Influence of lattice symmetry on ultrasound transmission through plates with subwavelength aperture arrays

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We study the transmission of sound waves through aluminum plates perforated with square and triangular hole arrays. We demonstrate both theoretically and experimentally that lattice symmetry affects the position of the Wood anomalies and the width of the transmission peaks. The angle and frequency dependence of sound transmission through perforated plates are thoroughly discussed. Finally, we observe unexpected anisotropic behavior in the long-wavelength Lamb-mode bands of perforated plates. © 2009 American Institute of Physics. [DOI: 10.1063/1.3196330]

Artificial metamaterial structures can exhibit extraordinary properties that are not found in naturally occurring systems. For example, photonic¹ and phononic² crystals can block the propagation of light and sound within frequency band gap regions. Other astonishing properties such as negative refraction have been recently demonstrated.³ In this context, much attention has been paid to metallic membranes perforated with subwavelength periodic hole arrays,^{4–6} for which extraordinary optical transmission (EOT) has been observed, in contrast to the poor transmission of individual apertures.⁷ The last decade has witnessed many efforts intended to clarify the physical origin and the interpretation of EOT, as well as to explore applications to sensing and optical processing.^{8,9}

The comparison between optics and acoustics has undergone successive revision after it was first discussed in the nineteenth century.^{10–12} Recently, some of the ideas developed for electromagnetic waves have been transferred to acoustics,^{13–16} emphasizing the similarities between subwavelength transmission in both cases.^{13–15} However, intrinsic differences separate the two kinds of waves. For example, and in contrast to optics, acoustic waves do not present a cutoff wavelength for the existence of guided modes.^{10,11,16,17} Also, sound can be extraordinarily shielded near the onset of diffraction.¹⁸ Furthermore, intrinsic acoustic modes (Lamb and Scholte–Stoneley waves) are conspicuous in thin plates, quite different from optics.¹⁹ Finally, we have recently shown¹⁸ that perforated plates are highly transparent to sound for both periodic and random distributions of holes.

In this letter, we study sound transmission through periodically perforated plates with square and triangular distributions of holes, using both theory and experiment. The plates are immersed in water. The periodic structures considered here consist of square and triangular periodic arrangements of circular holes, drilled on 200-mm-wide, 350-mmlong aluminum plates. The holes have a diameter d=3 mm. Both square and triangular lattices have a period a=5 mm. Three different plate thicknesses are investigated: t=2, 3,and 5 mm. In order to perform ultrasonic transmission measurements, we use the well-known ultrasonic immersion technique.¹⁸ The angle of incidence is varied by rotating the plate sample from 0° to 60° in steps of 1°. Moreover, we calculate transmission intensities by rigorously solving the sound wave equation within a hard-solid model,¹⁸ in which infinite impedance mismatch between the fluid and the plate is assumed. We show in Figs. 1(a)-1(f) a comparison between measured and calculated spectra for both square- and triangular-lattice hole arrays and for three different plate thicknesses. The calculated transmission spectra agree well with experimental ones, but for plate thickness t=2 and 3 mm, the calculated transmission is lower than the measured one over the largest wavelengths λ within the plotted region. This discrepancy originates in the effect of finite impedance ratio between the plate and the fluid.^{18,19} The transmission spectrum for t=5 mm follows quite closely the prediction of the hard-solid model.

Differences between square and triangular lattices can be clearly seen by comparing their respective spectra. The positions of the Wood anomaly (vertical arrows in Fig. 1, corresponding to transmission dips at $\lambda = a$ and $\lambda = \sqrt{3}a/2$ in the square and triangular lattice, respectively) reflect the different reciprocal-lattice vectors: $\mathbf{G}_{nm} = (2\pi/a)(n,m)$ and $\mathbf{G}_{nm} = (2\pi/a)[n-m,(n+m)/\sqrt{3}]$ for square and triangular arrays, respectively.

