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Decrease of the resonance bandwidth of micromechanical oscillators by phase control of the driving force

Javier Tamayo[a] and Laura M. Lechuga
Biosensor Group, Instituto de Microelectrónica de Madrid (CNM-CSIC), Isaac Newton 8 (PTM), Tres Cantos, 28760 Madrid, Spain

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A method for controlling the amplitude response of micromechanical oscillators is presented. The micromechanical oscillator is driven by two forces acting both in phase, a fixed sinusoidal force and a feedback force whose amplitude depends on the phase shift. This dependence exhibits a pronounced maximum when the phase shift is 90°, i.e., at the resonant frequency. Experiments performed with a microcantilever prove that this class of active control decreases the bandwidth of the amplitude response about two orders of magnitude. The noise of the microcantilever, mainly of a thermal nature, is not increased at resonance, and it is moderately increased at both sides of the amplitude peak. Moreover, the noise can be tuned by adjusting the ratio between the two driving forces. © *2003 American Institute of Physics.* [DOI: 10.1063/1.1571228]

There is an increasing interest in systems based on micromechanical oscillators (MMOs) and nanomechanical oscillators for communications, small force detection, and ultrasensitive biochemical sensors.[1] Thus, resonating microcantilevers are used for sensitive mapping of attractive and repulsive forces at nanometer scale in atomic force microscopy,[2] for ultrasensitive nanomechanical biosensors,[3] for charged-particle detection,[4] and in detectors of gravitational waves[5] to name a few applications. A commonly employed scheme is the excitation of the MMO by applying a sinusoidal driving force, and the monitoring of the oscillation changes due to the interaction between the MMO and the near environment. The measurement of the oscillation can be performed by two methods, one measures the amplitude and phase lag with respect to the driving force at the resonant frequency (or near), and the other method measures the resonant frequency, that is, the frequency where the oscillation is out of phase with respect to the driving force. Independently of the method employed, the sensitivity of the detection is proportional to the quality factor (Q), or more precisely, is inversely proportional to the MMO bandwidth response. In fact, both magnitudes are directly related through the expression $Q \cong 1.74 f_0/\Delta f_{1/2}$, where f_0 is the resonant frequency and $\Delta f_{1/2}$ is the bandwidth at which the amplitude decays to the half. The quality factor is determined by the internal losses of the MMO and the viscous damping between the MMO and the environment. The operation of MMOs in liquid environments and microfabrication in soft materials such as polymers are representative cases, in which the Q is low.[6,7] Moreover, microelectromechanical systems are evolving to nanometer dimensions emerging dissipation phenomena due to the high surface-to-volume ratio.[8,9]

Recently, methods have been developed to change the effective quality factor by using a mixed driving force composed of a fixed sinusoidal component and a feedback component proportional to the MMO oscillation shifted 90°.[6,10,11] For sinusoidal movement of the cantilever, the feedback driving force is proportional to the oscillator velocity, partly canceling the damping force and giving a higher effective quality factor (Q_{eff}). The price to pay for the enhanced sensitivity is that (i) the oscillation amplitude is also increased accordingly, so the amount of feedback that can be applied is limited by the maximum amplitude tolerated by the system, (ii) the thermal noise is amplified $(Q_{\text{eff}}/Q)^{1/2}$ times,[12,13] and (iii) the time needed to reach the steady oscillation is proportional to Q_{eff}, making the measurement slower.

Here, we present a different control to decrease the bandwidth of the MMO amplitude response, which does not necessarily increase the amplitude of the oscillation, it produces a tolerable increase of the oscillator noise that can be tuned, and it does not affect the MMO constant time to reach the steady oscillation. Briefly, the concept is to drive the cantilever with two forces, both in phase, one is a fixed sinusoidal force $(F_0 e^{i\omega t})$ that allows measuring the phase shift, the second is a feedback force $C e^{i\omega t}$ with an amplitude (C) that depends on the phase shift of the oscillation (φ), having a pronounced maximum when the phase shift is 90°, that is, at the resonant frequency. The frequency dependence of the amplitude is $A(\omega)=\chi(\omega)[F_0+C(\varphi)]$, where χ is the module of the harmonic oscillator transfer function and $\omega = 2\pi f$. The phase shift response of the harmonic oscillator is not affected as both driving forces are acting in phase.

To test the concept, we chose a system composed of a microcantilever (Digital Instruments, Veeco) whose displacement is measured with subnanometer resolution by using the well-known optical beam deflection method. The microcantilever was coated with a 25 nm thick layer of cobalt on both sides to allow magnetic excitation and the experiments were performed in air. The nominal spring constant (without coating) and the experimental resonant frequency were 0.05 N/m and 21440 Hz, respectively. The feedback driving force is produced by a function generator whose output is controlled by a personal computer. The transfer function chosen for the feedback was

[a]Author to whom correspondence should be addressed; electronic mail: jtamayo@imm.cnm.csic.es

