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A4-based tri-bimaximal mixing within inverse and linear seesaw schemes

M. Hirsch,[1,*] S. Morisi,[1,†] and J. W. F. Valle[1,‡]

[1] *AHEP Group, Institut de Física Corpuscular – C.S.I.C./Universitat de València*
Edificio Institutos de Paterna, Apt 22085, E–46071 Valencia, Spain

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We consider tri-bimaximal lepton mixing within low-scale seesaw schemes where light neutrino masses arise from TeV scale physics, potentially accessible at the Large Hadron Collider (LHC). Two examples are considered, based on the A_4 flavor symmetry realized within the inverse or the linear seesaw mechanisms. Both are highly predictive so that in both the light neutrino sector effectively depends only on three mass parameters and one Majorana phase, with no CP violation in neutrino oscillations. We find that the linear seesaw leads to a lower bound for neutrinoless double beta decay while the inverse seesaw does not. The models also lead to potentially sizeable decay rates for lepton flavor violating processes, tightly related by the assumed flavor symmetry.

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I. INTRODUCTION

Neutrino mass generation in the Standard Model is likely to come from a basic dimension-five operator that violates lepton number [1]. Little is known about the ultimate origin of this operator, including the nature of the underlying mechanism, its characteristic scale and/or flavor structure. Correspondingly, it has many possible realizations involving the exchange of scalar and/or fermions at the tree and/or radiative level [2].

In a broad class of models the exchange of heavy gauge singlet fermions induces neutrino masses via what is now called type-I seesaw [3, 4, 5, 6, 7]. An attractive mechanism called inverse seesaw has long been proposed as an alternative to the simplest type-I seesaw [8] (for other extended seesaw schemes see, e.g. [9, 10, 11]). In addition to the left-handed SM neutrinos ν in the inverse seesaw model ones introduces two $SU(3)\times SU(2)\times U(1)$ singlets ν^c, S. In the basis ν, ν^c, S the effective neutrino mass matrix is

$$M_\nu = \begin{pmatrix} 0 & M_D & 0 \\ M_D^T & 0 & M \\ 0 & M^T & 0 \end{pmatrix}, \qquad (1)$$

that can be simply justified by assuming a $U(1)_L$ global lepton number symmetry. Neutrinos get masses only when $U(1)_L$ is broken. The latter can be arranged to take place at a low scale, for example through the μSS mass term in the mass matrix given below,

$$M_\nu = \begin{pmatrix} 0 & M_D & 0 \\ M_D^T & 0 & M \\ 0 & M^T & \mu \end{pmatrix}, \qquad (2)$$

After $U(1)_L$ breaking the effective light neutrino mass matrix is given by

$$M_\nu = M_D {M^T}^{-1} \mu M^{-1} M_D^T. \qquad (3)$$

so that, when μ is small, M_ν is also small, even when M lies at the electroweak or TeV scale. In other words, the smallness of neutrino masses follows naturally since as $\mu \to 0$ the lepton number becomes a good symmetry [12] without need for superheavy physics.

The smallness of the parameter μ may also arise dynamically in sypersymmetric models and/or spontaneously in a Majoron-like scheme with $\mu \sim \langle\sigma\rangle$ where σ is a $SU(3)\times SU(2)\times U(1)$ singlet [13]. In the latter case, for sufficiently low values of $\langle\sigma\rangle$ there may be Majoron emission effects in neutrinoless double beta decay [14].

Recently another alternative seesaw scheme called linear seesaw has been suggested from $SO(10)$ [15]. Here we consider a simpler variant of this model based just on the framework of the $SU(3)\times SU(2)\times U(1)$ gauge structure.

