Hyperfine mixing in $b \to c$ semileptonic decay of doubly heavy baryons

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We qualitatively corroborate the results of W. Roberts and M. Pervin in Int. J. Mod. Phys. A 24, 2401 (2009) according to which hyperfine mixing greatly affects the decay widths of $b \to c$ semileptonic decays involving doubly heavy bc baryons. However, our predictions for the decay widths of the unmixed states differ from those reported in the work of Roberts and Pervin by a factor of 2, and this discrepancy translates to the mixed case. We further show that the predictions of heavy quark spin symmetry, might be used in the future to experimentally extract information on the admixtures in the actual physical bc baryons, in a model independent manner.

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I. INTRODUCTION

According to heavy quark spin symmetry (HQSS) [1], in the infinite heavy quark mass limit, one can select the heavy quark subsystem of a doubly heavy baryon to have a well defined total spin $S_h = 0, 1$. In Table I we show the ground state $J^\pi = \frac{1}{2}^+, \frac{3}{2}^+$ doubly heavy baryons classified so that S_h is well defined, and to which we shall refer to as the S_h-basis. Being ground states for the given quantum numbers a total orbital angular momentum $L = 0$ is naturally assumed.

Due to the finite value of the heavy quark masses, the hyperfine interaction between the light quark and any of the heavy quarks can admix both $S_h = 0$ and $S_h = 1$ spin components into the wave function. This mixing should be negligible for bb and cc doubly heavy baryons as the antisymmetry of the wave function would require radial excitations and/or higher orbital angular momentum in the $S_h = 0$ component. On the other hand, in the bc sector one expects the actual physical Ξ (Ω) particles to be admixtures of the Ξ_{bc}, Ξ'_{bc} (Ω_{bc}, Ω'_{bc}) states listed in Table I.

One can minimize the effect of hyperfine mixing for bc baryons by working in a different basis, that we shall call the qc-basis, in which it is the light quark q (q=u, d, s) and the c quark that couple to well defined total spin $S_{qc} = 0, 1$. For further use we shall denote the states in that basis as $\hat{\Xi}_{bc}$, $\hat{\Omega}_{bc}$ for $S_{qc} = 1$, and $\hat{\Xi}'_{bc}$, $\hat{\Omega}'_{bc}$ for $S_{qc} = 0$. In this latter basis hyperfine mixing is always inversely proportional to the b quark mass and it is thus negligible. This in fact means, that the hyperfine mixed bc states one would obtain after diagonalizing the mass Hamiltonian in the S_h-basis should be very close to these new qc-basis states. The qc-basis was used in the doubly heavy baryon mass determination of Ref. [2]. However, masses are very insensitive to hyperfine mixing and most mass calculations [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14] just ignore the mixing and use the S_h-basis.

Roberts and Pervin [15] have taken up the hyperfine mixing issue again, pointing out that it could greatly affect the decay widths of doubly heavy baryons. In Ref. [16], it was later noticed that the $b \to c$ semileptonic decay width for transitions involving the S_h-basis Ξ_{bc} (Ω_{bc}) state was so different from the corresponding one involving the Ξ'_{bc} (Ω'_{bc}) state, that experimental data, when available, could be used to extract information on the admixtures in the actual physical states. Following their own suggestion in Ref. [15], Roberts and Pervin have conducted a calculation in which they find that hyperfine mixing in the bc states has a tremendous impact on doubly heavy baryon $b \to c$ semileptonic decay widths [17]. In fact this kind of information was partially available in the literature. In Ref. [18], Faessler et al. evaluated the $\Gamma(\hat{\Xi}_{bc} \to \Xi_{cc}\, l\bar{\nu}_l)$ decay width obtaining a result that was a factor of around three smaller than the values obtained in Refs. [16, 19] for the same transition but now evaluated for the S_h-basis Ξ_{bc} state. This hinted to the relevance of hyperfine mixing, but it went largely unnoticed among the large differences present between the different theoretical predictions [16, 19, 20, 21, 22]. To our knowledge, Roberts and Pervin [17] have been the first ones to realize the importance of hyperfine mixing for decay properties. However, their work is not sufficiently known. Thus, for instance, the most recent calculation of the semileptonic decay widths of these baryons still ignores the effects of mixing [23].

