

An S_4 model for quarks and leptons with maximal atmospheric angle

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Abstract

We consider a model for quark and lepton masses and mixings based on S_4 flavor symmetry. The model contains six Higgs doublets where three of them give mass to the leptons and the other three gives mass to the quarks. Charged fermion and quark masses arise from renormalizable interactions while neutrino Majorana masses are generated through effective dimension five Weinberg operator. From the study of the minimization of the scalar potential we found a residual $\mu \leftrightarrow \tau$ symmetry in the neutrino sector predicting zero reactor angle and maximal atmospheric angle and for the quark sector we found a four-zero texture. We give a fit of the mass hierarchies and mixing angles in the quark sector.

1 Introduction

Quarks and leptons have very different mixing angles. A successful phenomenological ansatz for leptons has been proposed by Harrison, Perkins and Scott and is given by [1]

$$U_{\text{TBM}} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} \quad (1)$$

which corresponds to $\tan^2 \theta_{\text{atm}} = 1$, $\sin^2 \theta_{\text{Chooz}} = 0$ and $\tan^2 \theta_{\text{sol}} = 0.5$, providing a good first approximation to the values indicated by current neutrino oscillation data [2, 3]. The third massive eigenstate is maximally mixed between μ and τ states and the second eigenstate is trimaximally mixed between e , μ and τ . Therefore the mixing matrix in

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eq. (1) is called *tri-bimaximal* (TBM) mixing matrix. While the experimental mixing matrix for quarks is given by, see [4]

$$V_{\text{CKM}} = \begin{pmatrix} 0.9743 & 0.2252 & 0.0035 \\ 0.2251 & 0.97347 & 0.0412 \\ 0.00859 & 0.0404 & 0.999146 \end{pmatrix} \quad (2)$$

In spite of the experimental progress so far we have no a compelling theoretical evidence regarding the flavor problem, namely why we have mixings (1) and (2) and why fermions masses are hierarchical.

A possibility to solve the flavor problem is by assuming a symmetry between the three generations, extending the standard model with a flavor symmetry G_f . In past, when neutrino data was lacking, hypothesis on G_f could arise only from the quark sector. Successful ansatz for quarks was extended to the lepton sector, see for instance [5]. However recent discovery of large neutrino mixings suggest a different scenario. Successfully ansatz for the lepton sector can be extended to the quark sector. Tri-bimaximal lepton mixing can be simply derived by assuming A_4 flavor symmetry¹ [6] and other discrete flavor symmetries give nearly tri-bimaximal mixing [7, 8]. Example of extension with A_4 for the quark sector can be found in [9–11]. In this paper we consider the lepton and quark sectors simultaneously. There are two non Abelian discrete groups that are suitable for such a purpose, namely T' [12, 13] and S_4 [14] since they contain singlet, doublet and triplet irreducible representations. It seems reasonable to consider models where quarks transform as $2 + 1$ of G_f and leptons as 3 of G_f in order to obtain large mixing in the lepton sector and small mixing between first and second families in the quark sector with heavy top quark mass.

The group of permutation of four objects S_4 is the minimal flavor symmetry of the mass matrix M_l and M_ν yielding TBM as shown in [15]. However was recently clarified in [16] that the symmetries of M_l and M_ν are not also symmetries of the Lagrangian and thus S_4 is not special for TBM but it is simply one of many groups that can be used for TBM. Pioneer works using the symmetry group S_4 as a family symmetry and deducing predictions for masses and mixings of fermions are in Ref. [17]. An interesting feature of S_4 is that the neutrino mass matrix generated from a general dimension five Weinberg operator [18] like $LL\phi\phi/\Lambda$ where Λ is the cut-off scale, is diagonalized from TBM when ϕ is an A_4 -triplet $\phi = (\phi_1, \phi_2, \phi_3)$ that takes vev as $\langle\phi\rangle \sim (1, 1, 1)$. Differently in models with A_4 , the general dimension five operator $LL\phi\phi/\Lambda$ is not diagonalized from TBM, see [19]. The motivation is that in A_4 the contractions $(LL)_{1'}$ and $(LL)_{1''}$ break TBM, while in S_4 the TBM is preserved since the two A_4 representations $1'$ and $1''$ correspond to a one irreducible representation of S_4 , that is the doublet, and the contraction $(LL)_2$ preserve the TBM.

In this paper, we study a model based on the S_4 flavor symmetry, where the model is invariant under the $G_F = S_4 \times Z_3^q \times Z_2^q \times (Z_{2e} \times Z_{2\mu} \times Z_{2\tau})$ product. The quarks and charged lepton masses arise from renormalizable interactions while the Majorana neutrino masses arise from the general dimension five operator. We also study the Higgs potential invariant under the G_F symmetry and found the minimization conditions.

In the next section we introduce the model, in section 3 we study the minimization of the potential, in sections 4 and 5 we study the phenomenological consequence of our

¹ A_4 is the group of even permutations of four objects isomorphic to the group of symmetries of the tetrahedron.

