Future CMB cosmological constraints in a dark coupled universe

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Cosmic Microwave Background satellite missions as the on-going Planck experiment are expected to provide the strongest constraints on a wide set of cosmological parameters. Those constraints, however, could be weakened when the assumption of a cosmological constant as the dark energy component is removed. Here we show that it will indeed be the case when there exists a coupling among the dark energy and the dark matter fluids. In particular, the expected errors on key parameters as the cold dark matter density and the angular diameter distance at decoupling are significantly larger when a dark coupling is introduced. We show that it will be the case also for future satellite missions as EPIC, unless CMB lensing extraction is performed.

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I. INTRODUCTION

Current cosmological measurements point to a *flat* universe whose mass-energy includes 5% ordinary matter and 22% non-baryonic dark matter, but is dominated by the *dark energy* component, identified as the engine for the accelerated expansion [1–7]. The most economical description of the cosmological measurements attributes the dark energy to a Cosmological Constant (CC) in Einstein's equations, representing an invariable vacuum energy density. The equation of state of the dark energy component w in the CC case is constant and $w = -1$. However, from the quantum field approach, the bare prediction for the current vacuum energy density is ~ 120 orders of magnitude larger than the observed value. This situation is the so-called CC problem. In addition, there is no proposal which explains naturally why the matter and the vacuum energy densities give similar contributions to the universe's energy budget at this moment in the cosmic history. This is the so-called *why now?* problem, and a possible way to alleviate it is to assume a time varying, dynamical fluid. The quintessence option consists on a cosmic scalar field ϕ (called *quintessence* itself) which changes with time and varies across space, and it is slowly approaching its ground state. Also, the quintessence equation of state is generally not constant through cosmic time [8–13]. In principle, the quintessence field may couple to the other fields. In practice, observations strongly constrain the couplings to ordinary matter [14]. However, interactions within the dark sectors, i.e. between dark matter and dark energy, are still allowed. This could change significantly the universe and the density perturbation evolution, the latter being seeds for structure formation. For models equivalent to the one studied here, see *e.g.* Refs. [15–22].

In this paper we investigate how allowing for a feasible interacting dark matter and dark energy model will affect the cosmological constraints expected from future CMB experiments. The Planck satellite mission, for example, is expected to provide high quality constraints on several key parameters (see e.g. [23]). However, those forecasts are usually performed under the assumption that the dark energy component is either a cosmological constant or a fluid with constant, redshift independent equation of state $w = P/\rho$. It is therefore timely to investigate if the assumption of a more elaborate dark energy component with a coupling with the dark matter could have an impact on these constraints. Here we indeed focus on the future CMB data constraints on interacting dark matter-dark energy models, exploiting, in particular, the gravitational CMB lensing signal. The structure of the paper is as follows. Section II presents the background and the linear perturbations of the interacting dark matter-dark energy model explored here. Sections III and IV describe the CMB lensing extraction method and the future CMB data simulation used in our numerical analysis, respectively. We present our results in Sec. V and draw our conclusions in Sec. VI.

II. PRELIMINARIES

At the level of the background evolution equations, one can generally introduce a coupling between the dark matter and dark energy sectors as follows:

$$\dot{\bar{\rho}}_{dm} + 3\mathcal{H}\bar{\rho}_{dm} = a\bar{Q}, \qquad (1)$$
$$\dot{\bar{\rho}}_{de} + 3\mathcal{H}\bar{\rho}_{de}(1+w) = -a\bar{Q}, \qquad (2)$$

where the bars denotes background quantities, $\bar{\rho}_{dm}(\bar{\rho}_{de})$ refers to the dark matter (dark energy) energy density, the dot indicates derivative with respect to conformal time $d\tau = dt/a$ and $w = \bar{P}_{de}/\bar{\rho}_{de}$ is the dark-energy equation of state (P denotes the pressure). We take $\mathcal{H} = \dot{a}/a$ as the background expansion rate. We work in the context of a Friedman-Robertson-Walker (FRW) metric, assuming a flat universe and pressureless dark

matter $w_{dm} = \bar{P}_{dm}/\bar{\rho}_{dm} = 0$. $\bar{Q}$ encodes the dark coupling and drives the background energy exchange between dark matter and dark energy. In order to deduce the evolution of the background as well as the density and velocity perturbations in coupled models, we need an expression for the energy transfer at the level of of the stress-energy tensor:

