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Exclusive central production of heavy quarks at the LHC

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Abstract

We study the exclusive production of heavy flavors at central rapidities in hadron-hadron collisions within the k_T factorisation formalism. Since this involves regions of small Bjorken x in the unintegrated gluon densities, we include the next-to-leading order BFKL contributions working directly in transverse momentum representation. Our results are presented in a form suitable for Monte Carlo implementation.

1 Introduction

Scattering processes with at least one hard scale are typically well described using perturbative QCD in the framework of collinear factorisation. In this approach cross sections are written as a convolution of a purely perturbative partonic cross section with non-perturbative parton distribution functions. The latter follow the DGLAP evolution which describes their dependence on the hard perturbative scale. When the center of mass energy is very large compared to the perturbative hard scale, or a final state is fixed such that there are large rapidity differences among the emitted particles, an alternative high energy factorisation based on the Balitsky-Fadin-Kuraev-Lipatov (BFKL) evolution equation applies [1]. Here the hard subprocess is convoluted with the hadron structure using unintegrated gluon densities which include their k_T dependence in the small Bjorken x limit. This can be seen as the small x limit of k_T factorisation. In this case enhanced logarithmic in x contributions are resummed.

In the present letter we propose to take the exclusive production of heavy quark-antiquark pairs in the early data at the LHC as a test of this formalism and the use of unintegrated gluon densities. Large masses such as those of bottom or top quarks allow for a perturbative treatment. In the case of top quark pairs their masses are so large that the typical proved values of Bjorken x are not that small. In this case it is known that cross sections receive significant corrections from threshold logarithms (see [2, 3, 4, 5, 6, 7, 8, 9, 10] for recent results in this direction). The bottom quarks are lighter and therefore test regions of smaller values of x where the corresponding resummations find their natural environment. Previous investigations of heavy quark production similar to our present calculation where presented in [11]. What we will show in this letter is an alternative approach which operates with NLO unintegrated gluon densities in transverse momentum space, does not involve the use of anomalous dimensions, treats the kinematics of the quark-antiquark pair exclusively and is readily suitable for a Monte Carlo analysis which we will present elsewhere. Other works which we found of interest in the field of inclusive heavy flavor production are Refs. [11, 12, 13, 14, 15, 16, 17]. For the production of bottom pairs in particular we highlight Refs. [18, 19], where the reported agreement with experimental data at the Tevatron ranges from reasonable [18] to very good [19].

The fully exclusive study that we propose could be very useful at the LHC since it allows for the precise determination of the x values at which the unintegrated gluon densities are probed. This provides a good control on the accuracy of the approximations that we use in our calculation. At the LHC the dominant production process for both top and bottom pairs is given by gluon-gluon fusion. However, as we already pointed out, only the bottom pair production occurs at relatively small x providing the correct kinematics to apply high energy k_T factorisation. Top pair production, on the other hand, occurs at relatively large values of x due to the large top mass. Studies of its exclusive production would certainly require the matching of the present calculation with renormalization group evolution and can be therefore considered as a test of the capability to extend our high energy factorisation towards the region of large x. For our predictions we incorporate the NLO corrections to the BFKL evolution kernel [20]. A related study, devoted to the exclusive central production of jets in hadron-hadron collisions in k_T factorisation, was presented in Ref. [21].

After this brief Introduction, in Section 2, we present the general structure of the k_T factorised differential cross-section and calculate its different elements. In Section 3 we discuss the unintegrated gluon density in k_T space and its iterative structure. Finally, we write our Conclusions in Section 4.

2 The k_T factorised differential cross-section at NLO

To describe the differential cross-section for the exclusive production of a pair of heavy quarks within k_T factorisation it is convenient to introduce a Sudakov basis. To this end we define the light-like momenta p_1 and p_2 which coincide in the $s \to \infty$ limit with the momenta of the incoming protons p_A and p_B:

$$p_1 \;=\; p_A - \frac{m_P^2}{s} p_B, \qquad p_2 \;=\; p_B - \frac{m_P^2}{s} p_A, \tag{1}$$

with $s = (p_A + p_B)^2$ being the squared center of mass energy of the hadronic process. With these definitions, we can then work with the usual Sudakov decomposition of a general four momentum, *i.e.*

$$k = \alpha\, p_1 + \beta\, p_2 + k_\perp. \tag{2}$$

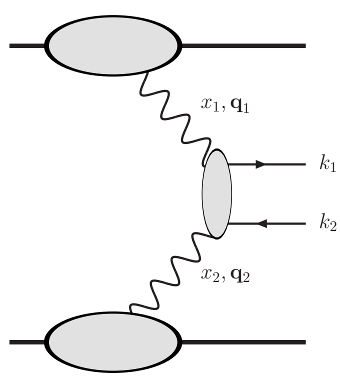

Figure 1: Central production of two heavy quarks in k_T factorisation

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The notation for the relevant momenta in the partonic hard subprocess is given in Fig. 1. In the BFKL formalism t-channel gluons carry a modified propagator which reggeises them. This propagator is associated to the momenta q_1 and q_2 in Fig. 1. These simplify in the high energy limit and can be written as

$$q_1 \;=\; x_1\, p_1 + q_{1,\perp}, \qquad\qquad q_2 \;=\; x_2\, p_2 + q_{2,\perp}. \tag{3}$$