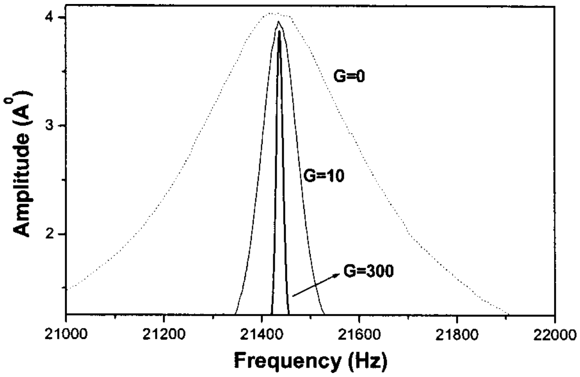

FIG. 1. Amplitude of a microcantilever as function of the driving frequency without (dotted line, $G=0$) and with phase control for G equal to 10 and 300. The measurements were performed in air.

$$C(\varphi)=F_{fb}e^{-G\cos^2\varphi}. \quad (1)$$

This function displays a pronounced maximum at resonance ($\varphi=90°$) where the feedback driving force is F_{fb} and it exponentially decays at frequencies off resonance. For simplicity, this active control will be so-called phase control.

Figure 1 shows the experimental frequency dependence of the amplitude for two values of G. From the measurements without feedback (dotted line in Fig. 1) a quality factor of about 67 was determined, which corresponds with a bandwidth $\Delta f_{1/2}=560$ Hz. For the correct performance of the phase control, a small sinusoidal driving force (F_0) was applied that gave a root-mean-square (rms) amplitude of 0.12 nm at resonance, allowing the phase shift measurement. The bandwidth of the resonance peak decreased to 84 Hz and 14 Hz for G equal to 10 and 300, respectively. Note that this active control allows narrowing of the resonance peak by increasing G without increasing the oscillation amplitude, and hence it can works with small oscillation amplitudes of ~0.1 nm as it is shown in Fig. 1. The frequency dependence of the phase shift is not affected (data not shown). The effect of G on the oscillator bandwidth can be analyzed by making a few simplifications. First, $\chi(\omega)\cong\chi(\omega_0)=Q/k$, owing to the frequency variation of χ, is negligible with respect to $C(\varphi)$ at resonance, and second, $\cos(\varphi)$ around resonance can be approximated as $2Q(f-f_0)/f_0$. Thus, the bandwidth of the amplitude response can be approximated as $\Delta f_{1/2}/f_0 \approx 0.42/(G^{1/2}Q)$.

Active control of the MMO oscillation also brings an increase of the noise of the cantilever motion signal, as this is fed back as excitation force. The amplified noise can arise from the cantilever displacement sensor and amplification stages, and from the Brownian motion owing to the thermal coupling between the MMO and the surrounding medium. The noise of the cantilever was measured by acquiring 50 samples per frequency value, and calculating the rms value. Figure 2(a) shows the amplitude (A) and its noise (δA) as a function of the frequency without the control feedback. The noise is primarily partitioned around the resonant frequency, indicating that the dominant source of δA is the Brownian cantilever motion. In fact, the cantilever behaves as an harmonic oscillator, which responds to the driving force and also to the random thermal kicking forces whose spectral density is uniform. Given a measurement bandwidth (B), the

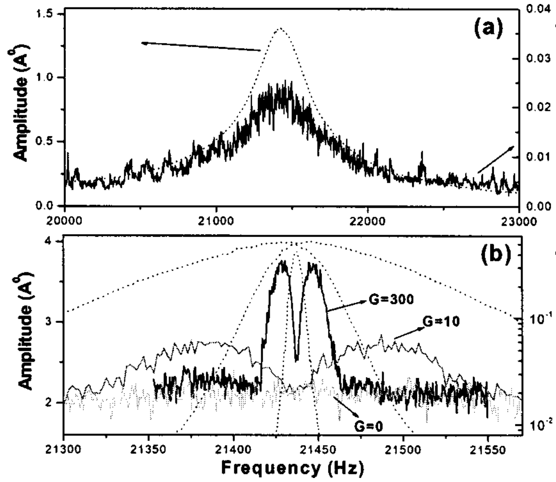

FIG. 2. (a) Average (dotted line) and rms (solid line) of the amplitude as a function of the driving frequency. (b) Average (dotted line) and rms (solid line) of the amplitude as a function of the driving frequency when the phase control is applied for G equal to 0 (that is without phase control) 10 and 300. Fifty measurements per driving frequency were taken to calculate the mean and rms values.

magnitude of the thermal force can be approximated as $\delta F_{\mathrm{th}}=(2kk_BTB/\pi Qf_0)^{1/2}$, where k is the spring constant, k_B is the Boltzmann constant, and T is the temperature.[12] Thus, the mean and rms values of the amplitude are the product of the module of the transfer function of the oscillator (χ) and the driving force and δF_{th} respectively. The amplitude noise exhibits a more complex behavior when the phase control is applied [Fig. 2(b)]. The noise is not significantly increased near resonance and it exhibits two maxima when the amplitude falls to about 40% of the peak value.