$$\nabla_\mu T^{\mu\nu}_{(a)} = Q^\nu_{(a)} \ , \qquad (3)$$

where $a = dm, de$, $T^{\mu\nu}$ refers to the energy-momentum tensor and $Q^\nu_{(a)}$ is the energy-momentum transfer between the dark matter and dark energy fluids. We consider

$$Q^{(dm)}_\nu = \xi H \rho_{de} u^{(dm)}_\nu = -Q^{(de)}_\nu \ , \qquad (4)$$

where ξ is a dimensionless coupling (considered constant, as well as w, in the present analysis). H and ρ_{de} refer to the total expansion rate and dark energy density, background plus perturbation, *i.e* $H = \mathcal{H}/a + \delta H$ and $\rho_{de} = \bar{\rho}_{de} + \delta\rho_{de}$ respectively. In the previous Eqs. (1) and (2), $\bar{Q}$ corresponds to $\xi\mathcal{H}\bar{\rho}_{de}/a$, see our choice of coupling in Eq. (4). Notice from Eq. (4) that $Q^{(a)}_\nu$ has been chosen parallel to the dark matter four velocity $u^{(dm)}_\nu$, in order to avoid momentum transfer in the rest frame of the dark matter component [16]. For this choice of energy exchange $Q^{(a)}_\nu$, positive (negative) values of the coupling ξ will lead to lower (higher) dark matter energy densities in the past than in the uncoupled $\xi = 0$ case. In the following, we restrict ourselves here to negative couplings and $w > 1$, which avoids instability problems in the dark energy perturbation equations, see Ref. [19].

The interacting model given by Eq. (4) has already been previously explored under several assumptions, see Refs. [16–19]. In those works, the linear perturbation analysis did not include perturbation of the expansion rate δH. Let us mention that the former is included in the numerical analysis presented here. The latter is quite relevant for the correct treatment of gauge invariant perturbation but it does not affect much the physical results. Details of the complete linear perturbation analysis will be presented in Ref. [24] including the specification of the initial conditions which have been chosen adiabatic for all the components except for the dark energy fluid, see Ref. [24]. For the numerical analysis presented here, we have modified the publicly available CAMB code [26], taking into account the presence of the dark coupling in both the background and the linear perturbation equations.

III. LENSING EXTRACTION

The analysis presented here includes, in addition to the primary CMB anisotropy angular power spectrum, the information from CMB lensing. Gravitational CMB lensing, as already shown (see e.g. [23, 25], can improve significantly the CMB constraints on several cosmological parameters, since it is strongly connected with the growth of perturbations and gravitational potentials at redshifts $z < 1$ and therefore, it can break important degeneracies. The lensing deflection field d can be related to the lensing potential ϕ as $d = \nabla\phi$ [27]. In harmonic space, the deflection and lensing potential multipoles follows

$$d^m_l = -i\sqrt{l(l+1)}\phi^m_l, \qquad (5)$$

and therefore, the power spectra $C^{dd}_l \equiv \langle d^m_l d^{m*}_l \rangle$ and $C^{\phi\phi}_l \equiv \langle \phi^m_l \phi^{m*}_l \rangle$ are related through

$$C^{dd}_l = l(l+1)C^{\phi\phi}_l. \qquad (6)$$

Lensing introduces a correlation between different CMB multipoles (that otherwise would be fully uncorrelated) through the relation

$$\left\langle a^m_l b^{m'}_{l'} \right\rangle = (-1)^m \delta^{m'}_m \delta^{l'}_l C^{ab}_l + \sum_{LM} \Xi^{mm'M}_{l\ l'\ L} \phi^M_L \ , \qquad (7)$$

where a and b are the T, E, B modes and Ξ is a linear combination of the unlensed power spectra $\tilde{C}^{ab}_l$ (see [28] for details).