*Electronic address: mahirsch@ific.uv.es
†Electronic address: morisi@ific.uv.es
‡Electronic address: valle@ific.uv.es

In the basis ν, ν^c, S the effective neutrino mass matrix is

$$M_\nu = \begin{pmatrix} 0 & M_D & M_L \\ M_D^T & 0 & M \\ M_L^T & M^T & 0 \end{pmatrix}. \quad (4)$$

Here the lepton number is broken by the $M_L\,\nu S$ term, and the effective light neutrino mass is given by

$$M_\nu = M_D(M_L M^{-1})^T + (M_L M^{-1}) M_D{}^T. \quad (5)$$

In addition to indications of non-vanishing neutrino mass, neutrino oscillation experiments [16, 17, 18, 19, 20] indicate a puzzling structure [21] of the elements of the lepton mixing matrix, at variance with the quark mixing angles.

In this paper we consider the possibility of predicting lepton mixing angles from first principles, in the framework of the inverse or linear seesaw mechanisms to generate light neutrino masses. An attractive phenomenological ansatz for leptons mixing [22] is the tribimaximal (TBM) one

$$U_{\mathrm{HPS}} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} \quad (6)$$

which is equivalent to the following values for the lepton mixing angles: $\tan^2\theta_{\mathrm{atm}} = 1$, $\sin^2\theta_{\mathrm{Chooz}} = 0$ and $\tan^2\theta_{\mathrm{sol}} = 0.5$, providing a good first approximation to the values indicated by current neutrino oscillation data.

Below we give two simple A_4 flavor symmetry realizations of the TBM lepton mixing pattern within the above seesaw schemes. For example, for the inverse seesaw case possible schemes are summarized in Table I.

cases	1)	2)	3)	4)	5)	6)	7)
M_D	M_0	I	I	I	M_0	M_0	M_0
M	I	M_0	I	M_0	I	M_0	M_0
μ	I	I	M_0	M_0	M_0	I	M_0

TABLE I: Possible TBM inverse seesaw schemes.

Recall that A_4 is the group of the even permutations of four objects. Such a symmetry was introduced to yield $\tan^2\theta_{\mathrm{atm}} = 1$ and $\sin^2\theta_{\mathrm{Chooz}} = 0$ [23, 24]. Most recently A_4 has also been used to derive $\tan^2\theta_{\mathrm{sol}} = 0.5$ [25]. The group A_4 has 12 elements and is isomorphic to the group of the symmetries of the tetrahedron, with four irreducible representations, three distinct singlets 1, $1'$ and $1''$ and one triplet 3. For their multiplications see for instance Ref. [25].

If the charged lepton matrix M_l is diagonalized on the left by the magic matrix U_ω

$$U_\omega = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}, \quad (7)$$

(with $\omega \equiv \exp i\pi/3$) we have tri-bimaximal mixing if the light neutrino mass matrix has the structure

$$M_0 = \begin{pmatrix} A & 0 & 0 \\ 0 & B & C \\ 0 & C & B \end{pmatrix}. \quad (8)$$

We note that M_0^{-1} has the same structure as M_0. This implies that, taking any one (or more) of the three M_D, M, μ matrices as having the M_0 structure, with the remaining ones proportional to the identity matrix I one obtains a light neutrino mass matrix of TBM-type, namely

$$\begin{pmatrix} x & y & y \\ y & x+z & y-z \\ y & y-z & x+z \end{pmatrix}, \quad (9)$$

leading to many potential ways to obtain the TBM mixing pattern within an inverse seesaw mechanism. In table I we list all possible tri-bimaximal schemes.

II. TRI-BIMAXIMAL INVERSE SEESAW

As illustrative example we consider the case with

$$M_D \propto M_0, \qquad M \propto I, \qquad \mu \propto I \quad (10)$$

Below we will give a flavor model for such a case. When we go to the basis where charged leptons are diagonal Eq. (7), we have $M_D \propto U_\omega M_0$, $M \propto I$, $\mu \propto I$.

