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Collective flavor transitions of supernova neutrinos

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We give a very brief overview of collective effects in neutrino oscillations in core collapse supernovae where refractive effects of neutrinos on themselves can considerably modify flavor oscillations, with possible repercussions for future supernova neutrino detection. We discuss synchronized and bipolar oscillations, the role of energy and angular neutrino modes, as well as three-flavor effects. We close with a short summary and some open questions.

1. Introduction

Neutrinos interact via the weak interactions and can thus cause a refractive effect on each other. Under most circumstances, the resulting self-interaction potential is much smaller than the vacuum oscillation term or the potential induced by ordinary matter. However, in the early cooling phase of core collapse supernovae, the density of neutrinos streaming off the hot nascent neutron star is sufficiently high to cause non-linear phenomena that can have practical importance. This was realized only recently in a series of papers of which we here cite only a few [1,2,3,4,5,6,7,8,9,10,11,12,13,14].

To describe collective effects it is useful to describe the neutrinos with flavor density matrices for each momentum mode. They are defined in terms of the annihilation operators a_i for neutrinos and $\bar{a}_j$ for antineutrinos in a given momentum mode $\mathbf{p}$ [15,16,17],

$$(\varrho_{\mathbf{p}})_{ij} = \langle a_i^\dagger a_j \rangle_{\mathbf{p}} \quad \text{and} \quad (\bar{\varrho}_{\mathbf{p}})_{ij} = \langle \bar{a}_j^\dagger \, \bar{a}_i \rangle_{\mathbf{p}} \,. \tag{1}$$

their equations of motion can be written as a commutator of an effective Hamiltonian with the density matrices,

$$\begin{aligned} \partial_t \varrho_{\mathbf{p}} \;=\; & -i\Big[\Omega_{\mathbf{p}} + \sqrt{2}\, G_{\rm F} L + \\ & +\sqrt{2}\, G_{\rm F} \int \frac{{\rm d}^3\mathbf{q}}{(2\pi)^3}\, (\varrho_{\mathbf{q}} - \bar{\varrho}_{\mathbf{q}})\, (1-\cos\theta_{\mathbf{pq}}), \varrho_{\mathbf{p}}\Big] \,. \end{aligned} \tag{2}$$

Here, $G_{\rm F}$ is the Fermi constant, in the mass basis the matrix of vacuum oscillation frequencies for relativistic neutrinos is $\Omega_{\mathbf{p}} = {\rm diag}(m_1^2, m_2^2, m_3^2)/2p$ with $p = |\mathbf{p}|$, and in the flavor basis $L = {\rm diag}(n_e, n_\mu, n_\tau)$, where n_e, n_μ, n_τ are the charged lepton number densities (particle minus antiparticle density) responsible for the matter induced potential. When neutrino self-interactions are negligible, a compensation of the vacuum and matter induced terms in the diagonal part of the Hamiltonian in Eq. (2) leads to the well known Mikheyev-Smirnov-Wolfenstein (MSW) effect [18,19]. An equation analogous to Eq. (2) holds for antineutrinos with the sign of the vacuum term $\Omega_{\mathbf{p}}$ reversed. The last term in Eq. (2) describes forward scattering on the neutrino background where the isotropic density term can lead to self-maintained coherence, whereas the flux term proportional to the cosine of the angle $\theta_{\mathbf{pq}}$ between momenta $\mathbf{p}$ and $\mathbf{q}$ can lead to self-induced decoherence. The flux of charged leptons has been assumed to vanish.

In the two-flavor case it is convenient to parametrize vacuum, matter and self-interaction terms by the frequencies

$$\begin{aligned}\omega_p &= \frac{|\Delta m^2|}{2p}\,; \\ \lambda(r) &= \sqrt{2}\,G_{\rm F} n_e(r)\,; \\ \mu(r) &= \sqrt{2}\,G_{\rm F}\left[F_{\bar\nu_e}(r) - F_{\bar\nu_x}(r)\right]\langle 1-\cos\theta_{\mathbf{pq}}\rangle\ ,\end{aligned} \qquad (3)$$

with F_i are the fluxes of the relevant neutrino species i, and ν_x stands for any one of ν_μ, ν_τ, $\bar\nu_\mu$ or $\bar\nu_\tau$. The oscillation phenomena discussed in the following depend crucially on the relative size of these three frequencies.

