

Direct Determination of the Solar Neutrino Fluxes from Solar Neutrino Data

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Abstract

We determine the solar neutrino fluxes from a global analysis of the solar and terrestrial neutrino data in the framework of three-neutrino mixing. Using a Bayesian approach we reconstruct the posterior probability distribution function for the eight normalization parameters of the solar neutrino fluxes plus the relevant masses and mixing with and without imposing the luminosity constraint. This is done by means of a Markov Chain Monte Carlo employing the Metropolis-Hastings algorithm. We also describe how these results can be applied to test the predictions of the Standard Solar Models. Our results show that, at present, both models with low and high metallicity can describe the data with good statistical global agreement.

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I. INTRODUCTION

The idea that the Sun generates power through nuclear fusion in its core was first suggested in 1919 by Sir Arthur Eddington, who pointed out that the nuclear energy stored in the Sun could explain the apparent age of the Solar System. In 1939, Hans Bethe described in an epochal paper [1] two nuclear fusion mechanisms by which main sequence stars like the Sun could produce the energy necessary to power their observed luminosities. The two mechanisms have become known as the pp-chain and the CNO-cycle [2]. For both chains the basic energy source is the burning of four protons to form an alpha particle, two positrons and two neutrinos. In the pp-chain fusion reactions among elements lighter than $A = 8$ produce a characteristic set of neutrino fluxes, whose spectral energy shapes are known but whose fluxes must be calculated with a detailed solar model. In the CNO-cycle the ^{12}C acts as a catalyst, while the ^{13}N and ^{15}O beta decays provide the primary source of neutrinos.

In order to precisely determine the rates of the different reactions in the two chains which are responsible for the final neutrino fluxes and their energy spectrum, a detailed knowledge of the Sun and its evolution is needed. Standard Solar Models (SSM's) [3, 4, 5, 6, 7, 8, 9] describe the properties of the Sun and its evolution after entering the main sequence. The models are based on a set of observational parameters: the surface luminosity of the Sun, its age, radius, and mass, and on several basic assumptions: spherical symmetry, hydrostatic and thermal equilibrium, equation of state, and present surface abundances of heavy elements. Over the past five decades, the solar models were steadily refined as the result of increased observational and experimental information about the input parameters (such as nuclear reaction rates and the surface abundances of different elements), more accurate calculations of constituent quantities (such as radiative opacity and equation of state), the inclusion of new physical effects (such as element diffusion), and the development of faster computers and more precise stellar evolution codes.

Despite the progress of the theory, only neutrinos, with their extremely small interaction cross sections, can enable us to see into the interior of a star and thus verify directly our understanding of the Sun [10]. Indeed from the earliest days of solar neutrino research this test was a primary goal of the solar neutrino experiments. However this task was made difficult by the increasing discrepancy between the predictions of the SSM's and the solar neutrino observations. This so-called "solar neutrino problem" [11, 12] was finally solved by the modification of the Standard Model of particle physics with the inclusion of neutrino masses and mixing which allowed for flavor conversion in the leptonic sector. As a consequence neutrino can change their flavor from the production point in the Sun to their detection in the Earth, and the flavor transition probability is energy dependent [13, 14]. This mechanism is known as the LMA-MSW solution to the solar neutrino problem, and affects both the overall number of events in solar neutrino experiments and the relative contribution expected from the different components of the solar neutrino spectrum. Because of these complications, it was necessary to assume the SSM predictions for all the solar neutrino fluxes and their

uncertainties in order to determine reasonably constrained values for neutrino masses and mixing. The upcoming of the real-time experiments Super-Kamiokande and SNO and the independent determination of the flavor oscillation probability using reactor antineutrinos at KamLAND opened up the possibility of extracting the solar neutrino fluxes and their uncertainties directly from the data [9, 15, 16, 17, 18, 19]. In general, in these works some set of simplifying assumptions were made in order to reduce the number of free parameters to be determined.

In parallel to the increased precision of the SSM-independent determination of the neutrino flavor parameters, a new puzzle has emerged in the consistency of SSM's [20]. Till recently SSM's have had notable successes in predicting other observations. In particular, quantities measured by helioseismology such as the radial distributions of sound speeds and densities [5, 6, 7, 8] showed good agreement with the predictions of the SSM calculations and provided accurate information on the solar interior. A key element to this agreement were the input values of abundances of heavy elements on the surface of Sun [21]. However, recent detailed determination of the abundances of the heavy elements on the solar surface lead to lower values [22, 23]. A SSM which incorporates such lower metallicities fails at explaining the helioseismological observations [20]. Changes in the Sun modeling, in particular of the less known convective zone, are not able to account for this discrepancy [24].

So far there has not been a successful solution of this puzzle. Thus the situation is that at present, there is no fully consistent SSM. This lead to the construction of two different sets of SSM's, one (labeled "GS") based on the older solar abundances [21] leading to high metallicity, and one (labeled "AGS") assuming lower metallicity as inferred from more recent determination of the solar abundances [22, 23]. In Ref. [9] the solar fluxes from such two models, BPS08(GS) and BPS08(AGS), were detailed, obtained with updated versions of the solar model calculations in Ref. [8]. In a very recent work [25] an update of the BPS08(AGS) solar model (now relabeled "AGSS09") has been constructed using the latest determination of the compositions [23] as well as some improvement in the equation of state. For what concerns the overall normalization of solar neutrino fluxes, the predictions of this new model are very close to those of BPS08(AGS).

In this work we perform a solar model independent analysis of the solar and terrestrial neutrino data in the framework of three-neutrino masses and mixing. The aim of the analysis is to simultaneously determine the flavor parameters and all the solar neutrino fluxes with a minimum set of theoretical priors. In Sec. II we present the method employed, the data included in the analysis and the minimum set of physical assumptions used in this study. The results of the analysis are given in Sec. III, where we show the reconstructed posterior probability distribution function for the eight normalization parameters of the solar neutrino fluxes. The effect of the luminosity constraint [26] on these results is explicitly shown. We also discuss the role of the Borexino results and its potential for improvement. We use the results of this analysis to statistically test to what degree the present solar neutrino data can discriminate between the two SSM's. Finally in Sec. IV we summarize our conclusions.

II. DATA ANALYSIS

In the analysis of solar neutrino experiments we include the total rates from the radiochemical experiments, Chlorine [27], Gallex/GNO [28] and SAGE [28]. For real-time experiments in the energy range of ^8B neutrinos we include the 44 data points of the electron scattering (ES) Super-Kamiokande phase I (SK-I) energy-zenith spectrum [29], the 34 data points of the day-night spectrum from SNO-I [30], the separate day and night rates for neutral current (NC) and ES events and the day-night energy-spectrum for charge current (CC) events from SNO-II (a total of 38 data points) [31], the three rates for CC, ES and NC from SNO-III [32], and the 7 points of the high-energy spectrum from the 246 live days of Borexino [33] (we denote this last data set as Borexino-HE).¹ Finally, we include the main set of the 192 days of Borexino data (which we denote as Borexino-LE) [35] in two different forms as described in Sec. III A and detailed in Appendix A. In brief, in one analysis we use the total event rates from ^7Be neutrinos as extracted by the Borexino collaboration from their energy spectrum. In the other we fit ourselves the Borexino energy spectrum in the energy range above 365 keV, corresponding to a total of 160 data points.

In the framework of three neutrino masses and mixing the expected values for these solar neutrino observables depend on the parameters Δm_{21}^2 , θ_{12} , and θ_{13} , and on the normalizations of the eight solar fluxes.

Besides solar experiments, we also include the latest results from the long baseline reactor experiment KamLAND [36, 37], which in the framework of three neutrino mixing also yield information on the parameters Δm_{21}^2 , θ_{12} , and θ_{13} . In addition, we include the information on θ_{13} obtained after marginalizing over Δm_{31}^2 and θ_{23} the results from the complete SK-I and SK-II atmospheric neutrino data sets (see the Appendix of Ref. [37] for full details on our analysis), the CHOOZ reactor experiment [38], K2K [39], the latest Minos ν_μ disappearance data corresponding to an exposure of 3.4×10^{20} p.o.t. [40], and the first Minos $\nu_\mu \rightarrow \nu_e$ appearance data presented in Ref. [41]. Details of the oscillation analysis of these observables will be presented elsewhere [42].

In what follows for convenience we will use as normalization parameters for the solar fluxes the reduced quantities:

$$f_i = \frac{\Phi_i}{\Phi_i^{\text{BPS08(GS)}}} \quad (1)$$

with $i = \text{pp}, ^7\text{Be}, \text{pep}, ^{13}\text{N}, ^{15}\text{O}, ^{17}\text{F}, ^8\text{B}$, and hep.

