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TOPOGRAPHY EFFECTS ON THE DISPLACEMENTS AND GRAVITY CHANGES DUE TO MAGMA INTRUSION

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Abstract: The paper describes a method for including topographic effects in a thermo-visco-elastic model. An approximate methodology for the consideration of topography in the computation of thermo-viscoelastic displacement and gravity changes was used. It gave us a relatively general and simple solution useful for solving the inverse problem. Our results show that for volcanic areas with an important relief, the perturbation of the thermo-viscoelastic solution (deformation and total gravity anomaly) due to topography can be quite significant. In consequence, neglect of topographic effects can give a rise to significant errors in the estimation of surface displacements and gravity changes, and therefore in the estimation of the intrusion characteristics obtained solving the inverse problem.

Key words: ground deformation modelling, topographic effect.

Introduction

A magma intrusion in the Earth's crust will cause effects (for example deformation and gravity changes) related to its mass as well to the pressurization of the chamber due to overfilling or temperature changes. The deformation due to an expanding or contracting magma chamber has frequently been modeled as a dilatation source in an elastic halfspace. The simplest example is the Mogi's model (Mogi 1958). The source of deformation is hypothesized as a hydrostatic pressure source, embedded in a homogeneous elastic half-space. The pressure source is regarded as a strain nucleus, that is a point like a source with radial expansion, that is similar to the inflation of a spherical cavity. A finite source can be satisfactorily approximated by a point source, provided that the source dimensions are small with respect to source depth. Other models have been used to examine different types of deformation sources. For instance, McTigue (1987) has derived an approximate analytical solution similar to the Mogi model that includes higher-order terms to represent the finite shape of a spherical body; Davis (1986) has developed approximate solutions for ellipsoidal magma chambers.

The theory of the thermoelastic phenomena shows that thermoelastic stresses and deformations can arise in an elastic continuum if an inhomogeneous temperature field exists in the media (see e.g. Nowacki 1962). Thus we can expect various thermoelastic phenomena to occur in some regions like volcanic areas, with an anomalous behaviour of the heat flow. In the geodynamic theory, it is well-known that thermo-elastic strains and stresses play a considerable role in the stress state of the lithosphere and its dynamics, specially in localities with pronounced geothermal anomalies (Combs & Hadley 1977; Teisseyre 1986). For this reason, Hvoždara & Rosa (1979, 1980) carried out a theoretical analysis of thermo-elastic deformations of a homogeneous half-space due to a point or linear source of heat, located at a particular depth in the half-space. They proved that thermo-elastic stresses are expansive in type and that they considerably disturb the normal lithostatic stress, specially near the surface of the half-space. Hvoždara & Brimich (1991) presented basic formulae and the results of numerical calculations for the simplified mathematical models of two important effects due to magmatic bodies in the Earth's lithosphere: a) static thermoelastic deformations, b) static elastic deformations due to upward pressure. The magmatic body is approximated by a finite volume source of heat in the first model and by a concentrated vertical force in the second one. The formulae for gravity anomaly due to non-uniform extension connected with thermo-elastic deformations were derived in Hvoždara & Brimich (1995).