The light neutrino mass matrix arises from the inverse seesaw relation in eq. (3) and we have

$$M_\nu \sim U_\omega M_0 M_0^T U_\omega^T \quad (11)$$

which is of TBM-type (9). For example, for the case of real M_0 we have only three mass parameters in the model, two of which are determined by neutrino oscillations [21] and the third is related to the overall scale of neutrino mass that can be probed in tritium and double beta decays. In the general case one can see that there is no CP violation in neutrino oscillations, so that only a Majorana phase survives. This is in sharp contrast with the generic form of the inverse seesaw, which has CP violation even in the massless neutrino limit [26].

The matter fields are assigned as in table II.

	L	l^c	ν^c	S	h	ξ,ϕ	$\xi'\phi'$
$SU_L(2)$	2	1	1	1	2	2	2
Z_3	ω	ω	1	1	1	ω^2	ω
A_4	3	3	3	3	1	1,3	1,3

TABLE II: Matter assignment for inverse seesaw model.

The renormalizable [1] Lagrangian invariant under the symmetry $A_4 \times Z_3$ is

$$\begin{aligned}\mathcal{L} = {Y_D}^k_{ij}\, L_i\, \nu^c_j(\phi_k + \xi) + M_{ij}\, \nu^c_i S_j + \\ \mu_{ij} S_i S_j + Y^k_{l_{ij}}\, L_i\, l^c_j(\phi'_k + \xi')\end{aligned} \tag{12}$$

where from A_4-contractions we have that the couplings are given in Eq. (10), $\mu = v_\mu I$, $M = v_M I$. However when ξ takes a vacuum expectation value (vev) and

$$\langle\phi\rangle \sim (1,0,0). \tag{13}$$

we have in general

$$M_D = \begin{pmatrix} a & 0 & 0 \\ 0 & a & b_1 \\ 0 & b_2 & a \end{pmatrix}. \tag{14}$$

In contrast to M_0 such a matrix is not symmetric. Here we assume the *ad hoc* relation $b_1 = b_2 = b$. Such a relation can be obtained in the context of an $SO(10)$ model or by assuming S_4 flavor symmetry instead of A_4 [2].

The light neutrino mass eigenvalues are

$$\{m_1, m_2, m_3\} = \frac{v_\mu}{v_M^2}\{(a+b)^2, a^2, -(a-b)^2\}. \tag{15}$$

When also ξ' takes a vev along

$$\langle\phi'\rangle \sim (1,1,1). \tag{16}$$

we have

$$M_l = \begin{pmatrix} \alpha & \beta & \gamma \\ \gamma & \alpha & \beta \\ \beta & \gamma & \alpha \end{pmatrix} = U_\omega \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\nu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} U_\omega^\dagger. \tag{17}$$

Therefore the charged lepton mass matrix is diagonalized on the left by the magic matrix U_ω as required.

We note that when the Higgs doublets ϕ and ϕ' take nonzero vevs, the A_4 symmetry breaks spontaneously into its two subgroups, namely Z_2 and Z_3, respectively. The consequence of such a *misalignment* is to have a large mixing in the neutrino sector. The problem how to get such a misalignment has been studied in many contexts [27].

III. TRI-BIMAXIMAL LINEAR SEESAW

We now consider the case of the linear seesaw, see eqs. (4) and (5). As for the inverse seesaw, there are different possible choices for M_D, M, M_L that can lead to the TBM structure. We take as example the case with

$$M_D \propto M_0, \qquad M \propto I, \qquad M_L \propto I. \tag{18}$$

When we go to the basis where charged leptons are diagonal (7), we have $M_D \propto U_\omega M_0$, $M \propto I$, $M_L \propto U_\omega$.

From eq. (5) the light neutrinos mass matrix is given as

$$M_\nu \sim U_\omega M_0 U_\omega^T + U_\omega M_0^T U_\omega^T. \tag{19}$$

[1] Here we have introduced several Higgs doublets. We can equivalently avoid having many Higgs doublets by introducing corresponding scalar electroweak singlet *flavon* fields.

