Few-Body Systems 0, 1–6 (2009)

Few-Body Systems

Four-quark stability*

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Abstract. The physics of charm has become one of the best laboratories exposing the limitations of the naive constituent quark model and also giving hints into a more mature description of meson spectroscopy, beyond the simple quark–antiquark configurations. In this talk we review some recent studies of multiquark components in the charm sector and discuss in particular exotic and non-exotic four-quark systems, both with pairwise and many-body forces.

More than thirty years after the so-called November revolution [1], heavy hadron spectroscopy remains a challenge. The formerly comfortable world of heavy mesons is shaken by new results [2]. This started in 2003 with the discovery of the $D^*_{s0}(2317)$ and $D_{s1}(2460)$ mesons in the open-charm sector. These positive-parity states have masses lighter than expected from quark models, and also smaller widths. Out of the many proposed explanations, the unquenching of the naive quark model has been successful [3]. When a $(q\bar{q})$ pair occurs in a P-wave but can couple to hadron pairs in S-wave, the latter configuration distorts the $(q\bar{q})$ picture. Therefore, the 0^+ and 1^+ $(c\bar{s})$ states predicted above the $DK(D^*K)$ thresholds couple to the continuum. This mixes meson–meson components in the wave function, an idea advocated long ago to explain the spectrum and properties of light-scalar mesons [4].

This possibility of $(c\bar{s}n\bar{n})$ (n stands for a light quark) components in D^*_s has open the discussion about the presence of compact $(c\bar{c}n\bar{n})$ four-quark states in the charmonium spectroscopy. Some states recently found in the hidden-charm sector may fit in the simple quark-model description as $(c\bar{c})$ pairs (e.g., $X(3940)$, $Y(3940)$, and $Z(3940)$ as radially excited χ_{c0}, χ_{c1}, and χ_{c2}), but others appear to be more elusive, in particular $X(3872)$, $Z(4430)^+$, and $Y(4260)$. The debate on the nature of these states is open, with special emphasis on the $X(3872)$. Since it

*Article based on the presentations by J. Vijande and J.-M. Richard at the Fifth Workshop on Critical Stability, Erice, Sicily, Received February 2, 2009; Accepted February 9, 2009.

was first reported by Belle in 2003 [5], it has gradually become the flagship of the new armada of states whose properties make their identification as traditional ($c\bar{c}$) states unlikely. An average mass of 3871.2 ± 0.5 MeV and a narrow width of less than 2.3 MeV have been reported for the $X(3872)$. Note the vicinity of this state to the $D^0\overline{D}^{*0}$ threshold, $M(D^0\,\overline{D}^{*0}) = 3871.2 \pm 1.2$ MeV. With respect to the $X(3872)$ quantum numbers, although some caution is still required until better statistic is obtained [6], an isoscalar $J^{PC} = 1^{++}$ state seems to be the best candidate to describe the properties of the $X(3872)$.

Another hot sector, at least for theorists, includes the ($cc\bar{n}\bar{n}$) states, which are manifestly exotic with charm 2 and baryon number 0. Should they lie below the threshold for dissociation into two ordinary hadrons, they would be narrow and show up clearly in the experimental spectrum. There are already estimates of the production rates indicating they could be produced and detected at present (and future) experimental facilities [7]. The stability of such ($QQ\bar{q}\bar{q}$) states has been discussed since the early 80s [8], and there is a consensus that stability is reached when the mass ratio $M(Q)/m(q)$ becomes large enough. See, e.g., [9] for Refs. This effect is also found in QCD sum rules [10]. This improved binding when M/m increases is due to the same mechanism by which the hydrogen molecule (p, p, e^-, e^-) is much more bound than the positronium molecule (e^+, e^+, e^-, e^-). What matters is not the Coulomb character of the potential, but its property to remain identical when the masses change. In quark physics, this property is named *flavour independence*. It is reasonably well satisfied, with departures mainly due to spin-dependent corrections.

The question is whether stability is already possible for ($cc\bar{n}\bar{n}$) or requires heavier quarks. In Ref. [9], a marginal binding was found for a specific potential for which earlier studies found no binding. This illustrates how difficult are such four-body calculations.