	\bar{L}	e_R	μ_R	τ_R	\bar{Q}_D	\bar{Q}_S	u_{RD}	u_{RS}	d_{RD}	d_{RS}	ϕ	H_D	H_s
$SU(2)$	2	1	1	1	2	2	1	1	1	1	2	2	2
S_4	3_1	1_1	1_1	1_1	2	1	2	1	2	1	3_1	2	1_1
Z_3^q	1	1	1	1	ω	ω	ω^2	ω^2	ω	ω	1	ω	ω
Z_2^q	+	+	+	+	+	-	+	-	+	-	+	-	+

Table 1: Quark, Lepton and scalar multiplet structure of our model, see text.

model for leptons and quarks respectively, and in section 6 we give the conclusions.

2 The Model

The model consist on the flavor symmetry $G_F = S_4 \times Z_3^q \times Z_2^q \times (Z_{2e} \times Z_{2\mu} \times Z_{2\tau})$, but in order to avoid unnecessary confusions, we will split the treatment for quarks and leptons.

Consider the model defined in Table (1). The left-handed doublets transform as a triplet of S_4 , namely $L = (L_e, L_\mu, L_\tau) \sim 3_1$ and the right-handed fields e_R, μ_R, τ_R as singlets of S_4 . In the quark sector we assume the third family to transform as a singlet of S_4 and the first and second families as a doublet, $Q_D = (Q_1, Q_2)$ and $q_D^c = (q_{R1}, q_{R2})$. We have six Standard Model Higgs doublets, $\phi = (\phi_1, \phi_2, \phi_3)$ transforming as a triplet under S_4 , $H_D = (H_1, H_2)$ and H_s transforming as a doublet and a singlet respectively under the S_4 flavor symmetry. Only the quark sector is charged with respect to the Z_3^q and Z_2^q symmetries. As a consequence of $S_4 \times Z_3^q \times Z_2^q$ assignment, the scalar field ϕ interact only with charged leptons and $H_{D,S}$ only with the quarks at the renormalizable level.

In order to have diagonal charged lepton mass matrix we also assume extra auxiliary symmetries $Z_{2e} \times Z_{2\mu} \times Z_{2\tau}$. In particular each right-handed field l_a^c is charged under the corresponding Z_{2a} with $a = e, \mu, \tau$ as well as each component of ϕ . As in [20–22] the $(Z_2)^3$ symmetries glue each l_i^c with the corresponding ϕ_i . We will show below that $(Z_2)^3$ remove off-diagonal terms in the charged lepton sector. Since the $(Z_2)^3$ symmetries do not commute with S_4 we have to take the semidirect product² of S_4 with $(Z_{2e} \times Z_{2\mu} \times Z_{2\tau})$. The assignment of the charged leptons and the scalar field ϕ with respect to $S_4 \times (Z_{2e} \times Z_{2\mu} \times Z_{2\tau})$ can be summarized in the following table.

leptons	L	e^c	μ^c	τ^c	ϕ_1	ϕ_2	ϕ_3
S_4	3_1	1_1	1_1	1_1	3_1		
Z_{2e}	+	-	+	+	-	+	+
$Z_{2\mu}$	+	+	-	+	+	+	-
$Z_{2\tau}$	+	+	+	-	+	-	+

The $S_4 \times Z_3^q \times Z_2^q \times (Z_{2e} \times Z_{2\mu} \times Z_{2\tau})$ invariant Lagrangian reads

$$\mathcal{L} = \mathcal{L}_l + \mathcal{L}_M + \mathcal{L}_q$$

²For the use of semi-direct product in model building see for instance [22, 23].

where

$$\mathcal{L}_l = y_e \bar{L}_e e_R \phi_1 + y_\mu \bar{L}_\mu \mu_R \phi_3 + y_\tau \bar{L}_\tau \tau_R \phi_2; \quad (3)$$

$$\begin{aligned} \mathcal{L}_M &= \frac{\lambda_1}{\Lambda} (L_i \tilde{\phi})_1 (L_j \tilde{\phi})_1 + \frac{\lambda_2}{\Lambda} (L_i \tilde{\phi})_2 (L_j \tilde{\phi})_2 + \frac{\lambda_3}{\Lambda} (L_i \tilde{\phi})_{3_1} (L_j \tilde{\phi})_{3_1} + \frac{\lambda'_3}{\Lambda} (L_i \tilde{\phi})_{3_2} (L_j \tilde{\phi})_{3_2} + \\ &+ \frac{\lambda_4}{\Lambda} (L_i L_j)_1 (\tilde{\phi} \tilde{\phi})_1 + \frac{\lambda_5}{\Lambda} (L_i L_j)_2 (\tilde{\phi} \tilde{\phi})_2 + \frac{\lambda_6}{\Lambda} (L_i L_j)_{3_1} (\tilde{\phi} \tilde{\phi})_{3_1}; \end{aligned} \quad (4)$$

