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Semileptonic decays of light quarks beyond the Standard Model

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We describe non-standard contributions to semileptonic processes in a model independent way in terms of an $SU(2)_L \times U(1)_Y$ invariant effective lagrangian at the weak scale, from which we derive the low-energy effective lagrangian governing muon and beta decays. We find that the deviation from Cabibbo universality, $\Delta_{\rm CKM} \equiv |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1$, receives contributions from four effective operators. The phenomenological bound $\Delta_{\rm CKM} = (-1 \pm 6) \times 10^{-4}$ provides strong constraints on all four operators, corresponding to an effective scale $\Lambda > 11$ TeV (90% CL). Depending on the operator, this constraint is at the same level or better then the Z pole observables. Conversely, precision electroweak constraints alone would allow universality violations as large as $\Delta_{\rm CKM} = -0.01$ (90% CL). An observed $\Delta_{\rm CKM} \neq 0$ at this level could be explained in terms of a single four-fermion operator which is relatively poorly constrained by electroweak precision measurements.

I. INTRODUCTION

Precise lifetime and branching ratio measurements [1] combined with improved theoretical control of hadronic matrix elements and radiative corrections make semileptonic decays of light quarks a deep probe of the nature of weak interactions [2, 3]. In particular, the determination of the elements V_{ud} and V_{us} of the Cabibbo-Kobayashi-Maskawa (CKM) [4, 5] quark mixing matrix is approaching the 0.025% and 0.5% level, respectively. Such precise knowledge of V_{ud} and V_{us} enables tests of Cabibbo universality, equivalent to the CKM unitarity condition[1] $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$, at the level of 0.001 or better. Assuming that new physics contributions scale as $\alpha/\pi(M_W^2/\Lambda^2)$, the unitarity test probes energy scales Λ on the order of the TeV, which will be directly probed at the LHC.

While the consequences of Cabibbo universality tests on Standard Model (SM) extensions have been considered in a number of explicit (mostly supersymmetric) scenarios [6, 7, 8, 9], a model-independent analysis of semileptonic processes beyond the SM is missing. The goal of this investigation is to analyze in a model-independent effective theory setup new physics contributions to low energy charged-current (CC) processes. The resulting framework allows

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[1] $V_{ub} \sim 10^{-3}$ contributes negligibly to this relation.

us to assess in a fairly general way the impact of semileptonic processes in constraining and discriminating SM extensions. We shall pay special attention to purely leptonic and semileptonic decays of light hadrons used to extract the CKM elements V_{ud} and V_{us}.

Assuming the existence of a mass gap between the SM and its extension, we parameterize the effect of new degrees of freedom and interactions beyond the SM via a series of higher dimensional operators constructed with the low-energy SM fields. If the SM extension is weakly coupled, the resulting TeV-scale effective lagrangian linearly realizes the electro-weak (EW) symmetry $SU(2)_L \times U(1)_Y$ and contains a SM-like Higgs doublet [10]. This method is quite general and allows us to study the implications of precision measurements on a large class of models. In particular, the effective theory approach allows us to understand in a model-independent way (i) the significance of Cabibbo universality constraints compared to other precision measurements (for example, could we expect sizable deviations from universality in light of no deviation from the SM in precision tests at the Z pole?); (ii) the correlations between possible deviations from universality and other precision observables, not always simple to identify in a specific model analysis.

This article is organized as follows. In Section II we review the form of the most general weak scale effective lagrangian including operators up to dimension six, contributing to precision electroweak measurements *and* semileptonic decays. In Section III we derive the low-energy ($O(1)$ GeV) effective lagrangian describing purely leptonic and semileptonic CC interaction. We discuss the flavor structure of the relevant effective couplings in Section IV. In Section V we give an overview of the phenomenology of V_{ud} and V_{us} beyond the SM, and derive the relation between universality violations and other precision measurements at the operator level. Section VI is devoted to a quantitative analysis of the interplay between the universality constraint and other precision measurements, while Section VII contains our conclusions.

II. WEAK SCALE EFFECTIVE LAGRANGIAN

As discussed in the introduction, our aim is to analyze in a model-independent framework new physics contributions to both precision electroweak observables and beta decays. Given the successes of the SM at energies up to the electroweak scale $v \sim 100$ GeV, we adopt here the point of view that the SM is the low-energy limit of a more fundamental theory. Specifically, we assume that: (i) there is a gap between the weak scale v and the scale Λ where new degrees of freedom appear; (ii) the SM extension at the weak scale is weakly coupled, so the EW symmetry $SU(2)_L \times U(1)_Y$ is linearly realized and the low-energy theory contains a SM-like Higgs doublet [10]. Analyses of EW precision data in nonlinear realizations of EW symmetry can be found in the literature [11, 12, 13, 14]. In the spirit of the effective field theory approach, we integrate out all the heavy fields and describe physics at the weak scale (and below) by means of an effective non-renormalizable lagrangian of the form:

$$\mathcal{L}^{(\mathrm{eff})} = \mathcal{L}_{\mathrm{SM}} + \frac{1}{\Lambda}\mathcal{L}_5 + \frac{1}{\Lambda^2}\mathcal{L}_6 + \frac{1}{\Lambda^3}\mathcal{L}_7 + \dots \tag{1}$$

$$\mathcal{L}_n = \sum_i \alpha_i^{(n)} \, O_i^{(n)} \, , \tag{2}$$

where Λ is the characteristic scale of the new physics and $\mathcal{O}_i^{(n)}$ are local gauge-invariant operators of dimension n built out of SM fields. Assuming that right-handed neutrinos do not appear as low-energy degrees of freedom, the building blocks to construct local operators are the gauge fields G_μ^A, W_μ^a, B_μ, corresponding to $SU(3) \times SU(2)_L \times U(1)_Y$, the five fermionic gauge multiplets,

$$l^i = \begin{pmatrix} \nu_L^i \\ e_L^i \end{pmatrix} \qquad e^i = e_R^i \qquad q^i = \begin{pmatrix} u_L^i \\ d_L^i \end{pmatrix} \qquad u^i = u_R^i \qquad d^i = d_R^i \ , \tag{3}$$

the Higgs doublet φ

$$\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \ , \tag{4}$$

and the covariant derivative

$$D_\mu = I\, \partial_\mu \, - \, i g_s \frac{\lambda^A}{2} G_\mu^A \, - \, i g \frac{\sigma^a}{2} W_\mu^a \, - \, i g' Y B_\mu \ . \tag{5}$$

In the above expression λ^A are the $SU(3)$ Gell-Mann matrices, σ^a are the $SU(2)$ Pauli matrices, g_s, g, g' are the gauge couplings and Y is the hypercharge of a given multiplet.

In our analysis we will not consider operators that violate total lepton and baryon number (we assume they are suppressed by a scale much higher than $\Lambda \sim$ TeV [15]). Under the above assumptions, it can be shown [10] that the first corrections to the SM lagrangian are of dimension six. A complete set of dimension-six operators is given in the pioneering work of Buchmüller and Wyler (BW) [10][2]. Truncating the expansion at this order we have

$$\mathcal{L}_{BW}^{(\mathrm{eff})} = \mathcal{L}_{\mathrm{SM}} + \sum_{i=1}^{77} \frac{\alpha_i}{\Lambda^2}\ O_i \ . \tag{6}$$

For operators involving quarks and leptons, both the coefficients α_i and the operators O_i carry flavor indices. When needed, we will make the flavor indices explicit, using the notation $[\alpha_i]_{abcd}$ for four-fermion operators.

The above effective lagrangian allows one to parameterize non-standard corrections to any observable involving SM particles. The contribution from the dimension six operators involve terms proportional to v^2/Λ^2 and E^2/Λ^2, where $v = \langle \varphi^0 \rangle \simeq 174\,\mathrm{GeV}$ is the vacuum expectation value (VEV) of the Higgs field and E is the characteristic energy scale of a given process. In order to be consistent with the truncation of (1) we will work at linear order in the above ratios.

We are interested in the minimal subset of the BW basis that contribute at tree level to CP-conserving electroweak precision observables *and* beta decays. Upon imposing these requirements (see Appendix A) we end up with a basis involving twenty-five operators. In selecting the operators, flavor symmetries played no role (in fact at this level the coefficients α_i can carry any flavor structure). However, in order to organize the subsequent phenomenological analysis, it is useful to classify the operators according to their behavior under the $U(3)^5$ flavor symmetry of the SM gauge lagrangian (the freedom to perform $U(3)$ transformations in family space for each of the five fermionic gauge multiplets, listed in Eq. 3).

[2] In the original list of BW there are eighty operators, but it can be shown that it can be reduced to seventy-seven (see Appendix A).

A. $U(3)^5$ invariant operators

The operators that contain only vectors and scalars are

$$O_{WB} = (\varphi^\dagger \sigma^a \varphi) W^a_{\mu\nu} B^{\mu\nu}, \quad O^{(3)}_\varphi = |\varphi^\dagger D_\mu \varphi|^2 \ . \tag{7}$$

There are eleven four-fermion operators:

$$O^{(1)}_{ll} = \frac{1}{2}(\bar{l}\gamma^\mu l)(\bar{l}\gamma_\mu l), \quad O^{(3)}_{ll} = \frac{1}{2}(\bar{l}\gamma^\mu \sigma^a l)(\bar{l}\gamma_\mu \sigma^a l) \tag{8}$$

$$O^{(1)}_{lq} = (\bar{l}\gamma^\mu l)(\bar{q}\gamma_\mu q), \quad O^{(3)}_{lq} = (\bar{l}\gamma^\mu \sigma^a l)(\bar{q}\gamma_\mu \sigma^a q), \tag{9}$$

$$O_{le} = (\bar{l}\gamma^\mu l)(\bar{e}\gamma_\mu e), \quad O_{qe} = (\bar{q}\gamma^\mu q)(\bar{e}\gamma_\mu e), \tag{10}$$

$$O_{lu} = (\bar{l}\gamma^\mu l)(\bar{u}\gamma_\mu u), \quad O_{ld} = (\bar{l}\gamma^\mu l)(\bar{d}\gamma_\mu d), \tag{11}$$

$$O_{ee} = \frac{1}{2}(\bar{e}\gamma^\mu e)(\bar{e}\gamma_\mu e), \quad O_{eu} = (\bar{e}\gamma^\mu e)(\bar{u}\gamma_\mu u), \quad O_{ed} = (\bar{e}\gamma^\mu e)(\bar{d}\gamma_\mu d). \tag{12}$$

Some comments are in order. In principle, in order to avoid redundancy (see discussion in Appendix A), one must discard either $O^{(3)}_{ll}$ or $O^{(1)}_{ll}$. However, here we have followed the common practice to work with both operators and consider only flavor structures factorized according to fermion bilinears. Moreover, we use the structure $\bar{L}\gamma_\mu L \cdot \bar{R}\gamma^\mu R$ in operators (10), instead of their Fierz transformed $\bar{L}R \cdot \bar{R}L$, that BW use. They are related by a factor (-2).