In another recent investigation, the four-body Schrödinger equation has been solved accurately using the hyperspherical harmonic (HH) formalism [11], with two standard quark models containing a linear confinement supplemented by a Fermi–Breit one-gluon exchange interaction (BCN), and also boson exchanges between the light quarks (CQC). The model parameters were tuned in the meson and baryon spectra. The results are given in Table 1, indicating the quantum numbers of the state studied, the maximum value of the grand angular momentum used in the HH expansion, $K_{\rm m}$, and the energy difference between the mass of the four-quark state, E_{4q}, and that of the lowest two-meson threshold calculated with the same potential model, Δ_E. For the ($cc\bar{n}\bar{n}$) system we have also calculated the radius of the four-quark state, R_{4q}, and its ratio to the sum of the radii of the lowest two-meson threshold, Δ_R.

Besides trying to unravel the possible existence of bound ($cc\bar{n}\bar{n}$) and ($c\bar{c}n\bar{n}$) states one should aspire to understand whether it is possible to differentiate between compact and molecular states. A molecular state may be understood as a four-quark state containing a single physical two-meson component, i.e., a unique singlet-singlet component in the colour wave function with well-defined spin and isospin quantum numbers. One could expect these states not being deeply bound and therefore having a size of the order of the two-meson system,

Table 1. $(c\bar{c}n\bar{n})$ (left) and $(cc\bar{n}\bar{n})$ (right) results.

$(c\bar{c}n\bar{n})$	CQC		BCN	
$J^{PC}(K_{\rm m})$	E_{4q}	Δ_E	E_{4q}	Δ_E
0^{++} (24)	3779	+34	3249	+75
0^{+-} (22)	4224	+64	3778	+140
1^{++} (20)	3786	+41	3808	+153
1^{+-} (22)	3728	+45	3319	+86
2^{++} (26)	3774	+29	3897	+23
2^{+-} (28)	4214	+54	4328	+32
1^{-+} (19)	3829	+84	3331	+157
1^{--} (19)	3969	+97	3732	+94
0^{-+} (17)	3839	+94	3760	+105
0^{--} (17)	3791	+108	3405	+172
2^{-+} (21)	3820	+75	3929	+55
2^{--} (21)	4054	+52	4092	+52

$(cc\bar{n}\bar{n})$	CQC			
$IJ^{P}(K_{\rm m})$	E_{4q}	Δ_E	R_{4q}	Δ_R
$0\,0^{+}$ (28)	4441	+15	0.624	>1
$0\,1^{+}$ (24)	3861	−76	0.367	0.808
$0\,2^{+}$ (30)	4526	+27	0.987	>1
$0\,0^{-}$ (21)	3996	+59	0.739	>1
$0\,1^{-}$ (21)	3938	+66	0.726	>1
$0\,2^{-}$ (21)	4052	+50	0.817	>1
$1\,0^{+}$ (28)	3905	+50	0.817	>1
$1\,1^{+}$ (24)	3972	+33	0.752	>1
$1\,2^{+}$ (30)	4025	+22	0.879	>1
$1\,0^{-}$ (21)	4004	+67	0.814	>1
$1\,1^{-}$ (21)	4427	+1	0.516	0.876
$1\,2^{-}$ (21)	4461	−38	0.465	0.766

i.e., $\Delta_R \sim 1$. Opposite to that, a compact state may be characterized by its involved structure on the colour space, its wave function containing different singlet-singlet components with non negligible probabilities. One would expect such states would be smaller than typical two-meson systems, i.e., $\Delta_R < 1$. Let us notice that while $\Delta_R > 1$ but finite would correspond to a meson-meson molecule $\Delta_R \stackrel{K\to\infty}{\longrightarrow} \infty$ would represent an unbound threshold.

As can be seen in Table 1 (left), in the case of the $(c\bar{c}n\bar{n})$ there appear no bound states for any set of quantum numbers, including the suggested assignment for the $X(3872)$. Independently of the quark–quark interaction and the quantum numbers considered, the system evolves to a well separated two-meson state. This is clearly seen in the energy, approaching the threshold made of two free mesons, and also in the probabilities of the different colour components of the wave function and in the radius [11]. Thus, in any manner one can claim for the existence of a bound state for the $(c\bar{c}n\bar{n})$ system.