The elastic and thermoelastic models described above have allowed an explanation of the measured geodetic data in many volcanic regions, particularly when movements occur on relatively short timescales. However, in many cases the elastic models seem to be unable to reproduce the observed uplifts unless unrealistic overpressures are considered and they are often unable to explain the simultaneously observed displacement and gravity changes (see e.g. Berrino et al. 1984; Jentzsch et al. 2001). The presence of incoherent materials and high temperatures produce a lower effective viscosity of the Earth's crust, making it necessary to consider inelastic properties of the media (Bonafede et al. 1986; Bonafede 1990; De Natale & Pingue 1993). Bonafede et al. (1986) and Bonafede (1990) worked out analytical solutions for the displacement and associated stress fields induced by a pressure point source in a viscoelastic half-space and showed that the viscoelastic response may reproduce the observed uplifts with plausible overpressure values. Several analytical models with inelastic properties have been proposed by different authors. Hvoždara (1992) considered a model of a viscoelastic half-space of the Kelvin type, with a point source of heat. This paper presents basic formulae for the stress and strain components of the deformation field. The thermo-viscoelastic deformation field due to a source of heat of prismatic shape embedded in a viscoelastic half-space and the formulae for the gravity changes due to the volume dilatation connected with the deformation field are derived in Brimich (2000). Folch et al. (2000) obtained and compared analytical and numerical solutions for ground displacement caused by an overpressurized magma chamber placed in a linear viscoelastic media composed by a layer over a half-space. Different parameters such as the size, depth or shape of the chamber, crustal rheologies or topography are considered and discussed. The effect of the topography is also considered. Fernández et al. (2001) presented a method extension of a deformation model previously developed to compute effects due to volcanic loading in elastic-gravitational layered media (Rundle 1982, 1983; Fernández & Rundle 1994; Fernández et al. 1997), for the computation of time-dependent deformation, potential and gravity changes due to magma intrusions in a layered viscoelastic medium. They assumed a plane Earth geometry consisting of welded elastic and viscoelastic layers overlying a viscoelastic half-space. They found that, in line with prior results obtained by other authors (see e.g. Bonafede et al. 1986), introducing viscoelastic properties in all of part of the medium can extend the displacements and gravity changes considerably, and therefore lower pressure increases are required to model given observed effects. In their results, the viscoelastic effects seem to depend mainly on the rheological properties of the layer (zone) where the intrusion is located, rather than on the rheology of the whole medium. They applied the model to the 1982–1984 uplift episode at Campi Flegrei modelling simultaneously observed vertical displacement and gravity changes. Their results clearly showed that for an appropriate interpretation of observed effects, it is necessary to consider the gravitational field in the inelastic theoretical models. This consideration can change the value and pattern of time-dependent deformation as well as the gravity changes, allowing us to explain cases of displacement without noticeable gravity changes or vice versa, cases with uplift and increment of gravity values, and others.

Many of the models with inelastic properties considered so far are analytical, assume point source of deformation and a flat, horizontal free surface. Volcanoes are commonly associated with significant topographic relief. The approximation of Earth's surface as flat and use of half-space solutions can lead to erroneous interpretation of the deformation data (see e.g. Cayol & Cornet 1998; Williams & Wadge 1998, 2000; Folch et al. 2000). Williams & Wadge (1998, 2000) and Cayol & Cornet (1998) pointed out that topography has a significant effect on predicted surface deformation by elastic models in regions of significant relief. Those authors pointed out that, in the elastic case, the interpretation of ground-surface displacements with half-space models can lead to erroneous estimations. Cayol & Cornet (1998) found that the steeper the volcano, the flatter the vertical displacement field. Folch et al. (2000) demonstrated that this result is dramatically emphasised in the viscoelastic case, where the topography changes in a very important way both the magnitude and the pattern of the displacement field. They also showed that neglect of the topographic effects may, in some cases, introduce an error greater than that implicit in the point source hypothesis. These are the reasons we want to study the effect of topography on the surface displacements and gravity changes obtained by the thermo-viscoelastic model described by Hvoždara (1992, 1998) and Brimich (2000).

Thus we can quantify the error produced in the thermo-visco-elastic solution. The effect of the topography is represented allowing point source depth to vary with the relief, thus we relaxed the restriction of a flat free surface. If the topographic perturbation is due primarily to the distance of the free surface from the magma chamber rather than the local shape of the free surface this type of solution should give a reasonable approximation (Williams & Wadge 1998).

If we use this methodology to introduce the topographic effects in the thermo-elastic and thermo-viscoelastic models we still get analytical solutions. The advantage of this assumption is something very clear, it allows us to obtain a relatively general and simple solution useful for solving the inverse problem (see e.g. Michalewicz 1994; Yu 1995; Yu et al. 1998; Tiampo et al. 2000). Numerical methods, such as the finite element or boundary element methods may be used to include the topographic effects when an accurate solution is desired for a particular deformation model, but such methods can be time-consuming in the length of time required to design a mesh and in the actual computation time.

