}^{(1)}$ =	$0.899\ \Omega'_{bc} + 0.437\ \Omega_{bc}$	$1/2^+$	7046
$\Xi_{bc}^{(2)}$ =	$-0.431\ \Xi'_{bc} + 0.902\ \Xi_{bc}$	$1/2^+$	6919	$\Omega_{bc}^{(2)}$ =	$-0.437\ \Omega'_{bc} + 0.899\ \Omega_{bc}$	$1/2^+$	7005

TABLE II. Physical spin-1/2 doubly heavy bc baryons. The admixture coefficients and the physical masses were obtained in Ref. [17].

II. ELECTROMAGNETIC DECAY

The electromagnetic decay width for the $B \to B'\gamma$ process is given by[1]

$$\Gamma = \frac{1}{2M}\int \frac{d^3P'}{(2\pi)^3\,2E'}\int \frac{d^3q}{(2\pi)^3\,2\omega}\ (2\pi)^4\delta^{(4)}(P-P'-q) \times \frac{1}{2J+1}\sum_s\ \sum_{s'}\sum_r \left(\mathcal{J}^{BB'\ \mu}_{s\,s'}(P,P')\,\varepsilon_{r\,\mu}(q)\right)\left(\mathcal{J}^{BB'\ \nu}_{s\,s'}(P,P')\,\varepsilon_{r\,\nu}(q)\right)^* \tag{2}$$

where $P=(M;\vec{0}\,)$, $P'=(E'=\sqrt{M'^2+\vec{P}'^2},\vec{P}')$ are respectively the four momenta of the initial and final baryons. J is the total spin of the initial baryon and s, s' are the spin third components of the initial and final baryons. $q=(\omega=|\vec{q}\,|,\vec{q}\,)$ is the final photon four momenta, being $\varepsilon_r(q)$ its polarization vector. Finally $\mathcal{J}^{BB'\ \mu}_{s\,s'}(P,P')$ stands for the electromagnetic current matrix element

$$\mathcal{J}^{BB'\ \mu}_{s\,s'}(P,P') = \left\langle B',\ s'\,\vec{P}'\ |J^\mu_{em}(0)|\ B,\ s\,\vec{P}=\vec{0}\right\rangle \tag{3}$$

with

$$J^\mu_{em}(0) = e\sum_q e_q\bar{\Psi}_q(0)\gamma^\mu\Psi_q(0)\ \ ;\ \ \frac{e^2}{4\pi}=\alpha_{em} \tag{4}$$

where the different e_q are the quark charges in units of the proton charge e, and α_{em} is the fine-structure constant.

Due to the conservation of the electromagnetic current we can take for real photons

$$\sum_r \varepsilon^\mu_r(q)(\varepsilon^\nu_r(q))^* \equiv -g^{\mu\nu} \tag{5}$$

and thus rewrite the total width as

$$\Gamma = \frac{1}{2M}\int \frac{d^3P'}{(2\pi)^3\,2E'}\int \frac{d^3q}{(2\pi)^3\,2\omega}\ (2\pi)^4\delta^{(4)}(P-P'-q) \times\frac{-1}{2J+1}\sum_s\ \sum_{s'}\mathcal{J}^{BB'\ \mu}_{s\,s'}(P,P')\left(\mathcal{J}^{BB'}_{s\,s'\ \mu}(P,P')\right)^* \tag{6}$$

The double sum in Eq. (6) is a Lorentz scalar and it can only depend on $P^2=M^2$, $P'^2=M'^2$ and $P\cdot P'=ME'$, with $E'=(M^2+{M'}^2)/2M$. As a result, all integrals can be done explicitly and we have the final expression

$$\Gamma = \frac{1}{8\pi M^2}\ \frac{M^2-M'^2}{2M}\ \ \frac{-1}{2J+1}\sum_s\ \sum_{s'}\mathcal{J}^{BB'\ \mu}_{s\,s'}(P,P')\left(\mathcal{J}^{BB'}_{s\,s'\ \mu}(P,P')\right)^*\Bigg|_{|\vec{q}\,|=\frac{M^2-M'^2}{2M}} \tag{7}$$

where, for the purpose of evaluation, we shall take $\vec{q}$ along the positive $Z-$axis.

A. Form factor decomposition of the electromagnetic current matrix elements

We will analyze $1/2\to 1/2$ and $3/2\to 1/2$ transitions.