¹ We have not included here the very recent results on the low energy threshold analysis of the combined SNO phase I and phase II [34]. These results provide information on the ^8B and hep fluxes and show no major difference with the results from their previous analysis, hence we expect that they will have no important impact on the results of the global analysis here presented. In particular we notice that their best fit determination of the ^8B flux as well as of the oscillation parameters are in perfect agreement with our results.

With this the theoretical predictions for the relevant observables (after marginalizing over θ_{23} and Δm_{23}^2) depend on 11 parameters: the three relevant oscillation parameters Δm_{21}^2 , θ_{12} , θ_{13} and the 8 reduced solar fluxes f_i .

With the data from the different data samples (D) and the theoretical predictions for them in terms of the relevant parameters $\vec{\omega} = (\Delta m_{21}^2, \theta_{12}, \theta_{13}, f_{\text{pp}}, \dots, f_{\text{hep}})$ we build the corresponding likelihood function

$$\mathcal{L}(\text{D}|\vec{\omega}) = \frac{1}{N} \exp \left[-\frac{1}{2} \chi^2(\text{D}|\vec{\omega}) \right] \quad (2)$$

where N is a normalization factor.

In Bayesian statistics our knowledge of $\vec{\omega}$ is summarized by the posterior probability distribution function (p.d.f.)

$$p(\vec{\omega}|\text{D}, \mathcal{P}) = \frac{\mathcal{L}(\text{D}|\vec{\omega}) \pi(\vec{\omega}|\mathcal{P})}{\int \mathcal{L}(\text{D}|\vec{\omega}') \pi(\vec{\omega}'|\mathcal{P}) d\vec{\omega}'} \quad (3)$$

$\pi(\vec{\omega}|\mathcal{P})$ is the prior probability density for the parameters. In our model independent analysis we assume a uniform prior probability over which we impose the following set of constraints to ensure consistency in the pp-chain and CNO-cycle, as well as some relations from nuclear physics:

- The fluxes must be positive:

$$f_i \geq 0. \quad (4)$$

- The physical requirement that the number of nuclear reactions that terminate the pp-chain does not exceed the number of initiating nuclear reactions implies [26, 43]:

$$\begin{aligned} \Phi_{7\text{Be}} + \Phi_{8\text{B}} &\leq \Phi_{\text{pp}} + \Phi_{\text{pep}} \\ \Rightarrow 8.49 \times 10^{-2} f_{7\text{Be}} + 9.95 \times 10^{-5} f_{8\text{B}} &\leq f_{\text{pp}} + 2.36 \times 10^{-3} f_{\text{pep}} \end{aligned} \quad (5)$$

- The $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction being the slowest process in the main branch of the CNO-cycle implies [43]

$$\Phi_{^{15}\text{O}} \leq \Phi_{^{13}\text{N}} \Rightarrow f_{^{15}\text{O}} \leq 1.34 f_{^{13}\text{N}}. \quad (6)$$

We have also imposed the CNO-II branch to be subdominant

$$\Phi_{^{17}\text{F}} \leq \Phi_{^{15}\text{O}} \Rightarrow f_{^{17}\text{F}} \leq 37 f_{^{15}\text{O}}. \quad (7)$$

- The ratio of the pep neutrino flux to the pp neutrino flux is fixed to high accuracy because they have the same nuclear matrix element. We have set the ratio equal to the average of the one in BPSO8(GS) and in BPSO8(AGS) with uncertainty given by the difference between the values in the two models

$$\frac{f_{\text{pep}}}{f_{\text{pp}}} = 1.008 \pm 0.010. \quad (8)$$

Flux	α_i (10 MeV)	a_i
pp	1.30987	0.7001
${}^7\text{Be}$	1.26008	0.05946
pep	1.19193	0.001654
${}^{13}\text{N}$	0.34577	0.003377
${}^{15}\text{O}$	2.15706	0.002521
${}^{17}\text{F}$	0.2363	1.13E-07
${}^8\text{B}$	0.66305	0.0000696
hep	0.37370	0.926E-07

TABLE I: The energy provided to the star by nuclear fusion reactions associated with the i^{th} neutrino flux, α_i , and the ratio of the i^{th} neutrino flux of the BPS08(GS) model to the characteristic solar photon flux, a_i (adapted from Ref. [26]).

The number of independent fluxes is reduced when imposing the so-called ‘luminosity constraint’, *i.e.*, the requirement that the sum of the thermal energy generation rates associated with each of the solar neutrino fluxes be equal to the solar luminosity [44]. In terms of the reduced fluxes it can be written as:

$$1 = \sum_{i=1}^8 \beta_i f_i \quad \text{with } \beta_i \equiv \frac{L_{\odot}^i}{L_{\odot}} = \left(\frac{\alpha_i}{10 \text{ MeV}} \right) a_i \quad (9)$$

where β_i is the fractional contribution to the total solar luminosity of the nuclear reactions responsible for the production of the $\Phi_i^{\text{BPS08(GS)}}$ neutrino flux. Here the constant α_i is the energy provided to the star by the nuclear fusion reactions associated with the i^{th} neutrino flux, and a_i is the ratio of this flux to the characteristic solar photon flux defined by $L_{\odot}/[4\pi(\text{A.U.})^2(10 \text{ MeV})]$. Ref. [26] presents a detailed derivation of Eq. (9) and the numerical values for the coefficients α_i and a_i which we reproduce for convenience in table I.

The analysis performed incorporating the priors in Eqs. (4)-(9) will be labeled as $\mathcal{P} = L_{\odot}$ and named ‘analysis with luminosity constraint’. When only Eqs. (4)-(8) are imposed we will name it ‘analysis without luminosity constraint’ $\mathcal{P} = \mathbb{I}_{\odot}$. Following standard techniques we reconstruct the posterior p.d.f. Eq. (3) using a Monte-Carlo algorithm; full details of our approach are given in Appendix B.

III. RESULTS

Our results for the analysis with luminosity constraint are displayed in Fig. 1 where we show the marginalized one-dimensional probability distributions $p(f_i|\mathcal{D}, L_{\odot})$ for the eight solar neutrino fluxes as well as the 90% and 99% CL two-dimensional allowed regions. The

