

The Edinburgh Mathematical Laboratory and Edmund Taylor Whittaker's role in the early development of numerical analysis in Britain

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Abstract

In 1912, Edmund Taylor Whittaker (1873–1956) was appointed to the Chair of Mathematics at the University of Edinburgh. The following year he opened the Edinburgh Mathematical Laboratory. The purpose of the Laboratory was practical instruction in topics which are now classed together as numerical analysis.

In this article I explore the inspiration, purpose, and impact of the Laboratory in the context of early 20th century British applied mathematics.

Zusammenfassung

Im Jahre 1912 wurde Edmund Taylor Whittaker (1873–1956) zum Direktor des mathematischen Instituts (Department) der University of Edinburgh ernannt. Ein Jahr später wurde als ein Teil des Departments das Edinburgh Mathematical Laboratory eröffnet. Die Zielstellung des Laboratoriums war praktischer Unterricht in Gebieten, die heute der Numerischen Analysis angehören. In diesem Artikel untersuche ich die Anregungen, die Ziele und die Wirkungen des Laboratoriums im Kontext der britischen angewandten Mathematik des frühen 20. Jahrhunderts.

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1. Introduction

In 1912, mathematician and astronomer, Edmund Taylor Whittaker (1873–1956) moved from Dunsink, where he was Royal Astronomer of Ireland, to Edinburgh, becoming Professor of Mathematics at the university.¹ The following year he opened the Edinburgh Mathematical Laboratory. The purpose of the Laboratory was to facilitate “practical instruction in numerical, graphical, and mechanical calculation and analysis” (Anon, 1913c, 205). The Laboratory would play an active part in the education of students at Edinburgh over the next forty-two years, when its operation became subsumed into the modern course of numerical analysis.

In the preceding years, regular courses or lectures in numerical techniques had been introduced in the German universities of Freiburg (1875), Leipzig (1882), and — most importantly (see below) — Göttingen (1904).² However, in Britain, as elsewhere in the world, such teaching was minimal. The use of a laboratory for teaching mathematics was also new.

This article will explore the founding and organisation of the Laboratory, and consider the extent to which it was an innovation. It will begin by discussing the inspiration for the Laboratory and describing its subsequent establishment and *modus operandi*. After highlighting some of the many publications, conferences, and other laboratories stimulated by its presence, the latter part of the article will move to examine the Laboratory’s influence on contemporary British mathematics.

2. Early laboratories

When Whittaker started his career in the 1890s, the use of laboratories for teaching science was still relatively new and the idea of using a laboratory for teaching mathematics, in Britain at least, was unheard of. Private laboratories belonging to individual scientists had long existed, but these tended to be small and for research purposes only. In approximately 1855, William Thomson, later Lord Kelvin (1824–1907), converted an old wine cellar in the basement of a building at the University of Glasgow into a laboratory, and in doing so opened the first physics laboratory for teaching in Britain (Gray, 1897, 486),(Gooday, 1990, 29). The early 1870s then saw the establishment of the Cavendish and Clarendon Laboratories at the Universities of Cambridge and Oxford respectively, and in 1878, Alexander Kennedy (1847–1928) opened the first British laboratory for teaching engineering at University College, London.

This was followed in the 1880s by Olaus Henrici’s Mechanics Laboratory for

¹Whittaker’s wife, Mary Whittaker (née Boyd) had family in Edinburgh, which may have influenced their decision to move to the city. For further information about Whittaker’s life and career, see (Maidment and McCartney, 2019).

²See (Meyer-Spasche, 2017, 2).

the training of engineers, at the Central Technical College, London.³ Henrici's laboratory was equipped with calculating machines, planimeters, integragraphs, integrators, and later a harmonic analyser, to solve problems in mechanics (Anon, 1896, 186).⁴ In 1900, another physics laboratory was opened at Owens College, Manchester (now University of Manchester), under the directorship of Arthur Schuster (1851–1934).

In 1903, the statistician Karl Pearson (1857–1936) set up the first of his laboratories, the Drapers' Biometric Laboratory, funded by a grant from the Worshipful Company of Drapers. The following year, with another grant from the Drapers, he set up an Astronomical Laboratory, and then in 1922, when he was no longer running the Astronomical Laboratory, he set up the Anthropometric Laboratory, both of these laboratories being subsidiary to the Biometric Laboratory. In addition to these, in 1907, he established the Galton Eugenics Laboratory. Pearson's laboratories, which were spaces for research rather than teaching, included instruments for mathematical and statistical calculations — the Biometric Laboratory had a 'calculating and integrating room' containing curve plotters, integrators, and calculating machines.⁵

3. The origins of 'numerical analysis'

It is difficult to pinpoint when numerical analysis became a field in its own right. Various techniques that now form the subject had been worked on since antiquity. Dominique Tournès explains that the techniques developed in astronomy, celestial mechanics, and rational mechanics led to the first professional applied mathematicians in the late 19th and early 20th centuries, which was approximately the time when the numerical analysis we know today became an autonomous discipline (Tournès, 2014a, 2).⁶

At the turn of the 20th century, the teaching of applied mathematics in Britain was of a high calibre. Leading scientists and mathematicians such as George Darwin, Joseph Larmor, Robert Ball, and James Jeans at Cambridge, Augustus Love at Oxford, and Horace Lamb in Manchester, taught a range of courses including the mathematical theory of electricity and magnetism, hydrodynamics, celestial mechanics, and hydrostatics. Numerical and graphical techniques were however, yet to become part of applied mathematics and were viewed as more of an appendix to pure mathematics.

During the same period there were efforts, both in Europe and in the United States, to reform mathematical education, notably by Felix Klein (1849–1925) in Germany, John Perry (1859–1920) in England, Émile Borel (1871–1956) in

³In 1910 the Central Technical College (later renamed the City and Guilds College) was incorporated into the newly founded Imperial College.

⁴For further information about Henrici's Laboratory and his harmonic analyser, see Barrow-Green (2017).

⁵For information on Pearson's laboratories, see Magnello (1999a) and Magnello (1999b).

⁶For information on the history of applied mathematics from the late 19th century and early 20th century see (Barrow-Green and Siegmund-Schultze, 2015, 66–72).

France,⁷ and Eliakim Moore (1862–1932) in America.⁸ In Britain, engineer and mathematician Perry initiated what subsequently became known as ‘The Perry Movement’ in the 1890s. Perry — who was Professor of Mathematics and Mechanics at the Royal College of Science and School of Mines (now Imperial College, London) from 1896 to 1913 — recognized that only a small minority of students would become great mathematicians, and argued that it would be more useful for students to be taught skills they could actually use in a non-academic career. Perry suggested that schools should adjust their focus away from the purely axiomatic teachings of Euclid and that universities should teach more of the utilitarian aspects of higher mathematics, including experimental geometry and practical mensuration. Perry also advocated for a laboratory approach to teaching, with teachers becoming more involved in students’ exercises. According to G D Mock, it was Perry’s suggestions “that led to the eventual dethronement of Euclid in England” (Mock, 1963), however it would appear to have been more successful at school level education.

As mentioned above, German universities were early to introduce regular courses in numerical and graphical methods, with Carl Runge leading the way in 1904 (see section 3.1). Even earlier than Runge, Charles Galopin-Schaub, at the University of Geneva in Switzerland, taught a course entitled ‘Calcul Approximatif’ in the late 1870s, but this covered less ground than those of Runge and Whittaker. In Italy a course in ‘numerical analysis’ appeared in the 1920s at the Regia Scuola di Ingegneria di Pisa by Gino Cassinis (1885–1964). According to (Brezinski and Wuytack, 2001, 11) — whose account contains invaluable information on the first courses in numerical analysis in several countries — this was in 1925, only 12 years after the opening of the Edinburgh Laboratory. Other countries were slower on the uptake, and it was not until the 1930s that similar courses began to be introduced elsewhere, a trend that continued through to the 1960s.

The term ‘numerical analysis’ is relatively recent and was not in common use when Whittaker opened his Laboratory. The term had been used earlier with alternative meanings, Lagrange’s 1798 ‘Essai d’analyse numérique sur la transformation des fractions’ (Lagrange, 1798) being a typical example, but it was not until the 1940s that it began to be widely used in the current sense. Early adopters include J H Curtiss who in 1947 set up the Institute for Numerical Analysis at the National Bureau of Standards in the US, and Douglas Rayner Hartree, whose textbook *Numerical Analysis* was published in 1952 (Hartree, 1952). However, there are earlier, less well known examples, such as J B Scarborough’s 1930 book *Numerical Mathematical Analysis* in which he states the object “is to set forth in a systematic manner and as clearly as possible the most important principles, methods and processes used for obtaining numerical results; and also methods and means for estimating the accuracy of such results” (Scarborough, 1930, vii).