$$\begin{aligned} \mathcal{L}_Y^q &= Y_2^d \bar{Q}_D d_{R_D} H_s + Y_3^d \bar{Q}_S d_{R_S} H_s + Y_4^d \bar{Q}_D d_{R_S} H_D + Y_5^d \bar{Q}_S q_{R_D} H_D + \\ &+ Y_2^u \bar{Q}_D u_{R_D} \tilde{H}_s + Y_3^u \bar{Q}_S u_{R_S} \tilde{H}_s + Y_4^u \bar{Q}_D u_{R_S} \tilde{H}_D + Y_5^u \bar{Q}_S u_{R_D} \tilde{H}_D, \end{aligned} \quad (5)$$

where Λ is a cut-off scale and $\tilde{\phi} = i\sigma_2 \phi^*$ and so on. According to the S_4 symmetry also the following off-diagonal terms in the lepton sectors are allowed

$$y_e (\bar{L}_\mu e_R \phi_3 + \bar{L}_\tau e_R \phi_2) + y_\mu (\bar{L}_e \mu_R \phi_1 + \bar{L}_\tau \mu_R \phi_2) + y_\tau (\bar{L}_e \tau_R \phi_1 + \bar{L}_\mu \tau_R \phi_3), \quad (6)$$

but these terms are not invariant under the $Z_{2e} \times Z_{2\mu} \times Z_{2\tau}$ symmetry.

3 The scalar potential

In our model there are 6 Higgs doublets that belong to one triplet $\phi = (\phi_1, \phi_2, \phi_3)$, one doublet $H_D = (H_1, H_2)$ and one singlet H_s representations of S_4 . In general the Higgs potential can be written as

$$V = V(\phi) + V(H_D, H_s) + V_{\text{int}}(\phi, H_D, H_s) \quad (7)$$

where $V(\phi)$ contains only S_4 -triplet, $V(H_D, H_s)$ contains both S_4 singlet and doublet scalars and $V_{\text{int}}(\phi, H_D, H_s)$ contains only quartic terms mixing the triplet with the doublet and the singlet. Moreover H_D and H_s transform with respect to $Z_3^q \times Z_2^q$ and each component ϕ_i of the S_4 triplet ϕ transforms with respect to $Z_{2e} \times Z_{2\mu} \times Z_{2\tau}$. Below we give the potential invariant under $S_4 \times Z_3^q \times Z_2^q$ and successively we consider the $Z_{2e} \times Z_{2\mu} \times Z_{2\tau}$ symmetry when we explicitly write each term in its components. The contributions to the Higgs Potential in eq. (7) invariant under the $S_4 \times Z_3^q \times Z_2^q$ symmetry are given by³

$$\begin{aligned} V(\phi) &= \mu [\phi^\dagger \phi]_{1_1} + \alpha ([\phi^\dagger \phi]_{1_1})^2 + l_2 [\phi^\dagger \phi]_2 [\phi^\dagger \phi]_2 + l_3 [\phi^\dagger \phi]_{3_1} [\phi^\dagger \phi]_{3_1} + \\ &+ l_4 [\phi^\dagger \phi]_{3_2} [\phi^\dagger \phi]_{3_2}, \\ V(H_D, H_s) &= \mu_2 [H_D^\dagger H_D]_{1_1} + \mu_3 [H_s^\dagger H_s] + \alpha_2 ([H_D^\dagger H_D]_{1_1})^2 + \alpha_3 ([H_D^\dagger H_D]_{1_2})^2 + \\ &+ \alpha_4 [H_D^\dagger H_D]_2 [H_D^\dagger H_D]_2 + l_5 [H_D^\dagger H_D]_{1_1} [H_s^\dagger H_s]_{1_1} + \alpha_5 [H_s^\dagger H_s]^2 + \\ &+ l_6 [H_D^\dagger H_D]_{1_1} [H_s^\dagger H_s]_{1_1}, \\ V_{\text{int}}(\phi, H_D, H_s) &= a_1 [\phi^\dagger \phi]_{1_1} [H_s^\dagger H_s]_{1_1} + b_1 [\phi^\dagger \phi]_{1_1} [H_D^\dagger H_D]_{1_1} + b_2 [\phi^\dagger \phi]_2 [H_D^\dagger H_D]_2 \\ &+ c_3 [H_D^\dagger \phi]_{3_1} [\phi^\dagger H_D]_{3_1} + c_4 [H_D^\dagger \phi]_{3_2} [\phi^\dagger H_D]_{3_2} \end{aligned} \quad (8)$$