On the other hand, the momenta of the produced heavy quarks have the following decomposition

$$k_i \;=\; \alpha_i\, p_1 + \beta_i\, p_2 + k_{i,\perp}, \qquad\qquad i \;=\; 1,2. \tag{4}$$

Taking into account the on-shellness of the produced quarks, the above Sudakov parameters can be expressed in terms of rapidities, transverse momenta and heavy quark masses M, *i.e.*

$$\alpha_i \;=\; \sqrt{\frac{M^2+\mathbf{k}_i^2}{s}}e^{\eta_i}, \qquad\qquad \beta_i \;=\; \sqrt{\frac{M^2+\mathbf{k}_i^2}{s}}e^{-\eta_i}, \qquad\qquad i=1,2. \tag{5}$$

Here η_1 (η_2) is the rapidity of the produced heavy quark (anti-quark) and $\mathbf{k}_i^2 = -k_{i,\perp}^2$ are the corresponding Euclidean squared transverse momenta.

Making use of the definitions

$$s_1 \;=\; (p_1+q_2)^2 = x_2\, s, \qquad\qquad s_2 \;=\; (p_2+q_1)^2 = x_1\, s, \tag{6}$$

which correspond to the center of mass energies of the upper and lower subamplitudes in Fig. 1, respectively, we can write the following expression for the differential cross-section of heavy quark production:

$$\begin{aligned}\frac{d^6\sigma}{d\eta_1 d\eta_2 d^2\mathbf{k}_1 d^2\mathbf{k}_2} &= \int_0^1 dx_1 \int_0^1 dx_2 \int \frac{d^2\mathbf{q}_1}{(2\pi)^3}\int \frac{d^2\mathbf{q}_2}{(2\pi)^3}\left[\int \frac{d^2\mathbf{q}_a}{2\pi}\frac{\Phi_A(\mathbf{q}_a)}{\mathbf{q}_a^2} f\left(\frac{s_1}{s_{0,1}},\mathbf{q}_a,\mathbf{q}_1\right)\right]\\ &\times \frac{|\Gamma_{\mathrm{RR}\to\mathrm{Q\bar{Q}}}(\mathbf{q}_1,\mathbf{q}_2;\mathbf{k}_1,\mathbf{k}_2,z)|^2}{\mathbf{q}_1^2\mathbf{q}_2^2}\left[\int \frac{d^2\mathbf{q}_b}{2\pi}\frac{\Phi_B(\mathbf{q}_b)}{\mathbf{q}_b^2} f\left(\frac{s_2}{s_{0,2}},\mathbf{q}_2,\mathbf{q}_b\right)\right]\\ &\times (2\pi)^4\delta^{(2)}(\mathbf{q}_1+\mathbf{q}_2-\mathbf{k}_1-\mathbf{k}_2)\,\delta(x_1-\alpha_1-\alpha_2)\,\delta(x_2-\beta_1-\beta_2).\end{aligned} \tag{7}$$

In this expression Φ_A and Φ_B denote the hadron impact factors, which are responsible for the coupling of the reggeised gluon to the proton A and B, respectively. $\Gamma_{\mathrm{RR}\to\mathrm{Q\bar{Q}}}$ indicates the high energy effective vertex coupling the two reggeised gluons to the heavy quark-antiquark pair with

$$z = \frac{\alpha_1}{x_1} = \frac{\sqrt{\mathbf{k}_1^2+M^2}}{\sqrt{\mathbf{k}_1^2+M^2}+\sqrt{\mathbf{k}_2^2+M^2}e^{\eta_2-\eta_1}} \tag{8}$$

being the fraction of the longitudinal momentum of the upper reggeised gluon along p_1, carried by the heavy quark. f denotes the BFKL gluon Green function with the following Mellin transform f_ω:

$$f\left(\frac{s_1}{s_{0,1}},\mathbf{q}_a,\mathbf{q}_1\right) = \int_{\mathcal{C}} \frac{d\omega}{2\pi i}\left(\frac{s_1}{s_{0,1}}\right)^{\omega} f_\omega(\mathbf{q}_a,\mathbf{q}_1), \tag{9}$$

where the contour of integration $\mathcal{C}$ lies parallel to the imaginary axis and to the right of all the singularities in f_ω. The resummation of high energy logarithms is achieved by iterating the BFKL integral equation for f_ω:

$$\omega f_\omega(\mathbf{q}_a,\mathbf{q}_1) = \delta^{(2)}(\mathbf{q}_a-\mathbf{q}_1) + \int d^2\mathbf{q}\, K_{\mathrm{BFKL}}(\mathbf{q}_a,\mathbf{q})\, f_\omega(\mathbf{q},\mathbf{q}_1). \tag{10}$$