There are seven operators containing two fermions that alter the couplings of fermions to the gauge bosons:

$$O^{(1)}_{\varphi l} = i(\varphi^\dagger D^\mu \varphi)(\bar{l}\gamma_\mu l) + \text{h.c.}, \quad O^{(3)}_{\varphi l} = i(h^\dagger D^\mu \sigma^a \varphi)(\bar{l}\gamma_\mu \sigma^a l) + \text{h.c.}, \tag{13}$$

$$O^{(1)}_{\varphi q} = i(\varphi^\dagger D^\mu \varphi)(\bar{q}\gamma_\mu q) + \text{h.c.}, \quad O^{(3)}_{\varphi q} = i(\varphi^\dagger D^\mu \sigma^a \varphi)(\bar{q}\gamma_\mu \sigma^a q) + \text{h.c.}, \tag{14}$$

$$O_{\varphi u} = i(\varphi^\dagger D^\mu \varphi)(\bar{u}\gamma_\mu u) + \text{h.c.}, \quad O_{\varphi d} = i(\varphi^\dagger D^\mu \varphi)(\bar{d}\gamma_\mu d) + \text{h.c.}, \tag{15}$$

$$O_{\varphi e} = i(\varphi^\dagger D^\mu \varphi)(\bar{e}\gamma_\mu e) + \text{h.c.} \tag{16}$$

Finally, there is one operator that modifies the triple gauge boson interactions

$$O_W = \epsilon^{abc}\, W^{a\nu}_\mu W^{b\lambda}_\nu W^{c\mu}_\lambda. \tag{17}$$

B. Non $U(3)^5$ invariant operators

Three are three four-fermion operators

$$O_{qde} = (\bar{\ell}e)(\bar{d}q) + \text{h.c.}, \tag{18}$$

$$O_{lq} = (\bar{l}_a e)\epsilon^{ab}(\bar{q}_b u) + \text{h.c.} \quad O^t_{lq} = (\bar{l}_a \sigma^{\mu\nu} e)\epsilon^{ab}(\bar{q}_b \sigma_{\mu\nu} u) + \text{h.c.} \tag{19}$$

and one operator with two fermions

$$O_{\varphi\varphi} = i(\varphi^T \epsilon D_\mu \varphi)(\bar{u}\gamma^\mu d) + \text{h.c.} \ , \tag{20}$$

which gives rise to a right handed charged current coupling.

The twenty-one $U(3)^5$ invariant operators contribute to precision EW measurements (see Ref. [16]), whereas only nine of the twenty-five operators contribute to the semileptonic decays, including all four $U(3)^5$ breaking operators.

We conclude this section with some remarks on our convention for the coefficients of the "flavored" operators: (i) in those operators that include the h.c. in their definition, the flavor matrix α will appear in the h.c.-part with a dagger; (ii) for the operators $O_{ll}^{(1,3)}$ and O_{ee}, because of the symmetry between the two bilinears, we impose $[\alpha]_{ijkl} = [\alpha]_{klij}$; (iii) in order to ensure the hermiticity of the operators (8)-(12) we impose $[\alpha]_{ijkl} = [\alpha]^*_{jilk}$. None of these conditions entails any loss of generality.

III. EFFECTIVE LAGRANGIAN FOR μ AND QUARK β DECAYS

Our task is to identify new physics contributions to low-energy CC processes. In order to achieve this goal, we need to derive from the the effective lagrangian at the weak scale (in which heavy gauge bosons and heavy fermions are still active degrees of freedom) a low-energy effective lagrangian describing muon and quark CC decays. The analysis involves several steps which we discuss in some detail, since a complete derivation is missing in the literature, as far as we know.

A. Choice of weak basis for fermions

At the level of weak scale effective lagrangian, we can use the $U(3)^5$ invariance to pick a particular basis for the fermionic fields. In general, a $U(3)^5$ transformation leaves the gauge part of the lagrangian invariant while affecting both the Yukawa couplings and the coefficients α_i of dimension six operators involving fermions. We perform a specific $U(3)^5$ transformation that diagonalizes the down-quark and charged lepton Yukawa matrices Y_D and Y_E and puts the up-type Yukawa matrix in the form $Y_U = V^\dagger Y_U^{\rm diag}$, where V is the CKM matrix. The flavored coefficients α_i correspond to this specific choice of weak basis for the fermion fields.

B. Electroweak symmetry breaking: transformation to propagating eigenstates

Once the Higgs acquires a VEV the quadratic part of the lagrangian for gauge bosons and fermions becomes non-diagonal, receiving contributions from both SM interactions and dimension six operators. In particular, the NP contributions induce kinetic mixing of the weak gauge bosons, in addition to the usual mass mixing. Therefore the next step is to perform a change of basis so that the new fields have canonically normalized kinetic term and definite masses.

Let us first discuss the gauge boson sector. We agree with the BW results on the definition of gauge field mass eigenstates and on the expressions for the physical masses (Ref. [10], section 4.1). However, we find small differences from their results in the couplings of the W

and Z to fermion pairs, which can be written as (ref. [10], section 4.2):

$$\begin{aligned}
\mathcal{L}_J &= \frac{g}{\sqrt{2}}\left(J^C_\mu W^{+\mu} + h.c.\right) + \frac{g}{\cos\theta^0_W} J^N_\mu Z^\mu && (21)\\
J^C_\mu &= \bar{\nu}_L\gamma_\mu\eta(\nu_L)e_L + \bar{u}_L\gamma_\mu\eta(u_L)d_L + \bar{u}_R\gamma_\mu\eta(u_R)d_R && (22)\\
J^N_\mu &= \bar{\nu}_L\gamma_\mu\epsilon(\nu_L)\nu_L + \bar{e}_L\gamma_\mu\epsilon(e_L)e_L + \bar{u}_L\gamma_\mu\epsilon(u_L)u_L + \bar{d}_L\gamma_\mu\epsilon(d_L)d_L \\
&\quad +\bar{e}_R\gamma_\mu\epsilon(e_R)e_R + \bar{u}_R\gamma_\mu\epsilon(u_R)u_R + \bar{d}_R\gamma_\mu\epsilon(d_R)d_R\ . && (23)
\end{aligned}$$

Here the ϵ's and η's are 3×3 matrices in flavor space. In the case of the charged current we find (BW do not have the $\dagger$ in $\alpha^{(3)}_{\varphi l}$ and $\alpha^{(3)}_{\varphi q}$)

$$\begin{aligned}
\eta(\nu_L) &= \mathbb{I} + 2\,\hat{\alpha}^{(3)\dagger}_{\varphi l} && (24)\\
\eta(u_L) &= \mathbb{I} + 2\,\hat{\alpha}^{(3)\dagger}_{\varphi q} && (25)\\
\eta(u_R) &= -\hat{\alpha}_{\varphi\varphi}\ , && (26)
\end{aligned}$$

where we have introduced the notation

$$\hat{\alpha}_X = \frac{v^2}{\Lambda^2}\,\alpha_X\ . \tag{27}$$

In the case of the neutral current (ϵ coefficients) we obtain the same results as BW except for the following replacement:

$$\hat{\alpha}_X \to \hat{\alpha}_X + \hat{\alpha}^\dagger_X \tag{28}$$

for $\alpha_X = \alpha^{(3)}_{\varphi l}, \alpha^{(1)}_{\varphi l}, \alpha^{(3)}_{\varphi q}, \alpha^{(1)}_{\varphi q}, \alpha_{\varphi e}, \alpha_{\varphi u}, \alpha_{\varphi d}$.

Finally, we need to diagonalize the fermion mass matrices. With our choice of weak basis for the fermions, the only step that is left is the diagonalization of the up-quark mass matrix, proportional to the Yukawa matrix $Y_U = V^\dagger Y_U^{\rm diag}$, where V is the CKM matrix. This can be accomplished by a $U(3)$ transformation of the u_L fields:

$$u_L \to V^\dagger u_L\ . \tag{29}$$

As a consequence, the charged current and neutral current couplings involving up quarks change as follows:

$$\begin{aligned}
\eta(u_L) &\to V\ \eta(u_L) \\
\epsilon(u_L) &\to V\ \epsilon(u_L)\ V^\dagger\ . && (30)
\end{aligned}$$

Similarly, appropriate insertions of the CKM matrix will appear in every operator that contains the u_L field.