³The term proportional to l_6 is of the form $(H^\dagger H_s)^2$

Once the symmetry $Z_{2e} \times Z_{2\mu} \times Z_{2\tau}$ is imposed, the contribution to the potential coming from $V_{\text{int}}(\phi, H_D, H_s)$ is reduced to

$$V_{\text{int}}(\phi, H_D, H_s) = \left[(b_1 + c_3 - c_4)(H_1^\dagger H_1 + H_2^\dagger H_2) + a_1 H_s^\dagger H_s \right] (\phi_1^\dagger \phi_1 + \phi_2^\dagger \phi_2 + \phi_3^\dagger \phi_3), \quad (9)$$

where we can reabsorb c_3 and c_4 in b_1 . Similarly we reduce the other terms after considering the symmetry $Z_{2e} \times Z_{2\mu} \times Z_{2\tau}$, then the full scalar potential V invariant under $S_4 \times Z_3^q \times Z_2^q \times (Z_{2e} \times Z_{2\mu} \times Z_{2\tau})$ is given by

$$\begin{aligned} V &= \mu(\phi_1\phi_1^\dagger + \phi_2\phi_2^\dagger + \phi_3\phi_3^\dagger) + (4l_3 + \alpha)(\phi_1\phi_1^\dagger)^2 + \\ &+ 2(l_2 + 5l_3 - l_4 + \alpha)\phi_2\phi_2^\dagger\phi_3\phi_3^\dagger + (l_3 + l_4 + \alpha)((\phi_2\phi_2^\dagger)^2 + (\phi_3\phi_3^\dagger)^2) + \\ &+ 2(l_2 - l_3 + l_4 + \alpha)\phi_1\phi_1^\dagger(\phi_3\phi_3^\dagger + \phi_2\phi_2^\dagger) + \\ &+ \left(H_1H_1^\dagger + H_2H_2^\dagger \right)^2 \alpha_2 + \left(-H_1H_1^\dagger + H_2H_2^\dagger \right)^2 \alpha_3 + 2H_1H_1^\dagger H_2H_2^\dagger \alpha_4 + \\ &+ \left(H_1H_1^\dagger + H_2H_2^\dagger \right) \mu_2 + \left(H_1H_1^\dagger + H_2H_2^\dagger \right) l_5 H_s H_s^\dagger + \\ &+ \mu_3 H_s H_s^\dagger + \alpha_5 (H_s H_s^\dagger)^2 + 2l_6 H_1^\dagger H_2^\dagger H_s H_s + h.c. \\ &+ b_1 (H_1^\dagger H_1 + H_2^\dagger H_2) (\phi_1\phi_1^\dagger + \phi_2\phi_2^\dagger + \phi_3\phi_3^\dagger) + \\ &+ a_1 H_s^\dagger H_s (\phi_1\phi_1^\dagger + \phi_2\phi_2^\dagger + \phi_3\phi_3^\dagger) \end{aligned} \quad (10)$$

In the case of real vev's, that is

$$\begin{aligned} \langle \phi \rangle &= (v_1, v_2, v_3) \\ \langle H_D \rangle &= (h_1, h_2) \\ \langle H_s \rangle &= v_s \end{aligned} \quad (11)$$

the equations of minimum are

$$\begin{aligned} \frac{\partial V}{\partial v_1} &= 2v_1 (b_1 (h_1^2 + h_2^2) + l_3 (8v_1^2 - 2(v_2^2 + v_3^2)) + a_1 v_s^2 + 2v_1^2 \alpha + 2(v_2^2 + v_3^2) (l_2 + l_4 + \alpha) + \mu) \\ &= 0, \\ \frac{\partial V}{\partial v_2} &= 2v_2 [b_1 (h_1^2 + h_2^2) + a_1 v_s^2 + \mu + \\ &+ 2(l_4 (v_1^2 + v_2^2 - v_3^2) + l_2 (v_1^2 + v_3^2) + l_3 (-v_1^2 + v_2^2 + 5v_3^2) + (v_1^2 + v_2^2 + v_3^2) \alpha)] \\ &= 0, \\ \frac{\partial V}{\partial v_3} &= 2v_3 [b_1 (h_1^2 + h_2^2) + a_1 v_s^2 + \mu \\ &+ 2(l_2 (v_1^2 + v_2^2) + l_4 (v_1^2 - v_2^2 + v_3^2) + l_3 (-v_1^2 + 5v_2^2 + v_3^2) + (v_1^2 + v_2^2 + v_3^2) \alpha) +] \\ &= 0, \end{aligned} \quad (12)$$

and

$$\begin{aligned}
\frac{\partial V}{\partial h_1} &= 2h_1 (b_1 (v_1^2 + v_2^2 + v_3^2) + l_5 v_s^2 + 2h_1^2(\alpha_2 + \alpha_3) + 2h_2^2(\alpha_2 - \alpha_3 + \alpha_4) + \mu_2) + 2l_6 h_2 v_s^2 = 0, \\
\frac{\partial V}{\partial h_2} &= 2h_2 (b_1 (v_1^2 + v_2^2 + v_3^2) + l_5 v_s^2 + 2h_2^2(\alpha_2 + \alpha_3) + 2h_1^2(\alpha_2 - \alpha_3 + \alpha_4) + \mu_2) + 2l_6 h_1 v_s^2 = 0, \\
\frac{\partial V}{\partial v_s} &= 2v_s ((h_1^2 + h_2^2) l_5 + a_1 (v_1^2 + v_2^2 + v_3^2) + 2v_s^2 \alpha_5 \mu_3 + 2l_6 h_1 h_2) = 0.
\end{aligned} \tag{13}$$