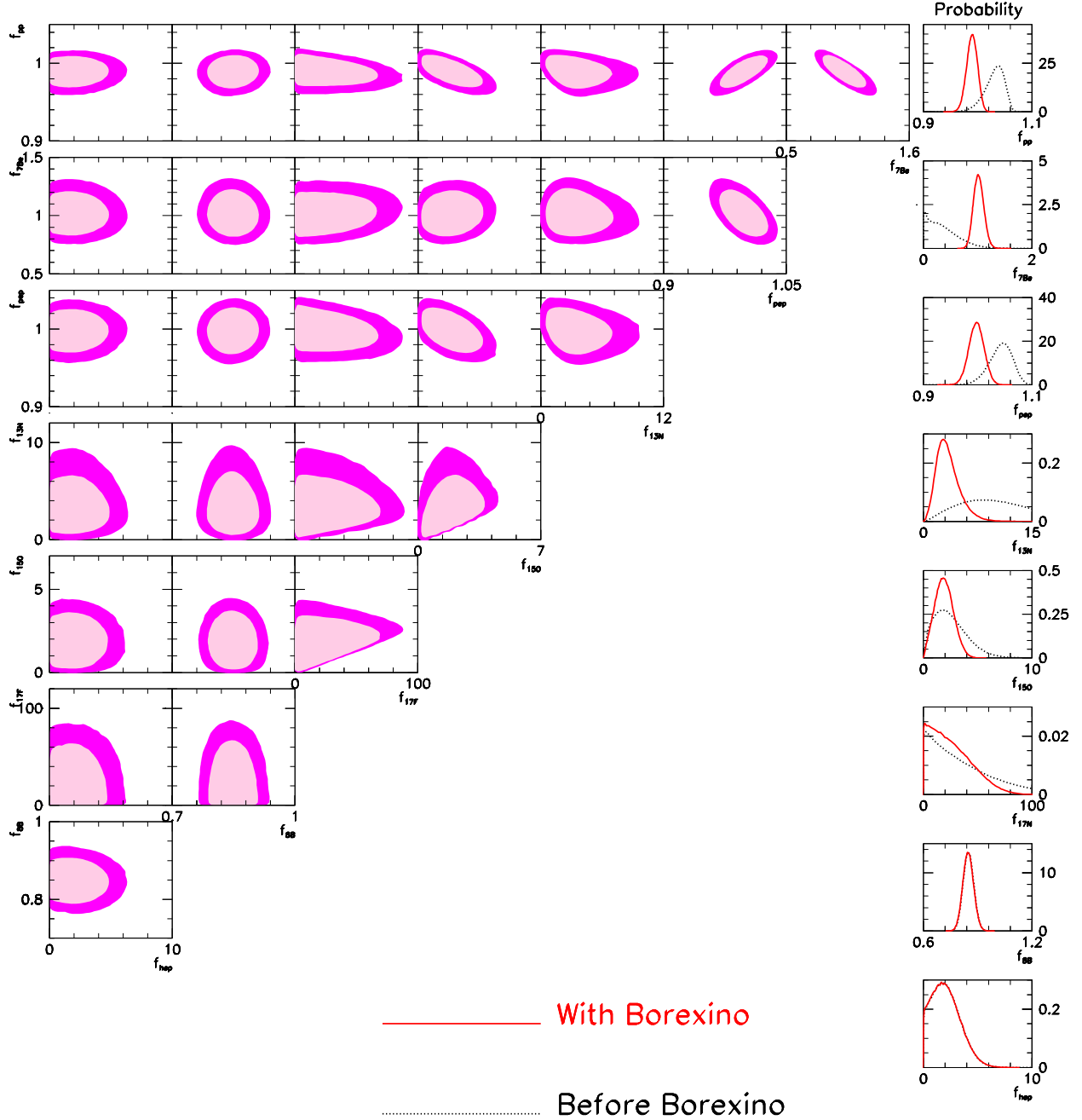


FIG. 1: Constraints from our global analysis on the solar neutrino fluxes. The curves on the most right panels show the marginalized one-dimensional probability distributions. The rest of the panels show the 90% and 99% CL two-dimensional credibility regions (see text for details). For comparison we also show the one-dimensional probability distributions before the inclusion of the Borexino spectral data.

corresponding ranges at 1σ [99% CL] on the oscillation parameters are:

$$\begin{aligned}\Delta m_{21}^2 &= 7.6 \pm 0.2 [\pm 0.5] \times 10^{-5} \text{ eV}^2, \\ \sin^2 \theta_{12} &= 0.33 \pm 0.02 [\pm 0.05], \\ \sin^2 \theta_{13} &= 0.02 \pm 0.012 [{}_{-0.02}^{+0.03}],\end{aligned}\tag{10}$$

while for the solar neutrino fluxes are:

$$\begin{aligned}f_{\text{pp}} &= 0.99_{-0.009}^{+0.010} [{}_{-0.030}^{+0.023}], & \Phi_{\text{pp}} &= 5.91_{-0.063}^{+0.057} [{}_{-0.16}^{+0.14}] \times 10^{10} \text{ cm}^{-2} \text{ s}^{-1}, \\ f_{7\text{Be}} &= 1.00_{-0.09}^{+0.10} [{}_{-0.21}^{+0.25}], & \Phi_{7\text{Be}} &= 5.08_{-0.43}^{+0.52} [{}_{-1.0}^{+1.3}] \times 10^9 \text{ cm}^{-2} \text{ s}^{-1}, \\ f_{\text{pep}} &= 1.00 \pm 0.014 [\pm 0.04], & \Phi_{\text{pep}} &= 1.41_{-0.020}^{+0.019} [{}_{-0.057}^{+0.054}] \times 10^8 \text{ cm}^{-2} \text{ s}^{-1}, \\ f_{13\text{N}} &= 2.7_{-1.2}^{+1.7} [{}_{-2.4}^{+5.6}], & \Phi_{13\text{N}} &= 7.8_{-3.4}^{+5.0} [{}_{-7.0}^{+16}] \times 10^8 \text{ cm}^{-2} \text{ s}^{-1}, \\ f_{15\text{O}} &= 1.8 \pm 0.9 [{}_{-1.8}^{+2.2}], & \Phi_{15\text{O}} &= 4.0_{-1.9}^{+1.8} [{}_{-3.8}^{+4.8}] \times 10^8 \text{ cm}^{-2} \text{ s}^{-1}, \\ f_{17\text{F}} &\leq 32 [72], & \Phi_{17\text{F}} &\leq 5.9 [43] \times 10^7 \text{ cm}^{-2} \text{ s}^{-1}, \\ f_{8\text{B}} &= 0.85 \pm 0.03 [\pm 0.08], & \Phi_{8\text{B}} &= 5.02_{-0.17}^{+0.18} [{}_{-0.42}^{+0.45}] \times 10^6 \text{ cm}^{-2} \text{ s}^{-1}, \\ f_{\text{hep}} &= 1.7_{-1.4}^{+1.3} [{}_{-1.7}^{+3.8}], & \Phi_{\text{hep}} &= 1.3 \pm 1.0 [{}_{-1.3}^{+3.0}] \times 10^4 \text{ cm}^{-2} \text{ s}^{-1}.\end{aligned}\tag{11}$$

We have also performed a Gaussian fit to the two-dimensional p.d.f. for the eight fluxes. We find that it can be best described by a Gaussian distribution with covariance matrix obtained by symmetrizing the 1σ ranges above and with error correlation matrix

	f_{pp}	$f_{7\text{Be}}$	f_{pep}	$f_{13\text{N}}$	$f_{15\text{O}}$	$f_{17\text{F}}$	$f_{8\text{B}}$	f_{hep}
f_{pp}	1	-0.81	0.74	-0.28	-0.64	-0.26	0.06	0.00
$f_{7\text{Be}}$		1	-0.58	-0.10	0.10	0.12	-0.05	0.00
f_{pep}			1	-0.22	-0.49	-0.20	0.04	0.01
$f_{13\text{N}}$				1	0.31	0.06	-0.02	0.00
$f_{15\text{O}}$					1	0.30	-0.03	0.00
$f_{17\text{F}}$						1	-0.02	0.00
$f_{8\text{B}}$							1	-0.04
f_{hep}								1

(12)

As seen in the Fig. 1 and in Eq. (12) the most important correlation appears between the pp and pep fluxes as expected from the relation (12). The correlation between the pp (and pep) and ^7Be flux is directly dictated by the luminosity constraint (see comparison with Fig. 2). The CNO fluxes are also correlated among themselves due to their similar energy spectrum. As a matter of fact the ^{15}O and ^{17}F fluxes are experimentally indistinguishable which implies that indeed what it is being constrained is their sum. Indeed equivalent results are obtained if only the sum is fitted. The, in principle, total anticorrelation is broken because of the positivity condition, (4), as well as the constraint (7).

These results imply the following share of the energy production in the pp-chain and the CNO-cycle

$$\frac{L_{\text{pp-chain}}}{L_{\odot}} = 0.986_{-0.006}^{+0.005} [{}_{-0.014}^{+0.011}], \quad \frac{L_{\text{CNO}}}{L_{\odot}} = 0.014_{-0.005}^{+0.006} [{}_{-0.011}^{+0.014}].\tag{13}$$