In order to obtain the deflection power spectrum from the observed C^{ab}_l, we have to invert Eq. (7), defining a quadratic estimator for the deflection field given by

$$d(a,b)^M_L = n^{ab}_L \sum_{ll'mm'} W(a,b)^{mm'M}_{l\ l'\ L} a^m_l b^{m'}_{l'} \ , \qquad (8)$$

where n^{ab}_L is a normalization factor needed to construct an unbiased estimator ($d(a,b)$ must satisfy Eq. (5)). This estimator has a variance:

$$\langle d(a,b)^{M*}_L d(a',b')^{M'}_{L'} \rangle \equiv \delta^{L'}_L \delta^{M'}_M (C^{dd}_L + N^{aa'bb'}_L) \qquad (9)$$

that depends on the choice of the weighting factor W and leads to a noise $N^{aa'bb'}_L$ on the deflection power spectrum C^{dd}_L obtained through this method. In the next section we describe the method followed to extract the lensing noise.

IV. FUTURE CMB DATA ANALYSIS

We evaluate the achievable constraints on the coupling parameter ξ by a COSMOMC analysis of future mock CMB datasets. The analysis method we adopt here is based on the publicly available Markov Chain Monte Carlo package `cosmomc` [29] with a convergence diagnostic using the Gelman and Rubin statistics. We sample the following seven-dimensional set of cosmological parameters, adopting flat priors on them: the baryon and cold dark matter densities $\Omega_b h^2$ and $\Omega_c h^2$, the ratio of the sound horizon to the angular diameter distance at

decoupling θ_s, the scalar spectral index n_s, the overall normalization of the spectrum A_s at $k = 0.002 \text{ Mpc}^{-1}$, the optical depth to reionization τ, and, finally, the coupling parameter ξ.

We create full mock CMB datasets (temperature, E–polarization mode and lensing deflection field) with noise properties consistent with Planck [30] and EPIC [?] experiments, see Tab. I for their specifications. The fiducial model is chosen to be the best-fit from the WMAP analysis of Ref. [1] with $\Omega_b h^2 = 0.0227$, $\Omega_c h^2 = 0.113$, $n_s = 0.963$, $\tau = 0.09$ and $\xi = 0$, fixing $w = -0.9$ for our numerical calculations.

Experiment	Channel	FWHM	$\Delta T/T$
Planck	70	14'	4.7
	100	10'	2.5
	143	7.1'	2.2
$f_{sky} = 0.85$			
EPIC	70	12'	0.05
	100	8.4'	0.05
	150	5.6'	0.06
$f_{sky} = 0.85$			

TABLE I: Planck and EPIC experimental specifications. Channel frequency is given in GHz, FWHM (Full-Width at Half-Maximum) in arc-minutes, and the temperature sensitivity per pixel in $\mu K/K$. The polarization sensitivity is $\Delta E/E = \Delta B/B = \sqrt{2}\Delta T/T$.

We consider for each channel a detector noise of $w^{-1} = (\theta\sigma)^2$, where θ is the FWHM (Full-Width at Half-Maximum) of the beam assuming a Gaussian profile and σ is the temperature sensitivity $\Delta T/T$ (see Tab. I for the polarization sensitivity). We therefore add to each C_ℓ fiducial spectra a noise spectrum given by:

$$N_\ell = w^{-1} \exp(l(l+1)/l_b^2), \quad (10)$$

where l_b is given by $l_b \equiv \sqrt{8\ln 2}/\theta$.

In this work, we use the method presented in [28] to construct the weighting factor W of Eq. (8). In that paper, the authors choose W to be a function of the power spectra C_l^{ab}, which include both CMB lensing and primary anisotropy contributions. This choice leads to five quadratic estimators, with $ab = TT, TE, EE, EB, TB$; the BB case is excluded because the method of Ref. [28] is only valid when the lensing contribution is negligible compared to the primary anisotropy, assumption that fails for the B modes in the case of Planck. In the case of EPIC we have decided to neglect the BB channel since it may be contaminated by unknown foregrounds and/or it may be used for foregrounds removal. The results presented here for the EPIC mission have therefore to be considered as conservative. The five quadratic estimators can be combined into a minimum variance estimator which provides the noise on the power spectrum of the

deflection field C_l^{dd}

$$N_l^{dd} = \frac{1}{\sum_{aa'bb'} (N_l^{aba'b'})^{-1}} . \quad (11)$$

We compute the minimum variance lensing noise for both Planck and EPIC experiments by means of a publicly available routine [32].

The datasets (which include the lensing deflection power spectrum) are analyzed with a full-sky exact likelihood routine [32].