From the second and third equations we obtain $v_2 = v_3 \equiv v$. From the first and second equation we found

$$v_1 = \frac{\sqrt{l_2 - 8l_3 + 2l_4} v_3}{\sqrt{l_2 - 5l_3 + l_4}} \equiv rv \tag{14}$$

We redefine $h_2 = \delta h_1$, and the remaining equations are rewritten as

$$\begin{aligned}
2v^2 (2(l_2 - l_3 + l_4 + 2l_3 r^2) + (2 + r^2)\alpha) + b_1 h_1^2 (1 + \delta^2) + \mu + a_1 \chi^2 &= 0 \\
b_1 (2 + r^2) v^2 + 2h_1^2 (\alpha_2 + \alpha_3 + (\alpha_2 - \alpha_3 + \alpha_4)\delta^2) + \mu_2 + (l_5 + l_6 \delta) \chi^2 &= 0 \\
(b_1 (2 + r^2) v^2 + 2h_1^2 (\alpha_2 - \alpha_3 + \alpha_4 + (\alpha_2 + \alpha_3)\delta^2) + \mu_2) + \frac{(l_6 + l_5 \delta) \chi^2}{\delta} &= 0 \tag{15} \\
a_1 (2 + r^2) v^2 + h_1^2 (l_5 + 2l_6 \delta + l_5 \delta^2) + \mu_3 + 2\alpha_5 \chi^2 b_1 h_1^2 + a_1 v_s^2 + \\
+ 2v_s^2 (l_2 (1 + r^2) + 2(3l_3 + \alpha) + r^2(-l_3 + l_4 + \alpha)) + \mu &= 0
\end{aligned}$$

From the second and third of these equations we found two solutions for δ , one is $\delta = \pm 1$, and the other is the one we are interested in

$$\delta = \frac{l_6 \chi^2}{2h_1^2 (2\alpha_3 - \alpha_4)}, \tag{16}$$

it is clear that in the limit $l_6 \rightarrow 0$, then $\delta \rightarrow 0$.

From the rest of equations we can determine v , v_s and h_1 . It is straightforward to compute the Hessian $\partial^2 V / \partial u_i \partial u_j$, where $u = (v_1, v_2, v_3, h_1, h_2, v_s)$, of the Higgs potential. We found that for a large region of the parameter space the Hessian is definite positive, therefore the solution we found is a real minimum. Summarizing the structure of the vev's, for the doublet and the singlet of S_4 we have $H_1 = h_1$, $H_2 = \delta h_1$ and $H_s = v_s$, so the alignment of the doublet is of the form

$$H_D \sim (1, \delta). \tag{17}$$

In the limit $l_6 = 0$, the alignment takes the form $H_D \sim (1, 0)$.

For the triplet of S_4 , $v_1 = rv$ and $v_2 = v_3 \equiv v$, so the alignment is of the form

$$\phi \sim (r, 1, 1). \tag{18}$$

Notice that in the case $l_4 = 3l_3$, implies $r = 1$, and the alignment takes the form $\phi \sim (1, 1, 1)$.

4 Lepton sector and maximal atmospheric angle

The charged lepton mass matrix is diagonal with masses proportional to the Yukawa couplings y_e , y_μ and y_τ .

When the ϕ Higgs doublet takes vev as, see equation (18),

$$\phi \sim (r, 1, 1),$$

the Majorana neutrino mass matrix takes the form

$$\begin{aligned} M_\nu &= l_1 \begin{pmatrix} r^2 & r & r \\ & 1 & 1 \\ & & 1 \end{pmatrix} + l_2 \begin{pmatrix} 2 & 1+r & 1+r \\ & 2r & 1+r^2 \\ & & 2r \end{pmatrix} + l_3 \begin{pmatrix} 2+4r^2 & -2-r & -2-r \\ & 1-4r & 5+r^2 \\ & & 1-4r \end{pmatrix} + \\ &+ l'_3 \begin{pmatrix} -2 & r & r \\ & 1 & -1-r^2 \\ & & 1 \end{pmatrix} + l_4 \begin{pmatrix} 1 & 0 & 0 \\ & 0 & 1 \\ & & 0 \end{pmatrix} + l_5 \begin{pmatrix} 0 & 1 & 1 \\ & 1 & 0 \\ & & 1 \end{pmatrix} \\ &+ l_6 \begin{pmatrix} 2(2r^2-2) & -2+2r & -2+2r \\ & 2(2-2r) & -2r^2+2 \\ & & 2(2-2r) \end{pmatrix} \equiv \begin{pmatrix} x & y & y \\ y & z & w \\ y & w & z \end{pmatrix} \end{aligned} \quad (19)$$

where $l_i = v^2 \lambda_i / \Lambda$ and $r = v_1 / v_3$. The matrix M_ν is $\mu \leftrightarrow \tau$ invariant therefore the atmospheric angle is maximal and the reactor angle is zero [24]. The solar angle depend from the parameter r and it is unpredicted. In the limit $r \rightarrow 1$ (see the potential) we have that $x + y = w + z$ and the solar angle is trimaximal.