In a general case where both the produced heavy quarks and the impact factors would provide a similar hard scale (*i.e.* if the protons were replaced by highly virtual photons, or jets with a high p_t were tagged in the forward/backward regions), a good choice for the energy scales $s_{0,i}$ would be given by $s_{0,1} = |\mathbf{q}_a|\sqrt{\Sigma}$ and $s_{0,2} = \sqrt{\Sigma}|\mathbf{q}_b|$, where

$$\Sigma \;=\; x_1\, x_2\, s \;=\; \hat{s} + (\mathbf{k}_1+\mathbf{k}_2)^2, \qquad\qquad \hat{s} \;=\; (k_1+k_2)^2. \tag{11}$$

$\hat{s}$ reads for the squared center of mass energy of the partonic process $g^* g^* \to Q\bar{Q}$. Such a choice naturally introduces the rapidities $\eta_{\tilde{A}}$ and $\eta_{\tilde{B}}$ of the emitted particles with momenta $p_{\tilde{A}}$ and $p_{\tilde{B}}$ since

$$\left(\frac{s_1}{s_{0,1}}\right)^\omega = e^{(\eta_{\tilde{A}} - \eta_{Q\bar{Q}})\omega}, \qquad \left(\frac{s_2}{s_{0,2}}\right)^\omega = e^{(\eta_{Q\bar{Q}} - \eta_{\tilde{B}})\omega}, \tag{12}$$

with the rapidity of the heavy quark system being given by

$$\eta_{Q\bar{Q}} = \frac{1}{2} \ln \frac{\alpha_1 + \alpha_2}{\beta_1 + \beta_2}. \tag{13}$$

If the BFKL Green function, the impact factors and the production vertex were known exactly at NLO, then the precise choice of the energy scales $s_{0,i}$ would turn out to be irrelevant since any dependence of the cross section on this scale would cancel at the same NLO accuracy. However, even in that case, when the kernel is exponentiated there is a residual dependence on $s_{0,i}$ which would correspond to NNLO and higher terms. A natural choice for $s_{0,i}$ is then that which reduces the size of those higher orders corrections to the minimum for a given observable.

In the case of interest for us in this letter there exists a hierarchy of scales with a large difference between the only hard scale provided by the invariant mass of the heavy quark pair system and the large transverse size of the incoming hadrons. Here the previous symmetric choice of scales is not appropriate as the scale of the heavy quark anti-quark system Σ is significantly larger than the transverse scales $\mathbf{q}_a^2$ and $\mathbf{q}_b^2$ associated to the scattered protons. A more natural choice for $s_{0,i}$ is given by Σ alone, *i.e.*

$$\left(\frac{s_1}{s_{0,1}}\right)^\omega = x_1^{-\omega}, \qquad \left(\frac{s_2}{s_{0,2}}\right)^\omega = x_2^{-\omega}. \tag{14}$$

This choice of the energy scale is common in deep inelastic scattering and leads to the concept of the unintegrated gluon density in a hadron. This represents the probability of resolving an off-shell gluon carrying a longitudinal momentum fraction x off the incoming hadron, together with a transverse momentum k_T.

As it is well-known, any choice of energy scale only matters at next-to-leading and higher orders since the LO approach is scale invariant. The LO unintegrated gluon density $g^{\rm LO}$ is defined as

$$g^{\rm LO}(x, \mathbf{k}) = \int \frac{d^2\mathbf{q}}{2\pi} \frac{\Phi_P(\mathbf{q})}{\mathbf{q}^2} f^{\rm LO}(x, \mathbf{q}, \mathbf{k}), \qquad \text{with} \qquad f^{\rm LO}(x, \mathbf{q}, \mathbf{k}) = \int_{\mathcal{C}} \frac{d\omega}{2\pi i} x^{-\omega} f_\omega^{\rm LO}(\mathbf{q}, \mathbf{k}), \tag{15}$$

where $f_\omega^{\rm LO}$ corresponds to the solution of the LO BFKL equation with kernel $K_{\rm BFKL} = K_{\rm BFKL}^{\rm LO}$. Contrary to the LO case, the next-to-leading order BFKL evolution is sensitive to changes in the energy scales $s_{0,i}$. As it was pointed out in [21], any shift of scales can be absorbed in the kernel, impact factors, and central production vertex. With the choice of energy scale as in Eq. (14) the NLO impact factors are modified by an extra logarithmic term of the form

$$\tilde{\Phi}_P^{\rm NLO}(\mathbf{q}) = \Phi_P^{\rm NLO}(\mathbf{q}) - \frac{\mathbf{q}^2}{2} \int d^2\mathbf{l}\, \frac{\Phi_P^{\rm LO}(\mathbf{l})}{\mathbf{l}^2}\, K_{\rm BFKL}^{\rm LO}(\mathbf{l}, \mathbf{q})\, \ln \frac{\mathbf{l}^2}{\mathbf{q}^2}. \tag{16}$$