A completely different behaviour is observed in Table 1 (right). Here, there are some particular quantum numbers where the energy is quickly stabilized below the theoretical threshold. Of particular interest is the 1^+ $cc\bar{n}\bar{n}$ state, whose existence was predicted more than twenty years ago [12]. There is a remarkable agreement on the existence of an isoscalar $J^P = 1^+$ $cc\bar{n}\bar{n}$ bound state using both BCN and CQC models, if not in its properties. For the CQC model the predicted binding energy is large, − 76 MeV, $\Delta_R < 1$, and a very involved structure of its wave function (the DD^* component of its wave function only accounts for the 50% of the total probability) what would fit into compact state. Opposite to that, the BCN model predicts a rather small binding, −7 MeV, and Δ_R is larger than 1, although finite. This state would naturally correspond to a meson-meson molecule.

Concerning the other two states that are below threshold in Table 1 a more careful analysis is required. Two-meson thresholds must be determined assuming quantum number conservation within exactly the same scheme used in the four–

quark calculation. Dealing with strongly interacting particles, the two-meson states should have well defined total angular momentum, parity, and a properly symmetrized wave function if two identical mesons are considered (coupled scheme). When noncentral forces are not taken into account, orbital angular momentum and total spin are also good quantum numbers (uncoupled scheme). We would like to emphasize that although we use central forces in our calculation the coupled scheme is the relevant one for observations, since a small non-central component in the potential is enough to produce a sizeable effect on the width of a state. These state are below the thresholds given by the uncoupled scheme but above the ones given within the coupled scheme what discard these quantum numbers as promising candidates for being observed experimentally.

Binding increases for larger M/m, but in the $(bb\bar{n}\bar{n})$ sector, there is no proliferation of bound states. We have studied all ground states of $(bb\bar{n}\bar{n})$ using the same interacting potentials as in the double-charm case. Only four bound states have been found, with quantum numbers $J^P(I) = 1^+(0)$, $0^+(0)$, $3^-(1)$, and $1^-(0)$. The first three ones correspond to compact states.

Now, one could question the validity of the potential models used in these estimates, or more precisely, of the extrapolation from mesons to baryons, and then to multiquark states. For the short-range terms, in particular one-gluon exchange, the additive rule

$$V = -\frac{3}{16}\sum_{i<j} \tilde{\lambda}_i^{(c)}.\tilde{\lambda}_j^{(c)}\, v(r_{ij}) \ , \tag{1}$$

is justified. Here $v(r)$ is the quark–antiquark potential governing mesons, and $\tilde{\lambda}_i^{(c)}$ is the colour generator. This is the non-Abelian version of the $1/r \to \sum q_i q_j / r_{ij}$ rule in atomic physics.

The confining part, however, is hardly of pairwise character. Several authors have proposed that the linearly rising potential σr of mesons (σ is the string tension) is generalised as

$$V = \sigma \min(d_1 + d_2 + d_3) \ , \tag{2}$$

where d_i is the distance from the i^{th} quark to a junction whose location is optimised, exactly as in the famous problem of Fermat and Torricelli. Unfortunately, the potential (2) differs little from the empirical ansatz (1) which here reduces to $\sigma(r_{12}+r_{23}+r_{31})/2$. Hence baryon spectroscopy cannot probe the three-body character of confinement.

In the case of two quarks and two antiquarks, the confining potential reads

$$V_4 = \min(V_f,\, V_s) \ , \tag{3}$$

given by the minimum of a flip-flop potential V_f and a Steiner-tree potential V_s, sometimes named “butterfly” (see Fig. 1). In V_f, each gluon flux goes from a quark to an antiquark. The second term corresponds to a minimal Steiner tree, with four terminals and two Steiner points. It is remarkable that this potential, which is supported by lattice QCD [13] is more attractive than the additive ansatz. This is illustrated in Ref. [14], where the four-body problem is solved

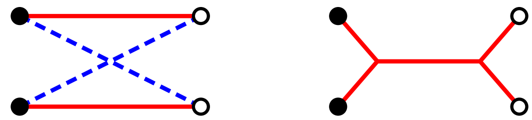

Figure 1. String model for four quarks: flip-flop (left) and Steiner-tree (right), an alternative configuration that is favoured when the quarks (full disks) are well separated from the antiquarks (open circles).