In the present calculation, in which we use our non-relativistic approach described in Ref. [14], we try to confirm their results. As shown below, we qualitatively corroborate their findings as to the importance of hyperfine mixing for

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Baryon	Quark content (l=u,d)	S_h^π	J^π	Baryon	Quark content	S_h^π	J^π
Ξ_{cc}	{c c} l	1^+	$1/2^+$	Ω_{cc}	{c c} s	1^+	$1/2^+$
Ξ_{cc}^*	{c c} l	1^+	$3/2^+$	Ω_{cc}^*	{c c} s	1^+	$3/2^+$
Ξ_{bb}	{b b} l	1^+	$1/2^+$	Ω_{bb}	{b b} s	1^+	$1/2^+$
Ξ_{bb}^*	{b b} l	1^+	$3/2^+$	Ω_{bb}^*	{b b} s	1^+	$3/2^+$
Ξ_{bc}	{b c} l	1^+	$1/2^+$	Ω_{bc}	{b c} s	1^+	$1/2^+$
Ξ_{bc}^*	{b c} l	1^+	$3/2^+$	Ω_{bc}^*	{b c} s	1^+	$3/2^+$
Ξ_{bc}'	[b c] l	0^+	$1/2^+$	Ω_{bc}'	[b c] s	0^+	$1/2^+$

TABLE I: Quantum numbers and quark content for ground state doubly heavy baryons classified so that S_h (spin of the heavy quark subsystem) is well defined.

evaluating $b \to c$ semileptonic decay widths of doubly heavy baryons. On the other hand, we find large discrepancies between the two calculations for the actual decay width values. Their results for unmixed states in the S_h-basis are a factor of two smaller than ours. This discrepancy translates to the full mixed state calculation. Besides, we will also show how HQSS predictions for the $b \to c$ transition matrix elements might be used in the future to experimentally obtain information on the admixtures of the bc baryons, in a model independent manner.

II. RESULTS AND DISCUSSION

As in Refs. [14, 16], we shall work in the S_h-basis. We assume $L = 0$ and symmetric orbital wave functions throughout, neglecting thus hyperfine mixing for doubly bb and cc baryons. However we shall take into account hyperfine mixing in the bc sector. Our unmixed wave functions are obtained using a variational approach that is based on HQSS. A full account can be found in Ref. [14]. We shall use the AL1 potential of B. Silvestre-Brac and C. Semay [4, 24] that for the $q\bar{q}$ interaction reads

$$V_{q\bar{q}}(r) = -\frac{\kappa}{r} + \lambda r - \Lambda + \frac{2\pi}{3m_q m_{\bar{q}}}\kappa' \frac{e^{-r^2/x_0^2}}{\pi^{\frac{3}{2}} x_0^3}\,\vec{\sigma}_q\cdot\vec{\sigma}_{\bar{q}} \quad ; \quad x_0(m_q, m_{\bar{q}}) = A\left(\frac{2m_q m_{\bar{q}}}{m_q + m_{\bar{q}}}\right)^{-B}. \tag{1}$$

It contains a linear confinement term, plus Coulomb and hyperfine ($\vec{\sigma}\cdot\vec{\sigma}$) terms coming from one-gluon exchange. All free parameters in the potential had been adjusted to reproduce the light and heavy-light meson spectra. For its use in baryons we apply the usual prescription [4, 25]

$$V_{qq} = \frac{1}{2}\,V_{q\bar{q}} \tag{2}$$

This is fully justified for the one-gluon exchange part of the potential where the implicit color dependence accounts for the factor of two difference between the $q\bar{q}$ and the qq interaction. The prescription is also phenomenologically very successfully for the confinement part. As shown in the lattice QCD calculation of Ref. [27], the confinement part of the potential for a three quark system is proportional to the minimal total length $L_{\rm min}$ of the color flux tube linking the three quarks, and in fact one has $\frac{1}{2}\sum_{j<k}|\vec{r}_j - \vec{r}_k| \approx L_{\rm min}$. The theoretical uncertainties on masses and decay widths associated to the use of different potentials were found to be small in Ref. [14], where we presented results for five different potentials taken from Refs. [4, 24, 25][1].