⁷See (Borel, 1904).

⁸See (Moore, 1903).

115 3.1. Carl Runge and the development of numerical analysis in Germany

German mathematician Carl Runge (1856–1927), who was one of the pioneers of modern numerical analysis, would prove to be an important influence on Whittaker. With a career spanning pure mathematics, spectroscopy, alternate current engineering, astrophysics, and geodesy, Runge was in an ideal position to appreciate the necessity for better calculation techniques.

120 Although Runge pursued his interest in calculation techniques whilst Professor of Mathematics at the Technische Hochschule of Hannover (1886–1904),⁹ he made his greatest impact after his appointment as Professor of Applied Mathematics at the University of Göttingen in 1904.¹⁰ Whilst there Runge made 125 changes to the curriculum. He designed his mathematics courses similarly to the laboratory sessions in physics and chemistry, with “plotting tables and drawing boards, compasses, slide rules, four-digit logarithm tables, and mechanical calculators” (Tobies, 2012, 63). In 1912, Felix Klein, who had been at Göttingen since 1886, remarked on Runge’s influence:¹¹

130 ever since our colleague Runge has been among us, we have understood applied mathematics to denote the teaching of mathematical principles, namely numerical and graphical methods (Tobies, 2012, 60).

Although Runge was German, his mother was English and he was raised speaking English. This meant he was able to spread his ideas to the English-speaking world. His book, *Graphical Methods*, was originally published in English¹² and was based on lectures he gave as a visiting professor at Columbia University in New York City between 1909 and 1910 (Runge, 1912). It focuses on the use of graphical calculation in the application of mathematical processes, in particular as applied to arithmetic, complex numbers, and linear and polynomial functions. It also covered the graphical representation of functions of one or more variables, and the graphical methods of differential and integral calculus. A further book, *Vorlesungen Über Numerisches Rechnen (Lectures in Numerical Computation)*, co-authored with Hermann König and published the same year as Whittaker and Robinson’s *The Calculus of Observations* (see 140 section 7.1), explored numerical techniques, however this was never translated into English (Runge and König, 1924).

145 In 1912 Runge attended the International Congress of Mathematicians (ICM) in Cambridge, giving a talk on ‘The mathematical training of the physicist in

⁹According to Renate Tobies, Runge’s “programmatic article ‘Über angewandte Mathematik’ [On Applied Mathematics]”, published in 1894 and a letter to Klein (undated), where he “explicates his numerical methods for solving first order differential equations, according to which differential quotients are approximated by difference quotients – provided the foundation for a new style of applied mathematics” (Tobies, 2012, 61).

¹⁰Runge’s position at Göttingen was notable for being the first Chair in Applied Mathematics in a German university.

¹¹Taken from the minutes of the meeting at the Göttingen Association for the Promotion of Applied Physics and Mathematics on 30 November 1912, see (Tobies, 2012, 60).

¹²A German translation followed three years later (Runge, 1915).

the university'. Earlier Runge had been asked by the International Commission on the Teaching of Mathematics to conduct an international enquiry on the teaching of mathematics to students of physics. Having sent out a survey, he received responses from Italy, Austria, Germany, Switzerland, Netherlands, England, and the United States. At the Congress Runge reported on the results he had obtained and elaborated on his conclusions. This was followed by a lively discussion including contributions from several distinguished mathematicians.¹³

According to Runge:

The difficulty is that many professors of mathematics have never been in the habit of calculating numerically and seem to have an aversion to teaching their students. Many are not familiar with the handling of mathematical instruments, with the slide rule, the integrator, the planimeter, the calculating machine, and little mention is made of them in mathematical lectures. The student thus forms a wrong idea of the possibility of carrying out a mathematical operation. As long as he is only interested in mathematical theorems, this does not matter. But a physicist or an engineer cannot be satisfied with existence-theorems, he wants the actual numerical result for the given data he has before him (Runge, 1913, 600).

Runge continued by suggesting topics to be studied: Green's and Stokes' theorem in the integral calculus, practical Fourier series, vector analysis, method of least squares, calculus of finite differences, numerical solution of equations, numerical calculation of integrals, and numerical solution of differential equations (Runge, 1913, 599–600).

He argued that students should carry out mathematical exercises in a laboratory setting, where students would receive individual support as well as actual practice with mathematical instruments. Runge believed this was more conducive to learning than in a standard lecture format (Runge, 1913, 601).

4. The establishment of Whittaker's Laboratory

Prior to the setting up of Whittaker's Laboratory, the University of Edinburgh opened a department of Technical Mathematics, appointing Ellice Martin Horsburgh as the first lecturer in 1903. The department was especially for engineering students, and Horsburgh was asked to institute specialist mathematics courses (Whittaker, 1936, 272). Horsburgh recognised that engineering students required a grounding in numerical and graphical computation and offered two courses which included graphical methods, equations, logarithms, calculations in certain branches of algebra, trigonometry, analytical geometry, curve tracing, and parts of the differential and integral calculus with simple applications of

¹³Including (amongst others) C Bioche (France), C Bourlet (France), F Enriques (Italy), G A Gibson (Scotland), A G Greenhill (England), E W Hobson (England), J Larmor (England), A E H Love (England), P G S Stäckel (Germany), J J Thomson (England), and A G Webster (USA) (Runge, 1913, 603).

such objects.¹⁴ Students were expected to use *Chambers' Mathematical Tables* which included tables of logarithms, logarithmic sines, tangents and secants, as well as sines and cosines, all to seven decimal places (Chambers and Chambers, 1860).

190 The establishment of the Laboratory — which ran in addition to Horsburgh's course — was the first time that numerical and graphical techniques had become a focus for the University's mathematicians, in a setting where both teaching and research were carried out. However, although the Laboratory was run by the Mathematics Department, there is evidence that members of the Natural
195 Philosophy Department were also actively involved (see section 7.3). The purpose of the Laboratory was two-fold. Mathematics undergraduates would be better trained for a variety of careers, such as engineering or accounting, while researchers could use their mathematical skill to help improve techniques, previously left to the engineer, actuary, etc.

200 In May 1913 — shortly before the Laboratory was opened — a notice was placed in the *Mathematical Gazette* which provided details of the Laboratory course as well as a list of the new tables to be calculated.¹⁵ There are striking similarities between Runge's and Whittaker's courses, with the latter's including: accuracy of calculations, method of least squares, numerical solutions of
205 linear equations, curve fitting, practical Fourier analysis, periodogram analysis as well as other function analysis, construction of curves and surfaces, projections, map-making, graphical solution of numerical equations, numerical solution of integral and differential equations, and the use of instruments such as slide rules, arithmometers, planimeters, integragraphs, and harmonic analysers.
210 The tables to be calculated and used were tables of Legendre and Bessel functions, the gamma function, the error function (Gauss), and other transcendental functions, while new functions to be tabulated included automorphic functions, parabolic-cylinder functions and elliptic-cylinder functions (Anon, 1913d, 97–98).¹⁶

215 The layout of the Laboratory was tightly specified. The desks were 3 feet wide, 1 foot 9 inches from front to back and 2 feet 6.5 inches tall. Each desk had a locker for storing computing paper (so that it would not need to be folded), and a cupboard for books with a book rest. The idea was that the (human) computer would then be able to “command a large space and utilize it for books,
220 papers, drawing-board, arithmometer, or instruments” (Gibb, 1915, 1).

Each student was given a copy of Barlow's tables of squares, Crelle's cal-

¹⁴Edinburgh University Library Special Collections, Edinburgh University Calendar 1905/06. Information courtesy of June Barrow-Green.

¹⁵The same notice was translated into French for the *L'enseignement Mathématique* (Fehr, 1913, 251–252) and a shorter version was included in the *Bulletin of the American Mathematical Society* (Anon, 1913b, 323).

¹⁶The increasing amount of computation required in scientific research during this period generated a need for a wide variety of mathematical tables, many of which, including those of Legendre and Bessel functions, were produced or commissioned by the British Association for the Advancement of Science, see (Croarken, 2003, 235–263).

culating tables,¹⁷ as well as logarithm tables (to seven decimal places). They were also provided with computing paper which was “paper ruled into squares by rulings a quarter of an inch apart; each square is intended to hold two digits; the rulings should be very faint, so as not to catch the eye more than is necessary to guide the alignment of the calculation”, and computing forms for practical Fourier analysis (Whittaker and Robinson, 1924, vi).