We have three different eigenvalues:

$$\begin{aligned} m_1 &= \frac{1}{2} \left(w + x + z - \sqrt{w^2 + x^2 + 8y^2 + z^2 + 2wz - 2x(w+z)} \right) \\ m_2 &= \frac{1}{2} \left(w + x + z + \sqrt{w^2 + x^2 + 8y^2 + z^2 + 2wz - 2x(w+z)} \right) \\ m_3 &= z - w, \end{aligned} \quad (20)$$

and it is possible to reproduce the ratio $\alpha = \Delta m_{\text{sol}}^2 / \Delta m_{\text{atm}}^2$.

5 The quark sector

In order to have small mixings and a hierarchical mass pattern for quarks we associate the quarks Q_L and q_L^c in a $2 \oplus 1_1$ irreducible representation of S_4 . The assignment for particles of the model are shown in table 1. From the Yukawa Lagrangian in eq. (5), once the electroweak symmetry is broken we obtain the mass matrix for the quarks

$$M_q = \begin{pmatrix} 0 & Y_2 h_s & Y_4 h_2 \\ Y_2 h_s & 0 & Y_4 h_1 \\ Y_5 h_2 & Y_5 h_1 & Y_3 h_s \end{pmatrix}. \quad (21)$$

From the study of the scalar potential we have the alignment

$$\langle H_D \rangle \sim (1, \delta). \quad (22)$$

As we know the nearest neighbor interaction (NNI) form of the quarks matrices is in agreement with the experiments of quark masses and mixings, so δ must be very small. This limit is obtained by setting $l_6 = 0$ in the Higgs potential. With this fine tuning, the alignment for H_D is $H_D \sim (1, 0)$ and the quark mass matrices, in eq. (21) take the form

$$M_{u,d} = \begin{pmatrix} 0 & Y_2^{u,d} h_s & 0 \\ Y_2^{u,d} h_s & 0 & Y_4^{u,d} h_1 \\ 0 & Y_5^{u,d} h_1 & Y_3^{u,d} h_s \end{pmatrix}. \quad (23)$$

Such matrices are four-zero texture and have the form of a nearest neighbor interaction, first proposed by Weinberg [25] and then extended by Fritzsch [26] (see also [27] and references therein). The mass matrices in (23) have factorizable phases [28, 29], i.e. $M_{u,d} = P_L^{u,d} \tilde{M}_{u,d} P_R^{u,d}$, where \tilde{M} has the same structure as (23) but without phases. Only two combinations of phases will enter into the CKM matrix, $P = P_L^{u*} P_L^d = \text{diag}(1, e^{i\beta_{ud}}, e^{i\alpha_{ud}})$.

We rewrite the mass matrices $\tilde{M}_{u,d}$ as

$$\tilde{M}_{u,d} = m_{t,b} \begin{pmatrix} 0 & \frac{q_{u,d}}{y_{u,d}} & 0 \\ \frac{q_{u,d}}{y_{u,d}} & 0 & b_{u,d} \\ 0 & d_{u,d} & y_{u,d}^2 \end{pmatrix}. \quad (24)$$

where

$$\begin{aligned} b_{u,d} &= \sqrt{\frac{p_{u,d} + 1 - y_{u,d}^4 - R_{u,d}}{2} - \frac{q_{u,d}^2}{y_{u,d}^2}}, \\ d_{u,d} &= \sqrt{\frac{p_{u,d} + 1 - y_{u,d}^4 + R_{u,d}}{2} - \frac{q_{u,d}^2}{y_{u,d}^2}}, \end{aligned} \quad (25)$$

and

$$R_{u,d} \equiv \sqrt{(1 + p_{u,d} - y_{u,d}^4)^2 - 4(p_{u,d} + q_{u,d}^4) + 4q_{u,d}^2 y_{u,d}^2}. \quad (26)$$

where $p_{u,d}$ and $q_{u,d}$ defined by

$$\begin{aligned} p_u &= \frac{m_u m_c}{m_t^2} & p_d &= \frac{m_d m_s}{m_b^2} \\ q_u^2 &= \frac{m_u^2 + m_c^2}{m_t^2} & q_d^2 &= \frac{m_d^2 + m_s^2}{m_b^2}. \end{aligned} \quad (27)$$

In this case, we have 4 real parameters in each mass matrix, $q_{u,d}$, $b_{u,d}$, $d_{u,d}$ and $y_{u,d}$. These parameters are rewritten in term of the masses of the quarks and the free parameters $y_{u,d}$. Therefore in the mixing matrix appears, 6 masses, two real parameters $y_{u,d}$ and the relative phases $\alpha_{ud} = \alpha_u - \alpha_d$ and $\beta_{ud} = \beta_u - \beta_d$. The CKM matrix is then given by

$$V_{CKM} = O_u^T P O_d, \quad (28)$$

where $O_{u,d}$ are the orthogonal matrices that diagonalize $\tilde{M}_{u,d}$ via

$$O_{u,d}^T \tilde{M}_{u,d} \tilde{M}_{u,d}^T O_{u,d} = \text{diag}(m_{u,d}^2, m_{c,s}^2, m_{t,b}^2). \quad (29)$$