[2] The reason is that in A_4 $3 \times 3 = 1 + 1' + 1'' + 3_S + 3_A$ and we must take also the antisymmetric contraction for Dirac mass terms. S_4 is the group of permutation of four objects and $3 \times 3 = 1 + 2 + 3_1 + 3_2$ where 3_1 and 3_2 are distinct irreducible representations.

We note that in contrast to the inverse seesaw, in the linear seesaw case the light neutrino mass matrix M_ν in eq. (19) is of TBM type also when M_0 is given by eq. (14) without any *ad hoc* symmetry assumption. Again, as before, we note that for the case of real M_0 there are only three mass parameters, two of which can be traded by the neutrino oscillation mass splittings [21], with the remaining one fixing the overall neutrino mass scale. Even in the presence of complex phases in M_0 there is no CP violation in neutrino oscillations, and only a Majorana phase remains (see below).

As an illustrative example we describe a model based on A_4 flavor symmetry, in table III.

	L	l^c	ν^c	S	h	ξ,ϕ	$\xi'\phi'$
$SU_L(2)$	2	1	1	1	2	2	2
Z_3	ω	1	ω	ω^2	1	ω	ω^2
A_4	3	3	3	3	1	1,3	1,3

TABLE III: Matter assignment for linear seesaw model.

The invariant Lagrangian is

$$\mathcal{L} = Y_{D}{}_{ij}^{k}\, L_i\, \nu_j^c(\phi_k+\xi) + M_{ij}\, \nu_i^c S_j + Y_{L_{ij}} L_i h\, S_j + Y_{l_{ij}}^{k} L_i\, l_j^c(\phi_k'+\xi') \tag{20}$$

where the couplings are given as in Eq. (18).

After the scalar fields take vevs obeying the alignment conditions given in eqs. (13) and (16), the resulting Yukawa couplings are given by (18) and therefore the light neutrino mass matrix is diagonalized by TBM in the basis where charged leptons are diagonal as explained above.

The light neutrino eigenvalues are given by

$$\{m_1,m_2,m_3\} = \frac{v_L}{v_M}\{(a+b),a,-(a-b)\}. \tag{21}$$

IV. PHENOMENOLOGY

Above we have introduced two very simple models based respectively on inverse and linear seesaw mechanisms. Due to the assumed flavor symmetry they are highly restrictive. By construction, the lepton mixing matrix in both models is predicted to be tribimaximal and neutrino phenomenology is effectively described by just three mass parameters and a phase. Two of them are the neutrino squared-mass splittings well-determined in neutrino oscillations. The other mass parameter characterizes the absolute neutrino mass scale which will be probled in tritium and neutrinoless double beta decay searches, as well as cosmology.

As we have noted already, there is no CP violation in neutrino oscillations, and only one Majorana phase remains and affects the predictions for neutrinoless double beta decay (see below).

Neutrinoless double beta decay

Despite their similarity, one can distinguish these models phenomenologically since eqs (3) and (5) give rise to different neutrino mass spectra and this implies different expectations for neutrinoless double beta decay, as illustrated in Fig. 1 (similar predictions have been made within A4-symmetric type-I seesaw models, as shown, for example in Ref. [28]).

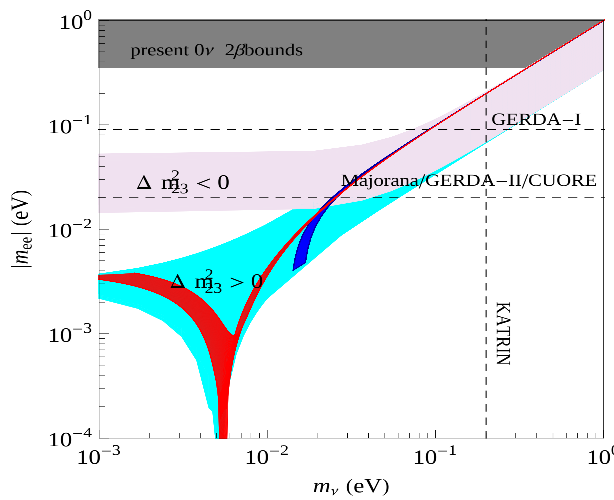

FIG. 1: Neutrinoless double beta decay parameter m_{ee} as a function of the lightest neutrino mass for inverse seesaw (red) and linear seesaw (blue). The cyan and purple bands represent respectively the generic regions allowed by current data with lepton mixing angles fixed to be the tri-bimaximal values. For references to experiments see [29].