The Laboratory also made use of some of the most modern calculating machines.¹⁸ These were large and heavy and therefore difficult to move around. They included:¹⁹

- Two types of arithmometer: an Archimedes and a Tate’s Arithmometer, based on the traditional arithmometer designed by Thomas de Colmar. They could perform direct addition and subtraction, as well as multiplication and division by a movable accumulator. The model of Archimedes that was used in the Laboratory was probably the Archimedes model C which measured 34x13x20cm and weighed 7kg, whereas the Tate’s Arithmometer was much bigger, with approximate measurements of 60x19x16cm (weight unknown).²⁰
- A Burroughs adding and listing machine: capable of direct addition, subtraction by adding the complement of the number,²¹ multiplication by repeated addition, and which printed (listed) the results on paper using ink and ribbon. This was a particularly heavy machine, the class one type — which was manufactured from 1885 — weighed 22kg and measured 27x38x31cm.
- Comptometers: which were similar to the Burroughs in that multiplication was done by repeated addition and subtraction by adding the complement of the number, but was a key-driven mechanical calculator, meaning that

¹⁷The first English edition of Crelle’s tables was published in 1897. The tables gave “The products of every two numbers from one to one thousand and their application to the multiplication and division of all numbers above one thousand” (Crelle, 1897, 1).

¹⁸The sum of £1000 was put aside from The Carnegie Trust for the cost of the Laboratory’s equipment. See Faculty of Science Minute Book, 1905–1912. Records of the University of Edinburgh, GB 0237 EUA IN1/ACA/SCI/1/9. Edinburgh University Library Special Collections. This was a large sum of money at the time, especially when considered in the context of the UK average annual wage which was then £70. The Carnegie Trust was founded in 1901 with US\$10 million gifted to the universities of Scotland by the Scottish-American industrialist Andrew Carnegie.

¹⁹The Laboratory exhibited its calculating machines at the Napier Tercentenary Celebrations of 1914 (Horsburgh, 1914, 75).

²⁰A rough estimate, based on the Archimedes, as well as other calculating machines would place it between 14–17kg.

²¹The ‘complement’ of a number is the number that needs to be added to make either 10 for a single digit number, 100 for a 2-digit number, 1000 for a 3-digit number and so on. The way the subtraction works is by adding the complement of the number you are subtracting, to whatever you are subtracting from, for example for 900–600, the complement of 600 is 400, which when added to 900 makes 1300, and then discarding the leading ‘1’ of 1300 to get the answer of 300.

250 instead of being operated by a crank, the numbers were added as soon as the keys were pressed. It was also able to do division by putting the dividend to the left end and performing repeated subtractions by using the complementary method. It is unclear which model the Laboratory used, however Model F — which was manufactured from 1915 — weighed 9kg and measured 24x37x15cm (Fig.1).

- 255 • Brunsvigas — one ordinary and one miniature: a calculator based on the Odhner Arithmometer, a development of the traditional arithmometer, designed by Willgodt Odhner, which used a pinwheel instead of a Leibniz wheel to reduce both the size and the cost.²² The dimensions of these machines varied by model, of which there were several. Again, it is unclear which the Laboratory used, however as an example, Brunsviga model J, 260 which was in production between 1907 to 1915, measured 43x23x17cm and weighed 16kg. The model M, which was a miniature Brunsviga produced between 1908 and 1927, measured 22x12x9.5cm and weighed 3.2kg.
- A Mercedes-Euklid: which used a lever set proportional rod and could do addition, subtraction, multiplication, and division. It weighed approxi- 265 mately 12kg and measured about 37x18x8cm (Fig.2).
- A Millionaire: this was the first commercially successful calculator to do direct multiplication, i.e. it did not rely on repeated addition. It could also do addition, subtraction, and division. The multiplicand or divisor was entered using sliders (up to 8 digits), while successive digits of the 270 multiplier or quotient were entered using a lever. It measured 67x32x19cm and weighed an impressive 35kg.

²²The more compact pinwheel mechanism used a set of wheels, each with an adjustable number of teeth, which could be set using a lever. This was then coupled with a counter, meaning that each rotation of the lever would add the specified number to the result. The older fashioned Leibniz wheel/stepped drum calculators contained cylinders with sets of teeth of incremental lengths, which made the overall calculating machine bigger.



Figure 1: Comptometer Adding Machine, Ridai Museum of Modern Science, Tokyo.



Figure 2: Mercedes-Euklid Calculating Machine Model 1, c.1910. Courtesy Bernd Gross, Wikimedia Commons.

5. Inspiration for the Laboratory

What was the rationale behind Whittaker’s decision to establish the Laboratory? A number of factors are pertinent, not least his interest in the practical techniques relating to astronomy. He held a passion for astronomy, something nurtured during his days as an undergraduate at Cambridge, and made tangible at the time by his success in securing the Sheepshanks Astronomical Exhibition.²³ After graduating Whittaker spent two years writing a report on behalf of the British Association on the progress of the solution of the problem of three bodies (Whittaker, 1899), with two further papers on orbital dynamics in 1902 (Whittaker, 1902b,c). In 1906 he was appointed Royal Astronomer of Ireland and Andrews Professor of Astronomy at Trinity College, Dublin. In this role Whittaker gave advanced lectures, published eight papers, six of which were on astronomy, as well as publishing work on the observations of variable stars. Later Whittaker used data collected from Dunsink Observatory for exercises at the Laboratory.

Another stimulus for establishment of the Laboratory at the time was that after taking up his position in Edinburgh, Whittaker found friendships with a number of actuaries,²⁴ later writing that his friends,²⁵ “told [him] of various mathematical problems that had arisen in their experience, mostly relating to interpolation, curve-fitting, and probability. Becoming interested in interpolation, [he] attempted to answer some fundamental questions regarding it”.^{26,27}

Furthermore, shortly after arriving in Edinburgh and prior to setting up the Laboratory, Whittaker attended the 1912 ICM in Cambridge.²⁸ Although Whittaker was unlikely to have attended Runge’s talk — he gave a talk himself in a parallel session but in another section — the similarity of the topics suggested by Runge in his address at the Congress to those taught at Whittaker’s Laboratory, as well as the laboratory format itself, strongly imply that Whittaker was influenced by the content of Runge’s talk which he would undoubtedly have heard about given his own interests and the attention the talk attracted at the Congress. It is also possible that he read the talk in manuscript form before it was published in the Proceedings (Runge, 1913). In addition, the French translation of the notice from the *Mathematical Gazette*, which appeared

²³The Sheepshanks Astronomical Exhibition was a three-year scholarship at £50 a year donated to the University in the name of astronomer Richard Sheepshanks, FRS, by his sister Anne after his death in 1855.

²⁴Edinburgh had been one of the chief centres for life assurance in Britain since the early 19th century.

²⁵One such friend was G J Lidstone, see (Whittaker and Robinson, 1924, vii), (Whittaker and Bartlett, 1968, 2).

²⁶This culminated in Whittaker’s paper ‘On the Functions represented by the Expansions of the Interpolation Theory’ (Whittaker, 1915). Later three more research papers came from problems suggested by actuarial friends; these were (Whittaker, 1918, 1923, 1924).

²⁷Information courtesy of the University of Toronto Thomas Fisher Rare Book Library, Edmund Taylor Whittaker Papers, Manuscript Collection 219, Box 11.

²⁸It is of interest to note that Whittaker was part of the international committee for the 1912 ICM.

in *L'enseignement Mathématique* in 1913, includes a final paragraph where it
305 relates the Laboratory in Edinburgh to the demand highlighted in the discussion
at the end of Runge's talk at the ICM (Fehr, 1913, 252). Further evidence for
Runge's influence comes from Alexander Aitken, Whittaker's student and later
colleague in Edinburgh, who in his obituary noted that Whittaker was "guided
in part by the example of Runge in Germany" (Aitken, 1956, 731).

310 As an astronomer as well as a mathematician, with a new-found interest
in the mathematical problems of actuaries, it seems likely that Runge's words
resonated with Whittaker — and just at the point when he had been placed in
charge of his own mathematics department. The difference between Whittaker
and his peers was that Whittaker not only recognised the potential in Runge's
315 words, but also was in a position to act on it.²⁹

Perhaps a final factor was that Whittaker was already familiar with the
laboratory set up. Whilst in Dublin, he was a vice-president of the University
Experimental Science Association, which held general meetings at least once
a month in the newly-opened physics laboratory of Trinity College. Its pur-
320 pose was the "encouragement of investigation in all branches of Experimental
Science".³⁰ Prior to this, while Whittaker was in Cambridge, it is probable
that he visited the Cavendish Laboratory. Opened in 1873 and completed in
1874, the Cavendish Laboratory incorporated a room dedicated to 'calculating
and drawing'. This room was for the practical calculations required in physics.
325 The Clarendon Laboratory at the University of Oxford also had a 'calculation
room' and as Andrew Warwick notes, "Whittaker's work in Edinburgh appears
as a transformation of the 'calculation room' into a separate laboratory in its
own right" (Warwick, 1995, 342). At both Cambridge and Oxford the calcu-
lating room was attached to the physics laboratory, whereas in Edinburgh the
330 Mathematical Laboratory was for everybody: physicists, engineers, and math-
ematicians alike.