C. Effective lagrangian for muon decay

The muon decay amplitude receives contributions from gauge boson exchange diagrams (with modified couplings) and from contact operators such as $O^{(1)}_{ll}$, $O^{(3)}_{ll}$, O_{le}. Since we work to first order in v^2/Λ^2, we do not need to consider diagrams contributing to $\mu \to e\bar{\nu}_\alpha\nu_\beta$ with the "wrong neutrino flavor", because they would correct the muon decay rate to $O(v^4/\Lambda^4)$. After integrating out the W and Z, the muon decay effective lagrangian reads:

$$\mathcal{L}_{\mu\to e\bar{\nu}_e\nu_\mu} = \frac{-g^2}{2m_W^2}\left[(1+\tilde{v}_L)\cdot \bar{e}_L\gamma_\mu\nu_{eL}\ \bar{\nu}_{\mu L}\gamma^\mu\mu_L \ + \ \tilde{s}_R\cdot \bar{e}_R\nu_{eL}\ \bar{\nu}_{\mu L}\mu_R\right] \ + \ h.c. \ , \tag{31}$$

where $m_W^2 = 1/2g^2v^2$ is the uncorrected W mass and

$$\tilde{v}_L \ = \ 2\ [\hat{\alpha}_{\varphi l}^{(3)}]_{11+22^*} - [\hat{\alpha}_{ll}^{(1)}]_{1221} - 2[\hat{\alpha}_{ll}^{(3)}]_{1122-\frac{1}{2}(1221)} \tag{32}$$

$$\tilde{s}_R \ = \ +2[\hat{\alpha}_{le}]_{2112} \ , \tag{33}$$

represent the correction to the standard $(V-A)\otimes(V-A)$ structure and the coupling associated with the new $(S-P)\otimes(S+P)$ structure, respectively.

D. Effective lagrangian for beta decays: $d_j \to u_i\,\ell^-\,\bar{\nu}_\ell$

The low-energy effective lagrangian for semileptonic transitions receives contributions from both W exchange diagrams (with modified W-fermion couplings) and the four-fermion operators $O_{lq}^{(3)}$, O_{qde}, O_{lq}, O_{lq}^t. As in the muon case, we neglect lepton flavor violating contributions (wrong neutrino flavor). The resulting low-energy effective lagrangian governing semileptonic transitions $d_j \to u_i\,\ell^-\,\bar{\nu}_\ell$ (for a given lepton flavor ℓ) reads:

$$\begin{aligned}\mathcal{L}_{d_j\to u_i\ell^-\bar{\nu}_\ell} \ = \ & \frac{-g^2}{2m_W^2}V_{ij}\Bigg[\Big(1+[v_L]_{\ell\ell ij}\Big)\ \bar{\ell}_L\gamma_\mu\nu_{\ell L}\ \bar{u}_L^i\gamma^\mu d_L^j \ + \ [v_R]_{\ell\ell ij}\ \bar{\ell}_L\gamma_\mu\nu_{\ell L}\ \bar{u}_R^i\gamma^\mu d_R^j \\ & + \ [s_L]_{\ell\ell ij}\ \bar{\ell}_R\nu_{\ell L}\ \bar{u}_R^i d_L^j \ + \ [s_R]_{\ell\ell ij}\ \bar{\ell}_R\nu_{\ell L}\ \bar{u}_L^i d_R^j \\ & + \ [t_L]_{\ell\ell ij}\ \bar{\ell}_R\sigma_{\mu\nu}\nu_{\ell L}\ \bar{u}_R^i\sigma^{\mu\nu}d_L^j\Bigg] \ + \ h.c. \ ,\end{aligned} \tag{34}$$

where

$$V_{ij}\cdot[v_L]_{\ell\ell ij} \ = \ 2\,V_{ij}\left[\hat{\alpha}_{\varphi l}^{(3)}\right]_{\ell\ell} + 2\,V_{im}\left[\hat{\alpha}_{\varphi q}^{(3)}\right]_{jm}^* - 2\,V_{im}\left[\hat{\alpha}_{lq}^{(3)}\right]_{\ell\ell mj} \tag{35}$$

$$V_{ij}\cdot[v_R]_{\ell\ell ij} \ = \ -\,[\hat{\alpha}_{\varphi\varphi}]_{ij} \tag{36}$$

$$V_{ij}\cdot[s_L]_{\ell\ell ij} \ = \ -\,[\hat{\alpha}_{lq}]_{\ell\ell ji}^* \tag{37}$$

$$V_{ij}\cdot[s_R]_{\ell\ell ij} \ = \ -V_{im}\,[\hat{\alpha}_{qde}]_{\ell\ell jm}^* \tag{38}$$

$$V_{ij}\cdot[t_L]_{\ell\ell ij} \ = \ -\,\left[\hat{\alpha}_{lq}^t\right]_{\ell\ell ji}^* \ . \tag{39}$$

In Eqs. (35-39) the repeated indices i,j,ℓ are not summed over, while the index m is.

IV. FLAVOR STRUCTURE OF THE EFFECTIVE COUPLINGS

So far we have presented our results for the effective lagrangian keeping generic flavor structures in the couplings $[\hat{\alpha}_X]_{abcd}$ (see Eqs. 32, 33, and 35 through 39). However, some of the operators considered in the analysis contribute to flavor changing neutral current (FCNC) processes, so that their flavor structure cannot be generic if the effective scale is around $\Lambda \sim$ TeV: the off-diagonal coefficients are experimentally constrained to be very small. While it is certainly possible that some operators (weakly constrained by FCNC) have generic structures, we would like to understand the FCNC suppression needed for many operators in terms of a symmetry principle. Therefore, we organize the discussion in terms of perturbations around the $U(3)^5$ flavor symmetry limit.

If the underlying new physics respects the $U(3)^5$ flavor symmetry of the SM gauge lagrangian, no problem arises from FCNC constraints. The largest contributions to the coefficients are flavor conserving and universal. Flavor breaking contributions arise through SM radiative corrections, due to insertions of Yukawa matrices that break the $U(3)^5$ symmetry. As a consequence, imposing exact $U(3)^5$ symmetry on the underlying model does not seem realistic. A weaker assumption, the Minimal Flavor Violation (MFV) hypothesis, requires that $U(3)^5$ is broken in the underlying model only by structures proportional to the SM Yukawa couplings [17, 18, 19, 20], and by the structures generating neutrino masses [21]. We will therefore organize our discussion in several stages:

1. first, assume dominance of $U(3)^5$ invariant operators;

2. consider effect of $U(3)^5$ breaking induced within MFV;

3. consider the effect of generic non-MFV flavor structures.

In order to proceed with this program, for the relevant operators we list below the flavor structures allowed within MFV. The notation is as follows: we denote by $\bar{\lambda}_{u,d,e}$ the diagonal Yukawa matrices; $\bar{m}_\nu$ represents the diagonal light neutrino mass matrix; V denotes the CKM matrix, while U is the PMNS [22] neutrino mixing matrix; v is the Higgs VEV and Λ_{LN} is the scale of lepton number violation, that appears in the definition of MFV in the lepton sector (we follow here the "minimal" scenario of Ref. [21]). With this notation, the leading "left-left" flavor structures in the quark and lepton sector read:

$$\Delta_{LL}^{(q)} = V^\dagger \bar{\lambda}_u^2 V \tag{40}$$

$$\Delta_{LL}^{(\ell)} = \frac{\Lambda_{\rm LN}^2}{v^4} U \bar{m}_\nu^2 U^\dagger . \tag{41}$$

We use Greek letters $\alpha, \beta, \rho, \sigma$ for the lepton flavor indices, while i, j for the quark flavor indices, and we neglect terms with more than two Yukawa insertions. Moreover, we denote by $\hat{\alpha}_X$, $\hat{\beta}_X$, and $\hat{\gamma}_X$ the numerical coefficients of $O(1) \times v^2/\Lambda^2$ that multiply the appropriate matrices in flavor space. For the operators that have a non-vanishing contribution in the $U(3)^5$ limit, we find:

$$[\hat{\alpha}_{\varphi l}^{(3)}]^{\alpha\beta} = \hat{\alpha}_{\varphi l}^{(3)} \delta^{\alpha\beta} + \hat{\beta}_{\varphi l}^{(3)} (\Delta_{LL}^{(\ell)})^{\alpha\beta} + \ldots \tag{42}$$

$$V^{im} \left[\hat{\alpha}_{\varphi q}^{(3)}\right]^{jm*} = \hat{\alpha}_{\varphi q}^{(3)} V^{ij} + \hat{\beta}_{\varphi q}^{(3)} (V \Delta_{LL}^{(q)})^{ij} + \ldots \tag{43}$$

$$V^{im} \left[\hat{\alpha}_{lq}^{(3)}\right]^{\alpha\beta mj} = \hat{\alpha}_{lq}^{(3)} \delta^{\alpha\beta} V^{ij} + \hat{\beta}_{lq}^{(3)} (\Delta_{LL}^{(\ell)})^{\alpha\beta} V^{ij} + \hat{\gamma}_{lq}^{(3)} \delta^{\alpha\beta} (V \Delta_{LL}^{(q)})^{ij} + \ldots \tag{44}$$

$$\left[\hat{\alpha}_{ll}^{(n)}\right]^{\alpha\beta\rho\sigma} = \hat{\alpha}_{ll}^{(n)} \delta^{\alpha\beta} \delta^{\rho\sigma} + \hat{\beta}_{ll}^{(n)} \left[\delta^{\alpha\beta} (\Delta_{LL}^{(\ell)})^{\rho\sigma} + (\Delta_{LL}^{(\ell)})^{\alpha\beta} \delta^{\rho\sigma}\right] + \ldots \tag{45}$$

$$[\hat{\alpha}_{le}]^{\alpha\beta\rho\sigma} = \hat{\alpha}_{le} \delta^{\alpha\beta}\delta^{\rho\sigma} + \hat{\beta}_{le} (\Delta_{LL}^{(\ell)})^{\alpha\beta} \delta^{\rho\sigma} + \ldots . \tag{46}$$

For the operators that vanish in the limit of exact $U(3)^5$ symmetry, we find:

$$\begin{aligned}
[\hat{\alpha}_{\varphi\varphi}]^{ij} &= \hat{\alpha}_{\varphi\varphi} \left(\bar{\lambda}_u \, V \, \bar{\lambda}_d\right)^{ij} + \ldots && (47) \\
V^{im} \, [\hat{\alpha}_{qde}]^{\alpha\beta jm*} &= \hat{\alpha}_{qde} \, \bar{\lambda}_e^{\alpha\beta} \, (V \bar{\lambda}_d)^{ij} + \hat{\beta}_{qde} \, (\bar{\lambda}_e \, \Delta_{LL}^{(\ell)})^{\alpha\beta} \, (V \bar{\lambda}_d)^{ij} \\
&+ \hat{\gamma}_{qde} \, \bar{\lambda}_e^{\alpha\beta} \, (V \Delta_{LL}^{(q)} \bar{\lambda}_d)^{ij} + \ldots && (48) \\
[\hat{\alpha}_{lq}]^{\alpha\beta ji*} &= \hat{\alpha}_{lq} \, \bar{\lambda}_e^{\alpha\beta} \, (\bar{\lambda}_u V)^{ij} + \hat{\beta}_{lq} \, (\bar{\lambda}_e \Delta_{LL}^{(\ell)})^{\alpha\beta} \, (\bar{\lambda}_u V)^{ij} \\
&+ \hat{\gamma}_{lq} \, \bar{\lambda}_e^{\alpha\beta} \, (\bar{\lambda}_u V \Delta_{LL}^{(q)})^{ij} + \ldots . && (49)
\end{aligned}$$

The coefficient of the tensor operator, $[\alpha_{lq}^{(t)}]$ has an expansion similar to the one of $[\alpha_{lq}]$.