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[1] Note the normalization of the baryon states should be such that

$$\left\langle B, s'\vec{P}'\,|B, s\vec{P}\right\rangle = \delta_{s\,s'}\,(2\pi)^3\,2E\,\delta^{(3)}(\vec{P}-\vec{P}') \tag{1}$$

1. Case $1/2 \to 1/2$

For $1/2 \to 1/2$ transitions we can write the following form factor decomposition

$$\begin{aligned}\mathcal{J}^{BB'\ \mu}_{s\,s'}(P,P') &= \left\langle B',\ s'\,\vec{P}' = -\vec{q}\,|J^{\mu}_{em}(0)|\ B,\ s\,\vec{P} = \vec{0}\right\rangle \\ &= \bar{u}'_{s'}(-\vec{q})\left[\left(\gamma^{\mu} - \frac{2(M-M')P'^{\mu}}{M^2-M'^2-q^2}\right)F_1 + \left(\frac{P^{\mu}}{M} - \frac{(M^2-M'^2+q^2)P'^{\mu}}{M(M^2-M'^2-q^2)}\right)F_2\right]u_s(\vec{0}) \end{aligned}\tag{8}$$

where $u(\vec{0})$, $\bar{u}'(-\vec{q})$ are the Dirac spinors (normalized to twice the fermion mass) for the initial and final baryon and F_1, F_2 are form factors that could only depend on the baryon masses and q^2. The above form factor decomposition trivially satisfies $q^{\mu}\mathcal{J}^{BB'}_{\mu} = 0$. For the present case we need the value of the form factors at $q^2 = 0$ ($|\vec{q}| = \frac{M^2-M'^2}{2M}$).

We shall have for the double sum in Eq. (7)

$$\begin{aligned}&\frac{-1}{2J+1}\sum_{s}\ \sum_{s'}\mathcal{J}^{BB'\ \mu}_{s\,s'}(P,P')\left(\mathcal{J}^{BB'}_{s\,s'\ \mu}(P,P')\right)^{*}\Big|_{|\vec{q}|=\frac{M^2-M'^2}{2M}} \\ &= -\frac{1}{2}Tr\left\{(\not{P}'+M')\left(\left(\gamma^{\mu} - \frac{2P'^{\mu}}{M+M'}\right)F_1 + \frac{q^{\mu}}{M}F_2\right)\right. \\ &\qquad \left.(\not{P}+M)\ \left(\left(\gamma_{\mu} - \frac{2P'_{\mu}}{M+M'}\right)F_1 + \frac{q_{\mu}}{M}F_2\right)\right\}\Big|_{|\vec{q}|=\frac{M^2-M'^2}{2M}} \\ &= -\frac{1}{2}Tr\left\{(\not{P}'+M')F_1\left(\gamma^{\mu} - \frac{2P'^{\mu}}{M+M'}\right)\right. \\ &\qquad \left.(\not{P}+M)\ \ F_1\left(\gamma_{\mu} - \frac{2P'_{\mu}}{M+M'}\right)\right\}\Big|_{|\vec{q}|=\frac{M^2-M'^2}{2M}} \\ &= 2\,(M-M')^2\ \ F_1^2\Big|_{|\vec{q}|=\frac{M^2-M'^2}{2M}}\end{aligned}\tag{9}$$

where in the second equality we have used current conservation.

The F_1 can be obtained as

$$F_1 = -\frac{1}{|\vec{q}|}\sqrt{\frac{E'+M'}{2M}}\,\mathcal{J}^{BB'\ 1}_{-1/2\ 1/2}(P,P')\tag{10}$$

where we have taken $\vec{q}$ along the positive $Z-$axis.

2. Case $3/2 \to 1/2$

For this case we could use

$$\mathcal{J}^{BB'\ \mu}_{s\,s'}(P,P') = \left\langle B',\ s'\,\vec{P}' = -\vec{q}\,|J^{\mu}_{em}(0)|\ B,\ s\,\vec{P} = \vec{0}\right\rangle = \bar{u}'_{s'}(-\vec{q})\widehat{\Gamma}^{\alpha\mu}\,u_{\alpha\ s}(\vec{0})\tag{11}$$

where $u_{\alpha}(\vec{0})$ is a Rarita-Schwinger spinor for the initial spin 3/2 baryon and $\widehat{\Gamma}^{\alpha\mu}$ is given by

$$\widehat{\Gamma}^{\alpha\mu} = \left(-\frac{C_3^V}{M'}\left(g^{\alpha\mu}\,\not{q} - q^{\alpha}\gamma^{\mu}\right) + \frac{C_4^V}{M'^2}\left(g^{\alpha\mu}\,q\cdot P - q^{\alpha}P^{\mu}\right) + \frac{C_5^V}{M'^2}\left(g^{\alpha\mu}\,q\cdot P' - q^{\alpha}P'^{\mu}\right)\right)\gamma_5\tag{12}$$