6. The Edinburgh Mathematical Colloquium

From the 4-8 August 1913, the first Edinburgh Mathematical Society (EMS)
Colloquium was held, a colloquium which, apart from interludes during the two
335 world wars, ran nearly every four years until 2003 (Fig.3). The idea for the first
Colloquium came from the demand for a summer course in the new Edinburgh
Mathematical Laboratory, which according to Cargill Gilston Knott in his report
on the Colloquium published in the *Mathematical Gazette*, had been expressed
by "several correspondents" (Knott, 1913, 165).³¹

²⁹There is no evidence to suggest that Whittaker came up against any resistance from the
university when securing funds for the Laboratory.

³⁰Trinity College Dublin University Calendar 1907–1908, p.55.

³¹Cargill Gilston Knott (1856–1922) was a lecturer in applied mathematics at the University
of Edinburgh between 1892 and 1920, then reader in applied mathematics between 1920 and
1922, as well as one of the founders of the Edinburgh Mathematical Society (est. 1883), which
ran the Colloquium.

340 As well as a course in the Laboratory, Whittaker decided that it would be
a good idea to include lectures on other areas of mathematics too.³² Thus
Arthur W Conway, from the National University of Ireland lectured on ‘The
Theory of Relativity and New Physical Ideas of Space and Time’, and Dun-
can MacLaren Young Sommerville from the University of St. Andrews lectured
345 on ‘Non-Euclidean Geometry and the Foundations of Geometry’. For Whit-
taker’s part, he lectured on ‘Practical Harmonic Analysis and Periodogram
Analysis:³³ an Illustration of Mathematical Laboratory Practice’, guiding his
‘students’ (who were mostly university professors and secondary school teach-
ers)³⁴ with a lecture each day. Whittaker demonstrated periodogram analysis
350 using measurements taken from Dunsink of the brightness of the variable star
RW Cassiopeia.³⁵ His ‘students’ were tasked with investigating the laws gov-
erning the curve representing the observed data and under his guidance found
that many could be decomposed into distinct oscillations (harmonics), which
could then be used to predict the observed quantity’s future behaviour. On
355 the final day of the Colloquium, Whittaker explained how Mader’s harmonic
analyser can be used to investigate the harmonics of a curve mechanically, and
also demonstrated calculating machines such as the Brunsviga.^{36,37}

In all there were 76 attendees of the Colloquium, 20 of whom were professors
or lecturers at universities, 50 were secondary school teachers, and the others
360 were employed in the practical application of mathematics, census, ordnance
survey, and meteorology departments.

³²The EMS issued a press release detailing each of the five days of the Colloquium (Anon, 1913a).

³³Periodogram analysis is the process of identifying the periodic constituents of a function.

³⁴Apart from Conway, all the attendees were either from or working in Scotland at the time of the Colloquium. A list of attendees can be found on the MacTutor History of Mathematics Archive. https://mathshistory.st-andrews.ac.uk/EMS/photo_1913/

³⁵Between September 1909 and March 1911, whilst Royal Astronomer of Ireland, Whittaker and his assistant Charles Martin observed the variable star RW Cassiopeia taking ninety-eight plates using the 15-inch photographic reflector at Dunsink Observatory.

³⁶Otto Mader (1880–1944) was a German engineer who in 1909 invented a harmonic analyser — a mechanical device capable of measuring the amplitudes of the sinusoidal components of a periodic function — which used gears and a pointer tracer to trace a curve. For a manual which gives a full description of how Mader’s harmonic analyser works, see <https://ub.fnwi.uva.nl/computermuseum/harmonanal.html>

³⁷Information courtesy of the MacTutor History of Mathematics Archive. http://www-history.mcs.st-and.ac.uk/ems/Colloquium_1913_1.html



Figure 3: Edinburgh Mathematical Society Colloquium 1913. Front row L to R: Whittaker, Arthur Conway, Alexander Burgess (President of the EMS), Duncan Sommerville and Peter Comrie (Secretary of the EMS).

6.1. Reception of the Colloquium

In his report of the Colloquium, Knott³⁸ described it as a resounding success. All members had enjoyed themselves to the full, with each of the lecturers surpassing themselves (Knott, 1913, 165–166). Another account was included in *The Scotsman*, which again described the Colloquium’s success and also noted that it was the first of its kind in Great Britain (Anon, 1913a, 8). Although these articles might contain an element of hyperbole — Knott was a university colleague of Whittaker’s, and the entries in *The Scotsman* are anonymous — the Colloquium was clearly successful, since another was scheduled for the following year. Furthermore, the 1914 Edinburgh Mathematical Colloquium — which once again had lectures by Whittaker in the Laboratory — had nearly 100 attendees, almost 20 more than in 1913 (Anon, 1914, 11). The 1914 Colloquium also attracted the internationally renowned French engineer and mathematician Maurice d’Ocagne,³⁹ the inventor of nomography.⁴⁰

Although the Colloquium was once again successful, only three days later, on 4 August 1914, Britain declared war on Germany and subsequently the Colloquium was put on hold, not recommencing until 1926.

³⁸See footnote 31.

³⁹Maurice d’Ocagne (1862–1938) was Professor of Geometry at the École Polytechnique at the time of the 1914 Edinburgh Mathematical Colloquium (Itard, 2008, 170).

⁴⁰Nomography is the science of constructing two-dimensional diagrams in order to compute approximate values of a function. For more information on nomography see Tournès (2014b).

380 7. Publications from the Laboratory

7.1. *The Calculus of Observations*

The most successful publication to emerge from the Laboratory is *The Calculus of Observations, A Treatise on Numerical Mathematics* which was written by Whittaker and his colleague in Edinburgh, George Robinson. The first edition was published in 1924, with further editions published in 1926, 1940, and 1944, as well as an American edition in 1967 (Whittaker and Robinson, 1967).

The treatise represents the courses of lectures given during 1913–1923 by Whittaker to undergraduate and graduate students in the Laboratory and was “regarded as a manual of the teaching and practice of the Laboratory” (Whittaker and Robinson, 1924, v). The manuscript was prepared for publication by Robinson, who was also responsible for verifying the calculations and providing examples. One feature that marks the book out from earlier publications, is its preference to arithmetical, as opposed to graphical methods in the solution of problems. As explained in the preface, after the establishment of the Laboratory, a trial was made of as many known methods as possible. During the ten years that had elapsed, Laboratory staff had all but abandoned graphical methods, which were deemed inferior. Another novelty of the book is that it contains theorems and methods derived from actuarial sources, the inspiration for which came from Whittaker’s friend the actuary George Lidstone whose work is often cited. *The Calculus of Observations* also has an ‘Index of Names’ — which with 222 entries — demonstrates how thorough Whittaker and Robinson were in its compilation.⁴¹ Footnotes with commentaries refer the reader to numerous publications stretching across time and space, making the book a valuable source of reference. A particularly interesting example comes from Whittaker’s discussion of the now well-known Adams-Bashforth method of numerical integration originally given in (Bashforth and Adams, 1883) but which until Whittaker’s publication had lain unknown to British mathematicians (Whittaker and Robinson, 1924, Chapter 14). Whittaker, who had probably been alerted to the method by the Norwegian Carl Størmer when they were both at the 1920 ICM in Strasbourg, made reference to earlier publications on the method by the Russian mathematician Aleksey Krylov, thus revealing that a Russian mathematician was better read than his British counterparts!⁴²

⁴¹Among those whose work is mentioned are several mathematicians with whom Whittaker worked, further exemplifying the collaborative nature of work within the Laboratory. These include Aitken, Brodetsky, Carse, Knott, Smeal, and Shearer. A similar thoroughness with regard to citing literature can be seen in Whittaker’s other publications, notably *Modern Analysis* (Whittaker, 1902a) and *Analytical Dynamics* (Whittaker, 1904), reflecting Whittaker’s strong interest in knowledge of the history of the subject.