Thermo-viscoelastic deformation model

Elastic and thermoelastic models have allowed an explanation of the measured geodetic data in many volcanic regions, particularly when movements occur on relatively short time-scales. The time evolution of heating of the halfspace (lithosphere) and associated deformation with it can be mathematically calculated by means of the theory of thermo-viscoelastic deformation. We consider a non-steady point source of heat located at depth x in the viscoelastic halfspace $z > 0$. For the uncoupled thermo-viscoelastic problem, the temperature disturbance field $T(x,y,z,t)$ due to this source must obey the equation (Nowacki 1962):

$$\lambda_T \nabla^2 T + w\delta(x)\delta(y)\delta(z-\zeta)H(t) = c_p \rho \frac{\partial T}{\partial t} \qquad (1)$$

where λ_T is heat conductivity, c_p is specific heat under constant pressure, ρ is the material density, w is the power of the heat source, δ is the Dirac function, $H(t)$ is Heavside's unit step function:

$H(t) = 0$ for $t < 0$,
$H(t) = 1$ for $t > 0$.

If the surface of the half-space is kept at a constant temperature, which can be taken to be zero, then we have the boundary condition on the surface $z = 0$:

$$T(x,y,z,t)\big|_{z=0} = 0 \qquad (2)$$

Considering the initial temperature disturbance in all points of the half-space as zero, we obtain the initial condition for $t = 0$:

$$T(x,y,z,t)\big|_{t=0}=0 \tag{3}$$

Then, the solution of equation (1) under the boundary and initial conditions, is obtained in the form (Carslaw & Jaeger 1959):

$$T(r,z,t)=\frac{w}{4\pi\lambda_T}\left\{R_1^{-1}erfc\left(\frac{R_1}{\sqrt{4\kappa t}}\right)-R_2^{-1}erfc\left(\frac{R_2}{\sqrt{4\kappa t}}\right)\right\} \tag{4}$$

where $R_1=[r^2+(z-\xi)^2]^{1/2}$, $R_2=[r^2+(z+\xi)^2]^{1/2}$, with $r=(x^2+y^2)^{1/2}$ being the horizontal distance from the polar axis z and $\kappa=\lambda_T/(c_p\rho)$. The complementary error function $erfc(s)$ is defined by:

$$erfc(s)=1-\frac{2}{\sqrt{\pi}}\int_0^s e^{-u^2}du \tag{5}$$

The time and space variable temperature disturbance causes variable stresses and displacements. Since the process of temperature change is much slower in comparison with the propagation time of elastic waves, it is sufficient to consider the static equilibrium equation for a viscoelastic body:

$$\sum_{j=1}^{3}\frac{\partial\sigma_{ij}}{\partial x_j}=0\quad i=1,2,3 \tag{6}$$

where σ_{ij} is the viscoelastic stress tensor. In the purely elastic case the components σ_{ij} are given by the Duhamel-Neumann relation, but in the viscoelastic case the stress-strain relations are given by more complicated formulae (Nowacki 1962).

In order to obtain the actual temporal behaviour of the displacements and stresses we have to Laplace transform these quantities. The detailed calculation was performed in Hvoždara (1992).

The calculation was performed for a Kelvin body, for which the generalized Duhamel-Neumann relation has the form of:

$$\sigma_{ij}(x_r,t)=2\mu\left(1+t^*\frac{\partial}{\partial t}\right)e_{ij}(x_r,t)+$$
$$+\delta_{ij}\left\{\frac{1}{3}\left[3K-2\mu\left(1+t^*\frac{\partial}{\partial t}\right)\Theta(x_r,t)\right]-3K\alpha_T T(x_r,t)\right\} \tag{7}$$

where e_{ij} is the strain tensor, μ is the modulus of rigidity (the Lamé's constant), $K=\lambda+2\mu/3$ is the bulk modulus, $t^*=\eta/\mu$ is decay time, η being the viscosity of material, α_T is the thermal coefficient of the linear expansion and $\Theta(x_r,t)$ is dilatation.