To evaluate the decay widths we shall use the model described in Ref. [14]. We work in a spectator approximation in which any of the b quarks in the initial state can decay into any of the c quarks in the final state. This, together with the right symmetry factor for doubly heavy (initial or final) baryon states with two equal heavy quarks, gives rise to an extra factor $\sqrt{2}$ in the amplitude compared to the similar $b \to c$ decays for baryons with just one heavy quark. This is in fact more important than it seems as many theoretical calculations have overlooked that factor, or got it wrong, altogether (See the erratum in Ref. [14]).

[1] All five potentials are conveniently tabulated in Ref. [26]. There, and for the first time, a variational approach based on HQSS was applied to solve the three-body problem in heavy baryons. A similar approach was used in Ref. [14] for doubly heavy baryons.

	This work	[15]	[7]	[10]	[28]	[29]		This work	[15]	[7]	[10]	[28]	[29]
$M_{\Xi_{cc}}$	3613	3676	3478	3620	3550	3510	$M_{\Omega_{cc}}$	3712	3815	3590	3778	3730	3719
$M_{\Xi^*_{cc}}$	3707	3753	3610	3727	3590	3548	$M_{\Omega^*_{cc}}$	3795	3876	3690	3872	3770	3746
$M_{\Xi_{bb}}$	10198	10340	10093	10202	10100	10130	$M_{\Omega_{bb}}$	10269	10454	10180	10359	10280	10422
$M_{\Xi^*_{bb}}$	10237	10367	10133	10237	10110	10144	$M_{\Omega^*_{bb}}$	10307	10486	10200	10389	10290	10432
$M_{\Xi_{bc}}$	6928	7020	6820	6933	6800	$6792^\dagger$	$M_{\Omega_{bc}}$	7013	7147	6910	7088	6980	$6999^\dagger$
$M_{\Xi'_{bc}}$	6958	7044	6850	6963	6870	$6825^\dagger$	$M_{\Omega'_{bc}}$	7038	7166	6930	7116	7050	$7022^\dagger$
$M_{\Xi^*_{bc}}$	6996	7078	6900	6980	6850	6827	$M_{\Omega^*_{bc}}$	7075	7191	6990	7130	7020	7024

TABLE II: Masses (MeV units) for S_h-basis unmixed states. Mixing has been taken into account only for the results marked with a † symbol.

	This work	[17]	[19]	[23]		This work	[17]	[19]	[23]
$\Gamma(\Xi^*_{bb} \to \Xi'_{bc}\, l\bar{\nu}_l)$	1.08	–	0.82	0.36 ± 0.10	$\Gamma(\Omega^*_{bb} \to \Omega'_{bc}\, l\bar{\nu}_l)$	1.14	–	0.85	0.42 ± 0.14
$\Gamma(\Xi^*_{bb} \to \Xi_{bc}\, l\bar{\nu}_l)$	0.36	–	0.28	0.14 ± 0.04	$\Gamma(\Omega^*_{bb} \to \Omega_{bc}\, l\bar{\nu}_l)$	0.38	–	0.29	0.15 ± 0.05
$\Gamma(\Xi_{bb} \to \Xi'_{bc}\, l\bar{\nu}_l)$	1.09	0.41	0.82	0.43 ± 0.12	$\Gamma(\Omega_{bb} \to \Omega'_{bc}\, l\bar{\nu}_l)$	1.16	0.51	0.83	0.48 ± 0.12
$\Gamma(\Xi_{bb} \to \Xi_{bc}\, l\bar{\nu}_l)$	2.00	0.69	1.63	0.80 ± 0.30	$\Gamma(\Omega_{bb} \to \Omega_{bc}\, l\bar{\nu}_l)$	2.15	0.92	1.70	0.86 ± 0.32
$\Gamma(\Xi'_{bc} \to \Xi_{cc}\, l\bar{\nu}_l)$	1.36	–	0.88	1.10 ± 0.32	$\Gamma(\Omega'_{bc} \to \Omega_{cc}\, l\bar{\nu}_l)$	1.36	–	0.95	0.98 ± 0.28
$\Gamma(\Xi_{bc} \to \Xi_{cc}\, l\bar{\nu}_l)$	2.57	1.38	2.30	2.10 ± 0.70	$\Gamma(\Omega_{bc} \to \Omega_{cc}\, l\bar{\nu}_l)$	2.58	1.54	2.48	1.88 ± 0.62
$\Gamma(\Xi'_{bc} \to \Xi^*_{cc}\, l\bar{\nu}_l)$	2.35	–	1.70	2.01 ± 0.62	$\Gamma(\Omega'_{bc} \to \Omega^*_{cc}\, l\bar{\nu}_l)$	2.35	–	1.83	1.93 ± 0.60
$\Gamma(\Xi_{bc} \to \Xi^*_{cc}\, l\bar{\nu}_l)$	0.75	0.52	0.72	0.64 ± 0.19	$\Gamma(\Omega_{bc} \to \Omega^*_{cc}\, l\bar{\nu}_l)$	0.76	0.56	0.74	0.62 ± 0.19