Except for the top quark, the Yukawa insertions typically involve a large suppression factor, as $\bar{\lambda}_i = m_i/v$. In the case of SM extensions containing two Higgs doublets, this scaling can be modified if there is a hierarchy between the vacuum expectation values v_u, v_d of the Higgs fields giving mass to the up- or down-type quarks, respectively. In this case, for large $\tan\beta \equiv v_u/v_d$ the Yukawa insertions scale as:

$$\begin{aligned}
\bar{\lambda}_u &= \frac{m_u}{v \, \sin\beta} \rightarrow \frac{m_u}{v} && (50) \\
\bar{\lambda}_d &= \frac{m_d}{v \, \cos\beta} \rightarrow \frac{m_d}{v} \, \tan\beta && (51) \\
\bar{\lambda}_\ell &= \frac{m_\ell}{v \, \cos\beta} \rightarrow \frac{m_\ell}{v} \, \tan\beta && (52)
\end{aligned}$$

V. PHENOMENOLOGY OF V_{ud} AND V_{us}: OVERVIEW

Using the general effective lagrangians of Eqs. (31) and (34) for charged current transitions, one can calculate the deviations from SM predictions in various semileptonic decays. In principle a rich phenomenology is possible. Helicity suppressed leptonic decays of mesons have been recently analyzed in Ref. [23]. Concerning semileptonic transitions, several reviews treat in some detail β decay differential distributions [24, 25]. Here we focus on the integrated decay rates, which give access to the CKM matrix elements V_{ud} and V_{us}: since both the SM prediction and the experimental measurements are reaching the sub-percent level, we expect these observables to provide strong constraints on NP operators.

V_{ud} and V_{us} can be determined with high precision in a number of channels. The degree of needed theoretical input varies, depending on which component of the weak current contributes to the hadronic matrix element. Roughly speaking, one can group the channels leading to $V_{ud,us}$ into three classes:

- semileptonic decays in which only the vector component of the weak current contributes. These are theoretically favorable in the Standard Model because the matrix elements of the vector current at zero momentum transfer are known in the $SU(2)$ ($SU(3)$) limit of equal light quark masses: $m_u = m_d$ ($= m_s$). Moreover, corrections to the symmetry limit are quadratic in $m_{s,d} - m_u$ [26, 27]. Super-allowed nuclear beta decays ($0^+ \to 0^+$), pion beta decay ($\pi^+ \to \pi^0 e^+ \nu_e$), and $K \to \pi \ell \nu$ decays belong to this class. The determination of $V_{ud,us}$ from these modes requires theoretical input on radiative corrections [28, 29, 30, 31, 32] and hadronic matrix elements via analytic methods [33], [34, 35, 36, 37], or lattice QCD methods [38, 39, 40, 41].

- semileptonic transitions in which both the vector and axial component of the weak current contribute. Neutron decay ($n \to pe\bar{\nu}$) and hyperon decays ($\Lambda \to pe\bar{\nu}$, ...) belong to this class. In this case the matrix elements of the axial current have to be determined experimentally [42].

  Inclusive τ lepton decays $\tau \to h\nu_\tau$ belong to this class (both V and A current contribute), and in this case the relevant matrix elements can be calculated theoretically via the Operator Product Expansion [43, 44].

- Leptonic transitions in which only the axial component of the weak current contributes. In this class one finds meson decays such as $\pi(K) \to \mu\nu$ but also exclusive τ decays such as $\tau \to \nu_\tau \pi(K)$. Experimentally one can determine the products $V_{ud} \cdot F_\pi$ and $V_{us} \cdot F_K$. With the advent of precision calculations of F_K/F_π in lattice QCD [45, 46, 47, 48, 49], this class of decays provides a useful constraint on the ratio V_{us}/V_{ud} [50].

Currently, the determination of V_{ud} is dominated by $0^+ \to 0^+$ super-allowed nuclear beta decays [33], while the best determination of V_{us} arises from $K \to \pi\ell\nu$ decays [3]. Experimental improvements in neutron decay and τ decays, as well as in lattice calculations of the decay constants will allow in the future competitive determinations from other channels. In light of this, we set out to perform a comprehensive analysis of possible new physics effects in the extraction of V_{ud} and V_{us}.

As outlined in the previous section, we start our analysis by assuming dominance of the $U(3)^5$ invariant operators. These are not constrained by FCNC and can have a relatively low effective scale Λ. In the $U(3)^5$ limit the phenomenology of CC processes greatly simplifies: all V_{ij} receive the same universal shift (coming from the same short distance structure). As a consequence, extractions of $V_{ud,us}$ from different channels (vector transitions, axial transitions, *etc.*) should agree within errors. Therefore, in this limit the new physics effects are entirely captured by the quantity

$$\Delta_{\rm CKM} \equiv |V_{ud}^{(\rm pheno)}|^2 + |V_{us}^{(\rm pheno)}|^2 + |V_{ub}^{(\rm pheno)}|^2 \ - \ 1 \ , \tag{53}$$

constructed from the $V_{ij}^{(\rm pheno)}$ elements extracted from semileptonic transitions using the standard procedure outlined below. We now make these points more explicit.

A. Extraction of V_{ij} and contributions to $\Delta_{\rm CKM}$ in the $U(3)^5$ limit

If we assume $U(3)^5$ invariance, only the SM operator survives in the muon decay lagrangian of Eq. (31), with [3]

$$\tilde{v}_L = 4\,\hat{\alpha}_{\varphi l}^{(3)} - 2\,\hat{\alpha}_{ll}^{(3)} \ . \tag{54}$$

Therefore, in this case the effect of new physics can be encoded into the following definition of the leptonic Fermi constant:

$$G_F^\mu = (G_F)^{(0)} \ (1 + \tilde{v}_L) \ , \tag{55}$$

[3] We disagree with the result of BW on the sign of $\hat{\alpha}_{ll}^{(3)}$.

where $G_F^{(0)} = g^2/(4\sqrt{2}m_W^2)$. Similarly, in the $U(3)^5$ symmetry limit, only the SM operator survives in the effective langrangian for semileptonic quark decays of Eq. (34), with coupling:

$$[v_L]_{\ell\ell ij} \ \to \ v_L \equiv 2\left(\hat{\alpha}_{\varphi l}^{(3)} + \hat{\alpha}_{\varphi q}^{(3)} - \hat{\alpha}_{lq}^{(3)}\right) \ . \tag{56}$$

As in the muon decay, the new physics can be encoded in a (different) shift to the effective semileptonic (SL) Fermi constant:

$$G_F^{\rm SL} = (G_F)^{(0)} \ (1 + v_L) \ . \tag{57}$$

The value of V_{ij} extracted from semileptonic decays is affected by this redefinition of the semileptonic Fermi constant and by the shift in the muon Fermi constant G_F^μ, to which one usually normalizes semileptonic transitions. In fact one has

$$\begin{aligned} V_{ij}^{(\rm pheno)} &= V_{ij} \ \frac{G_F^{\rm SL}}{G_F^\mu} = V_{ij} \ (1 + v_L - \tilde{v}_L) \\ &= V_{ij} \left[1 + 2\left(\hat{\alpha}_{ll}^{(3)} - \hat{\alpha}_{lq}^{(3)} - \hat{\alpha}_{\varphi l}^{(3)} + \hat{\alpha}_{\varphi q}^{(3)}\right)\right] \ . \end{aligned} \tag{58}$$

So in the $U(3)^5$ limit a common shift affects all the V_{ij} (from all channels). The only way to expose new physics contributions is to construct universality tests, in which the absolute normalization of V_{ij} matters. For light quark transitions this involves checking that the first row of the CKM matrix is a vector of unit length (see definition of $\Delta_{\rm CKM}$ in Eq. (53)). The new physics contributions to $\Delta_{\rm CKM}$ involve four operators of our basis and read:

$$\Delta_{\rm CKM} = 4\left(\hat{\alpha}_{ll}^{(3)} - \hat{\alpha}_{lq}^{(3)} - \hat{\alpha}_{\varphi l}^{(3)} + \hat{\alpha}_{\varphi q}^{(3)}\right) \ . \tag{59}$$

In specific SM extensions, the $\hat{\alpha}_i$ are functions of the underlying parameters. Therefore, through the above relation one can work out the constraints of quark-lepton universality tests on any weakly coupled SM extension.

B. Beyond $U(3)^5$

Corrections to the $U(3)^5$ limit can be introduced both within MFV and via generic flavor structures. In MFV, as evident from the results of Section IV, the coefficients parameterizing deviations from $U(3)^5$ are highly suppressed. This is true even when one considers the flavor diagonal elements of the effective couplings, due to the smallness of the Yukawa eigenvalues and the hierarchy of the CKM matrix elements. As a consequence, in MFV we expect the conclusions of the previous subsections to hold. The various CKM elements V_{ij} receive a common dominant shift plus suppressed channel-dependent corrections, so that Eq. (59) remains valid to a good approximation. In other words, both in the exact $U(3)^5$ limit and in MFV, $\Delta_{\rm CKM}$ probes the leading coefficients $\hat{\alpha}_X$ of the four operators $O_{\rm CKM} = \{O_{ll}^{(3)}, O_{lq}^{(3)}, O_{\varphi l}^{(3)}, O_{\varphi q}^{(3)}\}$.