V. RESULTS

Parameter	Planck	Planck Lens	EPIC	EPIC Lens
$\Delta(\Omega_b h^2)$	0.00015	0.00012	0.00004	0.00003
$\Delta(\Omega_c h^2)$	0.0297	0.0296	0.0295	0.016
$\Delta(\theta_s)$	0.00229	0.00215	0.00216	0.00102
$\Delta(\tau)$	0.0047	0.0041	0.0022	0.0021
$\Delta(n_s)$	0.0037	0.0029	0.0018	0.0013
$\Delta(\log[10^{10} A_s])$	0.016	0.012	0.005	0.004
$\Delta(H_0)$	2.34	2.16	2.17	1.33
$\Delta(\Omega_\Lambda)$	0.064	0.061	0.062	0.038
ξ	> -0.59	> -0.54	> -0.56	> -0.34

TABLE II: 68% c.l. errors on cosmological parameters. Upper limits on ξ are 95% c.l. constraints.

Parameter	Planck Lens	CMBPOL Lens
$\Delta(\Omega_b h^2)$	0.00013	0.00003
$\Delta(\Omega_c h^2)$	0.0010	0.0003
$\Delta(\theta_s)$	0.00026	0.00005
$\Delta(\tau)$	0.0042	0.0022
$\Delta(n_s)$	0.0031	0.0014
$\Delta(log[10^{10} As])$	0.013	0.005
$\Delta(H_0)$	0.53	0.12
$\Delta(\Omega_\Lambda)$	0.005	0.001

TABLE III: 68% c.l. errors on cosmological parameters from Planck and EPIC with lensing extraction in the standard non interacting case ($\xi = 0$).

Table II summarizes the errors from Planck and EPIC future data on the main cosmological parameters when a coupling ξ among the dark energy and dark matter fluids is introduced in the model. Notice that the errors on the cosmological parameters which are degenerate with ξ are larger than the errors that one would get for these parameters within a standard cosmology, where $\xi = 0$ (see Tab. III). In particular, in the case of $\Omega_c h^2$, H_0, Ω_Λ and θ_s, the errors we obtain here are one order of magnitude larger than the ones obtained with $\xi = 0$. For comparison, we show in Tab. II the parameter constraints both with and without lensing extraction. The cosmological parameter constraints from EPIC mock data are stronger than those coming from future Planck data when

the CMB lensing signal is exploited. The reason for that is because the EPIC experiment is expected to drastically reduce the noise in the CMB lensing extraction.
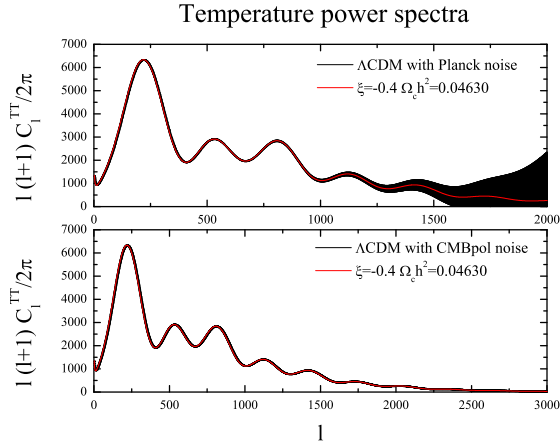

FIG. 1: Temperature power spectrum signal plus noise for Planck (top panel) and EPIC (bottom panel) experiments. The black curve depicts the ΛCDM model with $\Omega_c h^2 = 0.113$. The red curve illustrates a coupled model allowed by Planck data, with $\xi = -0.4$ and $\Omega_c h^2 = 0.0463$.
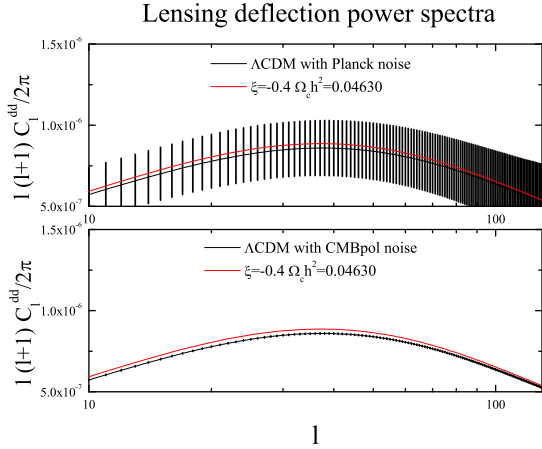

FIG. 2: Lensing deflection power spectrum signal plus noise for Planck (top panel) and EPIC (bottom panel) experiments. The black curve depicts the ΛCDM model with $\Omega_c h^2 = 0.113$. The red curve illustrates a coupled model allowed by Planck data, with $\xi = -0.4$ and $\Omega_c h^2 = 0.0463$.