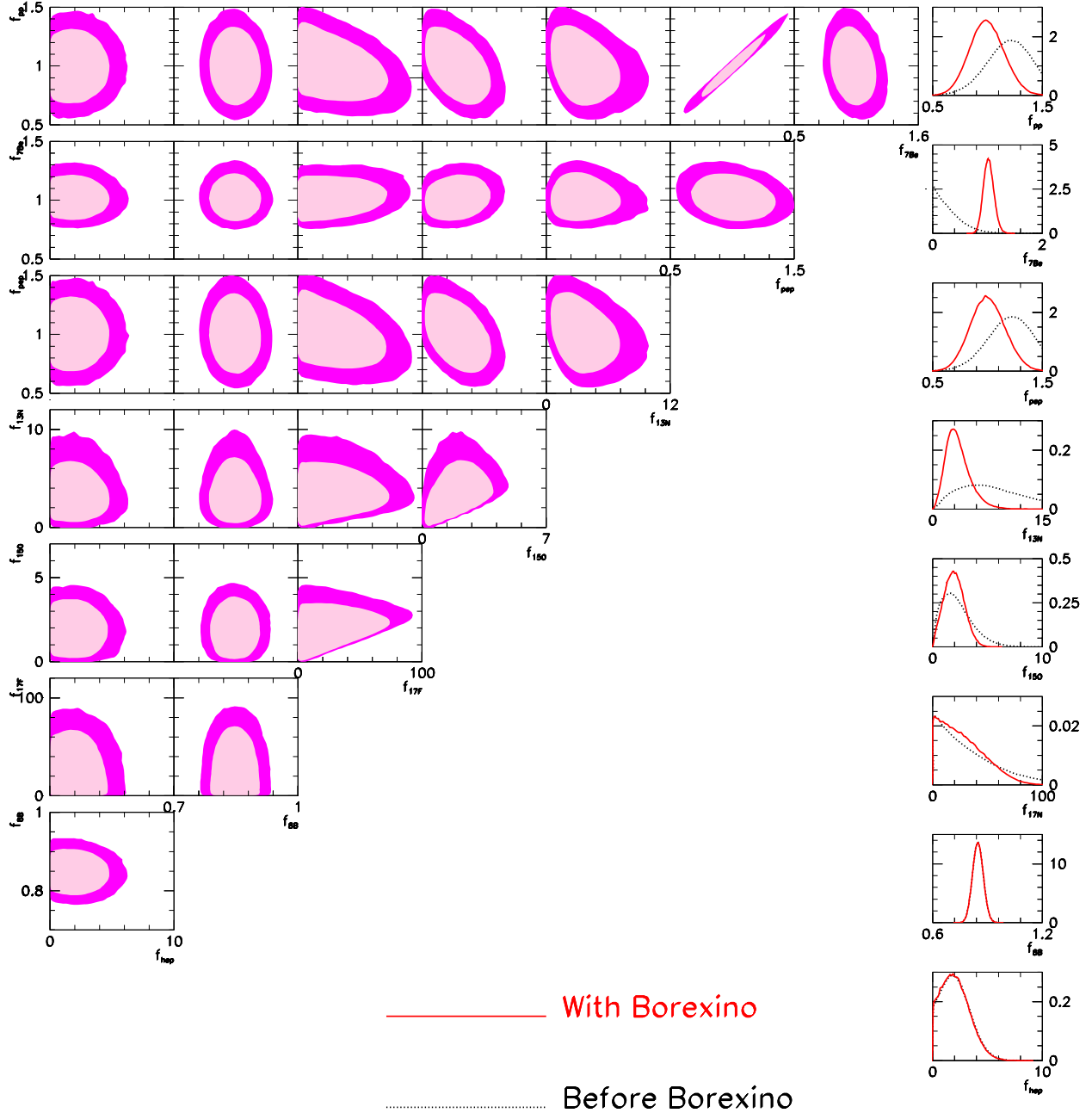


FIG. 2: Same as Fig. 1 but without the luminosity constraint, Eq. (9).

These are in perfect agreement with the SSM's which predict that $L_{\text{CNO}}/L_{\odot} \leq 1\%$ at the 3σ level.

In order to check the consistency of our results we have performed the same analysis without imposing the luminosity constraint, Eq. (9). The corresponding results for $p(f_i|D, \mathcal{L}_{\odot})$ and the two-dimensional allowed regions are shown in Fig. 2. As expected, it is the pp

flux the one which is mostly affected by the release of this constraint. This is so because, as seen in Table I, it is the pp reaction which gives most contribution to the solar energy production. Imposing the luminosity constraint on the pp flux only, already implies that this flux cannot exceed by than 9% the SSM prediction. Conversely releasing this constraint allow for a much larger pp flux. The pep flux is also severely affected as a consequence of the strong correlation with the pp flux, Eq. (8). In a smaller scale the CNO fluxes are also affected. This is mostly an indirect effect due to the modified contribution of the pp and pep fluxes to Gallium and Chlorine experiments which leads to a modification of the allowed contribution of the CNO fluxes to these experiments. Thus in this case we get:

$$\begin{aligned}
f_{\text{pp}} = f_{\text{pep}} &= 0.98_{-0.15}^{+0.16} [_{-0.40}^{+0.47}], \\
f_{7\text{Be}} &= 1.01_{-0.09}^{+0.1} [_{-0.22}^{+0.27}], \\
f_{13\text{N}} &= 2.7_{-1.3}^{+1.8} [_{-2.5}^{+5.7}], \\
f_{15\text{O}} &= 1.9 \pm 1 [_{-1.9}^{+2.3}], \\
f_{17\text{F}} &\leq 34[79].
\end{aligned}
\tag{14}$$

The determination of the ^8B and hep fluxes (as well as the oscillation parameters) is basically unaffected by the luminosity constraint.

Indeed the idea that the Sun shines because of nuclear fusion reactions can be tested accurately by comparing the observed photon luminosity of the Sun with the luminosity inferred from measurements of solar neutrino fluxes. We find that, without imposing the luminosity constraint the energy production in the pp-chain and the CNO-cycle are given by:

$$\frac{L_{\text{pp-chain}}}{L_{\odot}} = 0.98_{-0.14}^{+0.15} [\pm 0.40], \quad \frac{L_{\text{CNO}}}{L_{\odot}} = 0.015_{-0.007}^{+0.005} [_{-0.014}^{+0.013}].
\tag{15}$$

Comparing Eqs. (13) and (15) we see that the imposition of the luminosity constraint has limited impact on the bound on the amount of energy produced in the CNO-cycle while, as discussed above, the amount of energy in the pp-chain can now significantly exceed the total allowed by the luminosity constraint. Altogether we find that the present constraint for the ratio of the neutrino-inferred solar luminosity, $L_{\odot}(\text{neutrino-inferred})$, to the photon luminosity, L_{\odot} is:

$$\frac{L_{\odot}(\text{neutrino-inferred})}{L_{\odot}} = 1.00 \pm 0.14 [_{-0.34}^{+0.37}].
\tag{16}$$

Thus we find that, at present, the neutrino inferred luminosity perfectly agrees with the one directly determined and this agreement is known with a 1σ uncertainty of 15%.

A. The Role and Potential of Borexino

As seen in Figs. 1 and 2 the inclusion of Borexino has a very important impact in the determination of the ^7Be , pep, the CNO, and indirectly the pp fluxes. As mentioned above

and described in Appendix A, in our analysis we have fitted the 160 data points of the Borexino energy spectrum in the 365–2000 keV energy range, leaving as free parameters the normalization of the ^{11}C , ^{14}C , ^{210}Bi and ^{85}Kr backgrounds, the three relevant oscillation parameters, and the normalization of *all* the solar neutrino fluxes. In contrast, the Borexino collaboration fits the spectrum in the full energy range from ~ 200 –2000 keV, and allows for free normalizations of the ^{11}C , ^{10}C , $^{210}\text{Bi}+\text{CNO}$ and ^{85}Kr backgrounds as well as ^{14}C (which introduces an overwhelming background but is only relevant for events below 250 keV, hence it does not contribute to our analysis). Besides the normalization of these background components, only the ^7Be flux normalization is fitted to the data, and no direct information on the normalization of the other solar fluxes is extracted. In particular, the CNO fluxes are added to the ^{210}Bi background and fitted as a unique “background”, while the other solar fluxes are fixed to the BPS08(GS) SSM and the oscillation parameters are fixed to the best fit point of the global pre-Borexino analysis. With this procedure they determine the interaction rate for the 0.862 MeV ^7Be to be $49 \pm 3_{\text{stat}} \pm 4_{\text{sys}}$, which corresponds to $f_{^7\text{Be}} = 1.02 \pm 0.1$. Given the precision of the data and the energy spectrum of the irreducible backgrounds, their procedure is perfectly acceptable for the purpose of extracting the ^7Be normalization alone. However, in this work we are interested in testing the full set of SSM fluxes, not just ^7Be , and to this aim the consistent procedure is to allow for independent normalizations of all the solar fluxes.

In order to illustrate the impact of these two different approaches on the determination of the SSM fluxes, we have repeated our analysis using as unique data input from Borexino-LE the total interaction rate for ^7Be neutrinos quoted by the collaboration. The results are shown in Fig. 3. As seen in the figure the constraints imposed on the ^7Be flux after the inclusion of Borexino are basically the same, irrespective of whether one includes the full Borexino energy spectrum in the range 365–2000 keV or just the total ^7Be event rate extracted by the Borexino collaboration. However, the best fit and allowed ranges for the CNO fluxes are not the same. This proves that despite the unknown level ^{210}Bi contamination the Borexino spectral data can still provide useful information on CNO fluxes; when using only the total ^7Be event rate this information is lost and the constraints on CNO arise exclusively from the Gallium and Chlorine experiments.