For the time dependence of displacements u and stresses σ on the surface of the viscoelastic half-space, we have the following formulae (Hvoždara 1992):

$$u_r(r,0,t)=\frac{Qr}{\pi}\int_0^t V(t-\tau)S_1(r,\tau)d\tau,$$

$$u_z(r,0,t)=-\frac{Q}{2\pi}\left\{\zeta R_0^{-3}b(t)+\frac{2\zeta}{\sqrt{\pi}}\int_0^t b(t-\tau)\tau^{-1}(4\kappa\tau)^{-\frac{3}{2}}e^{\left(\frac{-R_0^2}{4\kappa\tau}\right)}d\tau-\right.$$
$$\left.-\int_0^t W(t-\tau)S_2(r,t)d\tau\right\}, \tag{8}$$

$$\sigma_{rr}(r,0,t)=\frac{2Q}{\pi}\left\{\int_0^t B(t-\tau)S_0(r,\tau)d\tau-\int_0^t U(t-\tau)S_1(r,\tau)d\tau\right\},$$

$$\sigma_{\varphi\varphi}(r,0,t)=\frac{2Q}{\pi}\left\{\int_0^t N(t-\tau)S_0(r,\tau)d\tau+\int_0^t U(t-\tau)S_1(r,\tau)d\tau\right\},$$

where

$$Q=\frac{w\kappa}{\lambda_T},\qquad R_0=(r^2+\zeta^2)^{1/2},$$
$$U(t)=B(t)-N(t),$$
$$V(t)=B(t)-M(t),$$
$$W(t)=B(t)-2M(t),$$
$$B(t)=\alpha_1\left[t-\beta_1\kappa_1^{-1}\left(1-e^{\kappa_1 t}\right)\right],$$
$$N(t)=\alpha_1\alpha_3\kappa_3^{-1}\left[t-\left(\kappa_3^{-1}+\beta_1/\kappa_1\right)\left(1-e^{-\kappa_3 t}\right)+\beta_1\kappa_3(\kappa_1\kappa_3-\kappa_1^2)^{-1}\left(e^{-\kappa_1 t}-e^{-\kappa_3 t}\right)\right],$$
$$b(t)=\alpha_2\left[t-\kappa_1^{-1}\left(1-e^{-\kappa_1 t}\right)\right],$$
$$M(t)=\alpha_2\alpha_3\kappa_3^{-1}\left[t-\left(\kappa_1^{-1}+\kappa_3^{-1}\right)\left(1-e^{-\kappa_3 t}\right)+\kappa_3(\kappa_1\kappa_3-\kappa_1^2)^{-1}\left(e^{-\kappa_1 t}-e^{-\kappa_3 t}\right)\right],$$

$$\alpha_1=\mu\alpha_2,$$
$$\alpha_2=9K\alpha_T/(3K+4\mu),$$
$$\alpha_3=9K/(2\mu t^*),$$
$$\kappa_1=(3K+4\mu)/(4\mu t^*),$$
$$\kappa_3=(3K+4\mu)/(\mu t^*),$$
$$\beta_1=1-\kappa_1 t^*,$$

$$S_0(r,t)=R_0^{-3}\left(3\zeta^2R_0^{-2}-1\right)\delta(t)-\frac{2\zeta}{t\beta^4}e^{\left(-\frac{R_0^2+\zeta^2}{2\beta^2}\right)}\left[\left(1-\frac{r^2}{\beta^2}\right)I_0\left(\frac{r^2}{2\beta^2}\right)+\frac{r^2}{\beta^2}I_1\left(\frac{r^2}{2\beta^2}\right)\right],$$

$$S_1(r,t)=R_0^{-3}\delta(t)-\frac{\zeta}{t\beta^4}e^{\left(-\frac{R_0^2+\zeta^2}{2\beta^2}\right)}\left[I_0\left(\frac{r^2}{2\beta^2}\right)-I_1\left(\frac{r^2}{2\beta^2}\right)\right],$$

$$S_2(r,t)=\zeta R_0^{-3}\delta(t)-\frac{2\zeta}{t\beta^3\sqrt{\pi}}e^{\left(-\frac{R_0^2}{\beta^2}\right)},$$

$\beta=\sqrt{4\kappa t}$, $I_1=\frac{dI_0(s)}{ds}$ is a modified Bessel function of the first kind, order 1, I_0 is the same, order 0, $\delta(t)$ is the Dirac function.