In a generic non-MFV framework, the channel-dependent shifts to V_{ij} could be appreciable, so that $\Delta_{\rm CKM}$ would depend on the channels used to extract $V_{ud,us}$. Therefore, comparing the values of V_{us} and V_{ud} (or their ratios) extracted from different channels

Classification	Standard Notation	Measurement	Reference
Atomic parity	$Q_W(Cs)$	Weak charge in Cs	[51]
violation (Q_W)	$Q_W(Tl)$	Weak charge in Tl	[52]
DIS	g_L^2, g_R^2	ν_μ-nucleon scattering from NuTeV	[53]
	R^ν	ν_μ-nucleon scattering from CDHS and CHARM	[54, 55]
	κ	ν_μ-nucleon scattering from CCFR	[56]
	$g_V^{\nu e}, g_A^{\nu e}$	ν-e scattering from CHARM II	[57]
Zline	Γ_Z	Total Z width	[58, 59]
(lepton and	σ_0	e^+e^- hadronic cross section at Z pole	[58, 59]
light quark)	$R_f^0 (f = e, \mu, \tau)$	Ratios of lepton decay rates	[58, 59]
	$A_{FB}^{0,f} (f = e, \mu, \tau)$	Forward-backward lepton asymmetries	[58, 59]
pol	$A_f (f = e, \mu, \tau)$	Polarized lepton asymmetries	[58, 59]
bc	$R_f^0 (f = b, c)$	Ratios of hadronic decay rates	[58, 59]
(heavy quark)	$A_{FB}^{0,f} (f = b, c)$	Forward-backward hadronic asymmetries	[58, 59]
	$A_f (f = b, c)$	Polarized hadronic asymmetries	[58, 59]
LEPII Fermion	$\sigma_f (f = q, \mu, \tau)$	Total cross sections for $e^+e^- \to f\overline{f}$	[58, 59]
production	$A_{FB}^f (f = \mu, \tau)$	Forward-backward asymmetries for $e^+e^- \to f\overline{f}$	[58, 59]
eOPAL	$d\sigma_e / d\cos\theta$	Differential cross section for $e^+e^- \to e^+e^-$	[60]
WL3	$d\sigma_W / d\cos\theta$	Differential cross section for $e^+e^- \to W^+W^-$	[61]
MW	M_W	W mass	[58, 59, 62]
Q_{FB}	$\sin^2\theta_{eff}^{lept}$	Hadronic charge asymmetry	[58, 59]

TABLE I: Measurements included in this analysis. This summary table was taken directly from Table I of [16] and repeated here for convenience. We added some details in the classification column as well as additional experimental references.

gives us a handle on $U(3)^5$ breaking structures beyond MFV. We will discuss this in a separate publication, where we will analyze the new physics contributions to the ratios $V_{ud}^{0^+\to 0^+}/V_{ud}^{n\to pe\bar{\nu}}$, $V_{us}^{K\to\pi\ell\nu}/V_{ud}^{0^+\to 0^+}$, $V_{us}^{K\to\mu\nu}/V_{ud}^{\pi\to\mu\nu}$, and $(V_{us}//V_{ud})^{\tau\to\nu\, h}$ from both inclusive and exclusive channels. In summary, we organize our analysis in two somewhat orthogonal parts, as follows:

- In the rest of this work we focus on the phenomenology of $\Delta_{\rm CKM}$ and its relation to other precision measurements. This analysis applies to models of TeV scale physics with approximate $U(3)^5$ invariance, in which flavor breaking is suppressed by a symmetry principle (as in MFV) or by the hierarchy $\Lambda_{\rm flavor} \gg$ TeV

- In a subsequent publication we will explore in detail the constraints arising by comparing the values of V_{us} (V_{ud}) extracted from different channels. These constraints probe the $U(3)^5$ breaking structures, to which other precision measurements (especially at high energy) are essentially insensitive.

VI. Δ_{CKM} VERSUS PRECISION ELECTROWEAK MEASUREMENTS

In the limit of approximate $U(3)^5$ invariance, we have shown in Eq. (59) that Δ_{CKM} constraints a specific combination of the coefficients $\hat{\alpha}_{ll}^{(3)}, \hat{\alpha}_{lq}^{(3)}, \hat{\alpha}_{\varphi l}^{(3)}, \hat{\alpha}_{\varphi q}^{(3)}$. Each of these coefficients also contributes to other low- and high-energy precision electroweak measurements [16], together with the remaining seventeen operators that make up the $U(3)^5$ invariant sector of our TeV scale effective lagrangian (see Sect. II A). Therefore, we can now address concrete questions such as: what is the maximal deviation $|\Delta_{\mathrm{CKM}}|$ allowed once all the precision electroweak constraints have been taken into account? Which observables provide the strongest constraints on the operators contributing to Δ_{CKM}? How does the inclusion of Δ_{CKM} affect the fit to precision electroweak measurements? Should a deviation $\Delta_{\mathrm{CKM}} \neq 0$ be established, in what other precision observables should we expect a tension with the SM prediction? At what level?

Our task greatly benefits from the work of Han and Skiba (HS) [16], who studied the constraints on the same set of twenty-one $U(3)^5$ invariant operators via a global fit to precision electroweak data. We employ a modified version of their publicly available fitting code in what follows. The analysis utilizes the experimental data summarized in Table I. The procedure involves constructing the χ^2 function for the observables listed in Table I, which contains 237 generally correlated terms. Indicating with $X^i_{\mathrm{th}}(\hat{\alpha}_k)$ the theoretical prediction for observable X^i (including SM plus radiative correction plus first order shift in $\hat{\alpha}_k = \alpha_k v^2/\Lambda^2$), and with X^i_{exp} the experimental value, the χ^2 reads

$$\chi^2(\hat{\alpha}_k) = \sum_{i,j} \left(X^i_{\mathrm{th}}(\hat{\alpha}_k) - X^i_{\mathrm{exp}} \right) \left(\sigma^2 \right)^{-1}_{ij} \left(X^j_{\mathrm{th}}(\hat{\alpha}_k) - X^j_{\mathrm{exp}} \right) \tag{60}$$

where $\sigma^2_{ij} = \sigma_i \; \rho_{ij} \; \sigma_j$ is expressed in terms of the combined theoretical and experimental standard deviation σ_i and the correlation matrix ρ_{ij}. For more details, we refer to Ref. [16]. In our numerical analysis we essentially use the code of HS[4] and minimally extend it by including the Δ_{CKM} constraint in the χ^2 function. Given the phenomenological input $V_{ud} = 0.97425(22)$ [33], $V_{us} = 0.2252(9)$ [63], we obtain the constraint $\Delta_{\mathrm{CKM}} = (-1 \pm 6) \times 10^{-4}$ [63]. Δ_{CKM} has essentially no correlation with the other precision measurements, due to the small fractional uncertainty in the Fermi constant.

We perform two different analyses, one in which all operators O_X are allowed to contribute, and one in which only a single operator at a time has non vanishing coefficient. These two regimes represent extreme model scenarios and possess different characteristics. In the global analysis, due to the large number of parameters, cancellations can dilute the impact of specific observables: the burden of satisfying a tight constraint from a given observable can be "shared" by several operators. On the other hand, within the single-operator analysis one may easily find correlations between different sets of measurements. We think of the single operator analysis as a survey of a simplified class of models, in which only one dominant effective operator is generated.

---

[4] We prefer to quote final results in terms of the dimensionless ratios $\hat{\alpha}_k = \alpha_k v^2/\Lambda^2$ ($v \simeq 174$ GeV) instead of $a_k = 1/\Lambda_k^2$ as in HS.

