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Reanalysis of pion pion phase shifts
from $K \to \pi\pi$ decays

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Abstract

We re-investigate the impact of isospin violation for extracting the s-wave $\pi\pi$ scattering phase shift difference $\delta_0(M_K) - \delta_2(M_K)$ from $K \to \pi\pi$ decays. Compared to our previous analysis in 2003, more precise experimental data and improved knowledge of low-energy constants are used. In addition, we employ a more robust data-driven method to obtain the phase shift difference $\delta_0(M_K) - \delta_2(M_K) = (52.5 \pm 0.8_{\text{exp}} \pm 2.8_{\text{theor}})°$.

1. If isospin were conserved the Fermi-Watson final-state interaction theorem would allow to extract the s-wave pion pion phase shift difference $\delta_0(M_K) - \delta_2(M_K)$ directly from $K \to \pi\pi$ decay rates. However, the $K \to \pi\pi$ amplitudes are sensitive to isospin violation including electromagnetic corrections, especially the $I = 2$ amplitude A_2. This is due to the large ratio $A_0/A_2 \sim 22$ (octet enhancement in nonleptonic weak decays) that enhances isospin-violating corrections to A_2.

It has been a long-standing problem to reconcile the phase shift difference extracted from $K \to \pi\pi$ decays with other determinations of pion pion phase shifts. This problem has become especially acute after the precise determination of $\pi\pi$ phase shifts from combining dispersion theory with chiral perturbation theory [1]. Our previous analysis of isospin violation in $K \to \pi\pi$ decays [2] (similar results were obtained in Ref. [3]) led to $\delta_0(M_K) - \delta_2(M_K) = (60.8 \pm 2.2_{\,\mathrm{exp}} \pm 3.1_{\,\mathrm{theor}})^\circ$, substantially bigger than the dispersion theoretical result [1] $\delta_0(M_K) - \delta_2(M_K) = (47.7 \pm 1.5)^\circ$.

We have decided to reanalyse the problem for several reasons.

- The experimental situation has substantially improved since 2003 for both the K^+ and K_S lifetimes and for the branching ratios of $K \to \pi\pi$ decays [4–6]. It was already observed in Ref. [7] that the new experimental information reduces the phase shift difference by more than three degrees (with higher statistical significance), bringing it closer to the dispersion theoretical value.

- New information has also become available on some of the low-energy constants (LECs) involved, both in the strong [8] and in the electromagnetic sector [9]. The effect on the phase shift difference is difficult to quantify but it is definitely smaller than the uncertainty assigned to the LECs in Ref. [2]. As one example, the new estimates of electromagnetic LECs [9] lead to a shift of the lowest-order $\pi^0 - \eta$ mixing parameter $\varepsilon^{(2)}$ from $1.06 \cdot 10^{-2}$ to $1.29 \cdot 10^{-2}$ [10]. This shift reduces the phase shift difference by 0.2°.

- As will be detailed below, the theoretical analysis of Ref. [2] can also be improved by relying to a lesser extent on the NLO calculation of $K \to \pi\pi$ decay amplitudes. Using less information potentially increases the uncertainty but this will be compensated by a less biased comparison with the data. The resulting estimate of isospin violation is expected to be more robust than the original estimate and it leads to a further decrease of the phase shift difference by nearly two degrees.

- Based on a method proposed in Ref. [11] making use of the optical theorem, effects of $O(e^2 p^4)$ were partially accounted for in Ref. [2]. Since a complete calculation of such effects is beyond present capabilities, we have decided to keep track of the associated theoretical uncertainty, but without including the partial corrections in the mean value for the phase shift difference.

2. We start by recalling the procedure of Ref. [2] for extracting the phase shift difference. The amplitudes $A_{+-} = A(K^0 \to \pi^+\pi^-)$, $A_{00} = A(K^0 \to \pi^0\pi^0)$ and $A_{+0} = A(K^+ \to \pi^+\pi^0)$ are parametrized as

$$
\begin{aligned}
A_{+-} &= A_0\, e^{i\chi_0} + \frac{1}{\sqrt{2}}\, A_2\, e^{i\chi_2} \\
A_{00} &= A_0\, e^{i\chi_0} - \sqrt{2}\, A_2\, e^{i\chi_2} \\
A_{+0} &= \frac{3}{2}\, A_2^+\, e^{i\chi_2^+}\ .
\end{aligned}
\tag{1}
$$

In the absence of CP violation, the amplitudes A_0, A_2, A_2^+ are real and positive by definition. In the isospin limit, $A_2 = A_2^+$ and the phases χ_I coincide with the strong $\pi\pi$ phase shifts δ_I at the kaon mass.