TABLE III: Semileptonic decay widths (in units of 10^{-14} GeV) for S_h-basis unmixed states. We have used $|V_{cb}| = 0.0413$. l stands for $l = e, \mu$.

A. Masses: unmixed results

In Table II we present the results we obtain for the masses of S_h-basis unmixed states. We have corrected numerical inaccuracies present in our former work in Ref. [14] and the masses we give now deviate slightly from the ones reported there. As a result, small changes will affect the decay widths to be discussed below. We also show the results obtained by Roberts and Pervin in Ref. [15]. They use a non-relativistic approach in which the orbital wave functions are expanded in a harmonic oscillator basis. Their results are always larger than ours by $50 \sim 180$ MeV. For the sake of comparison we also show the results of the non-relativistic calculation in Ref. [7], that give results that are about 100 MeV smaller that ours, and the ones obtained in three different relativistic approaches [10, 28, 29]. On the experimental side the SELEX Collaboration claimed evidence for the Ξ^+_{cc} baryon, in the $\Lambda^+_c K^- \pi^+$ and pD^+K^- decay modes, with a mass of $M_{\Xi^+_{cc}} = 3519 \pm 1$ MeV/c^2 [30]. No other experimental collaboration has found evidence for doubly charmed baryons so far and, at present, the Ξ^+_{cc} has only a one star status.

B. Semileptonic $b \to c$ decays: unmixed results

In Table III we show the results for the $b \to c$ semileptonic decay widths of doubly heavy baryons for unmixed states in the S_h-basis, and assuming massless fermions in the final state, thus only valid for decays into e, μ but not into τ. Our results are roughly a factor of two larger than the ones obtained by Roberts and Pervin in Ref. [17]. This discrepancy can not be attributed to the small mass differences between the two calculations as the energy involved in $b \to c$ decays is very large. We also show the decay widths obtained in two different relativistic approaches [19, 23]. Our results are in a global fair agreement with the ones in Ref. [19]. The agreement is also good with Ref. [23] but only for transitions with a bc baryon in the initial state. For transitions with a bb baryon in the initial state, hence a bc baryon in the final state, their results are a factor of two smaller than ours. All four calculations comply with the HQSS constraints among decay width ratios derived in Ref. [16] for unmixed states in the S_h-basis. As only ratios are involved, those relations can not be used to elucidate which of the non-relativistic calculation is preferable. Besides, as the Isgur-Wise functions are different, the ratios concern separately $bb \to bc$ decays and $bc \to cc$ decays so they can not be used to see which relativistic calculation, if any, is more correct for the case of the $bb \to bc$ transitions. This constitutes an open problem.

An interesting feature of the decay widths shown in Table III is that they are very different for transitions involving

Ξ_{bc} or Ξ'_{bc} (idem Ω_{bc} or Ω'_{bc}). Thus, one expects that the decay widths involving the actual physical bc states could be very different from the ones quoted in Table III, provided hyperfine mixing in the bc baryons is non negligible. In fact, we anticipate this to be the case as for bc states one expects physical Ξ and Ω to be close to the qc-basis states for which, apart from a global phase, one has (in what follows $B \equiv \Xi$ or Ω)

$$\begin{aligned} \hat{B}_{bc} &= \frac{\sqrt{3}}{2} B'_{bc} + \frac{1}{2} B_{bc}, \\ \hat{B}'_{bc} &= -\frac{1}{2} B'_{bc} + \frac{\sqrt{3}}{2} B_{bc}. \end{aligned} \tag{3}$$