Figures 2(a)–2(f) show measured transmission intensity maps as a function of both the reduced frequency $\omega a/2\pi c$ and the normalized parallel wave vector $k_{\parallel}a/\pi$ along the principal directions of the Brillouin zone (ΓX and ΓM for squared lattices, and ΓK and ΓM for triangular lattices) for all three plate thicknesses. Here, c=1480 m/s is the speed of sound in water. The Wood anomalies are included for reference as white dashed curves, representing the equation $|\mathbf{k}_{\parallel}+\mathbf{G}_{nm}|=\omega/c$ for different values of the Miller indices (n,m). The transmission is dipped right at the Wood anomalies.¹⁹ Comparison between results obtained for both lattices leads to the following conclusions:

(1) In all cases, the transmission dips at the Wood anoma-

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FIG. 1. Measured (solid curves) and calculated (dashed curves) normal-incidence sound transmission of aluminum plates perforated with holes forming a square lattice (a)–(c) and a triangular lattice (d)–(f), for plate thickness t=2, 3, and 5 mm. The period is a=5 mm and the hole diameter is d=3 mm in all cases. The hard-solid approximation has been used in the calculations.

lies and transmission maxima appear at slightly lower frequency values with respect to the prediction of the hard solid model. In contrast to the optical case,²⁰ the dips deviate significantly with respect to the anomalies

in many cases. We interpret this discrepancy as originating in the finite impedance of the aluminum-water interface, and in the presence of intrinsic plate modes (see below), which can dramatically modify the



FIG. 2. (Color online) Measured sound transmission as a function of normalized parallel wave vector $k_{\parallel}a/\pi$ and frequency $\omega a/2\pi c$ for perforated aluminum plates with square (a)–(c) and triangular (d)–(f) lattice symmetry (see insets) and for different plate thicknesses *t*. The period is a=5 mm and the hole diameter is d=3 mm in all cases. (g)–(i) show the transmission for homogeneous isotropic aluminum plates.

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TABLE I. Phase speed c_s for the S_0 -like mode below the crossing with the Wood anomaly for both square and triangular lattices.

t (mm)	Lattice	Direction	c _s (m/s)
2	Square	ГХ	3495 ± 33
		ΓM	3020 ± 27
2	Triangular	ΓK	2994 ± 22
		ΓM	3521 ± 9
3	Square	ΓX	3150 ± 48
		ΓM	2748 ± 23
3	Triangular	ΓK	2747 ± 22
		ΓM	3217 ± 35

anomaly condition, as already observed in mutually agreeing theory and experiment for square arrays.¹⁹

- (2) A mode similar to the S_0 Lamb mode shows up [see Figs. 2(g) and 2(h)], which undergoes a strong interaction with the Wood anomalies, and that is well reproduced by full-wave calculations.¹⁹
- (3) For thin plates (t=2,3 mm), only the S_0 Lamb mode plays a role. However, the A_1 mode enters the measured dispersion region for thicker plates [t=5 mm, Fig. 2(i)]. The interference with Wood anomalies results in avoided crossings and dip-peak structures that are conspicuous in all measured data sets.
- (4)These avoided crossings are particularly clear in thin plates (t=2,3 mm) and they seem to occur at lower frequencies in square arrays compared to triangular ones. One can actually extract the phase velocity c_S of the S_0 -like mode below the crossing by fitting to a linear dispersion at low frequencies. The results (see Table I) reveal the importance of plate thickness and orientation of \mathbf{k}_{\parallel} . For both types of lattices, the phase velocity decreases when the thickness increases from 2 to 3 mm. Moreover, the S_0 -like mode travels faster through the ΓX direction with square lattices, and through the ΓM with triangular lattices. For thicker plates, the bands evolve in a complex way, which prevents us from extracting accurate values for the group velocity within the frequency range of our measurements [Figs. 2(c) and 2(f)].

In summary, we have studied the transmission spectrum of square and triangular lattice hole arrays both experimentally and theoretically. The lattice symmetry of the hole arrays affects the position of the Wood anomalies and the width of the transmission peaks. The angle-resolved transmission shows, for both square and triangular lattice hole arrays, the interaction between Wood anomalies and Lamb modes, giving rise to complex transmission patterns.

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