To NLO in the chiral expansion, the phases χ_0, χ_2 cannot be calculated reliably: the resulting phases are substantially too small. To obtain the strong phase shift difference $\delta_0(M_K) - \delta_2(M_K)$, a two-step procedure was employed [2]. Using the NLO expressions for the absolute values A_0, A_2, A_2^+, the lowest-order couplings g_8, g_{27} were determined from the experimental rates together with the phase shift difference $\chi_0 - \chi_2$. Comparing the NLO amplitudes with and without including isospin violation, the differences

$$
\gamma_I = \chi_I - \delta_I(M_K) \qquad (I = 0, 2)
\tag{2}
$$

were calculated to obtain the final phase shift difference $\delta_0(M_K) - \delta_2(M_K)$ in the isospin limit.

There are two related potential pitfalls associated with this procedure. Although the theoretical NLO expressions for the phases χ_I cannot be trusted they enter the dispersive and absorptive parts of the amplitudes A_0, A_2, A_2^+ implicitly. Moreover, although both the χ_I and the δ_I cannot be calculated reliably to NLO in the chiral expansion the differences γ_I were assumed to be trustworthy. Both steps are therefore subject to a theoretical bias that is difficult to control at the level of accuracy considered.

The main idea of the alternative procedure proposed here is to use only the isospin-violating parts of the NLO amplitudes as theory input and to determine $\delta_0(M_K) - \delta_2(M_K)$ directly from the data. In contrast to the chiral corrections for the full amplitudes, the isospin-violating corrections are much smaller and therefore less subject to the bias discussed in the previous paragraph.

The amplitudes are now parametrized as

$$
\begin{aligned}
A_{+-} &= \overline{A}_0\, e^{i\delta_0(M_K)} + \frac{1}{\sqrt{2}}\,\overline{A}_2\, e^{i\delta_2(M_K)} + \Delta A_{+-}^{\mathrm{IB}} \\
A_{00} &= \overline{A}_0\, e^{i\delta_0(M_K)} - \sqrt{2}\,\overline{A}_2\, e^{i\delta_2(M_K)} + \Delta A_{00}^{\mathrm{IB}} \\
A_{+0} &= \frac{3}{2}\,\overline{A}_2\, e^{i\delta_2(M_K)} + \Delta A_{+0}^{\mathrm{IB}}\ .
\end{aligned}
\tag{3}
$$

All isospin violation is contained in the amplitudes $\Delta A_{+-}^{\mathrm{IB}}, \Delta A_{00}^{\mathrm{IB}}, \Delta A_{+0}^{\mathrm{IB}}$. They can be extracted from the NLO amplitudes of Ref. [2]. Since isospin violation was neglected in

3

the 27-plet amplitudes because of the $\Delta I = 1/2$ rule [2, 3] the amplitudes ΔA_n^{IB} ($n = +-, 00, +0$) scale linearly with the lowest-order octet coupling g_8. They also depend on higher-order LECs in addition to the loop contributions.

The moduli of the amplitudes in the isospin limit[1] are denoted as $\overline{A}_0, \overline{A}_2$. We will not use the theoretical expressions for these amplitudes but instead determine them together with the phase shift difference $\delta_0(M_K) - \delta_2(M_K)$ directly from the rates. For this purpose, we write the moduli of the amplitudes (3) as

$$
\begin{aligned}
|A_{+-}| &= \left| \overline{A}_0 + \frac{1}{\sqrt{2}} \overline{A}_2 \, e^{i(\delta_2(M_K)-\delta_0(M_K))} + \Delta A_{+-}^{\mathrm{IB}} \, e^{-i\delta_0(M_K)} \right| \\
|A_{00}| &= \left| \overline{A}_0 - \sqrt{2}\, \overline{A}_2 \, e^{i(\delta_2(M_K)-\delta_0(M_K))} + \Delta A_{00}^{\mathrm{IB}} \, e^{-i\delta_0(M_K)} \right| \\
|A_{+0}| &= \left| \frac{3}{2} \overline{A}_2 \, e^{i(\delta_2(M_K)-\delta_0(M_K))} + \Delta A_{+0}^{\mathrm{IB}} \, e^{-i\delta_0(M_K)} \right| .
\end{aligned}
\tag{4}
$$

In order to determine $\overline{A}_0, \overline{A}_2$ and $\delta_0(M_K) - \delta_2(M_K)$ from the three rates, we therefore also need the $I = 0$ phase $\delta_0(M_K)$ as input. We use the value obtained in Ref. [1]:

$$
\delta_0(M_K) = (39.2 \pm 1.5)^\circ .
\tag{5}
$$

From the structure of the moduli (4) one may already anticipate that the precise value of $\delta_0(M_K)$ will have little impact on the phase shift difference.