2. Synchronized and Bipolar Oscillations

Since muon and tau neutrinos behave equally in core collapse supernovae up to an effective $\mu\tau$ potential briefly discussed in Sect. 5, neutrino oscillations can be described in two-flavor approximation. The oscillations are then driven by the atmospheric mass squared difference, $|\Delta m^2_{\rm atm}| = |m_3^2 - m_2^2| = 2.40^{+0.12}_{-0.11}\times 10^{-3}\,{\rm eV}^2$ and the angle $\sin\theta_{13} \leq 0.04$ [20]. For these parameters, $\omega_p \simeq 0.4\,{\rm km}^{-1}$ for a typical neutrino energy of 15 MeV. Both normal and inverted hierarchy are still allowed for this case.

Considering only a single two-flavor mode $\mathbf{p}$ one can expand into Pauli matrices $\boldsymbol{\sigma}$, $\varrho = (f + \mathbf{P}\cdot\boldsymbol{\sigma})/2$, analogously for $\bar\varrho$, $L = (n_0 + n_e\mathbf{L}\cdot\boldsymbol{\sigma})/2$, and $\omega = \left[\omega_0 + |\Delta m^2|\mathbf{B}\cdot\boldsymbol{\sigma}/(2p)\right]/2$. With the vacuum mixing angle θ, $\mathbf{B} = (\sin 2\theta, 0, -\cos 2\theta)$, the radial flavor evolution (we use units in which the speed of light is unity) can be described by [4]

$$\partial_r\mathbf{P} = \left[+\omega\mathbf{B} + \lambda(r)\mathbf{L} + \mu(r)(\mathbf{P}-\bar{\mathbf{P}})\right]\times\mathbf{P}\,, \quad (4)$$

and an analogous equation for $\bar{\mathbf{P}}$ with the sign of ω reversed. It is useful to define the vectors

$$\begin{aligned}\mathbf{Q} &\equiv \mathbf{P} + \bar{\mathbf{P}} - \frac{\omega}{\mu}\mathbf{B}\,; \quad \mathbf{q}\equiv\frac{\mathbf{Q}}{Q} \\ \mathbf{D} &\equiv \mathbf{P} - \bar{\mathbf{P}}\,.\end{aligned} \qquad (5)$$

In the absence of matter, $\lambda = 0$, and for slowly varying μ, the equations of motion (now written in terms of time derivatives) are

$$\dot{\mathbf{Q}} = \mu\mathbf{D}\times\mathbf{Q}\,; \quad \dot{\mathbf{D}} = \omega\mathbf{B}\times\mathbf{Q}\,, \qquad (6)$$

which implies

$$\begin{aligned}&|\mathbf{Q}| = {\rm const}\,; \qquad \sigma\equiv\mathbf{D}\times\mathbf{Q} = {\rm const} \\ &\mathbf{D} = \frac{\mathbf{q}\times\dot{\mathbf{q}}}{\mu} + \sigma\mathbf{q}\,.\end{aligned} \qquad (7)$$

These are the equations for a spinning top of spin σ, angular momentum $(\mathbf{q}\times\dot{\mathbf{q}})/\mu$ and moment of inertia $I = ml^2 = \mu^{-1}$. Its energy then has a kinetic and a potential part,

$$\begin{aligned}E &= E_{\rm kin} + E_{\rm pot} = \frac{\mu}{2}\mathbf{D}^2 + \omega(\mathbf{B}\cdot\mathbf{Q} + Q) \\ &= \frac{\dot{\mathbf{q}}^2}{2\mu} + \frac{\mu}{2}\sigma^2 + \omega(\mathbf{B}\cdot\mathbf{Q} + Q)\,.\end{aligned} \qquad (8)$$

This description is a good approximation in the regime where neutrinos are freely streaming. The supernova neutrino (number) fluxes are thought to obey the hierarchy $F_{\nu_e} > F_{\bar\nu_e} > F_{\nu_x}$. One can then adopt initial conditions at the neutrino sphere are typically $\mathbf{P} = (0,0,1+\varepsilon)$, $\bar{\mathbf{P}} = (0,0,1)$, where the asymmetry parameter in terms of the fluxes of different neutrino flavors is

$$\epsilon = \frac{F_{\nu_e} - F_{\nu_x}}{F_{\bar\nu_e} - F_{\bar\nu_x}} - 1 = \frac{F_{\nu_e} - F_{\bar\nu_e}}{F_{\bar\nu_e} - F_{\bar\nu_x}}\,. \qquad (9)$$

Eq. (8) then implies that the "flavor pendulum" is in a stable initial position for the normal hierarchy, $\theta \ll 1$, whereas for the inverted hierarchy $\tilde\theta = \pi/2 - \theta \ll 1$ the pendulum is initially in a maximum energy state. We will focus on this case in the following.