⁴²Whittaker wrote “Kriloff” for Krylov (Whittaker and Robinson, 1924, 363). The method was presented in (Bashforth and Adams, 1883) in the context of capillary action, a topic which attracted little attention in Britain at the time, in contrast to astronomy and ballistics which were much worked on and in the need of applied methods. That it was noticed by Krylov, who was primarily working in shipbuilding, points to the broad notion of applied mathematics held by the Russians, and particularly by Krylov. For more information, on the history of the

The book contains chapters on interpolation and difference formulae, the solution of algebraic and transcendental equations, numerical integration, normal
415 frequency distributions, the method of least squares, practical Fourier analysis, graduation or smoothing of data, correlation, the search for periodicities, and the numerical solution of differential equations.

When *The Calculus of Observations* was first published in 1924, it received glowing reviews. English algebraist, William Berwick, wrote that “no previ-
420 ous book in any language contains the matter collected here after ten years’ experience of dealing with numerical data” and that “the authors are to be heartily congratulated on successfully producing a book that gives the result of such pioneering work” (Berwick, 1924, 378). American number theorist, Derrick Lehmer, wrote that “the whole book presents evidence on every page of sound
425 scholarship and good practical judgement” (Lehmer, 1925, 182), and Danish statistician Arne Fisher considered that “Whittaker’s treatise represents one of the few worthwhile presentations of what is called mathematical statistics” (Fisher, 1924, 417). In Germany mathematicians Leon Lichtenstein and Heinrich Liebmann described it as remarkable and that it had been “written and fur-
430 nished with exemplary care” (Lichtenstein and Liebmann, 1929). The reviewers were keen however to speculate on possible improvements to the Laboratory. For instance, Lehmer suggests that its workers might not fully appreciate the role of an up to date calculating machine. It is possible that the machines which were available in 1924 had not been upgraded since before the War, however,
435 it is more likely that this impression would be given by the lack of machine demonstrations in the computations of *The Calculus of Observations*. The authors acknowledge this in the preface, where they admit that the use of such may save “time and labour”. At a time when such machines were still a luxury, the authors probably did not want to assume the reader had access to them.
440 Statistician, Major Greenwood⁴³ also questions Whittaker for not taking the “standpoint of the English biometric school”, i.e. the work of statistician Karl Pearson, at the Laboratory. But concedes that this is probably down to it being a mathematical, as opposed to a statistical laboratory (Greenwood, 1924, 293).

In 1967, Dover Publications issued a reprint of the 1944 edition for the Amer-
445 ican market. In the 23 years that had passed, the field of numerical analysis had developed rapidly, as had computer science, yet there remained demand for a reprint.⁴⁴ In 1969, it was reviewed by British mathematician and computer scientist, Jack Howlett, alongside two similar, more modern textbooks. Howlett, who was then working at the forefront of computer science,⁴⁵ unsurprisingly

Adams-Bashforth method, and the history of multistep methods for the numerical integration of ordinary differential equations in general, see (Tournès, 1998).

⁴³Major Greenwood (1880–1949) was a British epidemiologist and statistician who became the first Professor of Epidemiology and Vital Statistics at the London School of Hygiene and Tropical Medicine in 1928. (Major was his forename, not a military rank).

⁴⁴The 1967 Dover edition changed the subtitle to ‘an introduction to numerical analysis’, reflecting the now widespread use of the term.

⁴⁵At the time of writing the review, Howlett was Director of the Atlas Computing Labo-

450 found it “completely outmoded” stating that “it never gets anywhere near the electronic computer — hardly the hand operated desk calculator” (Howlett, 1969, 108). Although there were some critical reviews, the fact that *The Calculus of Observations* was still being considered alongside much newer textbooks demonstrates its longevity.

455 During the 1930s to 1950s, virtually all English language textbooks on numerical analysis either reference *The Calculus of Observations* or acknowledge it in the preface.⁴⁶ Unsurprisingly, the book was remembered fondly in the obituaries of Whittaker.

7.2. *A Short Course in Interpolation*

460 In 1923, a year prior to the publication of *The Calculus of Observations*, the first four chapters of the book were published in a separate treatise entitled *A Short Course in Interpolation*. It was “offered as a short exposition suitable for first-year undergraduates” (Whittaker and Robinson, 1923, v) and was published separately so it could be used on its own as a less advanced text.

465 It was reviewed in the *Mathematical Gazette* by Australian-British statistician William F Sheppard,⁴⁷ who remarked:

470 A curious omission is that of quadrature, which would seem naturally to go with interpolation. The omission is the more surprising, as the complete work does contain a chapter on “Numerical Integration and Summation”. The probable explanation is that the separate publication of the first four chapters was an afterthought, and that it was then not possible to include the further chapter, which occurs rather later in the book.

475 Apart from this omission, the book will be found very useful. It is perhaps rather severely mathematical: there is very little, indeed, to suggest that interpolation is applicable to anything but mathematical tables. And the order adopted is perhaps not the best for the beginner. But the mathematician will find the book a useful compendium, and he will be grateful for the very full treatment of interpolation from unequal intervals (Sheppard, 1924, 220–221).

480 By “afterthought”, Sheppard presumably meant that the authors originally had in mind a single substantial book and thought of publishing the first four chapters separately only after they had written them. His description of the book as “rather severely mathematical”, refers more to the presentation of the contents than to the content itself. His remark concerning the lack of a chapter

ratory in Oxfordshire, which was a major international computing laboratory providing computing services to scientists.

⁴⁶This includes six of the most well known books published in the 1950s: Hartree (1952), Householder (1953), Kopal (1955), Booth (1955), Bennett et al. (1956) and Hildebrand (1956)

⁴⁷Sheppard was born in Sydney in 1863, attending Brisbane Grammar School before completing his school education at Charterhouse School, Surrey (Aitken and Whittaker, 1937, 279). At the time of the review, Sheppard had retired, having worked as a civil servant whilst carrying out mathematical research, of which he is best known in statistics.

485 on quadrature is unlikely to have concerned the authors since they explicitly
described the book as an “excellent preparation for the Differential Calculus”
(Whittaker and Robinson, 1923, v). They had arranged the book in this way to
make it accessible to a broad range of students, in particular first-year under-
graduates. This restriction on the scope of the book undoubtedly contributed
490 to its “severely mathematical” appearance since without considering it as a first
step towards the calculus, there is little to explain why the chapters on inter-
polation were focused on difference tables, divided difference tables, and the
central-difference formula and not on other applications.

7.3. *Edinburgh Mathematical Tracts*

495 In 1915, six short books were published as part of the Edinburgh Mathemat-
ical Tracts series, edited by Whittaker. Of the six Tracts, four described courses
held in the Mathematical Laboratory and were authored by members of either
the mathematics or natural philosophy departments in Edinburgh. These were
the first, second, fourth, and fifth Tracts.⁴⁸

500 *A Course in Descriptive Geometry and Photogrammetry for the Mathematical Laboratory*

The first of the Tracts was written by Edinburgh graduate, Edward Lindsay
Ince (1891–1941). Ince graduated in 1913 with a First Class Honours degree in
mathematics and by 1915 was undertaking research on differential equations.

505 In the preface Ince writes that the Tract was designed for “non-technical
students” in the Laboratory, i.e. mathematics as opposed to engineering stu-
dents. In describing the contents of the book, Ince is explicit about his debt
to earlier European writers such as Eugène Charles Catalan, Xavier Antomari,
Gino Loria, and Gaspard Monge. He particularly singles out Monge’s *Géométrie*
510 *Descriptive* (1799), which he considered “a never-failing source of inspiration”
(Ince, 1915, Preface).

In the introduction, Ince not only gives a description of the content and
purpose of the book but also details the methods and tools used in the Math-
ematical Laboratory. For example, the descriptive geometry course required a
515 drawing board and T-square, compasses, dividers, protractor, and pricker (a
needle mounted on a wooden handle, used for drawing points on a diagram)
(Ince, 1915, 10–11). Topics covered include orthogonal projection of plane and
curved surfaces, perspective, and photogrammetry,⁴⁹ with exercises for the stu-
dent where they can practice using the tools. As with the other tracts written
520 for the Laboratory, Ince’s text suffers from the difficulties arising from trying to
fit a potentially substantive topic into a short book. This was reflected upon in

⁴⁸The two other Tracts were the third, *Relativity*, by Arthur Conway and the sixth, *An Introduction to the Theory of Automorphic Functions*, by Lester R Ford (Conway, 1915; Ford, 1915).