C_3^V, C_4^V, C_5^V are vector form factors that, as before, could only depend on baryon masses and q^2. For the double sum in Eq. (7) We have in this case

$$\begin{aligned}&\frac{-1}{2J+1}\sum_{s}\ \sum_{s'}\mathcal{J}^{BB'\ \mu}_{s\,s'}(P,P')\left(\mathcal{J}^{BB'}_{s\,s'\ \mu}(P,P')\right)^{*}\Big|_{|\vec{q}|=\frac{M^2-M'^2}{2M}} \\ &= -\frac{1}{4}Tr\left\{(\not{P}'+M')\widehat{\Gamma}^{\alpha\mu}(-1)(\not{P}+M)G_{\alpha\beta}\,\gamma^0(\widehat{\Gamma}^{\beta}{}_{\mu})^{\dagger}\,\gamma^0\right\}\Big|_{|\vec{q}|=\frac{M^2-M'^2}{2M}}\end{aligned}\tag{13}$$

with

$$G_{\alpha\beta} = g_{\alpha\beta} - \frac{1}{3}\gamma_\alpha\gamma_\beta - \frac{2}{3}\frac{P_\alpha P_\beta}{M^2} + \frac{1}{3}\frac{P_\alpha\gamma_\beta - P_\beta\gamma_\alpha}{M} \tag{14}$$

Taking $\vec{q}$ along the positive $Z-$axis, the C_3^V, C_4^V, C_5^V form factors can be obtained as

$$\begin{aligned}
C_3^V &= -\frac{M'}{|\vec{q}|}\sqrt{\frac{1}{2M(E'+M')}}\left(\frac{1}{\sqrt{2}}\,\mathcal{J}^{BB'\ 1}_{-3/2\ -1/2}(P,P') + \sqrt{\frac{3}{2}}\,\mathcal{J}^{BB'\ 1}_{-1/2\ 1/2}(P,P')\right)\\
C_4^V &= \frac{M'^2}{M|\vec{q}|^3}\sqrt{\frac{E'+M'}{2M}}\left(\ \sqrt{\frac{3}{2}}\frac{ME'-M'^2}{M-E'}\,\mathcal{J}^{BB'\ 3}_{1/2\ 1/2}(P,P') + \frac{1}{\sqrt{2}}\,(2E'-M')\,\mathcal{J}^{BB'\ 1}_{-3/2\ -1/2}(P,P')\right.\\
&\qquad\left. -\sqrt{\frac{3}{2}}M'\,\mathcal{J}^{BB'\ 1}_{-1/2\ 1/2}(P,P')\right)\\
C_5^V &= \frac{M'^2}{|\vec{q}|^3}\sqrt{\frac{E'+M'}{2M}}\left(-\sqrt{\frac{3}{2}}\,\mathcal{J}^{BB'\ 3}_{1/2\ 1/2}(P,P') - \frac{1}{\sqrt{2}}\,\mathcal{J}^{BB'\ 1}_{-3/2\ -1/2}(P,P') + \sqrt{\frac{3}{2}}\,\mathcal{J}^{BB'\ 1}_{-1/2\ 1/2}(P,P')\right)
\end{aligned} \tag{15}$$

Within our model we shall obtain (see next section)

$$\mathcal{J}^{BB'\ 3}_{1/2\ 1/2}(P,P') = 0 \ \ ; \ \ \mathcal{J}^{BB'\ 1}_{-3/2\ -1/2}(P,P') = \sqrt{3}\,\mathcal{J}^{BB'\ 1}_{-1/2\ 1/2}(P,P') \tag{16}$$

so that

$$C_5^V = 0 \ \ , \ \ C_4^V = -C_3^V\frac{M'}{M} \ \ , \ \ C_3^V = -\sqrt{\frac{3}{2}}\,\frac{1}{|\vec{q}|}\sqrt{\frac{2M'^2}{M(E'+M')}}\,\mathcal{J}^{BB'\ 1}_{-1/2\ 1/2}(P,P') \tag{17}$$

For that case, the trace in Eq. (13) can be evaluated to be

$$\frac{(M-M')^2\,(M+M')^4}{6M^2M'^2}\,(C_3^V)^2\ \Big|_{|\vec{q}|=\frac{M^2-M'^2}{2M}} \tag{18}$$

including the $-1/4$ factor.