The NLO kernel receives two additional contributions, corresponding to the incoming and outgoing reggeised gluons:

$$\tilde{K}_{\rm BFKL}^{\rm NLO}(\mathbf{l}_a, \mathbf{l}_b) = K_{\rm BFKL}^{\rm NLO}(\mathbf{l}_a, \mathbf{l}_b) - \frac{1}{2} \int d^2\mathbf{l}\, K_{\rm BFKL}^{\rm LO}(\mathbf{l}_a, \mathbf{l})\, K_{\rm BFKL}^{\rm LO}(\mathbf{l}, \mathbf{l}_b)\, \ln \frac{\mathbf{l}^2}{\mathbf{l}_b^2}. \tag{17}$$

The NLO $Q\bar{Q}$ production vertex also gets two types of corrections, corresponding to the two different evolution chains originating from the hadrons A and B:

$$
\begin{aligned}
|\tilde{\Gamma}^{\rm NLO}_{\rm RR\to Q\bar{Q}}(\mathbf{q}_1,\mathbf{q}_2;\mathbf{k}_1,\mathbf{k}_2,z)|^2 \;=\; & |\Gamma^{\rm NLO}_{\rm RR\to Q\bar{Q}}(\mathbf{q}_1,\mathbf{q}_2;\mathbf{k}_1,\mathbf{k}_2,z)|^2 \\
& - \frac{\mathbf{q}_1^2}{2}\int\frac{d^2\mathbf{l}}{\mathbf{l}^2}\,K^{\rm LO}_{\rm BFKL}(\mathbf{q}_1,\mathbf{l})\,|\Gamma^{\rm LO}_{\rm RR\to Q\bar{Q}}(\mathbf{l},\mathbf{q}_2;\mathbf{k}_1,\mathbf{k}_2,z)|^2\,\ln\frac{\mathbf{l}^2}{(\mathbf{q}_2+\mathbf{l})^2} \\
& - \frac{\mathbf{q}_2^2}{2}\int\frac{d^2\mathbf{l}}{\mathbf{l}^2}\,|\Gamma^{\rm LO}_{\rm RR\to Q\bar{Q}}(\mathbf{q}_1,\mathbf{l};\mathbf{k}_1,\mathbf{k}_2,z)|^2\,K^{\rm LO}_{\rm BFKL}(\mathbf{l},\mathbf{q}_2)\,\ln\frac{\mathbf{l}^2}{(\mathbf{q}_1+\mathbf{l})^2}.
\end{aligned}
\tag{18}
$$

With these modifications, the NLO unintegrated gluon density is defined as follows

$$
g^{\rm NLO}(x,\mathbf{k}) \;=\; \int\frac{d^2\mathbf{q}}{2\pi}\frac{\tilde{\Phi}_P(\mathbf{q})}{\mathbf{q}^2}\tilde{f}(x,\mathbf{q},\mathbf{k}), \qquad \tilde{f}(x,\mathbf{q},\mathbf{k}) \;=\; \int_{\mathcal{C}}\frac{d\omega}{2\pi i}x^{-\omega}\tilde{f}_\omega(\mathbf{q},\mathbf{k}), \tag{19}
$$

where $\tilde{f}_\omega$ obeys the modified NLO BFKL equation

$$
\omega\tilde{f}_\omega(\mathbf{q}_a,\mathbf{q}_1) = \delta^{(2)}(\mathbf{q}_a-\mathbf{q}_1) + \int d^2\mathbf{q}\tilde{K}_{\rm BFKL}(\mathbf{q}_a,\mathbf{q})\tilde{f}_\omega(\mathbf{q},\mathbf{q}_1) \tag{20}
$$

with a NLO kernel which we will discuss in Section 3:

$$
\tilde{K}_{\rm BFKL}(\mathbf{q}_a,\mathbf{q}) = K^{\rm LO}_{\rm BFKL}(\mathbf{q}_a,\mathbf{q}) + \tilde{K}^{\rm NLO}_{\rm BFKL}(\mathbf{q}_a,\mathbf{q}). \tag{21}
$$

Using these definitions, the differential cross section in Eq. (7) at NLO accuracy is given by the expression