As shown in Fig. 3, the inclusion of the complete Borexino-LE spectrum leads to an improvement (albeit not very significant) of the determination of the ^{13}N flux. Without Borexino this flux is mostly (and poorly) constrained by the Gallium experiment, and including the additional information from Borexino positively adds to its knowledge. We notice, however, that the best fit value of ^{13}N flux in either analysis is always higher than the prediction of any of the SSM’s (but fully compatible at better than 1.5σ). This behavior is driven by the Gallium rate which is slightly higher than expected in any of the SSM’s within the framework of three neutrino oscillations. A higher best fit value of ^{13}N can easily accommodate this observation without conflicting with any of the other experiments nor with the observed spectrum at Borexino. On the contrary, this is not the case for the ^{15}O

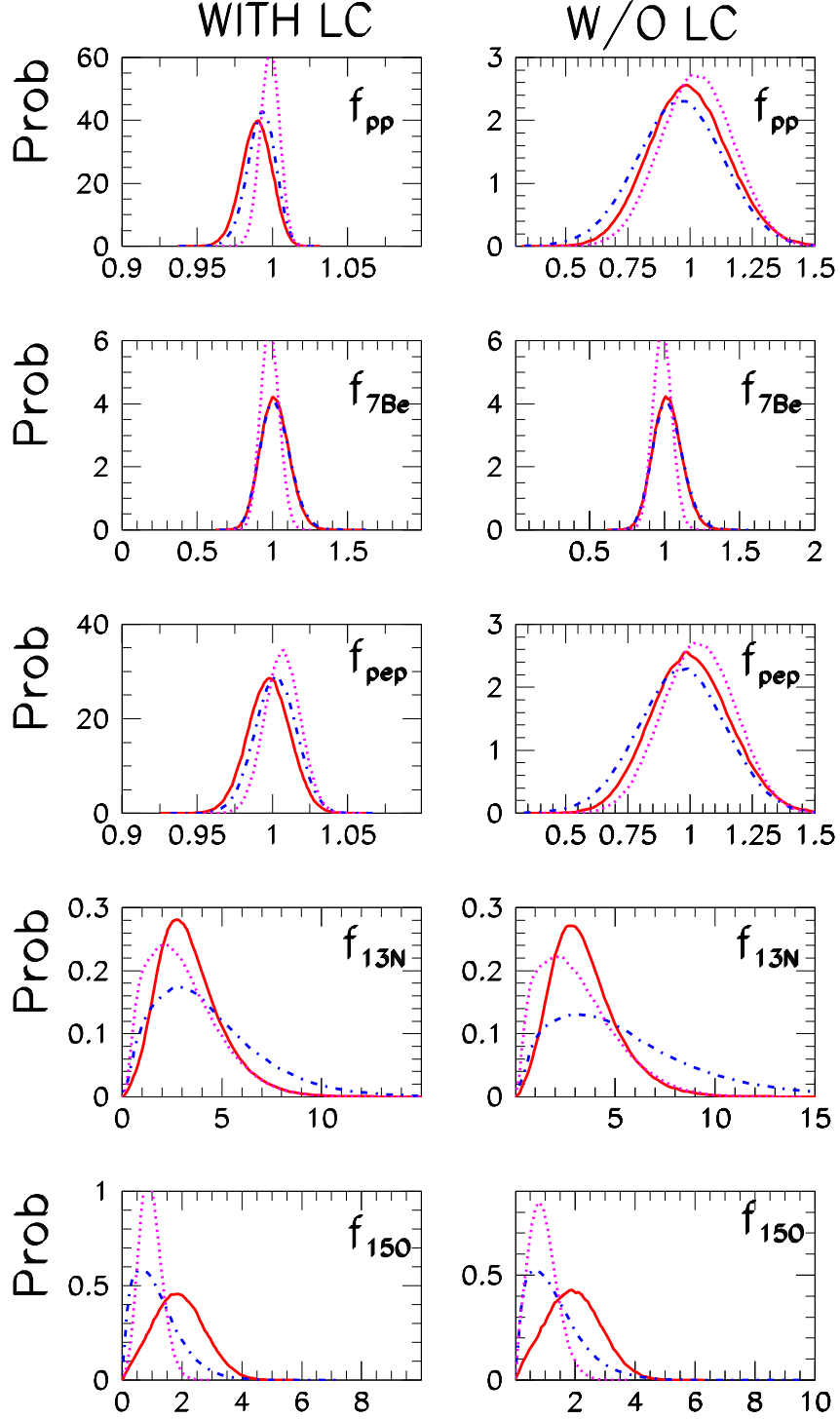


FIG. 3: Marginalized one-dimensional probability distributions for the fluxes contributing to Borexino-LE. The full line shows the determination obtained by fitting the Borexino spectrum data as described in Appendix A. The dashed-dotted line shows what the results would be if instead one had used the Borexino result for the extracted interaction rate of the 0.862 MeV ${}^7\text{Be}$ neutrinos. The dotted line represents the precision obtainable with the simulated “ideal” spectrum as described in Sec. III A.

flux. Adding the information from Borexino-LE spectrum leads to a (also small) worsening of the precision in the determination of this flux. We trace this apparently counter-intuitive result to the existing tension between the (low) Chlorine rate and the predicted rate within the framework of three neutrino oscillations. As a consequence Chlorine pushes towards lower values of the ^{15}O while we find that the spectrum of Borexino-LE prefers a higher amount of ^{15}O . When only the total event rate of ^7Be is used in the fit, the extracted ^{15}O flux is mostly driven by the Chlorine result. When including the Borexino-LE spectrum the tension results into a higher best fit for the ^{15}O flux but a worsening on the precision.

Given the fact that the CNO fluxes are those most different in between the two solar models, it is interesting to explore whether this tension can be resolved with future Borexino data and to what degree the CNO and pep fluxes can be better determined. Besides the accumulation of more statistics, Borexino in the near future aims at reducing the systematic uncertainties with the deployment of calibration sources in the detector [35]. Furthermore ideally if the ^{11}C background could be statistically subtracted, the pep and CNO fluxes could be directly accessed. In order to illustrate the potential of this possibility we have simulated an “ideal” spectrum of 85 bins in the energy range 365–1238 keV according to the expectations from the central values for the BPS08(GS) fluxes and the best fit point of oscillation and assuming that the ^{11}C has been fully subtracted while the other backgrounds are added under the same assumptions than in the present Borexino analysis. We have assumed double statistics and a reduction by a factor 3 of the systematic uncertainties. The results of the fit including this simulated spectrum are given in Fig. 3. As seen in the figure with this ideal experiment the precision of all the fluxes can be substantially improved (though for the CNO fluxes the precision is still far below the present uncertainties of the SSM’s) with the exception of the ^{13}N flux. This is a consequence of the tension between the higher value of ^{13}N preferred by the Gallium experiments and the SSM value used in the simulated spectrum. If we had simulated data corresponding to a higher value of ^{13}N flux, the precision in the determination of this flux would have also improved.

B. Comparison with the Standard Solar Model(s)

In Fig. 4 we show the marginalized one-dimensional probability distributions for the best determined solar fluxes in our analysis as compared to the predictions for the two SSM’s in Ref. [9].

In order to statistically compare our results with the SSM’s predictions we perform a significance test. In doing so we start by constructing a posterior probability distribution function for the solar fluxes as well as for a SSM central values, as the probability distribution from the data subject to a SSM model prior probability distribution function defined as

$$p(\vec{f}, \vec{f}_{SSM} | D, \text{SSM}) = \frac{\mathcal{L}(D | \vec{f}) \pi(\vec{f}, \vec{f}_{SSM} | \text{SSM})}{\int \mathcal{L}(D | \vec{f}) \pi(\vec{f}, \vec{f}_{SSM} | \text{SSM}) d\vec{f} d\vec{f}_{SSM}}. \quad (17)$$