One sees that, in contrast to the inverse seesaw, in the linear seesaw case there is a lower bound on the neutrinoless double beta decay rate despite the fact that we have a normal neutrino mass hierarchy. In contrast, the effect of the Majorana phase in the inverse seesaw can cause full cancellation in the decay rate.

Lepton flavor violating decays

In the inverse and linear seesaw models studied here, the neutrino mass matrix is a 9×9 symmetric matrix, see eqs (2) and (4). This is diagonalized by a corresponding unitary matrix $U_{\alpha\beta}$ of the same dimension, $\alpha,\ \beta = 1...9$, leading to three light Majorana eigenstates ν_i with $i = 1, 2, 3$ and six heavy ones N_j with $j = 4, .., 9$. The effective charged current weak interaction is characterized by a rectangular lepton mixing matrix $K_{i\alpha}$ [6].

$$\mathcal{L}_{CC} = \frac{g}{\sqrt{2}} K_{i\alpha} \overline{L}_i \gamma_\mu (1 + \gamma_5) N_\alpha \, W^\mu. \tag{22}$$

where $i = 1, 2, 3$ denote the left-handed charged leptons and α the neutrals. The contribution to the decay $l_i \to l_j \gamma$ arises at one loop (see for instance [30, 31]) from the exchanges of the six heavy right-handed Majorana neutrinos N_j which couple subdominantly to the charged leptons.

The well-known one loop contribution to this branching ratio is given by [32]

$$Br(l_i \to l_j \gamma) = \frac{\alpha^3 s_W^2}{256\pi^2} \frac{m_{l_i}^5}{M_W^4} \frac{1}{\Gamma_{l_i}} |G_{ij}|^2 \tag{23}$$

where

$$\begin{array}{l} G_{ij} = \sum_{k=4}^{9} K_{ik}^* K_{jk} G_\gamma \left(\frac{m_{N_k}^2}{M_W^2} \right) \\ G_\gamma(x) = -\frac{2x^3 + 5x^2 - x}{4(1 - x^3)} - \frac{3x^3}{2(1-x)^4} \ln x \end{array} \tag{24}$$

We note that, thanks to the admixture of the TeV states in the charged current weak interaction, this branching ratio can be sizeable even in the absence of supersymmetry [30]. Similar results hold for a class of LFV processes, including nuclear mu-e conversion [33]. As already noted, the rates for mu-e conversion and $\mu \to e\gamma$ are strongly correlated in this model. These are the most stringently constrained LFV decays.

The simplicity of their mass matrices, which are expressed in terms of very few parameters, makes the current models especially restrictive and this has an impact in the expected pattern of LFV decays. In contrast to the general case considered in [31, 33], here we can easily display the dependence of the $\mu \to e\gamma$ branching ratio on the new physics scale represented by the parameters $M \sim$ TeV and the parameters μ or v_L characterizing the low-scale violation of lepton number, since both are simply proportional to the identity matrix in flavor space. This is illustrated in Fig. 2.