3. We use the same experimental input as in Ref. [7], which is reproduced in Table 1.

Parameter	Value	Correlation	Reference
$\mathrm{BR}(K_S \to \pi^+\pi^-(\gamma))$	0.69196(51)	-0.9996	[4]
$\mathrm{BR}(K_S \to \pi^0\pi^0)$	0.30687(51)		
τ_S	0.08958(5) ns		[5]
$\mathrm{BR}(K^+ \to \pi^+\pi^0)$	0.2064(8)	-0.032	[6]
τ_+	12.384(19) ns		

Table 1: Experimental input taken from Ref. [7].

In addition to the experimental input, we need the isospin-violating amplitudes ΔA_n^{IB}. With the central values of the various LECs and displaying explicitly the linear dependence on the octet coupling g_8, we find

$$
\begin{aligned}
\Delta A_{+-}^{\mathrm{IB}} &= g_8 \, [2.25 - 0.83 \, i] \cdot 10^{-10} \ \ \mathrm{GeV} \\
\Delta A_{00}^{\mathrm{IB}} &= g_8 \, [-0.58 - 2.77 \, i] \cdot 10^{-10} \ \ \mathrm{GeV} \\
\Delta A_{+0}^{\mathrm{IB}} &= g_8 \, [-2.12 - 1.10 \, i] \cdot 10^{-10} \ \ \mathrm{GeV} .
\end{aligned}
\tag{6}
$$

---

[1]We define the isospin limit in terms of the neutral meson masses [2].

Updating the fit in Ref. [2] with the new data, we get a mean value $g_8 = 3.6$ (see also Ref. [7]). We are going to assign a 20% uncertainty to g_8, much bigger than the fit error.

Using the isospin-violating amplitudes (6) with $g_8 = 3.6$ and $\delta_0(M_K) = 39.2°$ [1], the experimental rates in Table 1 give rise to

$$
\begin{aligned}
\overline{A}_0 &= 2.7030(8) \cdot 10^{-7} \ \text{GeV} \\
\overline{A}_2 &= 0.1249(3) \cdot 10^{-7} \ \text{GeV} \\
\delta_0(M_K) - \delta_2(M_K) &= (52.54 \pm 0.83)° \ .
\end{aligned}
\tag{7}
$$

The errors are purely experimental but they take the correlations in Table 1 into account. The resulting correlations for the fitted quantities are small, at most -16% for the correlation between $\overline{A}_2$ and $\delta_0(M_K) - \delta_2(M_K)$.

The octet enhancement in $K \to \pi\pi$ decays is characterized by the amplitude ratio

$$
\frac{\overline{A}_0}{\overline{A}_2} = 21.63(4) \ ,
\tag{8}
$$

again with experimental error only.

4. For assessing the theoretical uncertainties, we concentrate on the phase shift difference.

The uncertainty induced by the error of $\delta_0(M_K)$ in (5) can be disposed of quickly. Varying the $I = 0$ phase shift as $\delta_0(M_K) = (39.2 \pm 3.0)°$ $(2\,\sigma)$ affects the phase shift difference only in the second decimal place for $\delta_0(M_K) - \delta_2(M_K)$. Of course, this has to do with the smallness of the isospin-violating amplitudes (6). The uncertainty in the phase shift difference is therefore completely negligible.

The overall scale of the isospin-violating amplitudes (6) is determined by the lowest-order coupling g_8. We assign a generous error of 20%, i.e., $g_8 = 3.6 \pm 0.8$. As already emphasized, this uncertainty is much larger than the fit error [2, 7] but it is meant to account also at least partially for effects of $O(e^2 p^4)$ (see below). Varying g_8 by 20% gives rise to an uncertainty $\pm 1.1°$ for the phase difference.

As in Ref. [2], we estimate the uncertainty associated with the various LECs by varying both the short-distance scale for the Wilson coefficients (0.77 GeV $\leq \mu_{\text{SD}} \leq$ 1.3 GeV) and the chiral renormalization scale (0.6 GeV $\leq \nu_\chi \leq$ 1 GeV). The central values in (7) correspond to $\mu_{\text{SD}} = 1$ GeV, $\nu_\chi = 0.77$ GeV. As already mentioned, the changes in the LECs from 2003 till today are well within the range expected from varying the two scales. It turns out that the error associated with the short-distance scale is asymmetrical: the phase difference happens to be minimal for $\mu_{\text{SD}} = 1$ GeV and the increase from varying μ_{SD} is at most 0.5°. From the dependence on the chiral renormalization scale, the error for the phase difference is bigger and nearly symmetrical around the central value for $\nu_\chi = 0.77$ GeV: the phase difference varies by $\pm 1.2°$.