Figures (1) and (2) illustrate the CMB temperature and lensing deflection spectra plus noise from Planck and EPIC experiments assuming two possible cosmologies: a ΛCDM universe with $\Omega_c h^2 = 0.113$ and a coupled model with $\xi = -0.4$ and $\Omega_c h^2 = 0.0463$. From measurements of the CMB unlensed temperature spectrum (see Fig. (1)), these two models will be degenerate, since they have identical spectra, albeit with different cold dark matter

densities, since a more negative coupling can be compensated with a lower $\Omega_c h^2$. As already pointed out in Ref. [19], in a universe with a negative coupling ξ, the matter content in the past can be higher than in the standard ΛCDM scenario due to an extra contribution proportional to the dark energy component, and therefore $\Omega_c h^2$ is strongly correlated with the coupling ξ.

Notice from Fig. (1) that neither Planck nor EPIC data will be able to distinguish among coupled and uncoupled models using only primary CMB anisotropy data. However, these two models predict distinguishable lensing potential spectra, see Fig. (2), and, while the Planck experiment will not have enough sensitivity to distinguish coupled versus uncoupled models, the EPIC experiment, with a greatly reduced noise on the CMB lensing extraction, will be able to test coupled models.
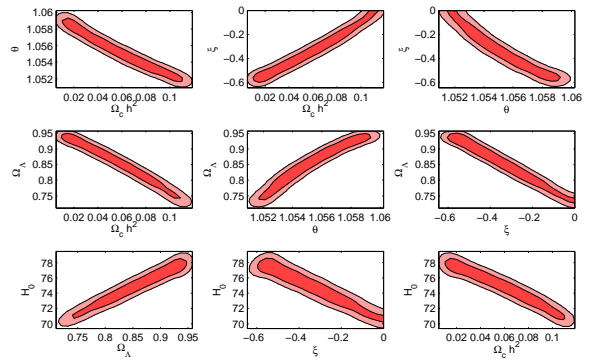

FIG. 3: The panels show the 68% and 95% confidence level contours combining the five most correlated parameters ($\Omega_c h^2$, θ_s, H_0, Ω_Λ and ξ) arising from a fit to mock Planck data without including lensing extraction in the analysis.
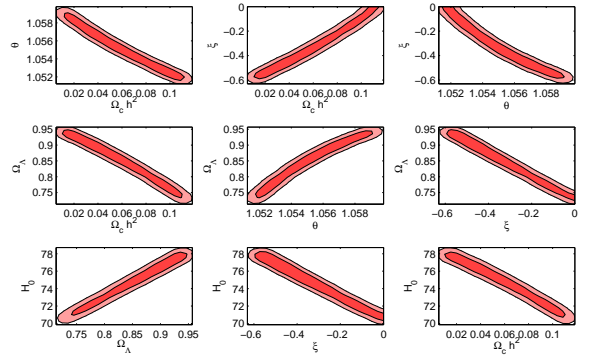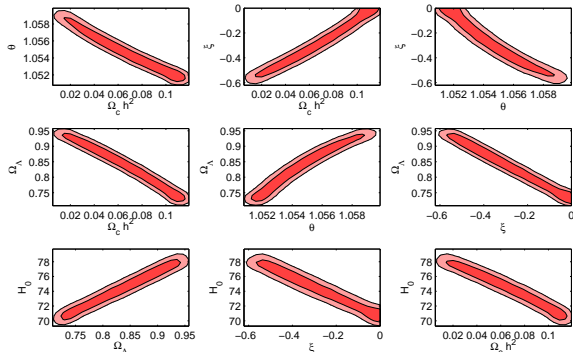

FIG. 4: The panels show the 68% and 95% confidence level contours combining the five most correlated parameters ($\Omega_c h^2$, θ_s, H_0, Ω_Λ and ξ) arising from a fit to mock EPIC data without including lensing extraction in the analysis.