In this case we can fit the quark mixing angles with a very good precision, with the values

$$\begin{aligned}
y_u &= 0.996333 & y_d &= 0.957981 \\
\alpha_{ud} &= -2.03052 & \beta_{ud} &= -1.49938 \\
m_u &= 2.35634 \text{ MeV} & m_c &= 1237.37 \text{ MeV} & m_t &= 174.276 \text{ GeV} \\
m_d &= 5.27743 \text{ MeV} & m_s &= 90.8056 \text{ MeV} & m_b &= 4243.63 \text{ MeV}
\end{aligned}
\tag{30}$$

we found

$$V^{\text{th}} = \begin{pmatrix} 0.974328 & 0.2251 & 0.00380224 \\ 0.22497 & 0.973513 & 0.0407534 \\ 0.00855318 & 0.0400268 & 0.999162 \end{pmatrix}
\tag{31}$$

which is in excellent agreement with the experimental values in eq. (2). The prediction for the Jarlskog invariant is $J = 3.1 \times 10^{-5}$ which is also in agreement with the experimental central value $J^{\text{exp}} = (2.92 \pm 0.15) \times 10^{-5}$. Note that one of the phases in P is redundant in the sense that to make a fit of quark mixings it is enough with one phase, as in [27].

6 Conclusions

We have studied a model based on the flavor symmetry S_4 for leptons and quarks where all charged fermion masses arise from renormalizable Lagrangian and neutrino mass matrix is induced from dimension five Weinberg operator. The model contains six Standard Model Higgs fields transforming respectively as a triplet, a doublet and a singlet under S_4 . We have studied the minimization of the full potential. We obtain a $\mu - \tau$ exchange invariant neutrino mass matrix predicting maximal atmospheric angle while the solar angle is undetermined. The model is compatible with normal, inverse and degenerate hierarchies for the neutrino masses. The quark mass matrices pattern have the form of a nearest neighbor interaction with four-zero texture. We give a numerical solution showing that it is possible to reproduce correctly the CKM mixing matrix within the experimental errors. It is well known that models with more than one Higgs $SU(2)$ doublet may in general, have tree level flavor changing neutral currents (FCNC) [31]. A complete analysis of this problem as well as a more deep analysis of the Higgs phenomenology, masses and decays widths, will go beyond the scope of the present paper, and we would like to leave this problems to a future work. An analysis of these problems was done in the scenario of an S_3 flavor symmetry in [32] where the strongest constraint on the scalar masses arises from the neutral K meson mixing.

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A The group S_4

S_4 is the finite group of the permutations of four objects (for a short introduction to S_4 , see for instance [30] and references therein). S_4 has 5 irreducible representations, two singlets 1_1 and 1_2 , a doublet 2 , and two triplets 3_1 and 3_2 .

The group S_4 is defined can be defined by two generators S and T that satisfy

$$S^4 = T^3 = (ST^2)^2 = \mathbb{1}. \quad (32)$$

In the basis of T diagonal the generators can be written for the different representations as

representation 1_1 : $S = 1, T = 1$

representation 1_2 : $S = -1, T = 1$

representation 2 : $S = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, T = \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}$

representation 3_1 : $S = \frac{1}{3} \begin{pmatrix} -1 & 2\omega & 2\omega^2 \\ 2\omega & 2\omega^2 & -1 \\ 2\omega^2 & -1 & 2\omega \end{pmatrix}, T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}$

representation 3_2 : $S = \frac{1}{3} \begin{pmatrix} 1 & -2\omega & -2\omega^2 \\ -2\omega & -2\omega^2 & 1 \\ -2\omega^2 & 1 & -2\omega \end{pmatrix}, T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}.$

In this basis the rules for the non trivial products of two irreducible representations are:

The multiplication rules with the 2-dimensional representation are the following:

$$2 \otimes 2 = 1_1 \oplus 1_2 \oplus 2 \quad \text{with} \quad \begin{cases} 1_1 \sim \alpha_1\beta_2 + \alpha_2\beta_1 \\ 1_2 \sim \alpha_1\beta_2 - \alpha_2\beta_1 \\ 2 \sim \begin{pmatrix} \alpha_2\beta_2 \\ \alpha_1\beta_1 \end{pmatrix} \end{cases}$$

$$2 \otimes 3_1 = 3_1 \oplus 3_2 \quad \text{with} \quad \begin{cases} 3_1 \sim \begin{pmatrix} \alpha_1\beta_2 + \alpha_2\beta_3 \\ \alpha_1\beta_3 + \alpha_2\beta_1 \\ \alpha_1\beta_1 + \alpha_2\beta_2 \end{pmatrix} \\ 3_2 \sim \begin{pmatrix} \alpha_1\beta_2 - \alpha_2\beta_3 \\ \alpha_1\beta_3 - \alpha_2\beta_1 \\ \alpha_1\beta_1 - \alpha_2\beta_2 \end{pmatrix} \end{cases}$$