III. NONRELATIVISTIC STATES AND MATRIX ELEMENTS EVALUATION

In this section we briefly describe our nonrelativistic states and the calculation of the electromagnetic current matrix elements within our model.

A. Nonrelativistic states

Our nonrelativistic states are constructed as

$$\begin{aligned}
\Big|B, s\,\vec{P}\Big\rangle_{NR} &= \int d^3Q_1 \int d^3Q_2 \sum_{\alpha_1,\alpha_2,\alpha_3} \hat{\psi}^{(B,s)}_{\alpha_1\,\alpha_2\,\alpha_3}(\,\vec{Q}_1,\vec{Q}_2\,)\ \frac{1}{(2\pi)^3\,\sqrt{2E_{f_1}2E_{f_2}2E_{f_3}}}\\
&\qquad \times\Big|\ \alpha_1\ \vec{p}_1 = \frac{m_{f_1}}{\overline{M}}\vec{P} + \vec{Q}_1\ \Big\rangle\Big|\ \alpha_2\ \vec{p}_2 = \frac{m_{f_2}}{\overline{M}}\vec{P} + \vec{Q}_2\ \Big\rangle\Big|\ \alpha_3\ \vec{p}_3 = \frac{m_{f_3}}{\overline{M}}\vec{P} - \vec{Q}_1 - \vec{Q}_2\ \Big\rangle
\end{aligned} \tag{19}$$

where α_j represents the spin (s), flavor (f) and color (c) quantum numbers ($\alpha \equiv (s,f,c)$) of the j-th quark, and $(E_{f_j}, \vec{p}_j)$, m_{f_j} are its four-momenta and mass. $\overline{M}$ is given by $\overline{M} = m_{f_1} + m_{f_2} + m_{f_3}$.

Quark states are normalized such that

$$\langle\ \alpha'\ \vec{p}\,'\ |\ \alpha\ \vec{p}\rangle = \delta_{\alpha'\,\alpha}\,(2\pi)^3\,2E_f\ \delta^{(3)}(\vec{p}\,' - \vec{p}) \tag{20}$$

In the transitions under study the baryons involved have $b\,c\,l$ quark content, where l represents a light quark u, d, s. We choose the wave functions such that quark 1 is a b, quark 2 is a c and quark 3 is the light one l.

$\hat{\psi}^{(B,s)}_{\alpha_1\,\alpha_2\,\alpha_3}(\,\vec{Q}_1,\vec{Q}_2\,)$ is the internal wave function in momentum space, being $\vec{Q}_1$ ($\vec{Q}_2$) the conjugate momenta to the relative position $\vec{r}_1$ ($\vec{r}_2$) between the light quark and the b (c) quark. This wave function is normalized as

$$\int d^3Q_1 \int d^3Q_2 \sum_{\alpha_1,\alpha_2,\alpha_3} \left(\hat{\psi}^{(B,s')}_{\alpha_1\,\alpha_2\,\alpha_3}(\,\vec{Q}_1,\vec{Q}_2\,)\right)^* \hat{\psi}^{(B,s)}_{\alpha_1\,\alpha_2\,\alpha_3}(\,\vec{Q}_1,\vec{Q}_2\,) = \delta_{s'\,s} \tag{21}$$

Thus, for our nonrelativistic baryon states we get

$$_{NR}\Big\langle\, B, s'\,\vec{P}\,'\,|\,B, s\,\vec{P}\,\Big\rangle_{NR} = \delta_{s'\,s}\,(2\pi)^3\,\delta^{(3)}(\vec{P}\,'-\vec{P}\,) \tag{22}$$

For unmixed states with a well defined S_h value the $\hat{\psi}^{(B,s)}_{\alpha_1\,\alpha_2\,\alpha_3}(\,\vec{Q}_1,\vec{Q}_2\,)$ wave function has the general form

$$\hat{\psi}^{(B,s)}_{\alpha_1\,\alpha_2\,\alpha_3}(\,\vec{Q}_1,\vec{Q}_2\,) = \frac{\varepsilon_{c_1\,c_2\,c_3}}{\sqrt{3!}}\ \widetilde{\phi}^B(\,\vec{Q}_1,\vec{Q}_2\,)\ \delta_{f_1\,b}\,\delta_{f_2\,c}\,\delta_{f_3\,l}(1/2,1/2,S_h;s_1,s_2,s_1+s_2)(S_h,1/2,J;s_1+s_2,s_3,s) \tag{23}$$

where $\frac{\varepsilon_{c_1\,c_2\,c_3}}{\sqrt{3!}}$ is the color wave function, with $\varepsilon_{c_1\,c_2\,c_3}$ the fully antisymmetric tensor in three (color) indices, and the $(j_1,j_2,j;m_1,m_2,m)$ are Clebsch-Gordan coefficients.

