Evaluation of non-ohmic losses with overlap integrals

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In the main text of the paper corresponding to the present document, WPP↔SPP conversion devices are considered. Reflection and radiation losses in such structures are evaluated by means of overlap integrals. In this Auxiliary Material section details of such procedure are provided.

It can be shown that the electromagnetic eigenmodes supported by a cylindrical structure (e.g., a wedge of constant height) are mutually orthogonal [1]. Let us denote such eigenmodes as

$$|\mathrm{n}\rangle = |\mathrm{n}(\mathbf{r}_\mathrm{T})\rangle = \{\mathbf{E}_\mathrm{n}(\mathbf{r}_\mathrm{T}), \mathbf{H}_\mathrm{n}(\mathbf{r}_\mathrm{T})\}, \tag{1}$$

where $n = \pm1, \pm2, \pm3, \ldots$. The fundamental mode is $n = \pm1$, and negative indices correspond to modes propagating in the negative z direction. $\{\mathbf{E}_\mathrm{n}, \mathbf{H}_\mathrm{n}\}$ stands for the electric and magnetic field, and $\mathbf{r}_\mathrm{T} = (x, y)$ are coordinates in the transverse plane. Eigenmode orthogonality reads

$$\langle \mathrm{n}|\mathrm{m}\rangle = \langle \mathrm{n}(\mathbf{r}_\mathrm{T})|\mathrm{m}(\mathbf{r}_\mathrm{T})\rangle = \iint_{XY\ \mathrm{plane}} \mathrm{d}x\mathrm{d}y\ \ \mathbf{e}_z \cdot \{\mathbf{E}_\mathrm{n}(\mathbf{r}_\mathrm{T}) \times \mathbf{H}^*_\mathrm{m}(\mathbf{r}_\mathrm{T})\} = \mathrm{sgn}(\mathrm{m})\delta_{|\mathrm{n}||\mathrm{m}|}, \tag{2}$$

where $\mathbf{e}_z$ is a unit vector along the longitudinal Z axis, the star denotes complex conjugate, and $\mathrm{sgn}(\cdot)$ stands for the sign function. Let us remark that: (i) The dependence on the z coordinate, $\exp(ik_\mathrm{n}z)$, has been omitted (k_n is the modal wave vector). (ii) Orthogonality applies both for guided and radiation modes (continuous indices should be used to label radiation modes, but we will avoid this to simplify notation). (iii) Counterpropagating modes with the same index (e.g., $|1\rangle$ and $|-1\rangle$) are not orthogonal. (iv) The scalar product of a mode with itself is proportional to the power carried in the longitudinal Z direction. (v) In general, the integral should be carried out in the infinite transverse XY plane. Nevertheless, when one of the modes is guided the integrand is non-negligible only in a finite part of the XY plane, due to transverse localization of the guided mode. Thus, in our computations of scalar products shown later, the integration area will be the transverse FDTD simulation window. (vi) In fiber and guided optics, orthogonality conditions are routinely used even when small losses are present.

For a general non-cylindrical structure (e.g., a wedge with height varying along the z coordinate), a generic solution $|\mathrm{f}(x, y, z)\rangle$ can be expanded in eigenmodes. For each z the eigenmodes corresponding to that particular transverse cross section, $|\mathrm{n}(x, y, z)\rangle$, should be used:

$$|\mathrm{f}(x, y, z)\rangle = \sum_\mathrm{n} a_\mathrm{n}(z)|\mathrm{n}(x, y, z)\rangle, \tag{3}$$

where the coefficients $a_\mathrm{n}(z)$ in the linear expansion are related to the projections (also termed overlaps) of the solution $|\mathrm{f}\rangle$ on the various eigenmodes $|\mathrm{n}\rangle$. For instance, the overlap with the fundamental WPP mode ($n = +1$) is

$$\langle \mathrm{f}|1\rangle(z) = \langle \mathrm{f}(x, y, z)|1(x, y, z)\rangle = \iint_{XY\ \mathrm{plane}} \mathrm{d}x\mathrm{d}y\ \ \mathbf{e}_z \cdot \{\mathbf{E}_\mathrm{f}(x, y, z) \times \mathbf{H}^*_1(x, y, z)\}. \tag{4}$$

When absorption is present, it is convenient to normalize both $|\mathrm{f}\rangle$ and $|1\rangle$ in a particular way that simplifies the bookkeeping of radiation leakage. Namely, at every transverse cross section, $z = \mathrm{const}$, the functions $|\mathrm{f}\rangle$ and $|1\rangle$ are normalized to unity in the chosen finite integration area. In the following we will plot the square of the overlap integral, $|\langle \mathrm{f}|1\rangle(z)|^2$, for the structures considered in the paper. Notice that, since $|1\rangle$ and $|-1\rangle$ are not orthogonal, this function may include an oscillating term whenever reflection occurs, due to the interference of both eigenmodes and the subsequent formation of a standing wave. On the other hand, the function should be constant for single mode propagation, no reflection, and negligible radiation losses (the mentioned constant is unity with the chosen normalization). This function is also smaller than unity when the linear expansion of $|\mathrm{f}\rangle$ includes other modes different from $|\pm1\rangle$. For our radiation evaluation purposes, the most important case of this possibility occurs when radiation is present in the chosen finite normalization area. In this situation $|\langle \mathrm{f}|1\rangle(z)|^2$ is smaller than unity. As the modes propagate and radiation leaves the normalization window, $|\langle \mathrm{f}|1\rangle(z)|^2$ tends to a unit value.