⁴⁹Photogrammetry is the art of obtaining metric representations of an object from photographs.

a review by American mathematician, Virgil Snyder. He wrote that “perhaps this is sufficient to comply with the avowed purpose of the Edinburgh Tracts, but to the reviewer it seems much too brief to be of greatest service”, although
525 he continued that “the book certainly succeeds in teaching the essential features of descriptive geometry in a remarkably small compass” (Snyder, 1916, p.365).

Although foremost a pure mathematician, Ince appreciated the demand for research to be applicable to physics and astronomy. His work on Mathieu Functions, which spanned much of the 1920s, involved him spending a large amount
530 of time constructing tables for this purpose (Whittaker, 1941, 141). Having held posts at Edinburgh, Leeds, and Liverpool, Ince spent the years 1932 to 1935 lecturing at Imperial College, London, where there was also a mathematical laboratory (see section 9.2). Ince’s final role as Head of the Department of Technical Mathematics at Edinburgh, oversaw the teaching of mathematics to
535 students in disciplines such as engineering, where the demand for computational mathematics was of great importance.

A Course in Interpolation and Numerical Integration for the Mathematical Laboratory

The second Tract was written by mathematician and astronomer David Gibb
540 (1883–1946). Gibb was a graduate of Edinburgh, who after a short stint as a school teacher became a lecturer in mathematics under Whittaker’s predecessor George Chrystal. Apart from working at the Royal Arsenal in Woolwich during the First World War, Gibb spent his whole career at Edinburgh. Like Whittaker, Gibb had an interest in actuarial science and for many years was an examiner
545 for the faculty of actuaries. Gibb also carried out research on the motion of the satellite of Neptune and the Fourier analysis of pulsations of variable stars (Smart, 1947; Whittaker, 1946).

Unlike Ince, Gibb does give a rationale for publishing a short book, explaining that since there are many claims on a student’s time, the Tract should be
550 kept to the most important theorems (Gibb, 1915, Preface). The material covered includes the calculus of finite differences, the formulae of interpolation, the construction and use of mathematical tables, and numerical integration.

The book received mixed reviews. In *The Mathematical Gazette*, James M Child⁵⁰ described the book as “abstruse” and criticized the author for not know-
555 ing certain elements of the mathematics. He also criticized Gibb for not using realistic intervals in a section on Simpson’s Rule (Child, 1917, 131). In the *Bulletin of the American Mathematical Society*, Maxime Bôcher (USA) was more sympathetic. Recognizing the problem of trying to compress “the preliminary theoretical matter into a very brief space without making it wholly unintelligible”, he was reluctant to criticize, although he did want more stress laid on
560 certain aspects of interpolation (Bôcher, 1916, 360).

⁵⁰Child may be familiar to historians of mathematics through his work on Leibniz’s manuscripts. At the time he wrote the review, he was a lecturer at Derby Technical College.

A Course in Fourier's Analysis and Periodogram Analysis for the Mathematical Laboratory

The third Tract to come from the Laboratory was written jointly by two
565 members of the natural philosophy department, George A Carse (1880–1950)
and George Shearer. Shearer was Assistant to the Professor of Natural Philo-
sophy, Charles Glover Barkla (1877–1944), having been awarded his MA from
Edinburgh in 1912.⁵¹ Carse started and ended his career at Edinburgh, having
570 spent the years 1904–1907 at Cambridge, working under the guidance of J J
Thomson in the Cavendish Laboratory, where he no doubt used the calculating
room. Like Gibb, he worked at the Royal Arsenal in Woolwich during the War.
Although Carse was foremost a physicist, he also lectured on statistics and had
a strong mathematical background.

The book focused on the practical elements of harmonic analysis, specif-
575 ically, Fourier series, periodogram analysis, and practical spherical harmonic
analysis, and was claimed by the authors to be the first adequate treatment
of periodogram analysis in any English text book (Carse and Shearer, 1915,
Preface).

George Ballard Mathews (1861–1922), who had been Senior Wrangler, i.e.
580 the top student of his year at Cambridge, in 1883 and was a lecturer at Uni-
versity College of North Wales, described the Tract as the most interesting of
the Laboratory tetrad, as it deals with less subject matter and is therefore
more comprehensive (Mathews, 1916, 618). John Borthwick Dale (1871–1953),
who was Assistant Professor in Mathematics at King's College London, thought
585 that the Tract would be of great value to students of astronomy and cosmical
physics.⁵² He did, however, note that Arthur Schuster, a founding figure in
the development of periodogram analysis, had not been properly acknowledged
(Dale, 1916, 223).

A Course in the Solution of Spherical Triangles for the Mathematical Laboratory

590 The fourth Tract to come from the Laboratory, which was on the solution
of spherical triangles — a topic which has practical applications in geodesy,
astronomy, and navigation — was written by Herbert Bell, another assistant in
the natural philosophy department.

The Tract was well received. Mathews described it as “excellent” (Math-
595 ews, 1916, 618) and A M Kenyon, in his review for the *Bulletin of the American
Mathematical Society*, wrote that it was clear and practical, keeping its ob-
ject in mind with a good proportion of definitions, exposition, and illustration
(Kenyon, 1917, 331).

⁵¹Information on Shearer has been hard to find. Based on the date of his MA (1912), he
was probably born c.1890.

⁵²Cosmical physics is the historical discipline which combined aspects from both the earth
sciences and astronomy. Research in cosmical physics usually featured terrestrial phenomena
which were connected to the solar system, such as geomagnetic storms or the aurora borealis,
and was predominant between 1850–1920 (Kragh, 2013, 1).

600 In summary, although the Tracts were in general considered too short —
they were intended to give students in the Laboratory the basics of the topic,
without overwhelming them with too much detail — they were used in both
Britain and the United States. As Mathews observed, Whittaker’s role in the
series was significant: “[M]athematicians owe a special debt to Prof. Whittaker
605 for the work he is doing in connection with this series; his encouragement and
help are acknowledged in handsome terms by several of the authors.” (Mathews,
1916, 618).

8. The Edinburgh Mathematical Laboratory and World War 1

Whittaker’s Laboratory had been open for a year before the First World
610 War began. Although Whittaker’s Laboratory did not partake in war work,
members of his Laboratory staff did. The war took its toll on the mathematics
department; many students were in service and travel was difficult.

In London, statistician Karl Pearson offered the services of his laboratory
staff to the government for the war effort. During the course of the war, Pear-
615 son and his staff worked for: the Board of Trade, producing unemployment
charts; the Royal Aircraft Factory, calculating torsional strain in the blades of
aeroplane propellers; the Admiralty Air Department, calculating bomb trajec-
tories; and the Ministry of Munitions, computing ballistic charts, high-angle
range tables, and fuze-scales. In June 1915, Whittaker learnt of an opening for
620 an Assistantship in Pearson’s laboratory, and recommended one of his recent
graduates, Andrew White Young (1891–1968), for the post.⁵³ On the 21 June
1915, Whittaker wrote to Pearson:

Mr Young has an excellent knowledge of the modern theory of DEs and
theory of functions, and, as a research student in the mathematical Labo-
625 ratory here, he has been trained in practical computation – interpolation,
subtabulation, numerical integration, numerical solutions of equations,
practical harmonic analysis, and periodogram analysis etc. I am accus-
tomed to lecture on correlation (making use of your papers), so he is not
unfamiliar with that subject, although his only big piece of practical work
630 hitherto has been the calculation of tables of the Mathieu functions, which
are not yet published. He is a good lecturer (I have heard him) and can
keep a class of Scotch students in order (not everybody can do this)...⁵⁴

Young joined Pearson’s staff and soon began working on war-related prob-
lems. However, after a promising start Young did not live up to expectations,

⁵³Young had graduated in 1913 with First Class honours in mathematics and natural philosophy, winning several honours as an undergraduate (Anon, 1915).

⁵⁴E T Whittaker to K Pearson, 21 June 1915. *Journal of the Galton Laboratory*. Karl Pearson Papers, 4/17. I am grateful to June Barrow-Green for bringing both this correspondence and the diary entries quoted to my attention.

635 and in a diary entry dated 25 May 1918, Pearson wrote that Young had become “very slack” and that he had “trusted too much to Young’s algebraic accuracy”.⁵⁵

Young was not the only member of Whittaker’s Laboratory to partake in war work. Ince became a member of Pearson’s staff in 1918 working for the
640 Ministry of Munitions. Pearson was impressed by Ince’s mathematical skill, though did not rate him as a computer.⁵⁶ As mentioned above, Carse and Gibb both worked at the Royal Arsenal in Woolwich, Carse as an expert on optical supplies and Gibb in the Ballistics Department of the Ordnance Committee, computing range tables for guns in the Navy and Army,⁵⁷ where he was reputed
645 to have been partly responsible for computing “the high-angle tables needed for firing over the Gallipoli Peninsula from the Aegean Sea to the Narrows” (Smart, 1947, p.28). No doubt his work computing tables in the Laboratory made him well suited to the task.