C. Results with mixing

To obtain the physical bc states in the S_h-basis we have to diagonalize the mass matrices for which the diagonal matrix elements are the corresponding masses in Table II. The hyperfine $\vec{\sigma} \cdot \vec{\sigma}$ term (see Eq.(1)) in the interaction between the light and any of the heavy quarks is responsible for the mixing as it gives rise to non vanishing non diagonal matrix elements. The values for the latter are respectively 18.3 MeV for Ξ baryons and 15.8 MeV for Ω baryons. After diagonalizing we get for the physical $\Xi^{(1)}_{bc}$ and $\Xi^{(2)}_{bc}$ states

$$\begin{aligned} \Xi^{(1)}_{bc} &= 0.902\ \Xi'_{bc} + 0.431\ \Xi_{bc} \ \ ; \ M_{\Xi^{(1)}_{bc}} = 6967\,\mathrm{MeV}, \\ \Xi^{(2)}_{bc} &= -0.431\ \Xi'_{bc} + 0.902\ \Xi_{bc} \ \ ; \ M_{\Xi^{(2)}_{bc}} = 6919\,\mathrm{MeV}, \end{aligned} \tag{4}$$

for Ξ baryons, and

$$\begin{aligned} \Omega^{(1)}_{bc} &= 0.899\ \Omega'_{bc} + 0.437\ \Omega_{bc} \ \ ; \ M_{\Omega^{(1)}_{bc}} = 7046\,\mathrm{MeV}, \\ \Omega^{(2)}_{bc} &= -0.437\ \Omega'_{bc} + 0.899\ \Omega_{bc} \ \ ; \ M_{\Omega^{(2)}_{bc}} = 7005\,\mathrm{MeV}, \end{aligned} \tag{5}$$

for Ω baryons. Theoretical uncertainties related to the determination of the variational wave functions may affect the last of the digits quoted above. As one sees, the masses of the physical states change only by a few MeV compared to the corresponding unmixed values in Table II. On the other hand the mixture is important and, as mentioned before, physical states are close to the qc-basis states (see Eq.(3)).

We have evaluated again the decay widths for the mixed states. These results are displayed in Table IV where, for better comparison, we also show in parenthesis the corresponding unmixed results. The changes we find in decay widths are very large, confirming the results of Roberts and Pervin in Ref. [17]. Qualitatively we find the same behavior, but the actual decay widths are very different. This is a reflection of the factor of two discrepancy present already for the unmixed case.

As noticed in Ref. [17], the widths for the decays $B^{(2)}_{bc} \to B^*_{cc}$ decrease considerably from their unmixed counterparts. In our calculation the decrease factor is 44 (54) for the $\Xi^{(2)}_{bc} \to \Xi^*_{cc}$ ($\Omega^{(2)}_{bc} \to \Omega^*_{cc}$) transition. This can be easily understood by taking into account that $B^{(2)}_{bc} \approx \hat{B}'_{bc}$. In the latter state, the light and c quarks are coupled to spin 0, whereas in the B^*_{cc} the light and any of the c quarks are in a relative spin 1 state. In any spectator calculation, as the ones here and in Ref. [17], the amplitude for the $\hat{B}'_{bc} \to B^*_{cc}$ transition cancels due to the orthogonality of the different spin states of the spectator quarks (qc pair) in the initial and final baryons. The fact that $B^{(2)}_{bc}$ slightly deviates from $\hat{B}'_{bc}$ produces a non zero, but small, decay width.

D. HQSS and mixing

The last result can be derived on more general grounds in the context of HQSS. In Ref. [31], using HQSS, the following hadronic amplitudes were found for states in the S_h-basis

$$\begin{aligned} \mathcal{A}(B_{bc} \to B^*_{cc})|_{\mathrm{HQSS}} &= \frac{1}{\sqrt{2}} \frac{-2}{\sqrt{3}} \eta\ \bar{u}'^{\mu}\ u, \\ \mathcal{A}(B'_{bc} \to B^*_{cc})|_{\mathrm{HQSS}} &= \frac{1}{\sqrt{2}} (-2)\, \eta\ \bar{u}'^{\mu}\ u, \end{aligned} \tag{6}$$