Stresses σ_{zz} and σ_{rz} are zero according to the known boundary condition of the theory of elasticity. The heat flow anomaly due to the temperature field is:

$$q_z(r,0,t)=\lambda_T\left[\frac{\partial\ T}{\partial\ z}\right]_{z=0}=$$
$$=\frac{w\zeta}{2\pi R_0^3}\left\{erfc\left[\frac{R_0^2}{4\kappa t}+R_0^2(\pi\kappa t)^{-1}\exp\left(\frac{-R_0^2}{4\kappa t}\right)\right]\right\} \tag{9}$$

In addition we can calculate the perturbation of gravity due to a point source of heat. There are two principal reasons for the gravity changes. The first one is the change of density ρ_0 by the increment of $\Delta\rho$ due to volumetric dilatation. The density change $\Delta\rho$ generates perturbation of the gravity potential, which for $z \geq 0$ obeys the Poisson equation. For $z < 0$ this potential satisfies the Laplace equation, that is, it is a harmonic function. Brimich (1998) obtained the following formula for the gravity anomaly:

$$\Delta g_{TVE}(r,0,t) = \frac{1}{2} G\rho_0 Q \left\{ W(t)\frac{-\zeta}{R_0^3} + 2\zeta \int_0^t W_2(t-\tau)\frac{e^{-R_0^2/(4\kappa\tau)}}{(2\kappa\tau)\sqrt{4\pi\kappa\tau^3}}\, d\tau \right\}, \quad (10)$$

where G = 6.67×10^{11} $kg^{-1}m^3s^{-2}$ is the gravity constant and

$$W_2(t) = \frac{9K\alpha_T}{6K+2\mu}\left[t - \kappa_3^{-1}(1-e^{-\kappa_3 t})\right].$$

The formula given by (10) determines the integral effect of the volumetric dilatation due to thermal expansion in a viscoelastic half-space.

The second reason for the gravity changes is the free-air change and Bouguer correction as an effect of vertical uplift of the surface above the source of heat. The gravity effect due to the upward doming of the surface of the Earth, that originally was the plane z = 0, is given by the sum of the free-air change of gravity and the Bouguer correction:

$$\Delta g_{FAB} = \left[-\gamma_{FA} + 2\pi G\rho_0\right] h(r), \quad (11)$$

where $h(r) = -u_z(0,r,t)$ is the doming, that is the vertical uplift, $\gamma_{FA} = 3.086\times10^{-6}$ ms^{-2}/m is the vertical gradient of normal gravity and $2\pi G\rho_0$ is the Bouguer correction.

The effect of topography

In this chapter the effect of the topography on the surface displacements and gravity changes obtained by the thermo-viscoelastic model described above is investigated. We propose a simple method of evaluating the topographic effects in a three-dimensional deformation model which consists of assuming a different source depth at each point for which a solution is desired. This methodology was introduced by Williams & Wadge (1998) and permits that we still have analytical solutions even if we relax the restriction of a free flat surface. The analytical solutions are useful for solving the inverse problem and avoiding inclusion of numerical models that can be time consuming. Therefore, we allow magma chamber depth to vary with topography, thus in the equations (8), (10) and (11) ζ is replaced by $\zeta' = \zeta + H$, where H is the point elevation, we want to obtain the viscoelastic deformation and gravity changes. If the topographic effect is due primarily to the distance of the free surface from magma chamber rather than the local shape of the free surface, this methodology comes near the actual case (Williams & Wadge 1998, 2000). To study the effect of the topography the relief of an area can be represented by a volcanic cone with height H and average slope of the flanks α. We consider the surface displacements and respective gravity changes caused by a point source of heat located beneath an axis-symmetric volcano with average slopes of their flanks of 0°, 15°, 20°, and 30°. The volcano models with slopes of 15° and 20° are representative of basaltic shield volcanoes, whereas the volcano models with slopes of 30° are representative of andesitic volcanoes (Cayol & Cornet 1998). Schematic illustration of the problem is given in Fig. 1. The effect of the topography is neglected when $\alpha = 0$ ($H = 0$). The rheological behaviour of the crust is represented by a homogeneous half-space of Kelvin's type with Lamé parameters λ and μ with the topography characterized by the same parameters. As a reference model we have used a point source of heat at the depth $\zeta = 2$ km, its intensity (power) $w = 2.6384\times10^7$ W in order to achieve the epicentral heat flow anomaly $q_z(0) = 42$ mW/m², since $q_z(0) = w(2\pi\,\zeta^2)^{-1}$. The density and elastic parameters of the half-space and the thermal parameters of the medium are shown in Table 1.