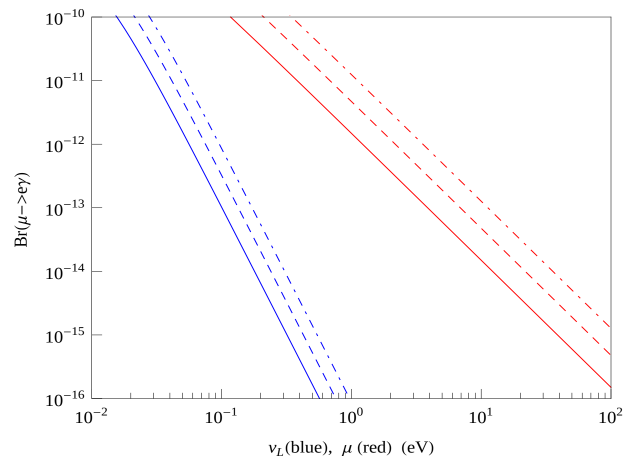

FIG. 2: $Br(\mu \to e\gamma)$ versus the lepton number violation scale: μ for the inverse seesaw (red color), and v_L for the linear seesaw (blue color). In both cases, M is fixed as $M = 100\, GeV$ (continous line), $M = 200\, GeV$ (dashed line) and $M = 1000\, GeV$ (dot-dashed line).

Note also that, in contrast to a generic inverse or linear seesaw model, in our A_4 based models the structure of the matrix G_{ij} is completely fixed, and this leads to predictions for ratios of LFV branching ratios. This can be seen easily as follows. Recall that we have only three mass parameters, two of which are determined by solar and atmospheric splittings, while the third is related to the overall scale of neutrino mass. The ratio

$$\alpha = \Delta m^2_{\rm sol} / \Delta m^2_{\rm atm}$$

is well determined by neutrino oscillation data [21].

For the inverse seesaw case we have from eq.(15)

$$\Delta m^2_{\rm atm} = \frac{v_\mu^2}{v_M^4} ((a-b)^4 - a^4) \tag{25}$$

$$\Delta m^2_{\rm sol} = \frac{v_\mu^2}{v_M^4} (a^4 - (a+b)^4) \tag{26}$$

then

$$\alpha = \frac{1 - (1-t)^4}{(1+t)^4 - 1} \tag{27}$$

where $t = -b/a$.

As mentioned, the main contributions to the LFV processes are those involving the heavy singlet neutrinos. Then the relevant elements of the lepton mixing matrix are $K_{ik} \sim M_D \cdot M^{-1}$, and as a result the G matrix of eq.

(24) is characterized by only two parameters,

$$G \sim U_\omega^T M_0^T M_0 U_\omega \tag{28}$$

and for inverse seesaw one finds:

$$G \sim \begin{pmatrix} a^2 + \frac{4ab}{3} + \frac{2b^2}{3} & -\frac{1}{3}b(2a+b) & -\frac{1}{3}b(2a+b) \\ -\frac{1}{3}b(2a+b) & \frac{1}{3}b(4a-b) & a^2 - \frac{2ab}{3} + \frac{2b^2}{3} \\ -\frac{1}{3}b(2a+b) & a^2 - \frac{2ab}{3} + \frac{2b^2}{3} & \frac{1}{3}b(4a-b) \end{pmatrix},$$

Taking ratios of branching ratios, prefactors cancel and one finds, for example for

$$\frac{Br(\tau \to \mu\gamma)}{Br(\tau \to e\gamma)} = \left(\frac{3+2t+2t^2}{2t-t^2}\right)^2, \tag{29}$$

where t is the solution of the eq. (27)

A similar procedure can be carried out for the linear seesaw, using eq.(21) for the light neutrino mass eigenvalues. One finds

$$\frac{Br(\tau \to \mu\gamma)}{Br(\tau \to e\gamma)} = \left(\frac{3+u}{u}\right)^2 , \tag{30}$$

where u is the solution of the equation

$$\frac{1-(1-u)^2}{(1+u)^2-1} = \alpha \ . \tag{31}$$

As a result of Eqs. (29) and (30) we obtain the predictions illustrated in Fig. 3. Note the different dependence on α.