As long as $\mu > 2\omega/(1-\sqrt{1+\varepsilon})^2$ the kinetic (self-interaction) term in Eq. (8) dominates and the oscillations are synchronized with a common frequency $\omega_{\rm synch} = (2+\varepsilon)\omega/\varepsilon$ around $\mathbf{B}$. For $\omega < \mu < 2\omega/(1-\sqrt{1+\varepsilon})^2$, bipolar oscillations take place with a frequency $\kappa = (2\omega\mu/(1+\varepsilon))^{1/2}$ in which $\mathbf{Q}$ swings between its initial position and a position in which it is parallel to $-\mathbf{B}$. This corresponds to a collective transition of $\nu_e\bar\nu_e$ to $\nu_x\bar\nu_x$ pairs, with a rate greatly speeded up compared to ordinary pair annihilation [1]. Furthermore, in the absence of matter $\mathbf{D}\cdot\mathbf{B} = (\mathbf{P}-\bar{\mathbf{P}})\cdot\mathbf{B}$

is strictly conserved [4], whereas in dense matter $(\mathbf{P}-\bar{\mathbf{P}})\cdot\mathbf{L}$ is approximately conserved. For small effective mixing angle, one thus has $P_z = \bar{P}_z + \epsilon$. Finally, when $\mu < \omega$ at large radii, vacuum oscillations ensue or, in the presence of matter with $\lambda \sim \omega$ the usual MSW effects can occur.

In the limit of small vacuum mixing angle, $\tilde{\theta} = \pi/2 - \theta \ll 1$, and for constant μ and λ, the time scale for bipolar conversion is

$$\tau_{\rm bipolar} \simeq -\kappa^{-1} \ln\left(\frac{\tilde{\theta}\kappa}{(\kappa^2+\lambda^2)^{1/2}}\right) . \quad (10)$$

The main effect of the matter term is thus to decrease the effective mixing angle to $\tilde{\theta}\kappa/(\kappa^2+\lambda^2)^{1/2}$ and thus delay the onset of bipolar oscillations. For smaller mixing angle it thus takes longer to tip over, the bipolar transition is less adiabatic and the nutation amplitude is larger. Since matter decreases the effective mixing angle, a high matter density also leads to a later onset of the bipolar transition and to a larger nutation amplitude [6].

For varying neutrino density and in the limit of small vacuum mixing angle, angular momentum and adiabatic energy conservation in the bipolar regime gives for the electron neutrino survival probability $P(\nu_e \to \nu_e) = \frac{1}{2}(1+P_z) \propto \mu(r)^{1/2}$.

Since the solar neutrino mass hierarchy is known to be normal, the bipolar conversion in the inverted atmospheric hierarchy is essentially pair conversion $\nu_e\bar{\nu}_e \to \nu_x\bar{\nu}_x$ from the second highest state m_1 to the lowest state m_3, where ν_x is a combination of ν_μ and ν_τ. We stress that for the inverted hierarchy, bipolar oscillations occur for arbitrarily small θ_{13}. This can be used as experimental test by observing a supernova with mega-ton detectors that are mostly sensitive to electron-antineutrinos via the reaction $\bar{\nu}_e + p \to n + e^+$ [21]: If either the atmospheric hierarchy is normal or if it is inverted with $\sin^2\theta_{13} \gtrsim 10^{-3}$, the bipolar transition is followed by an adiabatic MSW transition, and the electron anti-neutrino fluxes seen in two detectors behind and in front of the Earth are different. In contrast, if the atmospheric hierarchy is inverted and $\sin^2\theta_{13} \lesssim 10^{-5}$, the bipolar transition is followed by a non-adiabatic MSW transition, and the electron anti-neutrino fluxes seen in two such detectors are equal.