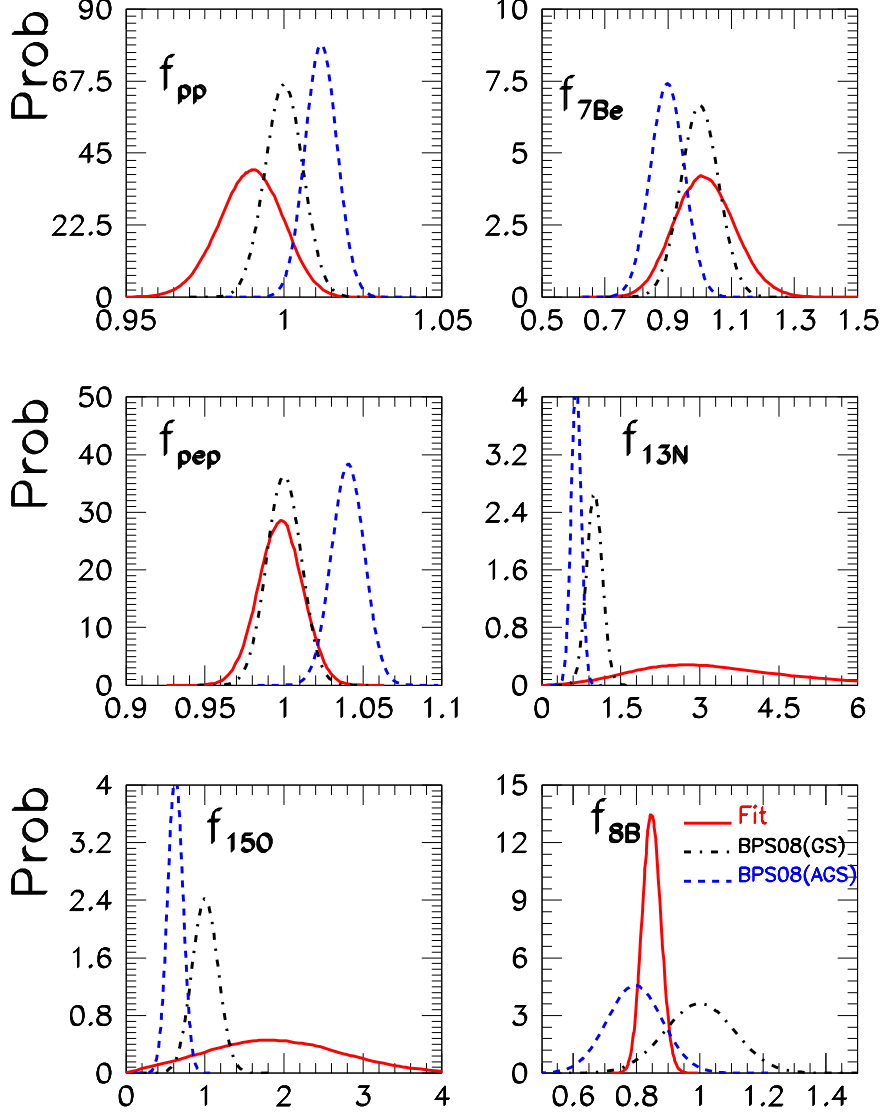


FIG. 4: Marginalized one-dimensional probability distributions for the best determined solar fluxes in our analysis as compared to the predictions for the two SSM's in Ref. [9].

where

$$-2 \ln \left[\pi(\vec{f}, \vec{f}_{\text{SSM}} | \text{SSM}) \right] = \sum_{i,j} (f_i - \bar{f}_{i,\text{SSM}}) V_{\text{SSM},ij}^{-1} (f_j - \bar{f}_{j,\text{SSM}}), \quad (18)$$

and $V_{\text{SSM},ij}$ is the SSM covariance matrix for the models. We build the covariance matrix from arbitrary models by interpolating the covariance matrix for the BPS08(GS) and BPS08(AGS) models given in Ref. [9, 45]. Since the covariance matrices for those models are very similar the results are not sensitive to this assumption. In order to improve over this approximation one would need a continuous model dependence of the flux covariance matrix which is currently unavailable.

The posterior probability distribution for a SSM characterized by the central values and

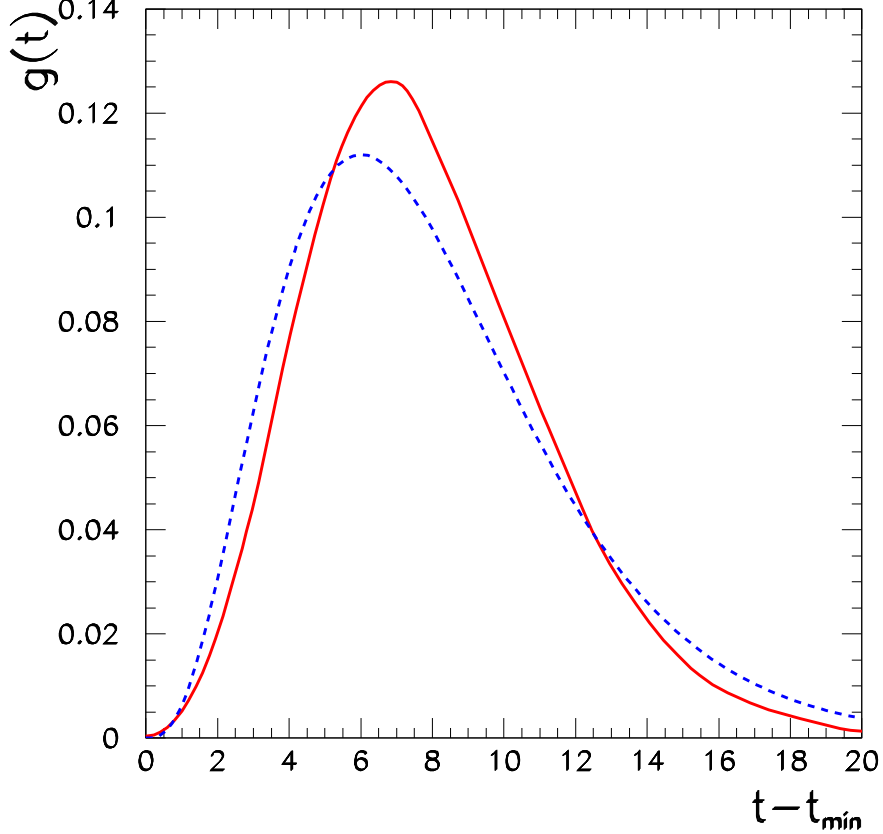


FIG. 5: Probability distribution function $g(t)$ (full line, see text for details). For comparison we show the corresponding distribution for a χ^2 p.d.f. with 8 degrees of freedom (dashed line).

that covariance matrix, subject to the constrained imposed by the Data is then

$$p(\vec{f}_{\text{SSM}}|\text{D}) = \int p(\vec{f}, \vec{f}_{\text{SSM}}|\text{D}, \text{SSM}) d\vec{f} \quad (19)$$

From $p(\vec{f}_{\text{SSM}}|\text{D})$ we define a probability distribution function for the statistics t as

$$g(t) = \int p(\vec{f}_{\text{SSM}}|\text{D}) \delta \left[t + 2 \ln \left(p(\vec{f}_{\text{SSM}}|\text{D}) \right) \right] d\vec{f}_{\text{SSM}} \quad (20)$$

By definition $g(t)$ is a function normalized to 1 in the interval $t_{\min} \leq t \leq \infty$. In Fig. 5 we plot the function $g(t)$. For comparison we show the corresponding distribution for a χ^2 p.d.f. with 8 degrees of freedom (dashed line).

The significance of the agreement between the data and what is expected under the assumption of each of the models is quantified by the P^{agr} value defined as the probability to find t in the region of equal or larger (lesser) compatibility with the data than the level of compatibility observed with each of the models

$$P_{\text{GS(AGS)}}^{\text{agr}} = \int_{t_{\text{GS(AGS)}}}^{t_{\text{max}}} g(t) dt \quad (21)$$

where $t_{\text{GS(AGS)}}$ is the value of the statistic obtained for the central value fluxes of the specific models:

$$t_{\text{GS(AGS)}} = -2 \ln \left[p(\vec{f}_{\text{GS(AGS)}} | \text{D}) \right]. \quad (22)$$

We find that the GS model has a lower t , $t_{\text{GS}} = 8.5$, while $t_{\text{AGS}} = 11.0$. With the probability distribution $g(t)$ shown in Fig. 5 this corresponds to $P_{\text{GS}}^{\text{agr}} = 43\%$ and $P_{\text{AGS}}^{\text{agr}} = 20\%$.

For comparison we have also constructed a χ^2 function comparing the best fit values of the fluxes in each of the models with those obtained in the analysis without the SSM priors and within the uncertainties given by the combined covariance matrix

$$\chi^2 = \sum_{ij} (\bar{f}_{\text{GS(AGS)},i} - \bar{f}_i^{\text{D}}) [V_{\text{GS(AGS)}} + V_{\text{D}}]_{ij}^{-1} (\bar{f}_{\text{GS(AGS)},j} - \bar{f}_j^{\text{D}}) \quad (23)$$

where V_{D} is the covariance matrix obtained by the best Gaussian approximation to the $p(f_i | \text{D}, L_{\odot})$ probability distribution function Eq. (12) and \bar{f}_j^{D} are the best fit fluxes from the data analysis without any SSM prior, Eq. (11). Should the distribution $p(\vec{f} | \text{D}, L_{\odot})$ be exactly Gaussian, both test would be equivalent. We find that this test maintains the better fit for the GS model $\chi_{\text{GS}}^2 = 5.2$ ($P_{\text{GS}}^{\text{agr}} = 74\%$) and $\chi_{\text{AGS}}^2 = 5.7$ ($P_{\text{AGS}}^{\text{agr}} = 68\%$) but yields higher probability for either model.