Table 2. Four–quark variational energy E_4 of $QQ\overline{qq}$ for the different confinement models (V_f stands for the flip-flop interaction, V_s for the Steiner-tree potential, and $V_4 = \min(V_f, V_s)$), compared to its threshold, and variational energy E_4' of $Q\overline{Q}q\overline{q}$ with the flip-flop model V_f, compared to its threshold T_4' as a function of the mass ratio.

M/m	E_4			T_4	E_4'	T_4'
	V_f	V_s	V_4		V_f	
1	4.644	5.886	4.639	4.676	4.644	4.676
2	4.211	5.300	4.206	4.248	4.313	4.194
3	4.037	5.031	4.032	4.086	4.193	3.959
4	3.941	4.868	3.936	3.998	4.117	3.811
5	3.880	4.754	3.873	3.942	4.060	3.705

with this confining term alone without short-range corrections. The results are displayed in Table 2. This four-body calculation is rather involved, as the potential at each point is obtained by a minimisation over several parameters. See Ref. [14] for technical details about the models and the numerical techniques used.

The results for the configurations $(QQ\bar{q}\bar{q})$ and $(Q\overline{Q}q\bar{q})$ are shown in Table 2 as function of the heavy-to-light mass ratio. Clearly, as M/m increases, a deeper binding is obtained for the flavour-exotic $(QQ\bar{q}\bar{q})$ system. For the hidden-flavour $(Q\overline{Q}q\bar{q})$, however, the stability deteriorates, becoming unbound for $M/m \gtrsim 1.2$.

More recently, the stability in this model has been demonstrated rigorously in the limit of very large M/m. The first step is to show that

$$V_4/\sigma \leq \frac{\sqrt{3}}{2}\left(|\boldsymbol{x}| + |\boldsymbol{y}|\right) + |\boldsymbol{z}| \ , \tag{4}$$

in terms of the Jacobi variables, $\boldsymbol{x} = \boldsymbol{r}_2 - \boldsymbol{r}_1$, $\boldsymbol{y} = \boldsymbol{r}_4 - \boldsymbol{r}_3$ and $\boldsymbol{z} = (\boldsymbol{r}_3 + \boldsymbol{r}_4 - \boldsymbol{r}_1 + \boldsymbol{r}_2)/2$, so that the Hamiltonian describing the relative motion is bounded by

$$H_b = \frac{\boldsymbol{p}_x^2}{M} + \sigma\frac{\sqrt{3}}{2}|\boldsymbol{x}| + \frac{\boldsymbol{p}_y^2}{m} + \sigma\frac{\sqrt{3}}{2}|\boldsymbol{y}| + \frac{\boldsymbol{p}_z^2}{4\mu} + \sigma|\boldsymbol{z}| \ , \tag{5}$$

(μ is the quark–antiquark reduced mass), which is *exactly solvable* for its ground state and gives binding for large M/m. Details will be published shortly [15].

To conclude, let us stress again the important difference between the two physical systems which have been considered. While for the $(c\bar{c}n\bar{n})$, there are two allowed physical *decay channels*, $(c\bar{c}) + (n\bar{n})$ and $(c\bar{n}) + (\bar{c}n)$, for the $(cc\bar{n}\bar{n})$ only one physical system contains the possible final states, $(c\bar{n})+(c\bar{n})$. Therefore, a $(c\bar{c}n\bar{n})$ four-quark state will hardly present bound states, because the system will reorder itself to become the lightest two-meson state, either $(c\bar{c})+(n\bar{n})$ or $(c\bar{n})+(\bar{c}n)$. In other words, if the attraction is provided by the interaction between particles i and j, it does also contribute to the asymptotic two-meson state. This does not happen for the $(cc\bar{n}\bar{n})$ if the interaction between, for example, the two quarks is strongly attractive. In this case there is no asymptotic two-meson state including such attraction, and therefore the system might bind.

Once all possible $(cc\bar{n}\bar{n})$, $(bb\bar{n}\bar{n})$ and $(c\bar{c}n\bar{n})$ quantum numbers have been exhausted very few alternatives remain. If additional bound four-quark states or higher configuration are experimentally found, then other mechanisms should be at work, for instance based on diquarks [4, 16, 17].

Acknowledgement. This work has been partially funded by the Spanish Ministerio de Educación y Ciencia and EU FEDER under Contract No. FPA2007-65748, by Junta de Castilla y León under Contract No. SA016A17, and by the Spanish Consolider-Ingenio 2010 Program CPAN (CSD2007-00042).

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