	This work mixed	This work unmixed	[17] mixed	[17] unmixed		This work mixed	This work unmixed	[17] mixed	[17] unmixed
$\Gamma(\Xi^*_{bb} \to \Xi^{(1)}_{bc}\, l\bar{\nu}_l)$	0.47	(1.08)	–	–	$\Gamma(\Omega^*_{bb} \to \Omega^{(1)}_{bc}\, l\bar{\nu}_l)$	0.48	(1.14)	–	–
$\Gamma(\Xi^*_{bb} \to \Xi^{(2)}_{bc}\, l\bar{\nu}_l)$	0.99	(0.36)	–	–	$\Gamma(\Omega^*_{bb} \to \Omega^{(2)}_{bc}\, l\bar{\nu}_l)$	1.06	(0.38)	–	–
$\Gamma(\Xi_{bb} \to \Xi^{(1)}_{bc}\, l\bar{\nu}_l)$	2.21	(1.09)	0.95	(0.41)	$\Gamma(\Omega_{bb} \to \Omega^{(1)}_{bc}\, l\bar{\nu}_l)$	2.36	(1.16)	0.99	(0.51)
$\Gamma(\Xi_{bb} \to \Xi^{(2)}_{bc}\, l\bar{\nu}_l)$	0.85	(2.00)	0.33	(0.69)	$\Gamma(\Omega_{bb} \to \Omega^{(2)}_{bc}\, l\bar{\nu}_l)$	0.91	(2.15)	0.30	(0.92)
$\Gamma(\Xi^{(1)}_{bc} \to \Xi_{cc}\, l\bar{\nu}_l)$	0.38	(1.36)	–	–	$\Gamma(\Omega^{(1)}_{bc} \to \Omega_{cc}\, l\bar{\nu}_l)$	0.37	(1.36)	–	–
$\Gamma(\Xi^{(2)}_{bc} \to \Xi_{cc}\, l\bar{\nu}_l)$	3.50	(2.57)	1.92	(1.38)	$\Gamma(\Omega^{(2)}_{bc} \to \Omega_{cc}\, l\bar{\nu}_l)$	3.52	(2.58)	1.99	(1.54)
$\Gamma(\Xi^{(1)}_{bc} \to \Xi^*_{cc}\, l\bar{\nu}_l)$	3.14	(2.35)	–	–	$\Gamma(\Omega^{(1)}_{bc} \to \Omega^*_{cc}\, l\bar{\nu}_l)$	3.14	(2.35)	–	–
$\Gamma(\Xi^{(2)}_{bc} \to \Xi^*_{cc}\, l\bar{\nu}_l)$	0.017	(0.75)	0.026	(0.52)	$\Gamma(\Omega^{(2)}_{bc} \to \Omega^*_{cc}\, l\bar{\nu}_l)$	0.014	(0.76)	0.013	(0.56)

TABLE IV: Semileptonic decay widths (in units of 10^{-14} GeV) for mixed states. For better comparison we also show in parenthesis the results for unmixed states. $l = e, \mu$. We have used $|V_{cb}| = 0.0413$.

where η is the universal Isgur-Wise function, different for Ξ and Ω decays, and u, u^μ are respectively the Dirac and Rarita-Schwinger spinors for the initial and final baryons. Combining these results together with Eq.(3) one immediately derives

$$\mathcal{A}(\hat{B}'_{bc} \to B^*_{cc})\Big|_{\rm HQSS} = 0 \tag{7}$$

implying $\Gamma(\hat{B}'_{bc} \to B^*_{cc}) = 0$ as before. For completeness, we give all the hadronic amplitudes for decays involving $\hat{B}_{bc}, \hat{B}'_{bc}$ that derive from the HQSS relations in Ref. [31]

$$\begin{aligned}
\mathcal{A}(\hat{B}_{bc} \to B_{cc})\Big|_{\rm HQSS} &= \frac{\eta}{\sqrt{2}}\,\bar{u}'(\gamma^\mu + \frac{1}{3}\,\gamma^\mu\gamma_5)\,u, \\
\mathcal{A}(\hat{B}'_{bc} \to B_{cc})\Big|_{\rm HQSS} &= \frac{\eta}{\sqrt{2}}\,\sqrt{3}\,\bar{u}'(\gamma^\mu - \gamma^\mu\gamma_5)\,u, \\
\mathcal{A}(\hat{B}_{bc} \to B^*_{cc})\Big|_{\rm HQSS} &= \frac{\eta}{\sqrt{2}}\,\frac{-4}{\sqrt{3}}\,\bar{u}'^\mu\,u.
\end{aligned} \tag{8}$$