Details on the calculation of the orbital wave function in coordinate space for each of the unmixed states involved in this study can be found in Refs. [13, 19].

B. Matrix elements evaluation

We evaluate the electromagnetic current matrix elements as

$$\begin{aligned}\mathcal{J}^{BB'\ \mu}_{s\,s'}(P,P') &= \Big\langle B',\ s'\,\vec{P}' = -\vec{q}\,|J^\mu_{em}(0)\,|\ B,\ s\,\vec{P}=\vec{0}\,\Big\rangle \\ &\equiv \sqrt{2M}\sqrt{2E'}\ {}_{NR}\Big\langle B',\ s'\,\vec{P}' = -\vec{q}\,|J^\mu_{em}(0)\,|\ B,\ s\,\vec{P}=\vec{0}\,\Big\rangle_{NR} \\ &= \sqrt{2M}\sqrt{2E'}\ \ \mathcal{J}^{BB'\ \mu}_{s\,s'}(\vec{q})\Big|_{NR}\end{aligned} \tag{24}$$

with

$$\begin{aligned}\mathcal{J}^{BB'\ \mu}_{s\,s'}(\vec{q})\Big|_{NR} &= \sum_j c^B_j \sum_k (c^{B'}_k)^* \int d^3Q_1 \int d^3Q_2\ \tilde{\phi}^B_j(\vec{Q}_1,\vec{Q}_2) \\ &\Bigg\{ \sum_{s_1,s_2} (1/2,1/2,S_{h\,j};s_1,s_2,s_1+s_2)(S_{h\,j},1/2,J;s_1+s_2,s-s_1-s_2,s) \\ &\quad (1/2,1/2,S'_{h\,k};s_1+s'-s,s_2,s_1+s_2+s'-s)(S'_{h\,k},1/2,J';s_1+s_2+s'-s,s-s_1-s_2,s') \\ &\quad \Bigg[-\frac{e}{3}\ \left(\tilde{\phi}^{B'}_k(\vec{Q}_1 - \frac{m_c+m_l}{\overline{M'}}\,\vec{q},\vec{Q}_2+\frac{m_c}{\overline{M'}}\,\vec{q})\right)^*\ \frac{\bar{u}_{b\ s_1+s'-s}(\vec{Q}_1-\vec{q})\ \gamma^\mu\ u_{b\,s_1}(\vec{Q}_1)}{\sqrt{2E_b(|\vec{Q}_1-\vec{q}\,|)2E_b(|\vec{Q}^I_1|)}} \\ &\qquad +\frac{2e}{3}\ (-1)^{S_{h\,j}-S'_{h\,k}}\ \left(\tilde{\phi}^{B'}_k(\vec{Q}_1+\frac{m_b}{\overline{M'}}\,\vec{q},\vec{Q}_2-\frac{m_b+m_l}{\overline{M'}}\,\vec{q})\right)^*\ \frac{\bar{u}_{c\ s_1+s'-s}(\vec{Q}_2-\vec{q})\ \gamma^\mu\ u_{c\,s_1}(\vec{Q}_2)}{\sqrt{2E_c(|\vec{Q}_2-\vec{q}\,|)2E_c(|\vec{Q}_2|)}}\ \Bigg] \\ &\quad +e_l\,e\ \left(\tilde{\phi}^{B'}_k(\vec{Q}_1+\frac{m_b}{\overline{M'}}\,\vec{q},\vec{Q}_2+\frac{m_c}{\overline{M'}}\,\vec{q})\right)^*\ \delta_{S_{h\,j}\,S'_{h\,k}} \\ &\qquad \sum_m (S_{h\,j},1/2,J;m,s-m,s)(S_{h\,j},1/2,J';m,s'-m,s') \\ &\qquad \times\frac{\bar{u}_{l\ s'-m}(-\vec{Q}_1-\vec{Q}_2-\vec{q})\ \gamma^\mu\ u_{l\,s-m}(-\vec{Q}_1-\vec{Q}_2)}{\sqrt{2E_l(|-\vec{Q}_1-\vec{Q}_2-\vec{q}\,|)2E_l(|-\vec{Q}_1-\vec{Q}_2|)}}\ \Bigg\}\end{aligned} \tag{25}$$

where we have used the one-body approximation. The first two terms are the contribution from the b and c quarks respectively, whereas the third term is the contribution from the light quark. e_l is the charge of the light quark in units of the proton charge e. Besides, we sum (sums on j, k) over the different contributions to the physical states and the c_j^B, $c_k^{B'}$ factors are the corresponding admixture coefficients. For the evaluation of the matrix elements we take $\vec{q}$ along the positive $Z-$axis.