3. Kinematic and Self-Induced Decoherence between Angular Modes

The momentum modes $\mathbf{p}$ essentially consist of energy and angular modes. For spherical symmetry one has the radius r as integration variable, such that

$$\begin{aligned} \mathbf{p} &\to (E \simeq |\mathbf{p}|, u \equiv \sin^2\theta_R) , \quad (11) \\ \varrho_{\mathbf{p}}(r) &\to \varrho_{E,u,r} , \end{aligned}$$

where θ_R is the emission angle relative to the radial direction at the neutrino sphere, $r = R$. The energy bins will lead to spectral splits [3,7,9,10], whereas angular bins can give rise to kinematic and self-induced decoherence [5,6]. To see this, we define the flux matrices

$$J_{E,u,r} \equiv \frac{E^2 \varrho_{E,u,r}}{2(2\pi)^2} , \quad (12)$$

With the radial velocity $v_{u,r} = v_{\mathbf{p},r} = \cos\theta_r = \sqrt{1-u(R/r)^2}$ one has the integral flux and number density matrices

$$\begin{aligned} J_r &\equiv \frac{r^2}{R^2}\int \frac{{\rm d}^3\mathbf{p}}{(2\pi)^3}\,\varrho_{\mathbf{p}} = \int_0^1 du \int_0^\infty dE\, J_{E,u,r} , \\ N_r &\equiv \int_0^1 du \int_0^\infty dE \frac{J_{E,u,r}}{v_{u,r}} . \quad (13) \end{aligned}$$

For an average matter velocity v_e one then has the general equations of motion:

$$\begin{aligned} i\partial_r J_{E,u,r} &= \left[\frac{\Omega_E + \lambda(r)L(1-v_{u,r}v_e)}{v_{u,r}} \right. \quad (14) \\ &\left. +\sqrt{2}\,G_{\rm F}\frac{R^2}{r^2}\left(\frac{N_r-\bar{N}_r}{v_{u,r}} - (J_r - \bar{J}_r)\right), J_{E,u,r}\right] , \end{aligned}$$

and analogously for antineutrinos with the opposite sign for Ω_E. Decoherence can occur due to the the $u-$dependence of the velocity $v_{u,r}$, either from the flux term in the self-interactions (self-induced decoherence) or kinematically in the matter terms. The matter term can be transformed away up to the $u-$dependence of $v_{u,r}$ [2,4]. The matter-induced multi-angle effect becomes important when

$$n_e = n_{e^-} - n_{e^+} \gtrsim n_{\bar{\nu}_e} . \quad (15)$$

If the matter density is very much larger than the neutrino density, the effective oscillation frequencies of different polarization vectors vary so greatly that they stay pinned to the $\mathbf{L}$ direction and no collective oscillations occur.

Numerical simulations have been performed to determine under which conditions decoherence occurs. For $\omega = 0.3$ km^{-1}, $\sin 2\tilde{\theta} = 10^{-3}$ and $\sin 2\theta = 10^{-3}$, the demarcation lines between coherence and decoherence for inverted and normal hierarchies, respectively, in the μ-ϵ-plane, can be approximated by [6]

$$\begin{aligned} \epsilon_{\rm IH} &\approx 0.225 + 0.027 \log_{10}\left(\frac{\mu}{10^6 \text{ km}^{-1}}\right), \\ \epsilon_{\rm NH} &\approx 0.172 + 0.087 \log_{10}\left(\frac{\mu}{10^6 \text{ km}^{-1}}\right). \end{aligned} \quad (16)$$

4. Energy Modes and Spectral Splits

Integrating Eq. (14) over E and u and writing for the two flavor case $J_r = [F_\nu - F_{\bar{\nu}} + \mathbf{D}\cdot\boldsymbol{\sigma}]/2$, in the limit of small mixing angles one gets [7]

$$\partial_r \mathbf{D} = \mathbf{e}_z \times \mathbf{M}\,, \quad (17)$$

where $\mathbf{M}$ is an integral of $J_{E,u,r}$ which is not interesting for our purposes. Thus, $(J_r - \bar{J}_r)_{22} - (J_r - \bar{J}_r)_{11} = F_{\nu_e} - F_{\bar{\nu}_e} - [F_{\nu_x} - F_{\bar{\nu}_x}] =$const, which corresponds to the approximate conservation of $(\mathbf{P} - \bar{\mathbf{P}})\cdot\mathbf{L}$ in the single mode approximation. Since the total lepton number is conserved, electron and $x-$lepton numbers are conserved separately.