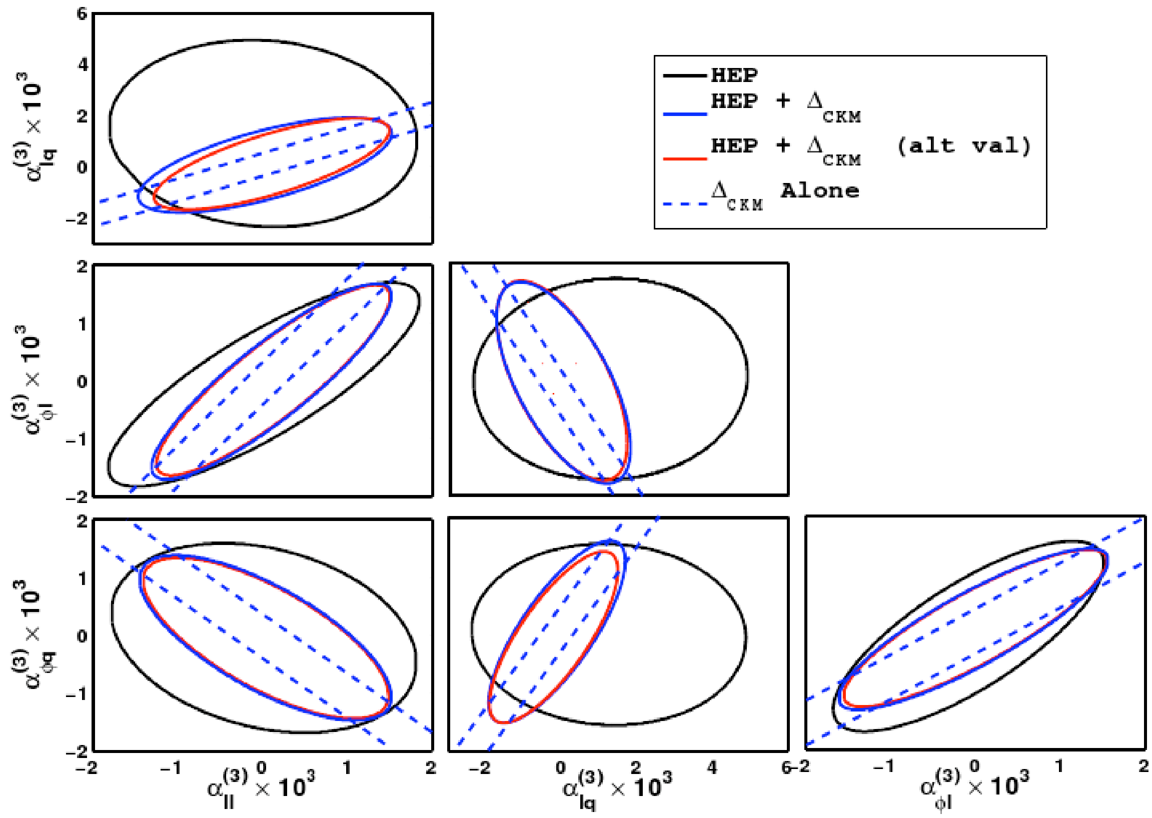

FIG. 1: 90% allowed regions for the coefficients $\hat{\alpha}^{(3)}_{ll}, \hat{\alpha}^{(3)}_{lq}, \hat{\alpha}^{(3)}_{\varphi l}, \hat{\alpha}^{(3)}_{\varphi q}$. These are projections from the 21 dimensional ellipsoid, obtained from the fitting code. We include the results for high energy observables alone (HEP, black unbroken curves), high energy data plus the current $\Delta_{\rm CKM}$ constraint (blue unbroken curve), high energy data plus the alternative value of $\Delta_{\rm CKM} = -0.0025 \pm 0.0006$ (red unbroken curve) and the bounds from the current $\Delta_{\rm CKM}$ alone (blue dashed curve).

A. Global analysis

In order to quantify the significance of the experimental CKM unitarity constraint, we first calculate the range of $\Delta_{\rm CKM}(\hat{\alpha}_k)$ allowed by existing bounds from all the precision electroweak measurements included in Table I. In terms of the best fit values and the covariance matrix of the $\hat{\alpha}_i$ [16] obtained from the fit to electroweak precision data, we find

$$-9.5 \times 10^{-3} \ \leq \ \Delta_{\rm CKM} \ \leq \ 0.1 \times 10^{-3} \qquad\qquad (90\% \text{ C.L.}) \ , \tag{61}$$

to be compared with the direct 90% C.L. bound $|\Delta_{\rm CKM}| \leq 1. \times 10^{-3}$. The first lesson from this exercise is that electroweak precision data leave ample room for a sizable non-zero $\Delta_{\rm CKM}$: the direct constraint is nearly an order of magnitude stronger than the indirect one! Therefore, one should include the $\Delta_{\rm CKM}$ constraint in global fits to the effective theory parameters.

The next question we address is: what is the impact of adding the $\Delta_{\rm CKM}$ constraint to the global electroweak fit? The chi-squared per degrees of freedom changes only marginally, from $\chi^2/d.o.f. = 180.12/215$ to $\chi^2/d.o.f. = 173.74/216$. We find that essentially the only impact is to modify the allowed regions for $\hat{\alpha}_{ll}^{(3)}, \hat{\alpha}_{lq}^{(3)}, \hat{\alpha}_{\varphi l}^{(3)}, \hat{\alpha}_{\varphi q}^{(3)}$. To illustrate this, in Figure 1, we display the projection of the twenty-one dimensional 90% confidence ellipsoid onto the relevant planes involving $\hat{\alpha}_{ll}^{(3)}, \hat{\alpha}_{lq}^{(3)}, \hat{\alpha}_{\varphi l}^{(3)}, \hat{\alpha}_{\varphi q}^{(3)}$. The black curves represent bounds before the inclusion of the $\Delta_{\rm CKM}$ constraint. The dashed blue lines outline the allowed regions found by considering only the effect of current $\Delta_{\rm CKM}$ bounds (Eq. 59): the regions are unbounded because large values of any of the $\hat{\alpha}_i$ may be canceled by a correspondingly large contribution of other operators. The situation changes when high energy observables are taken into account, as can be seen from the combined fit solid blue curve. Despite the relatively weak indirect $\Delta_{\rm CKM}$ constraints from high energy data, the unbounded parameter directions are cut off at the edge of the allowed black contour. In the orthogonal direction, the combined ellipse is shrunk significantly by the strong $\Delta_{\rm CKM}$ bound. Thus, the solid blue contour is rotated and contracted with respect to its parent black region. As evident from the figure, the main effect of including $\Delta_{\rm CKM}$ is to strengthen the constraints on the four-fermion operator $O_{lq}^{(3)}$.

At this stage we may also ask how would this picture change if a significant deviation from Cabibbo universality were to be observed. To answer this question, we show in Figure 1, the 90 % C.L. allowed regions (red solid curve) obtained by assuming a $\sim 4\,\sigma$ deviation, namely $\Delta_{\rm CKM} = -0.0025 \pm 0.0006$[5]. One can see that changing the central value of $\Delta_{\rm CKM}$ has only a minor effect on the allowed regions: the fit is driven by the comparatively small $\Delta_{\rm CKM}$ uncertainty, rather than its central value. While the fitting procedure tends to minimize the χ^2 contribution from $\Delta_{\rm CKM}$, this does not generate much tension with the remaining observables, as other operators can compensate the effect of potentially non-vanishint $\hat{\alpha}_i \subset \hat{\alpha}_{\rm CKM}$.

B. Single operator analysis

To gain a better understanding of the interplay between the $\Delta_{\rm CKM}$ constraint and other precision measurements, we embark on a single operator analysis. We assume that a single operator at a time dominates the new physics contribution and set all others to zero. A similar analysis (not including the CKM constraints) has been performed in [64]. We will only consider the operator set $O_{\rm CKM} = \{O_{ll}^{(3)}, O_{lq}^{(3)}, O_{\varphi l}^{(3)}, O_{\varphi q}^{(3)}\}$ that contributes to $\Delta_{\rm CKM}$, because for the other operators the analysis would coincide with that of Ref. [64]. In this simplified context we can ask questions about

(i) the relative strength of $\Delta_{\rm CKM}$ versus other precision electroweak measurements in constraining the non-zero $\hat{\alpha}_i$;

(ii) the size of correlations among SM deviations in various observables.

In order to address the first question above, for each coefficient $\hat{\alpha}_i \subset \hat{\alpha}_{\rm CKM}$ we derive the 90 % C.L. allowed intervals implied by: (a) the global fit to all precision electroweak

[5] This value has been chosen for illustrative purposes and could be realized if the central value of V_{us} from $K_{\ell 3}$ decays shifted down to $V_{us} = 0.2200$, which is preferred by current analytic estimates of the vector form factor (see Refs. [35, 36, 37]).

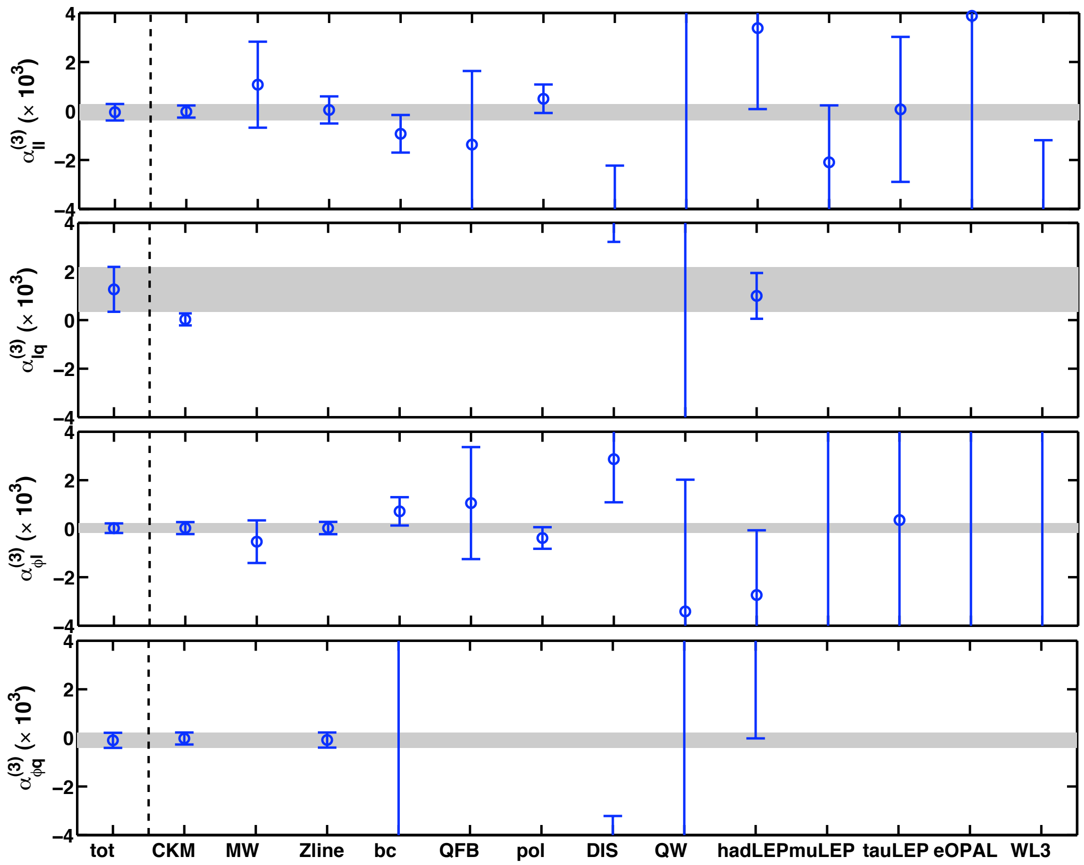

FIG. 2: The 90 % C.L. allowed regions for the coefficients $\hat{\alpha}_i$ within the single operator analysis. The first column displays the constraint from all precision observables except $\Delta_{\rm CKM}$. The second column displays the constraint coming exclusively from $\Delta_{\rm CKM}$. The remaining columns display the constraint derived from each subset of measurements listed in Table I.

measurements except $\Delta_{\rm CKM}$ (first column in Figure 2, also denoted by horizontal gray bands); (b) the $\Delta_{\rm CKM}$ constraint via Eq. (59) (second column in Figure 2); (c) each subset of measurements listed in Table I (remaining columns in Figure 2). Missing entries in Figure (2) signify that the measurement sets are independent of the selected operator. The plot nicely illustrates that, for the operators $O_i \subset O_{\rm CKM}$, the direct $\Delta_{\rm CKM}$ measurement provides constraints at the same level (for $\hat{\alpha}_{\varphi l}^{(3)}$) or better then the Z pole observables. Looking at the size of the constraints, we can immediately conclude that the operators $O_{ll}^{(3)}$, $O_{\varphi l}^{(3)}$, $O_{\varphi q}^{(3)}$, are quite tightly constrained by Z lineshape observables (fourth column in Figure 2), so that very little room is left for CKM unitarity violations. On the other hand, the operator $O_{lq}^{(3)}$ is relatively poorly constrained by electroweak precision data (LEP2 $e^+e^- \to q\bar{q}$ cross section provides the best constraint) and could account for significant deviations of $\Delta_{\rm CKM}$ from zero (first column of the second panel from top in Figure 2). In this case, the direct constraint is by far the tightest.