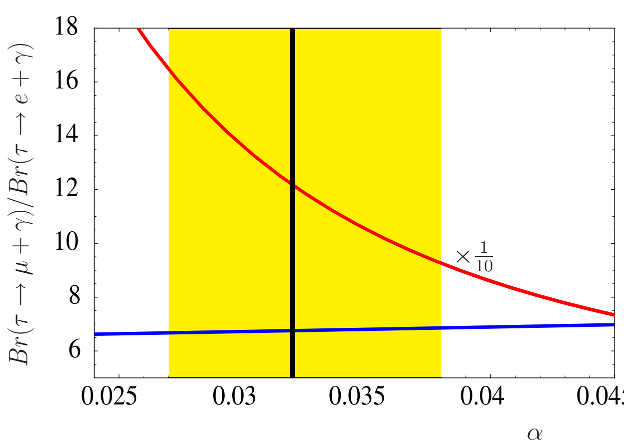

FIG. 3: $Br(\tau \to \mu\gamma)/Br(\tau \to e\gamma)$ vs α for inverse seesaw (red) and linear seesaw (blue). The vertical line indicates the best fit value for α, the band is the allowed 3σ C.L. range [21].

A basic symmetry property of the matrix G_{ij} is mu-tau symmetry, which implies that $G_{31} = G_{21}$, so that

$$\frac{Br(\tau \to e\gamma)}{Br(\mu \to e\gamma)} = \left(\frac{m_\tau}{m_\mu}\right)^5 \frac{\Gamma_\mu}{\Gamma_\tau} \approx 0.18, \tag{32}$$

for both linear and inverse seesaw. Given the current bounds on $\mu \to e\gamma$ we have $\mathrm{B}(\tau \to e\gamma) \lesssim 2\times10^{-12}$ placing a tremendous challenge for the search for lepton flavor violating tau decays for testing the prediction given in Fig. 3.

Before closing this section let us also mention that the TeV neutral heavy leptons are potentially accessible directly in accelerator experiments, see, for example, Ref. [34].

V. DISCUSSION

The inverse and linear seesaw mechanisms provide very interesting alternative scenarios to the type-I seesaw since the scale of the new fermions leading to neutrino mass can lie at the TeV scale, potentially accessible at the LHC. In this paper we have given two models based on the A_4 discrete flavor symmetry and realizing the successful tri-bimaximal ansatz for lepton mixing. We have introduced several Higgs doublets transforming as triplet and singlet representations of A_4. We have assumed that A_4 is spontaneously broken to Z_3 in the charged lepton sector and into Z_2 in the neutrino sector which yields the TBM lepton mixing pattern. Both models are highly predictive as they effectively depend only on three mass parameters and one Majorana phase, implying no CP violation in neutrino oscillations. In contrast to the inverse seesaw, the linear seesaw leads to a lower bound for neutrinoless double beta decay.

Among their other phenomenological features, the mixing of heavy neutrinos in the charged electroweak current leads to various lepton flavor violating decays such as $l_i \to l_j\gamma$ and $l_i \to l_j l_k l_k$. In contrast to standard type-I seesaw, here these rates can be sizeable even in the absence of supersymmetry. Moreover, the TBM mixing pattern leads to specific predictions for LFV decays as illustrated, for example, in Fig. 3. However, within our particular A_4 symmetry realizations, the TBM pattern also implies that $\mathrm{B}(\tau \to \mu\gamma) \lesssim 3\times10^{-10}$, well below current experimental sensitivities.

As a final comment, we have only described in this paper results that follow from exact symmetry realizations of the tri-bimaximal mixing pattern. It is possible, however, that the symmetry leading to TBM holds only at some high unification scale and deviations are induced. Possible radiative effects have been considered for example, in the framework of supergravity models in Ref. [35]. For example, in the presence of supersymmetry, broken by soft breaking terms that do not respect our flavor symmetry, one would have potentially important corrections that might enhance tau-violating processes with respect to the predictions presented here. Finally, let us also mention that, as already noted in [15] generic inverse and linear seesaw models may be embedded in an $SO(10)$ framework. The non-abelian flavor structure may be incorporated in these models in order to generate the TBM pattern discussed here, along the lines considered in Refs. [36] and [37]. These are issues that we hope to take up elsewhere.

Acknowledgments

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