We set the decay time for the Kelvin's type of the viscoelastic body as $t^* = 3.3\times10^{12}$ s, where $t^* = \eta/\mu$ and η is the mean viscosity of the crustal rocks. The time evolution of the horizontal and vertical displacements u_r, u_v and terms of total grav-

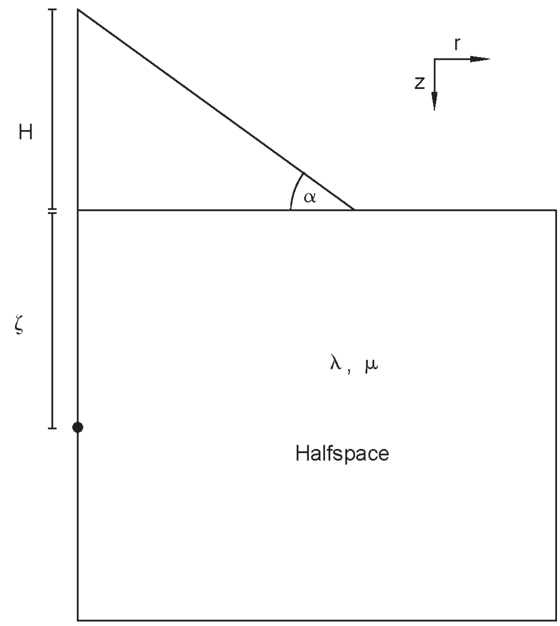

Fig. 1. Characteristics of the model used to determine the influence of the topography on the surface displacements.

Table 1: Properties of the homogeneous half-space considered in Figs. 2 to 4. Keys: λ and μ Lamé parameters, K Bulk modulus, ν Poisson ratio, ρ density, λ_T coefficient of heat conductivity, c_p specific heat under constant pressure, α_T coefficient of linear thermal expansion and κ coefficient of thermal conductivity.

λ	=	7.05×10^{10} Pa
μ	=	6.075×10^{10} Pa
K	=	1.11×10^{10} Pa
ν	=	0.26857
ρ	=	3 000 kg m^{-3}
λ_T	=	3 $Wm^{-1}K^{-1}$
c_p	=	840 $Jkg^{-1}K^{-1}$
α_T	=	10^{-6} K^{-1}
κ	=	1.1905×10^{-6} m^2s^{-1}

ity anomaly $\Delta g_{sum} = \Delta g_{TVE} + \Delta g_{FAB}$ was calculated for various times using multiples of the characteristic heat disturbance time $t_\kappa = \zeta^2 (4\kappa\tau)^{-1}$, which corresponds to the value e^{-1} of the known heat propagation factor $\exp(-\zeta^2/4\kappa\tau)$ in the epicentre of the heat source. The results for the depth $\zeta = 2$ km ($t_\kappa = 8.37 \times 10^9$ s) are presented in Figs. 2 to 4. The results are compared with the flat-surface solution given by the analytical method. The curves for $t/t_\kappa = 0.5, 1.0, 2.0, 3.0, 5.0, 7.0$ gradually approach the curves that were calculated by means of the formulae for the stationary thermo-elastic problem (Hvoždara & Brimich 1991). We can see that the displacements and gravitational anomalies approach their static values slowly, because of the viscoelastic behaviour of the half-space, which is mathematically expressed by the convolution integrals in the previous chapter. As is pointed out by other authors, the principal effect of topography is a reduction of vertical displacement and total gravity anomaly magnitudes due to the greater distance from the source to the free surface (the steeper the volcano, the flatter the displacement field and the gravity change). In Fig. 2 the changes of the vertical displacements caused by the topography are presented. Fig. 4 shows that the topography effect changes the pattern of the total gravity anomaly, too. Vertical displacements and total gravity changes are mostly influenced by the topographic effect, thus neglecting the topog-