In order to be able to evaluate the spin sums explicitly it is useful to use the following relations obtained assuming $\vec{q}$ to be along the positive $Z-$axis

$$
\begin{aligned}
\frac{1}{\sqrt{2E'2E}}\bar{u}_{s'}(\vec{p}\,'=\vec{p}-\vec{q})\gamma^0 u_s(\vec{p}) &= \sqrt{\frac{(E'+m)(E+m)}{2E'2E}}\chi^\dagger_{s'}\left(1+\frac{\vec{p}^{\,2}-|\vec{q}|p^3}{(E'+m)(E+m)}\right.\\
&\qquad\left. +i\frac{|\vec{q}|}{(E'+m)(E+m)}(\vec{\sigma}\times\vec{p})^3\right)\chi_s\\
&= \sqrt{\frac{(E'+m)(E+m)}{2E'2E}}\left[\left(1+\frac{\vec{p}^{\,2}-|\vec{q}|p^3}{(E'+m)(E+m)}\right)\delta_{s'\,s}\right.\\
&\qquad\left.+\frac{|\vec{q}|}{(E'+m)(E+m)}\Big(\,(-p^1+ip^2)\,\delta_{s'\,s+1}\,+(p^1+ip^2)\,\delta_{s'\,s-1}\Big)\right]
\end{aligned}
\tag{26}
$$

where we work in Pauli-Dirac representation and χ stands for a Pauli spinor. Similarly for the spatial components one has

$$
\begin{aligned}
\frac{1}{\sqrt{2E'2E}}\bar{u}_{s'}(\vec{p}\,'=\vec{p}-\vec{q})\gamma^j u_s(\vec{p}) &= \sqrt{\frac{(E'+m)(E+m)}{2E'2E}}\chi^\dagger_{s'}\left(\frac{\vec{p}^{\,j}}{E+m}+\frac{(\vec{p}-\vec{q})^j}{E'+m}\right.\\
&\qquad\left.+i\frac{E-E'}{(E'+m)(E+m)}(\vec{\sigma}\times\vec{p})^j-i\frac{1}{(E'+m)}(\vec{\sigma}\times\vec{q})^j\right)\chi_s\\
= \sqrt{\frac{(E'+m)(E+m)}{2E'2E}}\Bigg[&\left(\frac{\vec{p}^{\,j}}{E+m}+\frac{(\vec{p}-\vec{q})^j}{E'+m}+i\frac{E-E'}{(E'+m)(E+m)}(-p^2\delta_{j1}+p^1\delta_{j2})(\delta_{s\,1/2}-\delta_{s\,-1/2})\right)\delta_{s'\,s}\\
&+\delta_{j1}\frac{|\vec{q}|(E+m)-(E-E')p^3}{(E'+m)(E+m)}\,(\delta_{s'\,s-1}-\delta_{s'\,s+1})\\
&+i\delta_{j2}\frac{|\vec{q}|(E+m)-(E-E')p^3}{(E'+m)(E+m)}\,(\delta_{s'\,s+1}+\delta_{s'\,s-1})\\
&+\delta_{j3}\frac{E-E'}{(E'+m)(E+m)}((-p^1+ip^2)\delta_{s'\,s+1}+(p^1+ip^2)\delta_{s'\,s-1})\Bigg]
\end{aligned}
\tag{27}
$$

Using the above results in Eq.(25) it is now easy to see why for $3/2\to1/2$ transitions we find $\mathcal{J}^{BB'\,3}_{1/2\,1/2}(P,P')=0$ as a result of the orthogonality relations of the Clebsch-Gordan coefficients. Explicit evaluation of the spin sums also shows that for $3/2\to1/2$ transitions $\mathcal{J}^{BB'\,1}_{-3/2\,-1/2}(P,P')=\sqrt{3}\,\mathcal{J}^{BB'\,1}_{-1/2\,1/2}(P,P')$. This result can be obtained realizing that

$$
\begin{aligned}
&\sum_{s_1,s_2}(1/2,1/2,1;s_1,s_2,s_1+s_2)(1,1/2,3/2;s_1+s_2,s-s_1-s_2,s)\\
&\quad(1/2,1/2,S'_h;s_1+s'-s,s_2,s_1+s_2+s'-s)(S'_h,1/2,1/2;s_1+s_2+s'-s,s-s_1-s_2,s')\;\delta_{s'\;s+1}\\
&\qquad=-\frac{1}{\sqrt{2}}\left\langle[(1/2\otimes1/2)^{S'_h}\otimes1/2]^{1/2}_{s'}\right|\sigma^{(1)}_{+1}\left|[(1/2\otimes1/2)^1\otimes1/2]^{3/2}_s\right\rangle
\end{aligned}
\tag{28}
$$