Similarly one has

$$\begin{aligned}
\mathcal{A}(B_{bb} \to \hat{B}_{bc})\Big|_{\rm HQSS} &= \frac{\eta'}{\sqrt{2}}\,\bar{u}'(\gamma^\mu - \frac{5}{3}\,\gamma^\mu\gamma_5)\,u, \\
\mathcal{A}(B_{bb} \to \hat{B}'_{bc})\Big|_{\rm HQSS} &= \frac{\eta'}{\sqrt{2}}\,\sqrt{3}\,\bar{u}'(\gamma^\mu - \frac{1}{3}\gamma^\mu\gamma_5)\,u, \\
\mathcal{A}(B^*_{bb} \to \hat{B}_{bc})\Big|_{\rm HQSS} &= \frac{\sqrt{2}}{\sqrt{3}}\,\eta'\,\bar{u}'\,u^\mu, \\
\mathcal{A}(B^*_{bb} \to \hat{B}'_{bc})\Big|_{\rm HQSS} &= -\sqrt{2}\,\eta'\,\bar{u}'\,u^\mu.
\end{aligned} \tag{9}$$

where η' is different from η and, as before, we have a different η' for Ξ and Ω decays.

Following the steps and approximations in Ref. [16] one can derive HQSS-based relations for ratios involving decay widths of qc-basis states. Thus, for instance

$$R_1 = \frac{\Gamma(\hat{B}'_{bc} \to B_{cc})}{\frac{3}{4}\Gamma(\hat{B}_{bc} \to B^*_{cc}) + 3\Gamma(\hat{B}_{bc} \to B_{cc})}\Bigg|_{\rm HQSS} = 1, \tag{10}$$

and

$$\begin{aligned}
R_2 &= \frac{3\Gamma(B^*_{bb} \to \hat{B}_{bc})}{\Gamma(B^*_{bb} \to \hat{B}'_{bc})}\Bigg|_{\rm HQSS} = 1, \\
R_3 &= \frac{\frac{56}{9}\Gamma(B^*_{bb} \to \hat{B}_{bc}) + \frac{10}{27}\Gamma(B_{bb} \to \hat{B}'_{bc})}{\Gamma(B_{bb} \to \hat{B}_{bc})}\Bigg|_{\rm HQSS} = 1.
\end{aligned} \tag{11}$$

We can evaluate those ratios for the actual physical states substituting $\hat{B}_{bc}$ by $B^{(1)}_{bc}$ and $\hat{B}'_{bc}$ by $B^{(2)}_{bc}$ in Eqs.(10) and (11), and using the widths in Table IV. What one finds is $R_1 = 1.00$, $R_2 = 1.42$ and $R_3 = 1.47$. The sizeable deviation from 1 in the two latter cases is due to the difference between the physical mixing coefficients in Eqs. (4) and (5) and the ideal ones in Eq. (3), while the extraordinary agreement for R_1 seems to be purely accidental. To check that assertion we have played the game of evaluating the decay widths involving $\hat{B}_{bc}, \hat{B}'_{bc}$ states, assuming their masses to be respectively the ones for $B^{(1)}_{bc}, B^{(2)}_{bc}$. What we get for the ratios in this case is $R_1 = 1.07$, $R_2 = 0.996$ and $R_3 = 1.06$, all of them in good agreement with the HQSS-based predictions in Eqs.(10) and (11).

HQSS can predict the hadronic amplitudes of given states, for instance the ones in Ref. [16] for S_h-basis states, or the ones here that involve qc-basis states, but it does not tell which are the physical states. Thus in order to use HQSS to check width ratios for physical states, those states have to be known beforehand.