Should a non-zero $\Delta_{\rm CKM}$ be observed, in the single-operator framework it would be correlated to deviations from the SM expectation in other observables as well, since there is only one parameter in the problem (the coefficient $\hat{\alpha}_k$ of the dominant operator considered). We

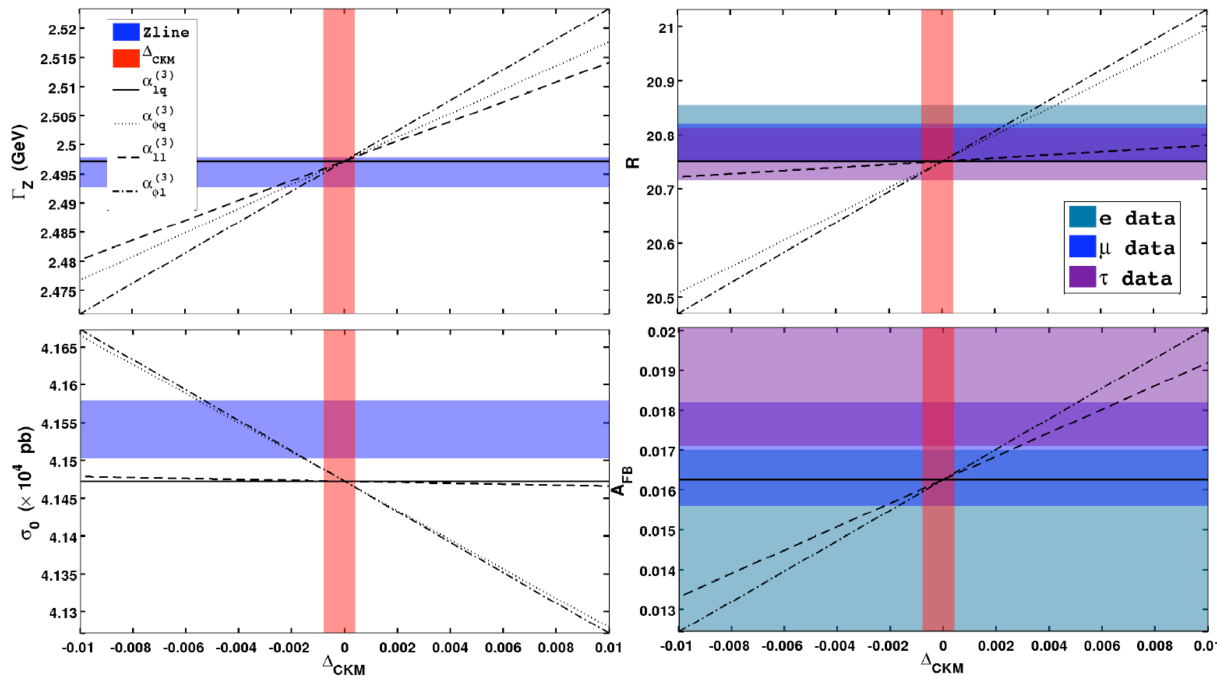

FIG. 3: Correlation of various Z pole observables with Δ_{CKM}. Operator $O_{lq}^{(3)}$ is not constrained by these measurements. The $O_{lq}^{(3)}$ and $O_{\varphi q}^{(3)}$ lines are degenerate in the A_{FB} panel. The 1σ bands for $\Delta_{\rm CKM}$ and Z pole measurements are shown in red and blue, respectively. The right panel bands are shaded differently to indicate e, μ and τ measurements separately. In the lower left panel $\sigma_0 = (12\pi\Gamma_{ee}\Gamma_{had})/(M_Z^2\Gamma_Z^2)$ parameterizes the maximum Z-pole cross-section for $e^+e^- \to$ had.

have studied quantitatively the expected correlation between $\Delta_{\rm CKM}$ and the most sensitive electroweak measurements. In Figures 3 and 4 we report the correlation between $\Delta_{\rm CKM}$ and Z pole observables. In these figures, each black line (solid or broken) corresponds to a given single-operator model, in which only one $\hat{\alpha}_k \neq 0$. Each point on the black line correspond to a particular value of $\hat{\alpha}_k$. A flat black line indicate that no correlation exists between the two observables considered. The red shaded bands indicate the current 1-σ Δ_{CKM} direct constraint, while the blue bands correspond to the 1-σ Z-pole observables. We use different blue shading to indicate various measurements included in the analysis. For example, the forward backward asymmetries (A_{FB}) and decay branching ratios (R) are shown in different color for each charged lepton flavor.

Figures 3 and 4 clearly illustrate how much we can move $\Delta_{\rm CKM}$ from zero before getting into some tension with Z pole precision measurements. Moreover, should a given $\Delta_{\rm CKM} \neq 0$ be measured, we can immediately read off in which direction other precision measurement should move, and by how much, within this class of models.

The model in which $O_{lq}^{(3)}$ is the dominant operator is somewhat special, as Z-pole observables do not put any constraint. In this model, correlations arise among the following four observables: $\Delta_{\rm CKM}$, the LEP2 $e^+e^- \to q\bar{q}$ cross section, neutrino DIS (in particular the NuTeV measurements of the ratios of NC to CC in $\nu_\mu - N$ DIS), and Atomic Parity Violation, which has only a very weak dependence on $\hat{\alpha}_{lq}^{(3)}$. The two tightest constraints arise from $\Delta_{\rm CKM}$ and LEP2. From the correlation plot in Figure 5 (upper panel, solid line)

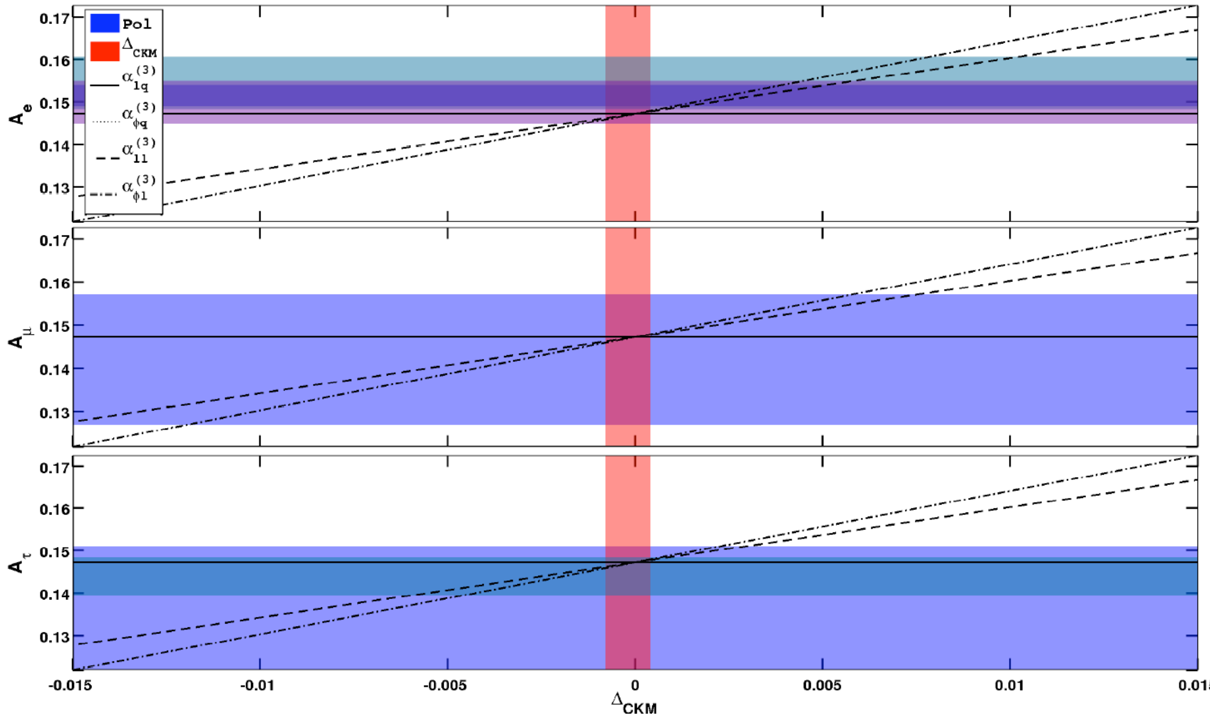

FIG. 4: Correlation of Z-pole polarized lepton asymmetries with $\Delta_{\rm CKM}$. Operators $O_{lq}^{(3)}$ and $O_{\varphi q}^{(3)}$ are not constrained by these measurements. The 1σ bands for $\Delta_{\rm CKM}$ and lepton asymmetries are shown in red and blue, respectively. Different blue shading correspond to different measurements.

one can see how LEP2 data in principle leave room for substantial quark-lepton universality violations, up to $|\Delta_{\rm CKM}| \sim 0.005$ at the 1-σ level. In the lower panel of Figure 5. we report the correlation plot between $\Delta_{\rm CKM}$ and the effective neutrino-nucleon coupling g_L^2 extracted from NuTeV data. The striking feature of this plot is that an explanation of the deviation between the SM prediction and the NuTeV measured range of g_L^2 in terms $O_{lq}^{(3)}$ (solid line) would require a $\Delta_{\rm CKM}$ at least 16σ below its current value.