$$2 \otimes 3_2 = 3_1 \oplus 3_2 \quad \text{with} \quad \begin{cases} 3_1 \sim \begin{pmatrix} \alpha_1\beta_2 - \alpha_2\beta_3 \\ \alpha_1\beta_3 - \alpha_2\beta_1 \\ \alpha_1\beta_1 - \alpha_2\beta_2 \end{pmatrix} \\ 3_2 \sim \begin{pmatrix} \alpha_1\beta_2 + \alpha_2\beta_3 \\ \alpha_1\beta_3 + \alpha_2\beta_1 \\ \alpha_1\beta_1 + \alpha_2\beta_2 \end{pmatrix} \end{cases}$$

The multiplication rules with the 3-dimensional representations are the following:

$$3_1 \otimes 3_1 = 3_2 \otimes 3_2 = 1_1 \oplus 2 \oplus 3_1 \oplus 3_2 \quad \text{with} \quad \left\{ \begin{array}{l} 1_1 \sim \alpha_1\beta_1 + \alpha_2\beta_3 + \alpha_3\beta_2 \\ 2 \sim \begin{pmatrix} \alpha_2\beta_2 + \alpha_1\beta_3 + \alpha_3\beta_1 \\ \alpha_3\beta_3 + \alpha_1\beta_2 + \alpha_2\beta_1 \end{pmatrix} \\ 3_1 \sim \begin{pmatrix} 2\alpha_1\beta_1 - \alpha_2\beta_3 - \alpha_3\beta_2 \\ 2\alpha_3\beta_3 - \alpha_1\beta_2 - \alpha_2\beta_1 \\ 2\alpha_2\beta_2 - \alpha_1\beta_3 - \alpha_3\beta_1 \end{pmatrix} \\ 3_2 \sim \begin{pmatrix} \alpha_2\beta_3 - \alpha_3\beta_2 \\ \alpha_1\beta_2 - \alpha_2\beta_1 \\ \alpha_3\beta_1 - \alpha_1\beta_3 \end{pmatrix} \end{array} \right.$$

$$3_1 \otimes 3_2 = 1_2 \oplus 2 \oplus 3_1 \oplus 3_2 \quad \text{with} \quad \left\{ \begin{array}{l} 1_2 \sim \alpha_1\beta_1 + \alpha_2\beta_3 + \alpha_3\beta_2 \\ 2 \sim \begin{pmatrix} \alpha_2\beta_2 + \alpha_1\beta_3 + \alpha_3\beta_1 \\ -\alpha_3\beta_3 - \alpha_1\beta_2 - \alpha_2\beta_1 \end{pmatrix} \\ 3_1 \sim \begin{pmatrix} \alpha_2\beta_3 - \alpha_3\beta_2 \\ \alpha_1\beta_2 - \alpha_2\beta_1 \\ \alpha_3\beta_1 - \alpha_1\beta_3 \end{pmatrix} \\ 3_2 \sim \begin{pmatrix} 2\alpha_1\beta_1 - \alpha_2\beta_3 - \alpha_3\beta_2 \\ 2\alpha_3\beta_3 - \alpha_1\beta_2 - \alpha_2\beta_1 \\ 2\alpha_2\beta_2 - \alpha_1\beta_3 - \alpha_3\beta_1 \end{pmatrix} \end{array} \right.$$

The products of two irreducible representations $\bar{A} \times B$ is different from the $A \times B$ since the T and the S generators are complex. When we multiply the complex conjugate of a two-dimensional representation with another two-dimensional representation, we have to interchange the indices $1 \leftrightarrow 2$ of the complex conjugate representation, that is, for instance in $\bar{2} \otimes 2$ we interchange $\alpha_1 \leftrightarrow \alpha_2$ and the product is given by

$$\bar{2} \otimes 2 = 1_1 \oplus 1_2 \oplus 2 \quad \text{with} \quad \left\{ \begin{array}{l} 1_1 \sim \alpha_2\beta_2 + \alpha_1\beta_1 \\ 1_2 \sim \alpha_2\beta_2 - \alpha_1\beta_1 \\ 2 \sim \begin{pmatrix} \alpha_1\beta_2 \\ \alpha_2\beta_1 \end{pmatrix} \end{array} \right.$$

When we multiply the complex conjugate of a three-dimensional representation with another representation, we have to interchange the indices $2 \leftrightarrow 3$ of the complex conjugate representation, that is, for instance in $\bar{3} \otimes 3$ we interchange $\alpha_2 \leftrightarrow \alpha_3$. Similarly in the case of $\bar{2} \otimes 3$ we have to interchange the indices $1 \leftrightarrow 2$ for the doublet, that is $\alpha_1 \leftrightarrow \alpha_2$.

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