However, HQSS predictions might be used to experimentally obtain information on the mixing angle for bc baryons in a model independent manner. Physical and S_h or qc-basis states will differ in a rotation, determined by a certain mixing angle. For instance one could write

$$\begin{pmatrix} B^{(1)}_{bc} \\ \\ B^{(2)}_{bc} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \hat{B}_{bc} \\ \\ \hat{B}'_{bc} \end{pmatrix}, \tag{12}$$

where one expects θ to be small. If the ratios

$$R_1^{\rm phys.} = \frac{\Gamma(B^{(2)}_{bc} \to B^*_{cc})}{\Gamma(B^{(1)}_{bc} \to B^*_{cc})} \quad , \quad R_2^{\rm phys.} = \frac{\Gamma(B^*_{bb} \to \hat{B}^{(1)}_{bc})}{\Gamma(B^*_{bb} \to \hat{B}^{(2)}_{bc})}, \tag{13}$$

were measured, we would have from the HQSS relations in Eqs.(7), (8) and (9)

$$\begin{aligned} R_1^{\rm phys.} &\sim (\tan\theta)^2 + \mathcal{O}(\frac{m_q, \Lambda_{QCD}}{m_c}), \\ R_2^{\rm phys.} &\sim \left(\frac{1-\sqrt{3}\tan\theta}{\sqrt{3}+\tan\theta}\right)^2 + \mathcal{O}(\frac{m_q, \Lambda_{QCD}}{m_c}), \end{aligned} \tag{14}$$

which would allow to determine[2] the mixing angle θ in a model independent manner, since HQSS is a proper QCD spin–flavor symmetry when the quark masses become much larger than the typical confinement scale, $\Lambda_{\rm QCD}$. To show that this idea works, we evaluate those ratios in the Ξ sector with our predictions for the decay widths. From them we get the mixing angle θ to be $|\theta| = 4.2^{\rm o}$ from $R_1^{\rm phys.}$ and $\theta = -4.6^{\rm o}$ from $R_2^{\rm phys.}$. In this latter case, we have taken the solution associated to a small mixing angle ($\tan\theta \leq 1/\sqrt{3}$). These estimates should be compared with $\theta = -4.5^{\rm o}$ as deduced from Eqs.(3) and (4). Similar results are obtained in the Ω sector. These ideas were firstly proposed in Ref. [16].

III. CONCLUSION

In the limit of charm and bottom masses infinitely large, though different, all spin–spin interactions can be neglected, and then there would exist a $J = 1/2$ baryon Ξ^∞_{bc} with a total degeneracy[3] of 4. A basis in this space can be constructed out of the states of the S_h-basis: $|J = 1/2, M = \pm 1/2; \Xi_{bc}\rangle$ and $|J = 1/2, M = \pm 1/2; \Xi'_{bc}\rangle$. The semileptonic $b \to c$ width of the Ξ^∞_{bc} baryon would then depend of the particular state (an arbitrary linear combination of the elements of the basis) in which it is prepared, in the same manner as for instance, the polarized $\Lambda \to N\pi$ decay width depends on the polarization of the initial hyperon. Thus, the HQSS relations found in [31] for the states of the S_h-basis can be used to derive HQSS relations for the transitions of any other Ξ^∞_{bc} baryon state (for instance Eqs.(7) and (8) for the case of states with well defined spin for the light–charm quark pair). Analogous considerations apply for the decays of the $bb-$baryons into a particular Ξ^∞_{bc} baryon state.

[2] For finite charm and bottom masses, there would be some short distance corrections, which probably largely cancel out in the ratio of widths.

[3] The discussion is similar for the case of Ω baryons.

In the real world, the charm and bottom masses, though large, are finite and the above degeneration is reduced, and thus one is left to that associated to third component of the total spin of the baryon. Because of the hyperfine interactions, the Ξ_{bc} and Ξ'_{bc} states are not eigenstates of the Hamiltonian, which is non-diagonal in the S_h-basis. We have diagonalized the Hamiltonian and found that the physical states are quite similar to those where the spin of the light–charm quark pair is well defined. This is because the charm quark mass is substantially smaller than that of the bottom one.

We have shown that this hyperfine mixing greatly affects the decay widths of $b \to c$ semileptonic decays involving doubly heavy bc baryons, and thus we have qualitatively corroborated the results of W. Roberts and M. Pervin of Ref. [17]. However, our predictions for the decay widths of the unmixed states differ from those reported in the work of Roberts and Pervin by a factor of 2, and this discrepancy translates to the mixed case.

Finally, we have discussed how the HQSS predictions for the semileptonic decay widths might be used in the future to experimentally extract information on the admixtures in the actual physical $bc-$baryon states.

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