VII. CONCLUSIONS

In this article we have investigated in a model-independent framework the impact of quark-lepton universality tests on probing physics beyond the Standard Model. We have identified a minimal set of twenty-five weak scale effective operators describing corrections beyond the SM to precision electroweak measurements and semileptonic decays. In terms of new physics corrections at the TeV scale, we have derived the low-energy effective lagrangians describing muon decay and beta decays, specifying both the most general flavor structure of the operators as well as the form allowed within Minimal Flavor Violation.

We have performed the phenomenological analysis assuming nearly flavor blind ($U(3)^5$ invariant) new physics interactions. In this framework flavor breaking is suppressed by a symmetry principle, such as the Minimal Flavor Violation hypothesis, or by the hierarchy $\Lambda_{\rm flavor} \gg$ TeV. We have shown that in this limit, the extraction of V_{ud} and V_{us} from any

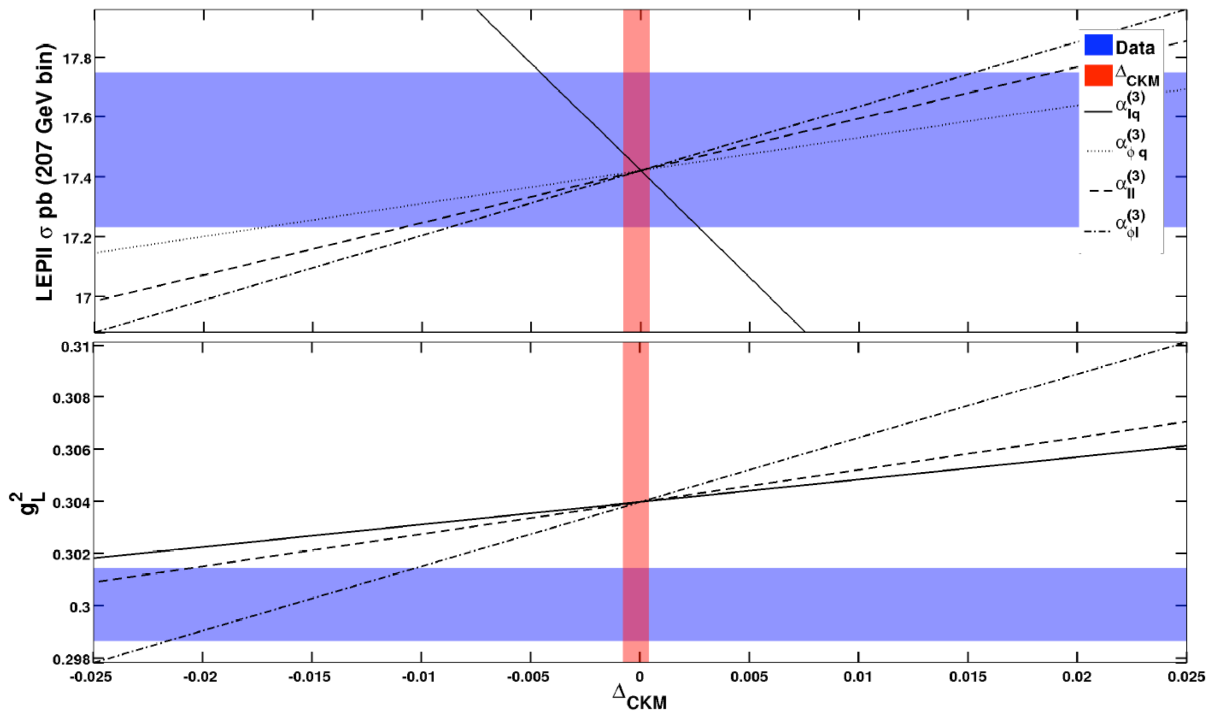

FIG. 5: Upper panel: correlation between $\Delta_{\rm CKM}$ and $\sigma(e^+e^- \to q\bar{q})(\sqrt{s} = 207$ GeV). Lower panel: correlation between $\Delta_{\rm CKM}$ and the effective neutrino-nucleon couplings g_L^2 measured by NuTeV. The 1σ bands for $\Delta_{\rm CKM}$ and the other observable are shown in red and blue, respectively.

channel should give the same result and the only significant probe of physics beyond the SM involves the quantity $\Delta_{\rm CKM} \equiv |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1$. In a subsequent publication we will explore the constraints arising by comparing the values of V_{us} (V_{ud}) extracted from different channels. These constraints probe those $U(3)^5$-breaking structures to which FCNC and other precision measurements are quite insensitive.

We have shown that in the $U(3)^5$ limit $\Delta_{\rm CKM}$ receives contributions from four short distance operators, namely $O_{\rm CKM} = \{O_{ll}^{(3)}, O_{lq}^{(3)}, O_{\varphi l}^{(3)}, O_{\varphi q}^{(3)}\}$, which also shift SM predictions in other precision observables. Using the result of Eq. 59, one can work out the constraints imposed by Cabibbo universality on any weakly coupled extension of the SM. Here we have focused on the model-independent interplay of $\Delta_{\rm CKM}$ with other precision measurements. The main conclusions of our analysis are:

- The $\Delta_{\rm CKM}$ constraint bounds the effective scale of all four operators $O_i \subset O_{\rm CKM}$ to be $\Lambda > 11$ TeV (90 % C.L.). For the operators $O_{ll}^{(3)}, O_{\varphi l}^{(3)}, O_{\varphi q}^{(3)}$ this constraint is at the same level as the Z-pole measurements. For the four-fermion operator $O_{lq}^{(3)}$, $\Delta_{\rm CKM}$ improves existing bounds from LEP2 by one order of magnitude.

- Another way to state this result is as follows: should the central values of V_{ud} and V_{us} move from the current values [3], precision electroweak data would leave room for sizable deviations from quark-lepton universality (roughly one order of magnitude above the current direct constraint). In a global analysis, the burden of driving a deviation from CKM unitarity could be shared by the four operators $O_i \subset O_{\rm CKM}$.

In a single operator analysis, essentially only the four-fermion operator $O_{lq}^{(3)}$ could be responsible for $\Delta_{\rm CKM} \neq 0$, as the others are tightly bound from Z-pole observables.

Our conclusions imply that the study of semileptonic processes and Cabibbo universality tests provide constraints on new physics beyond the SM that currently cannot be obtained from other electroweak precision tests and collider measurements.

APPENDIX A: DETAILS ON THE OPERATOR BASIS

In this appendix we discuss how to obtain from the BW operator basis the minimal subset describing CP-conserving electroweak precision observables and beta decays. We start with a few comments on the BW operator list, pointing out a few typos and omissions:

- The four-fermion operator $O_{lq}^t = (\bar{l}_a \sigma^{\mu\nu} e)\epsilon^{ab}(\bar{q}_b \sigma_{\mu\nu} u)$ must be added to the list (the ϵ tensor is used to contract weak SU(2) indices).
- The operators $O_{qq}^{(8,1)}, O_{qq}^{(8,3)}, O_{uu}^{(8)}$ and $O_{dd}^{(8)}$ can be eliminated using the Fierz transformation and the completeness relation of the Pauli (Gell-Mann) matrices: $\sum_I \tau^I_{ij}\tau^I_{kl} = -\delta_{ij}\delta_{kl} + 2\delta_{il}\delta_{kj}$;
- The dagger in the operator (3.55) should be replaced by a T (transpose symbol);
- The names O_{uG} and O_{dG} have been used twice in BW: operators (3.34, 3.36) and operators (3.61, 3.63).

As a result of the above observations, the complete list of dimension six operators involves seventy-seven operators.

Once the CP-assumption is taken into account, we have seventy-one operators in our effective lagrangian[6]. Moreover, we will not take into account the thirteen operators that involve only quark and gluon fields[7], because they will not appear in our observables (precision EW measurements and semileptonic decays) at the level we are working. Further operators that do not contribute to our observables are O_{qG}, O_{uG}, O_{dG}.

Since we are not considering processes involving the Higgs boson as an external particle, we can remove more operators from our list: $O_\varphi, O_{\partial\varphi}$ (they only involve scalar fields), and seven more operators[8] whose effect can be absorbed in a redefinition of the SM parameters g, g', g_s, v and the Yukawa couplings. In this way we end up with forty-six operators that can produce a linear correction to the SM-prediction of our observables. But a more detailed analysis of this list shows that twenty-one of them either do not produce linear corrections (because the interference with the SM vanishes) or produce effects suppressed by an additional factor (for example, low energy four-quark operators of dimension seven).

Finally we have the twenty-five operators listed in the text: twenty-one of them are invariant under the flavor symmetry $U(3)^5$ and contribute without suppression to the precision EW measurements [16]. The remaining four operators are non-invariant under $U(3)^5$.

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[6] The six operators removed are O_X with $X = \tilde{G}, \tilde{W}, \varphi\tilde{G}, \varphi\tilde{W}, \varphi\tilde{B}, \tilde{W}B$.

[7] O_X with $X = G, qq^{(1)}, qq^{(8)}, uu^{(1)}, dd^{(1)}, qq^{(1,1)}, qq^{(1,3)}, ud^{(1)}, ud^{(8)}, qu^{(1)}, qu^{(8)}, qd^{(1)}, qd^{(8)}$.

[8] O_X with $X = \varphi W, \varphi B, \varphi^{(1)}, \varphi G, e\varphi, u\varphi, d\varphi$

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