$$
\begin{aligned}
\frac{d^6\sigma}{d\eta_1 d\eta_2 d^2\mathbf{k}_1 d^2\mathbf{k}_2} =& \int_0^1 dx_1\int_0^1 dx_2\int\frac{d^2\mathbf{q}_1}{(2\pi)^3}\int\frac{d^2\mathbf{q}_2}{(2\pi)^3}\,g^{\rm NLO}(x_1,\mathbf{q}_1)\,g^{\rm NLO}(x_2,\mathbf{q}_2) \\
\times & \frac{|\Gamma_{\rm RR\to Q\bar{Q}}(\mathbf{q}_1,\mathbf{q}_2;\mathbf{k}_1,\mathbf{k}_2,z)|^2}{\mathbf{q}_1^2\mathbf{q}_2^2}\,\delta^{(2)}(\mathbf{q}_1+\mathbf{q}_2-\mathbf{k}_1-\mathbf{k}_2)\,\delta(x_1-\alpha_1-\alpha_2)\,\delta(x_2-\beta_1-\beta_2) \\
=& \int\frac{d^2\mathbf{q}_1}{(2\pi)^6}g^{\rm NLO}(\alpha_1+\alpha_2,\mathbf{q}_1)g^{\rm NLO}(\beta_1+\beta_2,\mathbf{k}_1+\mathbf{k}_2-\mathbf{q}_1)\frac{|\Gamma_{\rm RR\to Q\bar{Q}}(\mathbf{q}_1,\mathbf{k}_1+\mathbf{k}_2-\mathbf{q}_1;\mathbf{k}_1,\mathbf{k}_2,z)|^2}{\mathbf{q}_1^2(\mathbf{k}_1+\mathbf{k}_2-\mathbf{q}_1)^2}.
\end{aligned}
\tag{22}
$$

This expression can be interpreted as the convolution of the unintegrated gluon densities with a partonic differential cross section *i.e.*

$$
\frac{d^6\sigma}{d\eta_1 d\eta_2 d^2\mathbf{k}_1 d^2\mathbf{k}_2} = \int_0^1\frac{dx_1}{x_1}\int_0^1\frac{dx_2}{x_2}\int\frac{d^2\mathbf{q}_1}{2\pi^2}\int\frac{d^2\mathbf{q}_2}{2\pi^2}\,g^{\rm NLO}(x_1,\mathbf{q}_1)\,g^{\rm NLO}(x_2,\mathbf{q}_2)\,\frac{d^6\hat{\sigma}}{d\eta_1 d\eta_2 d^2\mathbf{k}_1 d^2\mathbf{k}_2} \tag{23}
$$

where

$$
d^6\hat{\sigma} \equiv \frac{1}{2\Sigma}|\mathcal{A}(q_1,q_2;k_1,k_2)|^2\frac{d\eta_1 d^2\mathbf{k}_1}{2(2\pi)^3}\frac{d\eta_2 d^2\mathbf{k}_2}{2(2\pi)^3}(2\pi)^4\delta^{(4)}(q_1+q_2-k_1-k_2). \tag{24}
$$

Here 2Σ is the flux factor and

$$
|\mathcal{A}(q_1,q_2;k_1,k_2)|^2 = \frac{\Sigma^2}{\mathbf{q}_1^2\mathbf{q}_2^2}|\Gamma_{\rm RR\to Q\bar{Q}}(\mathbf{q}_1,\mathbf{q}_2;\mathbf{k}_1,\mathbf{k}_2,z)|^2 \tag{25}
$$

is the squared matrix element for the production of a heavy $Q\bar{Q}$ pair from the fusion of two *transversely* polarised reggeised gluons. This means that their polarisations are chosen to satisfy

$$
\sum_\lambda \epsilon^\mu_{(\lambda)}(q_i)\epsilon^\nu_{(\lambda)}(q_i) \;=\; \frac{\mathbf{q}_i^\mu\mathbf{q}_i^\nu}{\mathbf{q}_i^2}, \qquad \text{with } i=1,2, \tag{26}
$$

and can be related up to the overall factor $\Sigma^2/\mathbf{q}_1^2\mathbf{q}_2^2$ in Eq. (25) to the usual longitudinally polarised reggeised gluons by means of a Ward-identity for the t-channel gluons.

At present the NLO corrections to the heavy quark production vertex $\Gamma_{\mathrm{RR}\to\mathrm{Q}\bar{\mathrm{Q}}}$ are not available and only the LO vertex is known [11]. This LO result can be written in the following form

$$|\Gamma^{\mathrm{LO}}_{\mathrm{RR}\to\mathrm{Q}\bar{\mathrm{Q}}}(\mathbf{q}_1,\mathbf{q}_2;\mathbf{k}_1,\mathbf{k}_2,z)|^2 = g^4\left(\frac{N_c}{2}A_1(\mathbf{q}_1,\mathbf{q}_2;\mathbf{k}_1,\mathbf{k}_2,z)+\frac{1}{2N_c}A_2(\mathbf{q}_1,\mathbf{q}_2;\mathbf{k}_1,\mathbf{k}_2,z)\right), \tag{27}$$