9. Mathematical laboratories elsewhere

650 9.1. University of Leeds

In 1923 a mathematical laboratory was opened at the University of Leeds with a course on ‘graphics and methods of numerical computation’.⁵⁸

The head of the department of mathematics, William P Milne (1881–1967), had attended the Edinburgh Mathematical Colloquium in 1913, whilst still a
655 school teacher at Clifton College, Bristol, and so had first-hand experience of using Whittaker’s Laboratory. In 1919, he had been appointed to the Chair of Mathematics at the University of Leeds, which gave him the opportunity to open a mathematical laboratory of his own.⁵⁹

Living in Bristol at the same time as Milne, having been appointed lecturer
660 in applied mathematics at the University in 1914, was Selig Brodetsky (1888–1954). Brodetsky’s main research area was aeronautics, but he also worked in optics and was interested in nomography. Milne, who wanted Brodetsky as a member of his staff at Leeds, offered him the role of lecturer in applied mathematics, with the promise of a rapid promotion to Reader and with the

⁵⁵In 1918 Young submitted a paper on an airplane propeller blade flexure and torsion problem for printing as a Drapers Technical Memoir and to the University for a DSc degree. However, Arthur Berry of the Royal Aircraft Factory in Farnborough spotted a flaw in Young’s analysis, leading to the paper being recalled. Karl Pearson, *Journal of the Galton Laboratory*, 25 May 1915. Karl Pearson Papers 4/17. For more information on Pearson and his work for the Ministry of Munitions see (Barrow-Green, 2015).

⁵⁶K Pearson, *Journal of the Galton Laboratory*, 25 May 1918. Karl Pearson Papers 4/17.

⁵⁷Another mathematician computing range tables for the Ballistics Department of the Ordnance Committee at Woolwich was the analyst J E Littlewood.

⁵⁸Special Collections, Leeds University Library, ‘University Calendars 1923–1924’, archive series: LUA/PUB/003/3/48, p.175.

⁵⁹Milne, who had been born and educated in Scotland, attending the University of Aberdeen as an undergraduate, held the Chair of Mathematics at Leeds until his retirement in 1946. He became a member of the Edinburgh Mathematical Society in 1910.

665 hope of introducing a Chair in Applied Mathematics soon after. This prospect
appealed to Brodetsky who joined Milne’s department in 1919 (Brodetsky, 1960,
91). The following year Brodetsky’s book, *A First Course in Nomography* was
published (Brodetsky, 1920).

670 Early in 1921, Milne and Brodetsky took a trip to Edinburgh where they
met Whittaker who showed them his Mathematical Laboratory.⁶⁰ Whilst there
they asked Whittaker if he knew of an appropriate candidate to run the math-
ematical laboratory at Leeds (Brodetsky, 1960, 98). Whittaker recommended
Australian mathematician, Glenny Smeal (1890–1974), who had been appointed
675 Assistant Lecturer in Mathematics at Edinburgh in 1914, with the “purpose of
specialising in the statistical and numerical side of Higher Mathematical Analy-
sis in the Mathematical Laboratory of Edinburgh University”⁶¹ and had already
published a paper, ‘On direct and inverse interpolation by divided differences’
(Smeal, 1919). Whittaker thought highly of Smeal, as is evident in his recom-
mendation:

680 ...in view of his special qualifications, he is now entrusted with most of
the higher Mathematical Laboratory work — in fact, with all that I do
not take myself: and lately I have tended more and more to hand over
my share to him. If he leaves Edinburgh, I shall have some difficulty in
replacing him: for I do not know of any one who has such an extensive
685 knowledge of numerical analysis and so much experience in teaching it.⁶¹

Smeal was appointed to the newly-created post of Assistant Lecturer in
Graphics and Computation at the University of Leeds in 1921, with the mathe-
matical laboratory opening two years later. In 1924, Smeal co-authored a paper
with Brodetsky in which they modified a result of Runge’s in order to make
690 Graeffe’s method for solving numerical algebraic equations more efficient for
higher degree equations with complex roots, particularly those of relevance to
practical aeronautics (Brodetsky and Smeal, 1924).⁶² Smeal retained his link
with Edinburgh throughout his career at Leeds through his membership of the
Edinburgh Mathematical Society which he had joined in 1914. He retired from
695 the University in 1946.

9.2. *Imperial College, London*

Another mathematical laboratory which had a direct link to Whittaker
through its instigator was the laboratory at Imperial College. In this case the
instigator, Hyman Levy (1889–1975), not only had a connection to Whittaker,
700 but he had also come under the influence of Runge.

⁶⁰After the death of Cargill Gilston Knott in 1921, Brodetsky’s contact with Whittaker
led to him acting as interim lecturer in applied mathematics, spending Monday to Wednesday
teaching at Leeds, and Thursday and Friday at Edinburgh. This lasted for fifteen weeks
at which point the newly appointed Professor of Natural Philosophy at Edinburgh, Charles
Galton Darwin (1887–1962), took over permanently (Brodetsky, 1960, 102).

⁶¹Special Collections, Leeds University Library, staff file of G. Smeal CRO MF 226.F38.

⁶²For Runge’s discussion of Graeffe’s method, see (Runge, 1900).

Levy was born and raised in Edinburgh and studied mathematics and physics at the University of Edinburgh, graduating in 1911 before Whittaker's arrival,⁶³ after which he was advised to continue his study at the University of Cambridge. However, disliking Cambridge's class bias, he chose instead to study
705 in Göttingen under Hilbert and Runge, where he would have attended Runge's course in numerical analysis. While in Göttingen, Levy maintained his association with Edinburgh⁶⁴ and in 1916 was made a Fellow of the Royal Society of Edinburgh with Whittaker as one of his proposers. After four years of war work at the National Physical Laboratory, Levy was appointed Assistant Professor at
710 Imperial College in 1920, being promoted to Professor three years later (Stewart, 2004). In 1923, the same year as Leeds, Imperial opened its own mathematical laboratory. That Levy had a significant role in the founding of the laboratory is evident from an article that he published in *The Mathematical Gazette* in 1925.

In the article he outlined the purpose of a mathematical laboratory, noting
715 in particular its usefulness in helping students over the difficulty of applying their knowledge to real-life problems. He also described how graphical methods can be used to help those who struggle with abstract mathematical ideas. In his discussion concerning what should be taught in a laboratory, in which he refers to the work of Runge and others, he detailed the following (Levy, 1925,
720 374–376):

- Tables of different types of functions
- The use of the slide rule, multiplying machine, and planimeter
- The properties of series
- Nomography and the grading of functions
- 725 • Finite differences
- Interpolation
- Extrapolation
- Numerical differentiation and integration
- Fourier series
- 730 • Partial differential equations
- Laws of error
- Curve-fitting

⁶³Levy graduated with an MA with First Class honours, winning a Ferguson Scholarship, an 1851 Exhibition, and a Carnegie Research Fellowship.

⁶⁴Levy was a member of the Edinburgh Mathematical Society and published in their *Proceedings*.

Levy concluded his article by arguing that a mathematical laboratory ought to strive to open new lines of enquiry, thus differentiating it from a calculating room, and he drew attention to the increasing demand for practical mathematics in industry, with growing numbers of government research laboratories and industrial research associations being opened. He did not specify what he meant by new lines of enquiry, but it is likely to have been similar to what was being developed in Edinburgh — the production of new mathematical tables, the improvement of calculating machines as well as original research in any of the topics of numerical analysis.

In 1923, a year prior to the publication of *The Calculus of Observations*, Levy published a translation of Horst von Sanden’s textbook on practical analysis, originally written in 1913 (von Sanden and Levy, 1923). Von Sanden had studied numerical techniques at Göttingen and obtained his doctorate under the supervision of Runge. In *Practical Analysis*, among other techniques, von Sanden included a detailed description of the Burkhardt calculating machine — another arithmometer — and a more brief description of the Millionaire.

Unlike the *Calculus of Observations*, which is described as a course manual, *Practical Analysis* is intended for a variety of readers and only covers a portion of the course at Göttingen. The preface advocates the teaching of numerical techniques at universities, but *Practical Analysis* is clearly intended for the teacher and technical worker too. This is evident in the way von Sanden compares methods, as it allows the reader to assess which is best for their use. In translating *Practical Analysis*, Levy added more examples making it more accessible. *The Calculus of Observations* also has examples at the end of each chapter intended as exercises for the student, with answers at the end of the book. The latter also contains the ‘Index of Names’ mentioned above, which refers to bibliographical footnotes throughout the text, providing the student with some historical background, as opposed to the rather daunting bibliography at the end of *Practical Analysis*.