The spectral split is governed by lepton number conservation for both flavors separately: Antineutrinos swap completely in a bipolar transition. To compensate, neutrinos can only swap above a certain energy because under typical supernova conditions

$$|F_{\nu_x} - F_{\nu_e}| > |F_{\bar{\nu}_x} - F_{\bar{\nu}_e}|\,. \quad (18)$$

The spectral splits are created by an adiabatic transition between the regime dominated by neutrino-self interactions and the low neutrino density regime [7]. If this transition is not completely adiabatic, as may be the case in a real supernova, the spectral split tends to be washed out [8]. Spectra splits may be observable in future observations of a galactic supernova explosion [14]. Such splits may also occur in the antineutrino sector [10,13], albeit at lower energies and it is not clear if such features are not washed out when taking into account angular modes [13].

5. Three-flavor Effects

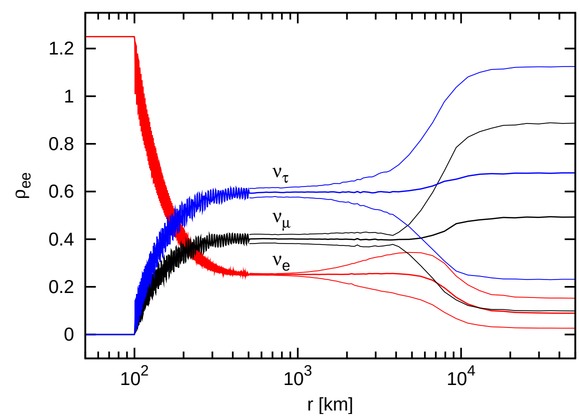

Figure 1. Radial evolution of the fluxes of ν_e (red), ν_μ (black) and ν_τ (blue) fluxes, normalized to the initial $\bar{\nu}_e$-flux, for a fixed neutrino energy $E = 20$ MeV and an inverted atmospheric hierarchy. For the neutrino parameters we use $\Delta m^2_{12} = \Delta m^2_{\rm sol} = 7.65\times10^{-5}$ eV^2, $\Delta m^2_{13} = \Delta m^2_{\rm atm} = 2.4\times10^{-3}$ eV^2, $\sin^2\theta_{12} = 0.304$, $\sin^2\theta_{13} = 0.01$, $\sin^2\theta_{23} = 0.4$, and a vanishing Dirac phase $\delta = 0$, all consistent with measurements [20]. The matter density profile $\lambda(r) = 4\times10^6\,(R/r)^3$km^{-1} was assumed, where the neutrinosphere is at $R = 10$ km. The self-interaction term is taken as $\mu(r) = 7\times10^5\,(R/r)^4/(2-(R/r)^2)$km^{-1}. After bipolar conversion, thick lines represent the average fluxes, whereas thin lines signify the envelopes of the fast flux oscillations.

In normal matter, μ and τ leptons appear only as virtual states in radiative corrections to neutral-current ν_μ and ν_τ scattering, causing a

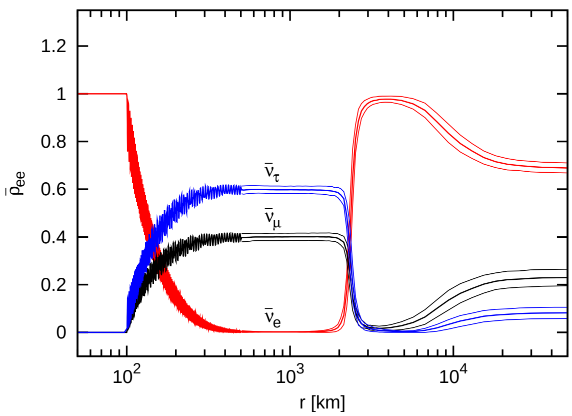

Figure 2. Same as Fig. 1, but for the antineutrino fluxes.

shift $\Delta V_{\mu\tau} = \sqrt{2}\, G_{\rm F} Y_\tau^{\rm eff} n_B$ between ν_μ and ν_τ, where n_B is the baryon density. It has the same effect on neutrino dispersion as real τ leptons with an abundance [22]

$$\begin{aligned} Y_\tau^{\rm eff} &= \frac{3\sqrt{2}\, G_{\rm F} m_\tau^2}{(2\pi)^2}\left[\ln\left(\frac{m_W^2}{m_\tau^2}\right) - 1 + \frac{Y_n}{3}\right] \\ &\simeq 2.7\times 10^{-5}\,, \end{aligned} \tag{19}$$

where n_e was assumed to equal the proton density and Y_n is the neutron fraction of n_B. This should be compared to the ordinary MSW potential $\Delta V = \sqrt{2}\, G_{\rm F} Y_e n_B$ where $Y_e = n_e/n_B$ is the electron fraction.