with

$$\begin{aligned}A_1(\mathbf{q}_1,\mathbf{q}_2;\mathbf{k}_1,\mathbf{k}_2,z) =& \frac{\mathbf{q}_1^2\mathbf{q}_2^2}{\hat{s}\,\Sigma}2\left(\frac{1}{\hat{t}-M^2}-\frac{1}{\hat{u}-M^2}\right)\left(\frac{(1-z)\mathbf{k}_1^2+M^2}{z}-\frac{z\mathbf{k}_2^2+M^2}{1-z}\right)\\ &-\left(\frac{[(\mathbf{q}_1-\mathbf{k}_1)^2+M^2][(\mathbf{q}_1-\mathbf{k}_2)^2+M^2]-(\mathbf{k}_1^2+M^2)(\mathbf{k}_2^2+M^2)}{(\hat{t}-M^2)(\hat{u}-M^2)}\right)^2\\ &+\left(\frac{(\mathbf{q}_1-\mathbf{k}_2)^2+M^2-\frac{z}{1-z}(\mathbf{k}_2^2+M^2)}{\hat{u}-M^2}+\frac{E(M^2)}{\hat{s}}\right)\\ &\qquad\times\left(\frac{(\mathbf{q}_1-\mathbf{k}_1)^2+M^2-\frac{1-z}{z}(\mathbf{k}_1^2+M^2)}{\hat{t}-M^2}-\frac{E(M^2)}{\hat{s}}\right),\end{aligned} \tag{28}$$

and

$$\begin{aligned}A_2(\mathbf{q}_1,\mathbf{q}_2;\mathbf{k}_1,\mathbf{k}_2,z) =& \left(\frac{[(\mathbf{q}_1-\mathbf{k}_1)^2+M^2][(\mathbf{q}_1-\mathbf{k}_2)^2+M^2]-(\mathbf{k}_1^2+M^2)(\mathbf{k}_2^2+M^2)}{(\hat{t}-M^2)(\hat{u}-M^2)}\right)^2\\ &-\frac{\mathbf{q}_1^2\mathbf{q}_2^2}{(\hat{t}-M^2)(\hat{u}-M^2)}.\end{aligned} \tag{29}$$

To write down Eqs. (28,29) we defined, apart from the variables introduced in Eqs. (8,11) and the partonic Mandelstam invariants

$$\hat{t} = (q_1-k_1)^2 = -\frac{1-z}{z}(\mathbf{k}_1^2+M^2)-(\mathbf{q}_1-\mathbf{k}_1)^2, \tag{30}$$

$$\hat{u} = (q_1-k_2)^2 = \frac{z}{1-z}(\mathbf{k}_2^2+M^2)-(\mathbf{q}_1-\mathbf{k}_2)^2, \tag{31}$$

the following set of transverse momenta

$$\mathbf{\Delta} = \mathbf{k}_1+\mathbf{k}_2, \qquad \mathbf{\Lambda} = (1-z)\mathbf{k}_1 - z\mathbf{k}_2, \tag{32}$$

which allow us to express Eq. (11) as

$$\Sigma = \hat{s}+\mathbf{\Delta}^2 = \frac{\mathbf{\Lambda}^2+M^2}{z(1-z)}+\mathbf{\Delta}^2. \tag{33}$$

Finally, we have also used

$$E(M^2) \equiv 2(2z-1)\mathbf{q}_1^2+2\mathbf{q}_1\cdot\mathbf{\Lambda}+\frac{1-2z}{z(1-z)}(\mathbf{\Lambda}^2+M^2)-\left[(2z-1)\mathbf{\Delta}^2+2\mathbf{\Lambda}\cdot\mathbf{\Delta}\right]\frac{\mathbf{q}_1^2}{\Sigma}. \tag{34}$$

The explicit form of the vertex in Eq. (27), keeping all the information on the outgoing $Q\bar{Q}$ system, will permit a comprehensive study of differential distributions in exclusive observables. For this we will also need to keep track of the multiple soft emission stemming from the gluon evolution. How to achieve this task is discussed in the following Section.

3 Multiparticle production and the unintegrated gluon density

The NLO unintegrated gluon densities in Eq. (19), which enter the differential cross section of Eq. (22), require both the NLO BFKL gluon Green function and the proton impact factor $\Phi_P(\mathbf{q})$. The latter is of non-perturbative origin, it can only be modelled and has to be extracted from the data. A possible simple choice for a model of the proton impact factor would be

$$\Phi_P(\mathbf{q}) \sim \left(\frac{\mathbf{q}^2}{\mathbf{q}^2+\Lambda^2}\right)^{\lambda}. \tag{35}$$

Here λ is a positive free parameter, while Λ is a momentum scale of the order of $\Lambda_{\rm QCD}$. A more sophisticated alternative to Eq. (35) has been presented in Ref. [22] where it was proposed to expand the proton impact factor over a set of orthogonal conformal invariant eigenfunctions.

The second building block of the unintegrated gluon densities is given by the NLO BFKL gluon Green function. An alternative formulation to the usual treatment in Mellin space was proposed in Ref. [23]. This form of solving the equation by iteration in momentum space has the advantage of dealing exactly with running coupling effects and incorporates the full azimuthal angle dependence of the soft multiparticle emission in multi-regge kinematics associated to the BFKL evolution. For a complete analysis of exclusive properties of multigluon final states associated to the production of a heavy $Q\bar{Q}$ pair we consider this to be the most convenient of the available methods of analysis of the BFKL Green function at NLO. By means of a phase space slicing parameter λ the virtual and real contributions are treated separately.