We consider here the structures related to the WPP↔SPP conversion device. We keep the same notation as in the paper, namely, structures I, II, and III correspond to wedges of constant height, linearly decreasing height, and an abrupt decreasing of height, respectively. Figure 1 renders the function $|\langle \mathrm{f}|1\rangle(z)|^2$ for the three mentioned structures. For the function associated to structure I (black line) we distinctly observe three phenomena: (i) small ripples, (ii) a value lower than unity for $z < z_\mathrm{t} = 2\,\mu$m, and (iii) a value about unity for $z > z_\mathrm{t}$. The ripples are due to the interference of the incoming WPP and a reflected (counterpropagating) WPP. The period of the oscillation is consistent with the WPP wave vector. From the amplitude of the ripples it can be computed that the reflection coefficient is 0.1%. This tiny reflection is not physical and it is due to spurious reflection of the WPP mode at the simulation boundary. The function being smaller than unity for $z < z_\mathrm{t}$ is due to the fact that, in our FDTD simulations, the source excites WPP modes and radiation modes. The displayed behavior of $|\langle \mathrm{f}|1\rangle(z)|^2$ shows that the contribution of radiation modes to the total field $|\mathrm{f}(x, y, z)\rangle$ is negligible (in the transverse simulation window) after the excitation transient (i.e., for $z > z_\mathrm{t}$). Finally, a value of the function about unity for $z > z_\mathrm{t}$ demonstrates that, after the excitation transient, radiation does not leak anymore (ohmic absorption is the only source of losses after the excitation transient in structure I).

The analysis of structure II (red line) is analogous: the reflection is still very small (0.2%) and not important for our purposes. This tiny reflection is most likely caused by the discontinuity (at $z_\mathrm{d} = 3.9\,\mu$m) in our conversion device. The radiation losses in the excitation transient are similar to those discussed for structure I. Finally, the function $|\langle \mathrm{f}|1\rangle(z)|^2$ is plotted as long as the WPP mode exists (i.e., for $z < z_\mathrm{c} = 5.8\,\mu$m). The graph demonstrates that radiation leakage induced by our conversion device is very small: less than 5% for $z_\mathrm{t} < z < z_\mathrm{c}$. These are the important results of the present document: **our WPP↔SPP conversion device produces very little amount of reflection and radiation up to the coordinate z_c where the WPP mode reaches the cutoff.**

The data corresponding to structure III (green dotted line) show large oscillations due to the reflection of the WPP mode at the abrupt height discontinuity ($z_{\mathrm{III}} = 4.6\,\mu$m). Reflection is estimated to be about 20% in this case.

In summary, reflection and radiation can be estimated with the help of overlap integrals. The performed tests show that the WPP↔SPP conversion device with varying height produces very small reflection and radiation losses.

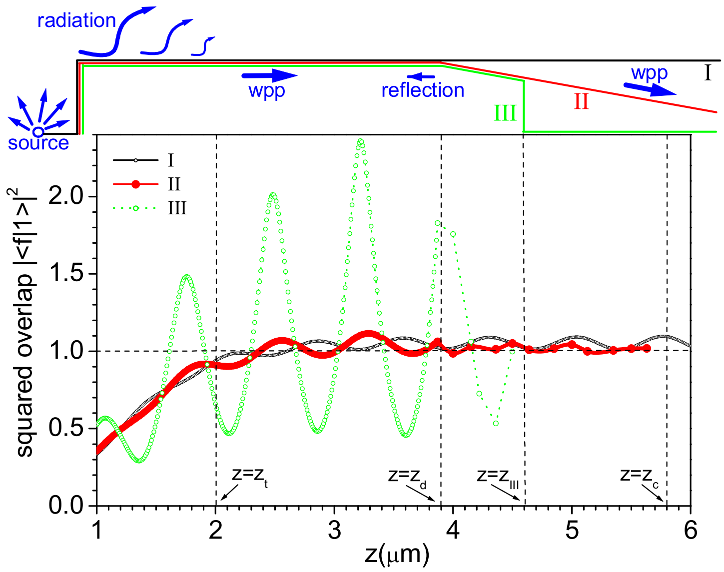

FIG. 1: Squared overlap integral $|\langle \mathrm{f}|1\rangle(z)|^2$ as a function of the longitudinal z coordinate for various structures. Black solid line: structure I, red line: structure II, green dotted line: structure III. The schematics on top of the graph shows the height profile for the three considered structures and the physical processes involved.

[1] A. W. Snyder and J. D. Love, *Optical Waveguide Theory* (Chapman and Hall, London, 1983).