Since the phase-controlled driving force is a function of the phase shift of the oscillation, there are now two sources of noise in the amplitude, one is the cantilever motion produced by the kicking thermal forces, and the other arises from the phase fluctuations. The amplitude is $A(\omega)=\chi(\omega)\times[F_0+F_{\mathrm{fb}}\exp(-\mathrm{G}\cos^2\varphi)]$, thereby, the amplitude noise can be split into two terms, $\delta A=\chi(\omega)\delta F_{\mathrm{th}}+(dA/d\varphi)\delta\varphi$. Since the dominant source of noise is the Brownian cantilever motion, the phase fluctuation $\delta\varphi$ is produced by the incoherent thermal forces δF_{th}, which act equally in and out of phase with respect to the driving force. Thus, $\delta\varphi$ is approximately equal to the ratio between δF_{th} and the amplitude of the driving force. Therefore, the amplification of the amplitude noise can be written as

$$\frac{\delta A_{\mathrm{on}}-\delta A_{\mathrm{off}}}{\delta A_{\mathrm{off}}}=\frac{2G|\cos\varphi\sin\varphi|}{\dfrac{F_0}{F_{\mathrm{fb}}}e^{G\cos^2\varphi}+1}, \quad (2)$$

where $\delta A_{\mathrm{on,off}}$ are the amplitude noise with and without phase control, respectively. The numerator of Eq. (2) is zero at resonance and approximately increases linearly with $(f-f_0)/f_0$ and G near resonance. However, the denominator shows at minimum at resonance, and exponentially increases with the square of $(f-f_0)/f_0$ and G near resonance, approximately. The interplay between these factors gives two symmetric maxima at both sides of the resonance peak. More interestingly, the increase of the noise, owing to the feedback driving force, is a function of G and F_0/F_{fb}. Thus, given a

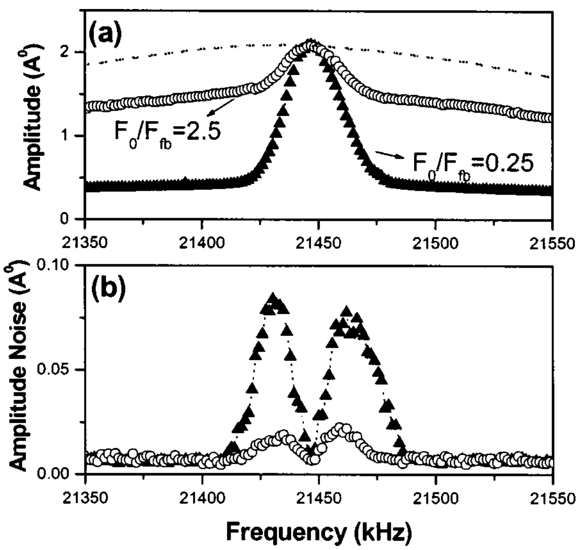

FIG. 3. Average amplitude (a) and its noise (b) as function of the driving frequency for a ratio between the amplitudes of the fixed and feedback driving forces (F_0/F_{fb}) of 0.25 (solid triangles) and 2.5 (circles). The total driving force (F_0+F_{fb}) was kept constant. The dotted line in (a) is the amplitude without phase control.

value of G that primarily determines the bandwidth of the amplitude response, the signal-to-noise ratio can be tuned by adjusting F_0 and F_{fb}.

To check the noise model described in last paragraph, the amplitude and its noise were measured for a predetermined G value, and different values of F_0 and F_{fb}. Figure 3 shows the amplitude and its noise for $G=100$ and for F_0/F_{fb} equal to 0.25 and 2.5, keeping constant the total driving force. A higher F_0/F_{fb} ratio produces a significant reduction of the noise with a slight widening of the amplitude peak. The height of the noise peaks at both sides of the resonant frequency falls about 4.5 times, and their width are also reduced. In other experiments, the amplitude noise did not change appreciably for different values of F_0 and F_{fb} keeping the ratio between them constant. This validates the model [Eq. (2)] proposed for the noise for phase control of MMO. The noise can be reduced by increasing the ratio F_0/F_{fb}, and the signal-to-noise ratio can be improved by increasing the amount of feedback (F_{fb}) while keeping the ratio between the two driving forces constant.

In conclusion, we have presented a different concept of the active control of MMOs, in which the amplitude of the driving signal is a function of the phase shift. This allows a significant decrease of the bandwidth of the amplitude response when the phase shift dependence of the amplitude of the driving force shows a pronounced maximum at 90° that is, at resonance. It highlights that the amplitude noise at resonance is not increased, and the noise off resonance can be tuned by controlling the ratio between the fixed and feedback driving forces. The application of this active control ranges from scanning probe microscopy and nanomechanical sensors, for the detection of small interactions that gives a resonant frequency shift, to communications for signal processing and filtering. Moreover, the concept can be extended, and more complex amplitude patterns can be obtained. Thus, in the case presented here, if the feedback driving force is subtracted from instead of added to the fixed driving force, an amplitude minimum at resonance would be obtained. This makes this class of controls potentially useful for smart actuation. For instance, the MMO can be placed in near proximity to a signal carrier such as electrical connecting element or an optical waveguide. An external stimulus that shift the resonant frequency would cancel or set into oscillation the MMO, switching or modulating the electrical or optical signal.[14]

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