As it has been previously explained, the BFKL kernel receives in the case of NLO unintegrated gluon densities an additional contribution, Eq. (17), due to the choice of the energy scales $s_{0,i}$. This affects the real emission contribution to the kernel but not the gluon Regge trajectory, ω_λ, which in this physical regularisation can be written as

$$\omega_\lambda(\mathbf{q}) = -\xi(|\mathbf{q}|\lambda)\ln\frac{\mathbf{q}^2}{\lambda^2} + \bar{\alpha}_s^2\frac{3}{2}\zeta(3) \quad \text{with} \quad \xi(X) = \bar{\alpha}_s + \frac{\bar{\alpha}_s^2}{4}\left(\frac{4}{3}-\frac{\pi^2}{3}+\frac{5}{3}\frac{\beta_0}{N_c}-\frac{\beta_0}{N_c}\ln\frac{X}{\mu^2}\right), \tag{36}$$

where $\bar{\alpha}_s = \alpha_s(\mu)N_c/\pi$ and $\beta_0 = (11N_c - 2n_f)/3$. μ is the renormalisation scale in the $\overline{\rm MS}$ scheme. λ can be understood as an effective gluon mass or as a lower cut-off for the transverse momenta of the emitted gluons.

The NLO real emission kernel $\tilde{K}_\lambda^{\rm real}$, which accounts for the emission of gluons or massless quarks in quasi-multi-regge kinematics, is given by the following sum

$$\tilde{K}_\lambda^{\rm real}(\mathbf{l}_a,\mathbf{l}_a+\mathbf{l}) = \frac{1}{\pi\mathbf{l}^2}\xi(\mathbf{l}^2)\theta(\mathbf{l}^2-\lambda^2) + \hat{K}^{\rm real}(\mathbf{l}_a,\mathbf{l}_a+\mathbf{l}) + K_{\rm coll}(\mathbf{l}_a,\mathbf{l}_a+\mathbf{l}), \tag{37}$$

where [20]

$$
\begin{aligned}
\hat{K}^{\rm real}(\mathbf{l}_a,\mathbf{l}_b) =& \frac{\bar{\alpha}_s^2}{4\pi}\Bigg\{-\frac{1}{(\mathbf{l}_a-\mathbf{l}_b)^2}\ln^2\frac{\mathbf{l}_a^2}{\mathbf{l}_b^2}+\left(1+\frac{n_f}{N_c^3}\right)\left(\frac{3(\mathbf{l}_a\cdot\mathbf{l}_b)^2-2\mathbf{l}_a^2\mathbf{l}_b^2}{16\mathbf{l}_a^2\mathbf{l}_b^2}\right)\\
&\times\left[\frac{2}{\mathbf{l}_a^2}+\frac{2}{\mathbf{l}_b^2}+\left(\frac{1}{\mathbf{l}_b^2}-\frac{1}{\mathbf{l}_a^2}\right)\ln\frac{\mathbf{l}_a^2}{\mathbf{l}_b^2}\right]\\
&-\left[3+\left(1+\frac{n_f}{N_c^3}\right)\left(1-\frac{(\mathbf{l}_a^2+\mathbf{l}_b^2)^2}{8\mathbf{l}_a^2\mathbf{l}_b^2}-\frac{(2\mathbf{l}_a^2\mathbf{l}_b^2-3\mathbf{l}_a^4-3\mathbf{l}_b^4)}{16\mathbf{l}_a^4\mathbf{l}_b^4}(\mathbf{l}_a\cdot\mathbf{l}_b)^2\right)\right]\\
&\times\int_0^\infty dx\frac{1}{\mathbf{l}_a^2+x^2\mathbf{l}_b^2}\ln\left|\frac{1+x}{1-x}\right|\\
&+\frac{2(\mathbf{l}_a^2-\mathbf{l}_b^2)}{(\mathbf{l}_a-\mathbf{l}_b)^2(\mathbf{l}_a+\mathbf{l}_b)^2}\left[\frac{1}{2}\ln\frac{\mathbf{l}_a^2}{\mathbf{l}_b^2}\ln\frac{\mathbf{l}_a^2\mathbf{l}_b^2(\mathbf{l}_a-\mathbf{l}_b)^4}{(\mathbf{l}_a^2+\mathbf{l}_b^2)^4}+\left(\int_0^{-\mathbf{l}_a^2/\mathbf{l}_b^2}-\int_0^{-\mathbf{l}_b^2/\mathbf{l}_a^2}\right)dt\frac{\ln(1-t)}{t}\right]\\
&-\left(1-\frac{(\mathbf{l}_a^2-\mathbf{l}_b^2)^2}{(\mathbf{l}_a-\mathbf{l}_b)^2(\mathbf{l}_a+\mathbf{l}_b)^2}\right)\left[\left(\int_0^1-\int_1^\infty\right)dz\frac{1}{(\mathbf{l}_b-z\mathbf{l}_a)^2}\ln\frac{(z\mathbf{l}_a)^2}{\mathbf{l}_b^2}\right]\Bigg\} \qquad (38)
\end{aligned}
$$