Finally, we consider higher-order effects of $O(e^2 p^4)$. In Ref. [2] we made use of the optical theorem and $\pi\pi$ scattering amplitudes to $O(e^2 p^2)$ [12,13] to estimate the absorptive part of

the $K \to \pi\pi$ amplitudes to $O(e^2 p^4)$. Based on that we quantified the $O(e^2 p^4)$ corrections to γ_2 (and to $\delta_0(M_K) - \delta_2(M_K)$) to be $+2.6°$ and included them in our final estimate. However, since a complete calculation of $K \to \pi\pi$ to $O(e^2 p^4)$ is not available, here we assume a more prudent attitude of not including this correction in the central value. We rather take this result as a first measure of the size of higher-order corrections. We have also reached a similar conclusion on the size of these effects with no reference to the optical theorem analysis. Within the method introduced in this letter, we have parametrized higher-order corrections via three different scale factors multiplying the isospin-breaking amplitudes $\Delta A^{\mathrm{IB}}_{+-,00,+0}$. We have repeated the fit for different choices of the scale factors ranging independently between 0.5 and 1.5, finding that the output $\delta_0(M_K) - \delta_2(M_K)$ changes by at most $2.2°$, which will be used as our final estimate of higher-order effects.

Altogether, our new procedure for confronting theory with experiment leads to the following final value for the phase shift difference in the isospin limit:

$$
\begin{aligned}
\delta_0(M_K) - \delta_2(M_K) &= (52.5 \pm 0.8_{\mathrm{exp}} \pm 1.1_{g_8}{}^{+0.5}_{-0.0}{}_{\mathrm{SD}} \pm 1.2_\chi \pm 2.2_{O(e^2 p^4)})° \\
&= (52.5 \pm 0.8_{\mathrm{exp}} \pm 2.8_{\mathrm{theor}})° \ .
\end{aligned}
\tag{9}
$$

5. If isospin violation is neglected (except in the physical pseudoscalar masses for phase space), i.e. taking $\Delta A^{\mathrm{IB}}_n = 0$, the fit to the experimental rates gives

$$
[\delta_0(M_K) - \delta_2(M_K)]_{\mathrm{Isospin}} = (47.3 \pm 1.0)°
\tag{10}
$$

and

$$
\left[\frac{\overline{A}_0}{\overline{A}_2}\right]_{\mathrm{Isospin}} = 22.41(5) \ .
\tag{11}
$$

The substantial experimental improvements achieved with the most recent kaon data have reduced the phase shift difference from the value $(48.6 \pm 2.6)°$ obtained in 2003 in the isospin limit [2] (and about $58°$ some 30 years ago [5]). The value (10) would be in perfect agreement with determinations from $\pi\pi$ scattering data:

$$
[\delta_0(M_K) - \delta_2(M_K)]_{\pi\pi} =
\begin{cases}
(47.7 \pm 1.5)° & [1] \\
(50.9 \pm 1.2)° & [14]
\end{cases}
\tag{12}
$$

However, owing to the large A_0/A_2 ratio in $K \to 2\pi$ decays, isospin-breaking corrections to the dominant $\Delta I = 1/2$ amplitude generate sizeable contributions to A_2 [compare the results (8) and (11)], modifying also the amplitude phases. Our 2003 analysis of isospin breaking in $K \to 2\pi$ decays concluded that these effects increase the phase shift difference significantly. Including some $O(e^2 p^4)$ corrections through the optical theorem, we found the result $\delta_0(M_K) - \delta_2(M_K) = (60.8 \pm 2.2_{\mathrm{exp}} \pm 3.1_{\mathrm{theor}})°$ [2]. The large difference with the $\pi\pi$ determinations (12) has been a pending puzzle since then.

In this letter we have reanalysed the $K \to 2\pi$ phase shift determination, taking advantage of the improved experimental situation. Moreover, we have modified the theoretical

analysis in order to be less sensitive to theoretical uncertainties. Since the absorptive contributions that generate the strong amplitude phases start to appear at the one-loop level, a NLO theoretical calculation of the amplitudes only provides the phase shifts at leading order, which are therefore subject to large uncertainties. To minimize theoretical errors, we have only used as theory input the calculation of the isospin-breaking contributions ΔA_n^{IB}. In this way, we can determine all other quantities (the phase shift difference and the amplitudes in the isospin limit) directly from a fit to the data. The residual theoretical uncertainties associated with the isospin-breaking contributions have been estimated conservatively in two different ways, as explained in the previous section. Our final result in Eq. (9) is lower than our 2003 determination. Both the new data (as already observed in Ref. [7]) and the different treatment of theory input tend to lower the resulting value: 3.3° from experiment and altogether 5° from theory. This updated determination of the phase shift difference from $K \to 2\pi$ decays turns out to be in agreement with the $\pi\pi$ results in (12), although with a larger uncertainty.

Acknowledgements

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