The potential Eq. (19) would lead to a MSW resonance at density $\rho \simeq 3\times 10^7\,{\rm g\,cm^{-3}}$, corresponding to $\lambda \simeq 10^4\,{\rm km}^{-1}$, provided that the radius at which this resonance occurs is beyond the radii at which collective oscillations occur. The ν_e and $\bar\nu_e$ survival probabilities would then be sensitive to the θ_{23} angle which governs $\nu_\mu - \nu_\tau$ mixing and such a situation could a priori arise in the accretion phase of an iron-core supernova [11].

However, the $\nu_\mu - \nu_\tau$ refractive effect is unlikely to play any practical role: At the required high matter densities the multi-angle matter effect is likely to trigger multi-angle decoherence such that the fluxes of the different flavors tend to be maximally equilibrated [12].

In Figs. 1 and 2 we show the radial dependence of neutrino and antineutrino fluxes for a typical example of three-flavor oscillations in single mode approximation. The asymmetry parameter $\varepsilon = 0.25$ from Eq. (9) is sufficiently large to prevent self-induced decoherence according to Eq. (16). Since $\mu(r) \gtrsim \lambda(r)(1 - v_{u,r}) \simeq \lambda(r)(R/r)^2/2$ outside the synchronized oscillation regime, matter induced decoherence should also be negligible according to Eq. (15). Only one radial mode was thus taken into account in this simulation. After the synchronized oscillation phase which lasts until $r \simeq 100\,{\rm km}$, a bipolar transition occurs which lasts until $r \simeq 300\,{\rm km}$. During this bipolar phase, the survival probability of electron antineutrinos falls off roughly as $\mu(r)^{1/2}$, as discussed in Sect. 2, whereas the electron neutrino survival probability approaches the value $\varepsilon = 0.25$, as dictated by approximate flavor conservation. If energy modes would be included in such a simulation, this would result in a spectral split such that the ν_e flux would not swap with the $\nu_\mu + \nu_\tau$ fluxes below a certain critical energy determined by approximate flavor conservation. Furthermore, a $\nu_\mu - \nu_\tau$ MSW transition would occur at $r \simeq 100\,{\rm km}$, were it not for the collective effects that prevent such a transition. Finally, an MSW resonance occurs at $r \simeq 2500\,{\rm km}$ for antineutrinos due to the assumed inverted hierarchy, after which vacuum oscillations remain.

6. Open Questions and Conclusions

We first summarize our main conclusions. Neutrino self-interactions can play a major role in the oscillations of neutrinos in core collapse supernovae. The one-mode approximation is meanwhile well understood and can be thought of as a spinning top. It often leads to a surprisingly accurate description of the full problem which in the simplest case of spherical symmetry requires the introduction of modes both for energy and for the direction of a given neutrino trajectory with respect to the radial direction. Asymmetry between neutrinos and anti-neutrinos leads to spectral splits in the context of energy modes and, if sufficiently large, prevents self-induced decoherence of angular modes. Matter effects can also lead to decoherence if the charged lepton density is larger than the neutrino density. This is also likely to mask any significant effects of the second-order difference between the ν_μ and ν_τ refractive index on resulting neutrino fluxes. If matter densities are very much larger than neutrino densities, the multi-mode flavor polarization vectors remain pinned to the direction corresponding to flavor eigenstates and no flavor conversion occurs.

There are still unresolved issues in collective neutrino oscillations, including their detailed numerical description in the absence of spherical symmetry, which can be relevant, for example, in the presence of hydrodynamic turbulence in the background of ordinary matter. It is, for example, currently not completely clear, even conceptually, how to describe damping due to the different matter profiles "seen" along different neutrino trajectories in this context. Another interesting question could be if their could be any significant dependence of collective oscillations on the Dirac phase δ [23].

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