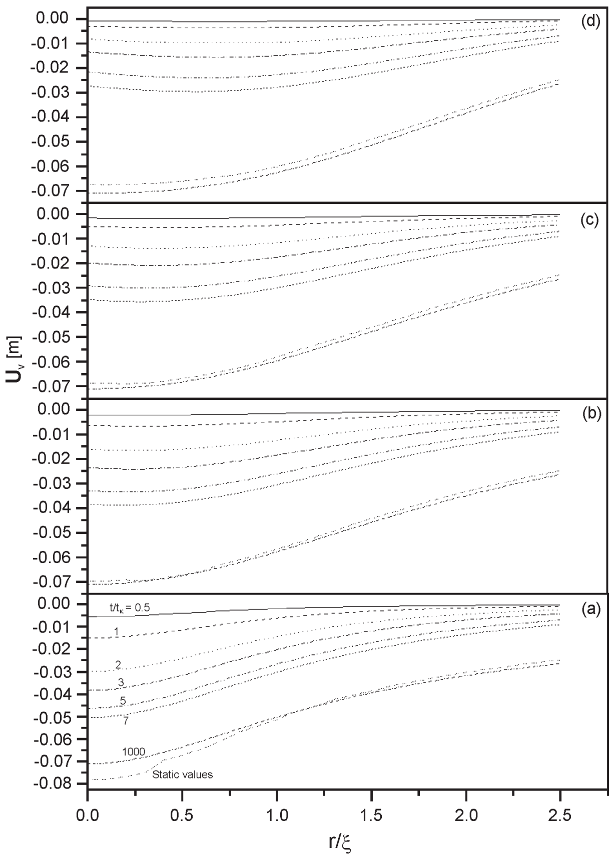

Fig. 2. Thermo-viscoelastic vertical displacement in meters computed for different time values, the source described in Table 1 and considering (a) a flat surface, and (b)–(d) axis-symmetric volcanic cone with an average slope of the flanks of 15°, 20° and 30° respectively. t_κ is the decay time defined in the text.

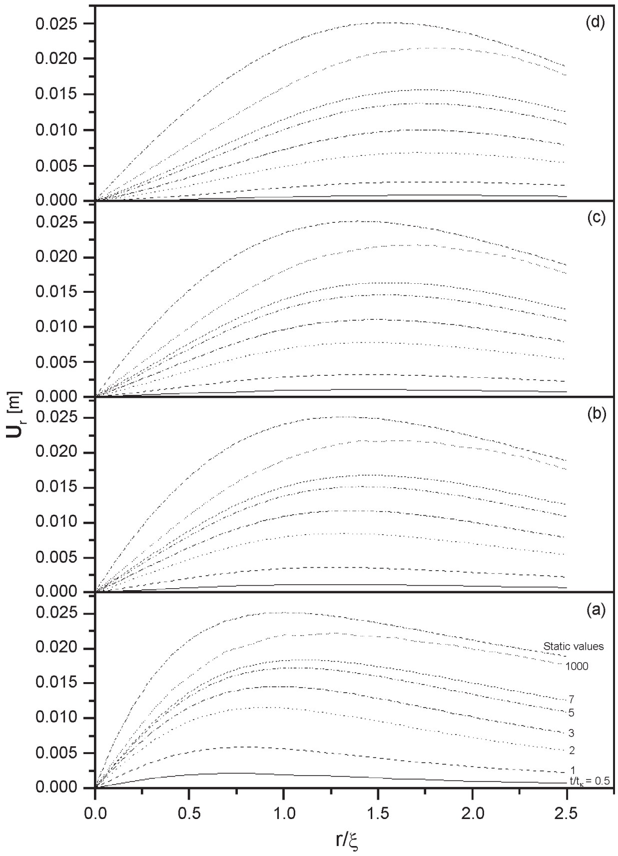

Fig. 3. Same as Fig. 2 but for radial displacements.