In other words while the fit shows a somehow better agreement with the model with higher metallicities the statistical difference is not significant. Besides the precision of the data, we notice that while the measurements of SNO and SK favor a lower ${}^8\text{B}$ flux as predicted by the low metallicity models, the determination of the ${}^7\text{Be}$ in Borexino and the corresponding determination of the pp flux from the luminosity constraint show better agreement with the GS predictions.

IV. SUMMARY

We have performed a solar model independent analysis of the solar and terrestrial neutrino data in the framework of three-neutrino oscillations. We have done so following a Bayesian approach in terms of a Markov Chain Monte Carlo using the Metropolis-Hastings algorithm. This allows for determination of the eleven-dimensional probability distribution function consistently incorporating the required set of theoretical priors.

Our results obtained when incorporating the requirement that the sum of the thermal energy generation rates associated with each of the solar neutrino fluxes be equal to the solar luminosity are given in Eq. (11) and Fig. 1. We have also tested this luminosity constraint with results given in Eq. (14) and Fig. 2. We find that at present the neutrino inferred luminosity perfectly agrees with the one directly determined and it is known with a 1σ uncertainty of 15%. The analysis also allows for testing the fractional production of energy in the pp-chain and the CNO-cycle. We find that the total amount of the solar luminosity produced in the CNO-cycle is bounded to be $L_{\text{CNO}}/L_{\odot} < 2.8\%$ at 99% CL.

Finally we have described the statistical test which can be performed to with these results to seed some light on the so-called solar composition problem which at the present arises in the construction of the solar standard model. We find that while the fit shows a somehow better agreement with the model with higher metallicities the statistical difference is not very significant. Besides the precision of the data, this result is also due to the fact that while the measurements of SNO and SK favor a lower ^8B flux as predicted by the low metallicity models, the determination of the ^7Be in Borexino and the corresponding determination of the pp flux from the luminosity constraint show better agreement with the predictions from the model incorporating a higher metallicity.

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APPENDIX A: ANALYSIS OF BOREXINO SPECTRA

In our analysis of Borexino we include both the low-energy (LE) data presented in Ref. [35], which are crucial for the reconstruction of the ^7Be line, as well as the high-energy (HE) data discussed in Ref. [33], which are mostly sensitive to the ^8B flux. For the low-energy part we extracted the 180 experimental data points and the corresponding statistical uncertainties from Fig. 2 of Ref. [35], checking explicitly that the statistical error σ_b^{stat} is just the square root of the number of events N_b^{ex} in each bin b (except in the region where statistical α 's subtraction had been performed). Similarly, for the high-energy part we extracted the 6 experimental data points and statistical uncertainties from Fig. 3 of Ref. [33]. For both data sets the theoretical prediction N_b^{th} for the bin b is calculated as follows:

$$N_b^{\text{th}}(\vec{\omega}, \vec{\xi}) = n_{\text{el}} T_b^{\text{run}} \sum_{\alpha} \int \frac{d\Phi_{\alpha}^{\text{det}}}{dE_{\nu}}(E_{\nu}|\vec{\omega}) \frac{d\sigma_{\alpha}}{dT_e}(E_{\nu}, T_e) R_b(T_e|\vec{\xi}) dE_{\nu} + N_b^{\text{bkg}}(\vec{\xi}) \quad (\text{A1})$$

where $\vec{\omega}$ describes both the neutrino oscillation parameters and the eight solar flux normalizations, and $\vec{\xi}$ is a set of variables parametrizing the systematic uncertainties as required by the ‘‘pulls’’ approach to χ^2 calculation. Here n_{el} is the number of electron targets in a fiducial mass of 78.5 tons with an electron/nucleon ratio of 11/20 for pseudocumene, and T_b^{run} is the total data-taking time which we set to 192 and 246 live days for LE and HE data, respectively. In the previous formula $d\sigma_{\alpha}/dT_e$ is the elastic scattering differential cross-section for neutrinos of type $\alpha \in \{e, \mu, \tau\}$, and $d\Phi_{\alpha}^{\text{det}}/dE_{\nu}$ is the corresponding flux of solar

neutrinos *at the detector* – hence it incorporates the neutrino oscillation probabilities. The detector response function $R_b(T_e|\vec{\xi})$ depends on the *true* electron kinetic energy T_e and on the three systematic variables ξ_{vol} , ξ_{scl} and ξ_{res} :

$$R_b(T_e|\vec{\xi}) = (1 + \pi_{\text{vol}} \xi_{\text{vol}}) \int_{T_b^{\text{min}}(1+\pi_{\text{scl}}^b \xi_{\text{scl}})}^{T_b^{\text{max}}(1+\pi_{\text{scl}}^b \xi_{\text{scl}})} \text{Gauss}[T_e - T', \sigma_T(1 + \pi_{\text{res}} \xi_{\text{res}})] dT' \quad (\text{A2})$$

where $\text{Gauss}(x, \sigma) \equiv \exp[-x^2/2\sigma^2]/\sqrt{2\pi}\sigma$ is the normal distribution function, while T_b^{min} and T_b^{max} are the boundaries of the *reconstructed* electron kinetic energy T' in the bin b . Note that we assumed an energy resolution $\sigma_T/T_e = 6\%/\sqrt{T_e [\text{MeV}]}$, rather than the “official” value $5\%/\sqrt{T_e [\text{MeV}]}$ quoted by the collaboration, since our choice lead to a perfect match of the ${}^7\text{Be}$ line shown in Fig. 2 of Ref. [35]. We verified that also the other solar fluxes plotted in Ref. [46] are carefully reproduced. As for the effects introduced by systematic uncertainties, we assumed $\pi_{\text{vol}} = 6\%$ for the fiducial mass ratio uncertainty, $\pi_{\text{scl}}^b = 2.4\%$ (1%) for the energy scale uncertainty in LE (HE) data, and an arbitrary $\pi_{\text{res}} = 10\%$ for the energy resolution uncertainty.

The backgrounds $N_b^{\text{bkg}}(\vec{\xi})$ which appear in Eq. (A1) only affect the low-energy data, and are not included in the calculation of the high-energy event rates. The ${}^{10}\text{C}$, ${}^{11}\text{C}$, ${}^{14}\text{C}$ and ${}^{85}\text{Kr}$ background shapes were taken from Fig. 2 of Ref. [35], whereas the ${}^{238}\text{U}$, ${}^{214}\text{Pb}$ and ${}^{210}\text{Bi}$ were extracted from slide 7 of Ref. [46]. We explicitly verified that with the normalizations as inferred in the figures the sum of all these backgrounds with the expected SSM fluxes precisely reproduces the “Fit” line shown in Fig. 2 of Ref. [35]. Note that because of the overwhelming ${}^{14}\text{C}$ at energies below ~ 250 keV our fit in this energy region is never good. We do not know if this is due to loss of numerical precision in our extraction of the ${}^{14}\text{C}$ shape or to the fact that there are additional free parameters to be fitted for this background. In any case, in order to avoid biasing our analysis by the low quality description on these data points we use only the 160 points of the spectrum above 365 keV. Hence the ${}^{14}\text{C}$ background is irrelevant. Following the procedure outlined in Ref. [35] the normalization of the ${}^{238}\text{U}$ and ${}^{214}\text{Pb}$ backgrounds are assumed to be known, whereas the normalizations of the ${}^{85}\text{Kr}$, ${}^{210}\text{Bi}$, ${}^{11}\text{C}$ and ${}^{10}\text{C}$ backgrounds are introduced as free parameters and fitted against the data – taking care to ensure their positivity. Hence:

$$N_b^{\text{bkg}}(\vec{\xi}) = N_b^{\text{U238}} + N_b^{\text{Pb214}} + N_b^{\text{Kr85}} \xi_{\text{Kr85}} + N_b^{\text{Bi210}} \xi_{\text{Bi210}} + N_b^{\text{C11}} \xi_{\text{C11}} + N_b^{\text{C10}} \xi_{\text{C10}}. \quad (\text{A3})$$