where $\left|[(1/2\otimes1/2)^S\otimes1/2]^J_s\right\rangle$ is the total spin state of three spin 1/2 particles coupled to total spin J and third component s, and $\sigma^{(1)}_{+1}$ is the $+1$ component of the spin operator of the first particle. Similarly

$$
\begin{aligned}
&\sum_m(1,1/2,3/2;m,s-m,s)(S'_h,1/2,1/2;m,s'-m,s')\;\delta_{1\,S'_h}\delta_{s'\;s+1}\\
&\qquad=-\frac{1}{\sqrt{2}}\left\langle[(1/2\otimes1/2)^{S'_h}\otimes1/2]^{1/2}_{s'}\right|\sigma^{(3)}_{+1}\left|[(1/2\otimes1/2)^1\otimes1/2]^{3/2}_s\right\rangle
\end{aligned}
\tag{29}
$$

with $\sigma^{(3)}_{+1}$ is the $+1$ component of the spin operator of the third particle. The Wigner-Eckart theorem now gives

$$
\begin{aligned}
&\left\langle[(1/2\otimes1/2)^{S'_h}\otimes1/2]^{1/2}_{-1/2}\right|\sigma^{(j)}_{+1}\left|[(1/2\otimes1/2)^1\otimes1/2]^{3/2}_{-3/2}\right\rangle\\
&\qquad=\sqrt{3}\;\left\langle[(1/2\otimes1/2)^{S'_h}\otimes1/2]^{1/2}_{1/2}\right|\sigma^{(j)}_{+1}\left|[(1/2\otimes1/2)^1\otimes1/2]^{3/2}_{-1/2}\right\rangle
\end{aligned}
\tag{30}
$$

	$\Gamma(10^{-8}$ GeV)		$\Gamma(10^{-8}$ GeV)
$\Xi^*_{bcu} \to \Xi'_{bcu}\gamma$	4.04	$\Omega^*_{bc} \to \Omega'_{bc}\gamma$	3.69
$\Xi^*_{bcd} \to \Xi'_{bcd}\gamma$	4.04		
$\Xi^*_{bcu} \to \Xi_{bcu}\gamma$	105	$\Omega^*_{bc} \to \Omega_{bc}\gamma$	20.9
$\Xi^*_{bcd} \to \Xi_{bcd}\gamma$	50.5		
$\Xi'_{bcu} \to \Xi_{bcu}\gamma$	0.992	$\Omega'_{bc} \to \Omega_{bc}\gamma$	0.568
$\Xi'_{bcd} \to \Xi_{bcd}\gamma$	0.992		

TABLE III. Electromagnetic decay widths (in units of 10^{-8} GeV) for unmixed states with a well defined S_h value.

	$\Gamma(10^{-8}$ GeV)		$\Gamma(10^{-8}$ GeV)
$\Xi^*_{bcu} \to \Xi^{(1)}_{bcu}\gamma$	6.05	$\Omega^*_{bc} \to \Omega^{(1)}_{bc}\gamma$	0.31
$\Xi^*_{bcd} \to \Xi^{(1)}_{bcd}\gamma$	0.12		
$\Xi^*_{bcu} \to \Xi^{(2)}_{bcu}\gamma$	73.9	$\Omega^*_{bc} \to \Omega^{(2)}_{bc}\gamma$	50.2
$\Xi^*_{bcd} \to \Xi^{(2)}_{bcd}\gamma$	103		
$\Xi^{(1)}_{bcu} \to \Xi^{(2)}_{bcu}\gamma$	12.4	$\Omega^{(1)}_{bc} \to \Omega^{(2)}_{bc}\gamma$	8.52
$\Xi^{(1)}_{bcd} \to \Xi^{(2)}_{bcd}\gamma$	20.9		