FIG. 5: The panels show the 68% and 95% confidence level contours combining the five most correlated parameters ($\Omega_c h^2$, θ_s, H_0, Ω_Λ and ξ) arising from a fit to mock Planck data including lensing extraction in the analysis.
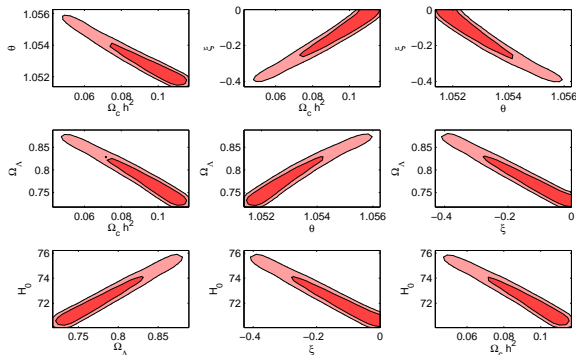

FIG. 6: The panels show the 68% and 95% confidence level contours combining the five most correlated parameters ($\Omega_c h^2$, θ_s, H_0, Ω_Λ and ξ) arising from a fit to mock EPIC data including lensing extraction in the analysis.

Figures (3) and (4) depict the 68% and 95% confidence level contours combining the five most correlated parameters arising from a fit to mock Planck and EPIC data respectively, without considering CMB lensing extraction. We can see that the cold dark matter density $\Omega_c h^2$, the Hubble constant H_0, the cosmological constant Ω_Λ, the sound horizon angle θ_s and the coupling ξ are all strongly correlated and neither Planck nor EPIC data will be able to break these degeneracies. Notice as well that, despite the technological advances of EPIC, the error on the cosmological parameters achieved by the Planck experiment will not be further improved by EPIC data if no lensing signal is considered.

Figures (5) and (6) show the 68% and 95% confidence level contours combining the five most correlated parameters arising from a fit to Planck and EPIC future data respectively, considering the information from CMB lens-

ing. Notice that the inclusion of lensing power spectrum improves drastically the EPIC constraints. However, the addition of CMB lensing information does not change Planck results. This is due to the fact that the lensing noise for EPIC is significantly lower than for the Planck experiment, and therefore EPIC data would be able to reject models that otherwise would be accepted by Planck, see Fig. (2).

VI. CONCLUSIONS

The current accelerated expansion of the universe is driven by the so-called dark energy. This negative pressure component could be interpreted as the vacuum energy density, or as a cosmic, dynamical scalar field. If a cosmic quintessence field is present in nature, it may couple to the other fields in nature. While the couplings of the quintessence field to ordinary matter are severely constrained, an energy exchange among the dark matter and dark energy sectors is allowed by current observations.

The major goals of the on-going Planck and the future EPIC experiments are to determine the nature of the dark energy component and to measure the remaining cosmological parameters with unprecedented precision. Several studies in the literature have been devoted to explore the performance of Planck and EPIC experiments in the dark energy scenario, see for instance Ref. [23]. In this paper we have explored the performance of Planck and EPIC experiments in alternative dark energy cosmologies, more concretely, in a universe with a coupling ξ among the dark energy and dark mater components [19].

We have generated mock data for the Planck and EPIC experiments. CMB gravitational lensing extraction has also been included in the analysis. The lensing noise has been computed by means of the minimum variance estimator method of Ref. [28]. The mock data have then been analyzed using MCMC techniques to compute the errors on the several cosmological parameters considered here. We find that relevant degeneracies are present among the coupling ξ and some other cosmological parameters, as the cold dark matter density $\Omega_c h^2$. Therefore, in the presence of a coupling, the expected Planck or EPIC errors on quantities as the cold dark matter energy density or the angular diameter distance at decoupling θ_s are one order of magnitude larger than in standard cosmologies with $\xi = 0$.

When gravitational CMB lensing extraction is included in the analysis, Planck results remain unchanged, due to the high level of lensing noise for this experiment. However, the EPIC mission, which will benefit from a much lower lensing noise level, can (a) provide tighter constraints on the cosmological parameters, even in the presence of a coupling, and (b) distinguish among coupled and uncoupled models that would look identical if they were fitted to Planck (lensed or unlensed) data.

VII.   ACKNOWLEDGMENTS

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