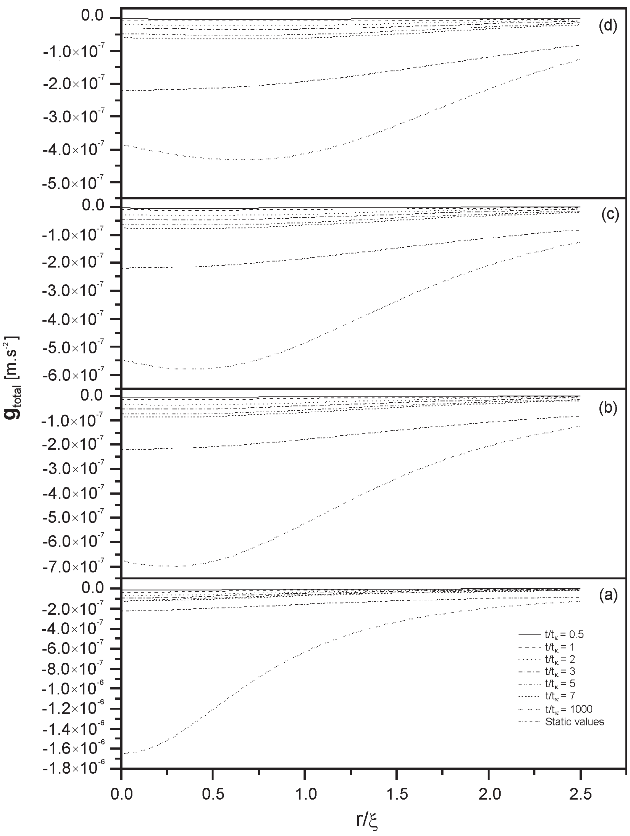

Fig. 4. Same as Fig. 2 but for total gravity changes. Units are m/s^2.

raphy may lead to a miss-interpretation of the volume change of the source. We observe in our results that, as Folch et al. (2000), the effects of the topography are dramatically emphasized in the viscoelastic case.

It is not the case for radial displacements (Fig. 3), where the effects of the considered topography obtained with the used approximate method are not very important. This result should be tested by comparing with numerical methods as pointed out by Williams & Wadge (1998) or similar to that used by Cayol & Cornet (1998) and Folch et al. (2000).

We observe in our results, like Folch et al. (2000) for a purely viscoelastic medium, that neglecting the topography generates distortions in a viscoelastic half-space of Kelvin type, which would lead to inaccuracies in the predicted displacements and gravity changes.

Conclusions

The thermo-elastic models used to interpret the anomalous behaviour of the heat flow in some volcanic regions, can be used particularly when movements associated with volcanic activity occur on relatively short timescales. However, the presence of incoherent materials and high temperatures produce a lower effective viscosity of the Earth's crust, making it necessary to consider inelastic properties of the media. That is the reason why Hvoždara (1992, 1998) and Brimich (2000) considered a thermo-viscoelastic half-space with a point source of heat to model displacements and gravity changes caused by a magma intrusion. The results show that the thermo-viscoelastic solution gradually approachs the solution obtained for the stationary problem (thermo-elastic solution). The models used to interpret the geodetic data measured in volcanic areas, typically compute the deformation field and gravity changes at the surface of an elastic half-space due to a point source at depth and assume that topography does not significantly affect the results. Considering previous results obtained by other authors for elastic (Williams & Wadge 1998, 2000; Cayol & Cornet 1998) and viscoelastic media (Folch et al. 2000), we have included topographic effects in the thermo-viscoelastic model. We have used an approximate methodology. This methodology permits us to have an analytical solution which allows us to solve the inverse problem. With the methodology described above, we can observe the reduction of vertical displacements in regions with higher topography due to the greater distance from the source of heat to the free surface. In volcanic areas of greater relief the perturbation of the thermo-viscoelastic solution (deformation and total gravity anomaly) due to topography can be quite significant. Therefore we have demonstrated that the topography may significantly affect the surface displacements and gravity changes computed for a magma chamber represented by a heat point source. Thus we can conclude that any model which neglects the topographic effect could cause a significant error in the estimation of surface displacements and gravity changes, or in the determination of the characteristics of the intrusion if we use the model to solve the inverse problem.

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