	$\Gamma(10^{-8}$ GeV)		$\Gamma(10^{-8}$ GeV)
$\Xi^*_{bcu} \to \Xi^{(1)}_{bcu}\gamma$	1.56	$\Omega^*_{bc} \to \Omega^{(1)}_{bc}\gamma$	0.415
$\Xi^*_{bcd} \to \Xi^{(1)}_{bcd}\gamma$	0.748		
$\Xi^*_{bcu} \to \Xi^{(2)}_{bcu}\gamma$	123	$\Omega^*_{bc} \to \Omega^{(2)}_{bc}\gamma$	24.2
$\Xi^*_{bcd} \to \Xi^{(2)}_{bcd}\gamma$	59.2		
$\Xi^{(1)}_{bcu} \to \Xi^{(2)}_{bcu}\gamma$	22.8	$\Omega^{(1)}_{bc} \to \Omega^{(2)}_{bc}\gamma$	3.78
$\Xi^{(1)}_{bcd} \to \Xi^{(2)}_{bcd}\gamma$	11.0		

TABLE IV. Electromagnetic decay widths (in units of 10^{-8} GeV) for physical states. We show the full calculation results (left panel) and results obtained considering only the contributions where the total spin of the heavy quark subsystem does not change (right panel).

IV. RESULTS AND CONCLUSIONS

In Table III we show the results for the e.m. decay widths evaluated with the unmixed states of Table I, while in the left panel of Table IV, the results using the physical spin-1/2 bc states of Table II, are given. The effects of mixing are relevant for all transitions. In particular we find

$$\begin{aligned}
\Gamma(\Xi^*_{bcd} \to \Xi^{(1)}_{bcd}\gamma) &\approx \frac{1}{33}\,\Gamma(\Xi^*_{bcd} \to \Xi'_{bcd}\gamma) \\
\Gamma(\Xi^{(1)}_{bcu} \to \Xi^{(2)}_{bcu}\gamma) &\approx 13\,\Gamma(\Xi'_{bcu} \to \Xi_{bcu}\gamma) \\
\Gamma(\Xi^{(1)}_{bcd} \to \Xi^{(2)}_{bcd}\gamma) &\approx 21\,\Gamma(\Xi'_{bcd} \to \Xi_{bcd}\gamma) \\
\Gamma(\Omega^*_{bcd} \to \Omega^{(1)}_{bcd}\gamma) &\approx \frac{1}{12}\,\Gamma(\Omega^*_{bcd} \to \Omega'_{bcd}\gamma) \\
\Gamma(\Omega^{(1)}_{bc} \to \Omega^{(2)}_{bc}\gamma) &\approx 15\,\Gamma(\Omega'_{bc} \to \Omega_{bc}\gamma)
\end{aligned} \tag{31}$$

which shows very clearly that hyperfine mixing can not be ignored when evaluating e.m. transitions involving spin-1/2 bc baryons. Besides, we observe that mixing breaks the degeneracy originally present in the unmixed case for most transitions involving bcu and bcd baryons.

We know that in the infinite heavy quark mass limit only the terms where the spin of the heavy quark subsystem does not change can contribute to the decay widths. This effect can already be seen in Table III in the fact that $\Gamma(\Xi^*_{bcl} \to \Xi_{bcl}\gamma) \gg \Gamma(\Xi^*_{bcl} \to \Xi'_{bcl}\gamma)$ or in the smallness of $\Gamma(\Xi'_{bcl} \to \Xi_{bcl}\gamma)$. Similar results are observed in the Ω sector.

To see how far from the ideal infinite heavy quark mass limit we are in this case, we show in the right panel of Table IV the results obtained with the physical spin-1/2 bc states, but considering only the contributions to the decay widths of the terms where the total spin of the heavy quark subsystem does not change. We see big changes when compared to the full results in the left panel of the same Table. As a result, and in contrast to the weak decay case discussed in Ref. [17], and to our prior expectations also outlined in this latter reference, heavy quark spin symmetry relations deduced in the infinitely heavy mass limit, are not accurate enough for the study of e.m. transitions involving doubly heavy baryons. Next-to-leading corrections turn out to be quite large as the differences between the two panels in Table IV indicate. Another important point here is that the decay widths are proportional to the factor $(M^2 - M'^2)(M - M')^2$ coming from phase space and the spin sums. As M is close to M', the decay widths are very sensitive to the actual baryon masses.

Finally, we would like to stress, once more, that the experimental measurement of e.m. widths will be extremely valuable in order to extract information on the hyperfine mixing of doubly heavy bc baryons, as the difference among the results of Table III (for unmixed states) and the left panel of Table IV (mixed states) clearly show. However, we should also point out that by looking only at e.m. transitions, it would not be possible to determine their actual mixing matrix without relying on a theoretical model. In this respect, the situation is more favorable in the case of the semileptonic weak decays of these baryons, as we discussed in Ref. [17], where leading order heavy quark symmetry relations turned out to be much more accurate.

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