Levy also co-authored two books on numerical mathematics: *Numerical Studies in Differential Equations* and some years later *Finite Difference Equations*.⁶⁵ The former combines the author’s experience from just over ten years work in the laboratory with that of previous mathematicians and includes the work of both Runge and Whittaker (Levy and Baggott, 1934), (Levy and Lessman, 1959).

9.3. Massachusetts Institute of Technology

Whittaker’s laboratory also had a direct influence on Joseph Lipka (1883–1924), a Polish-born mathematician working at the Massachusetts Institute of Technology. Whilst travelling in the summer of 1913, Lipka visited Whittaker’s laboratory and was sufficiently impressed to open his own the following year.

⁶⁵Levy was also the author of two books for the Nelson’s Aeroscience Manuals series: *Elementary Mathematics* (Levy, 1942) and *Elementary Statistics* (Levy, 1944).

In his laboratory course, Lipka offered “practical instruction in numerical, graphical and mechanical calculation and analysis as required in the engineering or applied mathematical sciences.” He included topics such as the determination of the accuracy of calculations (arithmetic and logarithmic); numerical solution of equations (including transcendental and differential equations); graphical methods in arithmetic, algebra, and calculus; curve-fitting; and the use of calculating instruments such as the slide rule, planimeter, and integragraphs.⁶⁶ In 1918, Lipka published a book, *Graphical and Mechanical Computation*, based on his laboratory course. It is in the preface that Lipka acknowledges his debt to Whittaker, writing that “he owes the idea of a Mathematical Laboratory to Professor E. T. Whittaker of the University of Edinburgh” (Lipka, 1918, iv). Three years later, *Graphical and Mechanical Computation* was republished in two volumes. The first volume, *Part I. Alignment Charts*, introduced scales and the slide rule with an exhaustive account of nomograms (Lipka, 1921a) and the second volume, *Part II. Experimental Data*, covered topics ranging from curve-fitting and harmonic analysis, to interpolation and numerical, graphical, and mechanical calculus (Lipka, 1921b).

The book was well received by reviewers. In the United States, Arthur R Crathorne commended it, noting a lack of similar American texts (Crathorne, 1922, 272). In Britain, Brodetsky wrote that it worked well as a foundation for a course in a mathematical laboratory and ought to be available in every mathematical, engineering, or technical library (Brodetsky, 1921, 617–618).

Lipka died in 1924, aged only forty-one, but the mathematical laboratory course continued after his death, under the supervision of Raymond Donald Douglass, being offered for the last time in the academic year 1953–1954.⁶⁷

9.4. Mathematics Division, National Physical Laboratory

In 1940, Hungarian born Jewish mathematician, Arthur Erdélyi (1908–1977) travelled to Edinburgh to join Whittaker’s department. Having already left Hungary due to restrictions on Jewish scholars, Erdélyi was then forced to leave Czechoslovakia (now Czech Republic), or face internment in a concentration camp. Erdélyi contacted Whittaker — the leading expert in his field — and appealed for help to secure the necessary £400 a year salary required to emigrate to the UK. It was difficult for Whittaker to raise the funds, but with the help of Brodetsky, Erdélyi successfully emigrated to Scotland in early 1939.⁶⁸

After settling in Edinburgh, Erdélyi became part of the war-effort as a consultant to the Admiralty Computing Service (ACS). The service had been set up in 1943 to “advise on and to carry out computational tasks for Admiralty

⁶⁶Information courtesy of the Course Catalogue of the Massachusetts Institute of Technology, 1914–1915, p.397, part of the *Bulletin of the Massachusetts Institute of Technology, Boston* published December 1914 available at <http://hdl.handle.net/1721.3/82755>.

⁶⁷Douglass co-authored *Elements of Nomography* with Douglas Payne Adams (Douglass and Adams, 1947).

⁶⁸For more information on Erdélyi and the plight of other German-speaking mathematicians fleeing to or from Czechoslovakia, see (Siegmond-Schultze, 2012).

810 establishments” (Croarken, 2005, 26). In this capacity Erdélyi, along with John
Todd and Donald Sadler, wrote a *Memorandum on the Centralization of Com-*
putation in a National Mathematical Laboratory, where they demonstrated the
demand for a National Mathematical Laboratory, which would be much bigger
than the ACS. This was sent to the Secretary of the Department of Scientific
815 and Industrial Research (DSIR), Sir Edward Appleton.

The director of the National Physical Laboratory (NPL), Sir Charles Galton
Darwin⁶⁹ had been requesting that the NPL set up a mathematics division,
but for statistics. Darwin’s contact with Appleton resulted in the creation
of the Mathematics Division of the NPL, which would provide both a centre
820 for statistics and a computing service, where new computational techniques
and machinery would be developed. The Division became a leading centre
for numerical analysis (Croarken, 2005, 23) as well as the home of the Pilot
ACE computer, based on a design by Alan Turing, who worked at the Division
between 1945 and 1947.

825 10. The Edinburgh Mathematical Laboratory after Whittaker

Alexander Aitken (1895–1967) — who succeeded Whittaker as head of the
mathematics department in 1946 — continued running the Mathematical Lab-
oratory course until 1960, when it was renamed ‘numerical analysis’.⁷⁰

In 1961, the University Grants Committee had a funding round for equip-
830 ment. This was referred to Aitken, who did not see the need for a computer
(Watson, 2009, 164). Despite this, the University set up its Computer Unit in
1963 under the leadership of Sidney Michaelson (1925–1991), who would become
the first Professor of Computer Science in Scotland.

As an undergraduate, Michaelson had studied mathematics at Imperial Col-
835 lege, London under Levy. During the 1950s and 1960s he worked in numerical
analysis before moving to Edinburgh to become director of the new Computer
Unit.

11. Conclusion

Although many of the ideas behind Whittaker’s innovations may be traced
840 back to Runge, without Whittaker Britain would not have become one of the
earliest places to implement the widespread teaching and research in numerical
analysis. Whittaker not only recognised the potential of Runge’s work, but built
on it, laying the groundwork for what was to come. Whittaker’s influence is ev-
ident both through the publications which emanated from the Laboratory, and
845 from the other laboratories which explicitly followed his example. In addition,

⁶⁹Darwin was the grandson of Charles Darwin and the Godson of Sir Francis Galton.

⁷⁰The university calendars suggest that this was simply a renaming of the course, with no
significant changes. At this point mention of the Laboratory ceases.

Whittaker's laboratory course was the catalyst for the first Edinburgh Mathematical Colloquium which brought mathematicians together so they could not only appreciate numerical analysis but also benefit from the opportunity to work in a tailor-made environment.

850 *The Calculus of Observations*, one of the first books in English to cover many of the topics of numerical analysis, contributed significantly to the spread of Whittaker's ideas and became the key reference in numerical analysis for future authors. During the 20th century, numerical analysis developed so rapidly that there was soon a plethora of textbooks available, but the new generation of texts
855 owed a debt to Whittaker which is widely acknowledged through references and citations.

In establishing his Mathematical Laboratory, Whittaker laid down a blueprint for mathematical laboratories to come. With the advent of the electronic computer and the increasing dominance of machines, mathematical laboratories
860 evolved into computing laboratories with eventually the name 'mathematical laboratory' falling into disuse. Typical of this shift was the Cambridge Mathematical Laboratory which opened in 1937 but in 1970 was renamed the Computer Laboratory.⁷¹

Through the introduction of a mathematical laboratory, Whittaker brought
865 numerical analysis to the attention of British and American mathematicians, and in doing so sped up its transition from being a tool used by, for instance, engineers, to the field it is today. He helped broaden British mathematics in a period increasingly dominated by abstract mathematics, and he encouraged his students to work on 'practical' problems, helping them gain perspective.
870 Whittaker died in 1956, at which point numerical analysis was really gaining momentum. Although Whittaker never saw the full extent to which his work had an impact on not only mathematics but also computing, astronomy, engineering and so much more, he nonetheless considered the Mathematical Laboratory as his greatest contribution to mathematical education.⁷²

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⁷¹Before the Cambridge Mathematical Laboratory became fully operational, World War II had started and the Laboratory was put to use for the war-effort. The Laboratory returned to university use after the war. For information about the Cambridge Mathematical Laboratory, see (Croarken, 1992).

⁷²See (Erdélyi, 1957, 53).

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