The $\chi^2(\vec{\omega})$ function is constructed in the usual way in the context of the pull method, by introducing standard penalties for the $\vec{\xi}$ variables (except for those parametrizing the free normalizations of the backgrounds) and marginalizing over them:

$$\chi^2(\vec{\omega}) = \min_{\vec{\xi}} \left\{ \sum_b \left[\frac{N_b^{\text{th}}(\vec{\omega}, \vec{\xi}) - N_b^{\text{ex}}}{\sigma_b^{\text{stat}}} \right]^2 + \xi_{\text{vol}}^2 + \xi_{\text{scl}}^2 + \xi_{\text{res}}^2 \right\}. \quad (\text{A4})$$

As a test of our procedure we first perform a fit under the same assumptions as Ref. [35]. This is, besides the background normalizations we only leave the ${}^7\text{Be}$ flux normalization

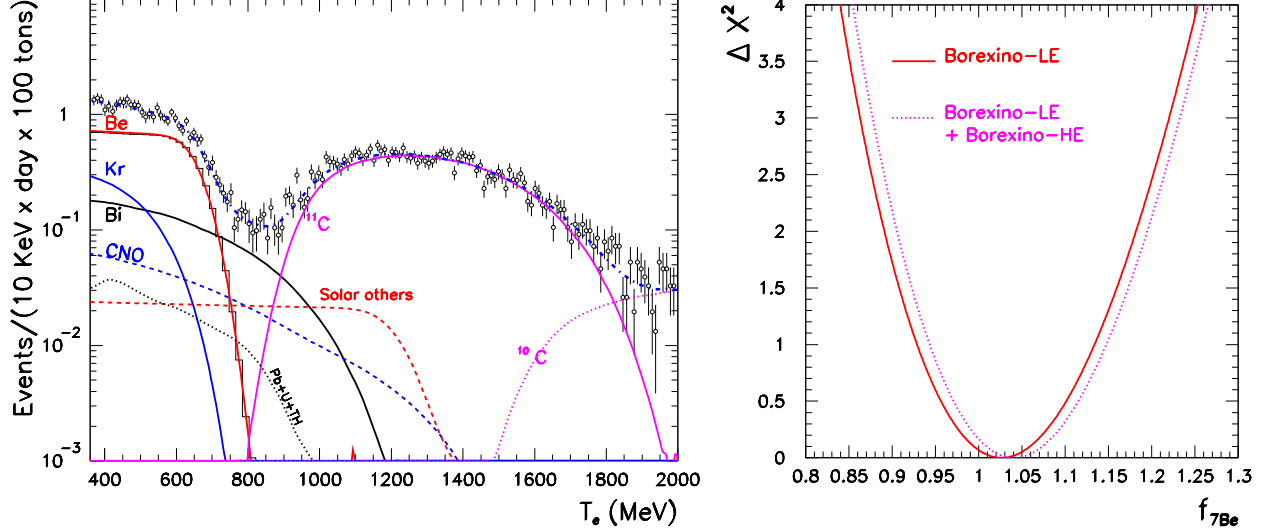


FIG. 6: Spectrum for the best fit point of our spectral fit to the Borexino-LE data in the energy region between 350–2000 KeV under the assumptions described in the Appendix (left) and $\Delta\chi^2$ as a function of the ${}^7\text{Be}$ flux for the different test analysis of Borexino data (right).

to be fitted to the data. The other solar fluxes are fixed to the BPS08(GS) SSM and the oscillation parameters are fixed to the best fit point of the global pre-Borexino analysis. The result of this test is shown in Fig. 6. Comparing the left panel with Fig. 2 of Ref. [35] one observes the perfect agreement in the best fit ${}^7\text{Be}$ flux spectra. Furthermore in the right panel we plot the $\Delta\chi^2$ for this test fit as a function of $f_{7\text{Be}}$. As seen in the figure our procedure leads to a determination of the ${}^7\text{Be}$ normalization in very good agreement with that obtained by Borexino $f_{7\text{Be}} = 1.02 \pm 0.1$. We also show in the right panel the curve for the combined analysis of Borexino-LE and Borexino-HE data under the same assumptions. As seen in the figure, the inclusion of the Borexino-HE tends to push the extracted value of $f_{7\text{Be}}$ towards a slightly higher value. This is due to the assumed correlation of the systematic uncertainties (in particular the one associated with the total fiducial volume) between LE and the HE. This small effect is diluted once both data sets are included in the global fit and the results are independent of the degree of correlation assumed between the systematic errors.

APPENDIX B: DETAILS OF THE MARKOV CHAIN MONTE CARLO

In this analysis we’ve used the Metropolis-Hasting algorithm including the adapting for the kernel function to increase the efficiency. The algorithm is defined as follows:

1. Given a parameter set $\vec{\omega}$, a new value $\vec{\omega}'$ is generated according a the transition kernel

$q(\vec{\omega}, \vec{\omega}')$.²

2. With $\vec{\omega}$ and $\vec{\omega}'$ we compute the value:

$$h = \min \left(1, \frac{\mathcal{L}(\mathbf{D}|\vec{\omega}') \pi(\vec{\omega}'|\mathcal{P})}{\mathcal{L}(\mathbf{D}|\vec{\omega}) \pi(\vec{\omega}|\mathcal{P})} \right). \quad (\text{B1})$$

3. A random number $0 \leq r \leq 1$ is generated and if $r \leq h$, $\vec{\omega}'$ is accepted in the chain.

4. Go back to step (1) starting with $\vec{\omega}'$ if it has been accepted or again with $\vec{\omega}$ if not.

All the points accepted in this algorithm constitute the Markov Monte-Carlo Chain $\{\vec{\omega}_\alpha\}$ with $\alpha = 1, \dots, N_{\text{tot}}$, where N_{tot} is the total number of points in the chain. The method ensures that, once convergence has been reached, the chain take values over the parameter space with frequency proportional to the posterior p.d.f.

Technically in order to reconstruct the posterior p.d.f. from the chain we discretize the parameter space by dividing in n_i subdivisions, $\Delta_i^{k_i}$ of length $\ell_i^{k_i}$ (with $1 \leq k_i < n_i$), the physically relevant range of each parameter ω_i . If we label $\Omega_{k_1 \dots k_m}$ the cell corresponding to subdivisions $\Delta_1^{k_1} \dots \Delta_m^{k_m}$ ($m = 10$ or 11 for the analysis with or without the luminosity constraint), we compute the value of the posterior p.d.f. as

$$p(\vec{\omega} \in \Omega_{k_1 \dots k_m} | \mathbf{D}, \mathcal{P}) = \frac{1}{V_{k_1 \dots k_m}} \frac{M_{k_1 \dots k_m}}{N_{\text{tot}}} \quad (\text{B2})$$

where $M_{k_1 \dots k_m}$ is the number of points in the chain with parameter values within the cell $\Omega_{k_1 \dots k_m}$ and $V_{k_1 \dots k_m} = \ell_1^{k_1} \times \dots \times \ell_m^{k_m}$ is the volume of the cell. In order for the procedure to rend a smooth p.d.f. a sufficiently large N_{tot} is needed.

The marginalized one-dimensional p.d.f.'s for the parameter ω_i is reconstructed as

$$p(\omega_i \in \Delta_i^{k_i} | \mathbf{D}, \mathcal{P})_{1\text{-dim}} = \frac{1}{\ell_i^{k_i}} \sum_{k_j \neq i=1}^{n_j} \frac{M_{k_1 \dots k_i \dots k_m}}{N_{\text{tot}}}. \quad (\text{B3})$$

Similarly the marginalized two-dimensional p.d.f.'s for the parameter $\omega_i \omega_j$ is

$$p(\omega_i \in \Delta_i^{k_i}, \omega_j \in \Delta_j^{k_j} | \mathbf{D}, \mathcal{P})_{2\text{-dim}} = \frac{1}{\ell_i^{k_i} \ell_j^{k_j}} \sum_{k_l \neq i, j=1}^{n_l} \frac{M_{k_1 \dots k_i \dots k_j \dots k_m}}{N_{\text{tot}}}. \quad (\text{B4})$$

From these, we obtain the two-dimensional credibility regions with a given CL as the region with smallest area and with CL integral posterior probability. In practice they are obtained as the regions surrounded by a two-dimensional isoprobability contour which contains the

² We start with a flat kernel and after the chain has a certain size, we use a kernel in terms of the covariance matrix V computed with the the points in the chain. If U is the matrix diagonalizing V and d_i are the eigenvalues, $\vec{\omega}' = \vec{\omega} + U \vec{\omega}$ with $\tilde{\omega}_i$ generated according to a distribution $|\tilde{\omega}_i|/d_i \times \exp(-\tilde{\omega}_i/d_i)$. The kernel is adapted, *i.e.*, the covariance matrix is recalculated, every several steps.

point of highest posterior probability and within which the integral posterior probability is CL.

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