and

$$
K_{\rm coll}(\mathbf{l}_a,\mathbf{l}_b) = -\frac{1}{2}\int d^2\mathbf{l}\,K_{\rm BFKL}^{\rm LO}(\mathbf{l}_a,\mathbf{l})\,K_{\rm BFKL}^{\rm LO}(\mathbf{l},\mathbf{l}_b)\,\ln\frac{\mathbf{l}^2}{\mathbf{l}_b^2}. \qquad (39)
$$

Following Ref. [23], this representation of the NLO BFKL kernel can be now used to solve iteratively the integral equation. The explicit solution for the gluon Green function then reads

$$
\begin{aligned}
f(x,\mathbf{q},\mathbf{k}) = x^{-\omega_\lambda(\mathbf{q})}\Bigg\{&\delta^{(2)}(\mathbf{q}-\mathbf{k})+\sum_{n=1}^{\infty}\prod_{i=1}^{n}\int d^2\mathbf{l}_i\left[\tilde{K}_\lambda^{\rm real}(\mathbf{q}+\sum_j^{i-1}\mathbf{l}_j,\mathbf{q}+\sum_j^{i}\mathbf{l}_j)\right.\\
&\left.\times\int_{x_{i-1}}^1\frac{dx_i}{x_i}x_i^{-\omega_\lambda(\mathbf{q}+\sum_{j=1}^{i}\mathbf{l}_j)+\omega_\lambda(\mathbf{q}+\sum_{j=1}^{i-1}\mathbf{l}_j)}\right]\delta^{(2)}(\mathbf{q}+\sum_{j=1}^{n}\mathbf{l}_j-\mathbf{k})\Bigg\}, \qquad (40)
\end{aligned}
$$

with $x_0 \equiv x$. Note that this representation can be now implemented in a Monte Carlo event generator where all the information about each of the emitted particles is recorded. At NLO each iteration of the kernel, or each of the terms in the sum of Eq. (40), corresponds to one or two emissions well separated in rapidity from previous and subsequent clusters of particles. Inserting this function in the formula for the differential distributions will generate our exclusive observables.

In the real emission kernel of Eq. (37), the two terms explicitly written in Eqs. (38,39) do not carry, apart from an overall $\bar{\alpha}_s^2(\mu^2)$ factor, any renormalisation scale dependence. This is different from the remaining part of the real emission kernel and the gluon trajectory which contain the function ξ of Eq. (36). This can be written as

$$
\xi(X) \;=\; \bar{\alpha}_s(\mu^2)\left(1-\frac{\bar{\alpha}_s(\mu^2)}{4N_c}\beta_0\ln\frac{X}{\mu^2}+\bar{\alpha}_s(\mu^2)S\right), \quad \text{with} \quad S \;=\; \frac{1}{12}\left(4-\pi^2+5\frac{\beta_0}{N_c}\right). \qquad (41)
$$

In this expression, the logarithmic term can be absorbed into a redefinition of the running of the coupling which corresponds to the replacement of $\bar{\alpha}_s(\mu^2)$ by $\bar{\alpha}_s(X)$. The remaining, non-logarithmic term, can be identified as a common factor which appears when dealing with resummations of soft gluons [24]. The term $\bar{\alpha}_s\,(1+\bar{\alpha}_s S)$ is proportional to the two-loop cusp anomalous dimension. Generally, the appearance of this term offers the possibility to change from the $\overline{\rm MS}$ renormalisation scheme to the *Gluon-Bremsstrahlung* (GB) scheme. Such a change corresponds to a shift of the Landau pole $\Lambda_{\rm GB} = \Lambda_{\overline{\rm MS}}\exp S\frac{2N_c}{\beta_0}$. Stability under this change of scheme offers a good tool to test our theoretical predictions.

4 Conclusions

In this letter we set the theoretical framework for a study of heavy flavor production in central regions of rapidity in hadron-hadron collisions using k_T factorisation at NLO. While the heavy flavor production vertex is kept at LO, the unintegrated gluon density is treated by taking into account the full NLO corrections. The latter contain both the full NLO BFKL evolution and the further corrections which arise due to the asymmetric choice of energy scales, inherent to hadronic cross-sections, and which can be understood as the onset of collinear evolution from the soft hadrons to the hard production vertex.

The NLO BFKL Green function which, convoluted with the (non-perturbative) proton impact factor, forms the NLO unintegrated gluon density has been presented in an iterative way which allows for a numerical evaluation using Monte-Carlo integration techniques. In future publications we will present results on this numerical implementation, together with fits of our unintegrated gluon density to deep inelastic data from HERA and predictions for heavy quark pair production at the Large Hadron Collider at CERN.

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