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SPATIAL AND SPATIO-TEMPORAL ENGLE-GRANGER REPRESENTATIONS, NETWORKS AND COMMON CORRELATED EFFECTS

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Spatial and Spatio-temporal Engle-Granger representations, Networks and Common Correlated Effects

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Abstract

We provide a way of representing spatial and temporal equilibria in terms of a Engle-Granger representation theorem in a panel setting. We use the mean group, common correlated effects estimator plus multiple testing to provide a set of weakly cross correlated correlations that we treat as spatial weights. We apply this model to the 324 local authorities of England, and show that our approach successfully mops up weak cross section correlations as well as strong cross sectional correlations. JEL Codes: C21, C22, C23, R3. Keywords: Spatio-temporal Engle-Granger Theorems; cross sectional dependence.

1 Introduction

The (temporal) Engle-Granger representation theorem provides clarity on the role of long run equilibrium, partial adjustment to disequilibrium and short-run dynamics. Here, we develop Engle-Granger representations, first for a spatial or network framework, and then for a spatio-temporal framework, to study short and long-run dynamics of spatial or network panel data over time.

Both representations offer simple interpretation as error correction models analogous to the temporal case. In pure cross-section, there is potential partial adjustment to a spatial equilibrium. Within a spatio-temporal setting, and under some simplifying assumptions, there is partial adjustment to two equilibrium relationships, one in the time dimension and one in the cross-sectional dimension either in space or within a network.[1] This simplification relies on two homo-

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[1]Hereafter when we refer to the spatial dimension we mean a network as well. Networks can be spatial but there are many networks between households, peers and firms that are not spatial in a geographical sense. However, the network architecture itself can be viewed as connections in an abstract spatial domain.

geneity assumptions. The first is the familiar pooled mean group assumption (Shin et al., 1999) of a homogeneous temporal equilibrium relationship across all the panel units, and the second is an analogous cross-section equilibrium homogeneity assumption over time.

Furthermore, our work highlights how the spatial equilibrium can be modelled using, for example, common correlated effects (Pesaran, 2006) which incorporates strong cross-section dependence. Much of the existing literature treats strong dependence, modelled using common correlated effects (CCE) or cross-section averages, as nuisance parameters. Our spatio-temporal ECM shows how these strong dependence effects can be structurally interpreted.

Once strong dependence is adequately modelled, weak dependence rests within the short run dynamics, for which the current literature provides several estimates of spatial weights matrices. In general, the spatial weights matrix is not fully identified (Bhattacharjee and Jensen-Butler, 2013), but it can be estimated under alternate identifying assumptions (Bhattacharjee and Jensen-Butler, 2013; Bhattacharjee and Holly, 2013; Bailey, Holly and Pesaran, 2016). Applying our spatio-temporal model to house prices at the local authority level in England, we find evidence of temporal cointegration and spatial cointegration, as well as substantial short run dynamics which we model by multiple testing on cross-section correlations (Bailey, Holly and Pesaran, 2016).

The mathematical notation for the remainder is as follows: lowercase letters refer to scalars ($y_{i,t}$), bold lowercase ($\mathbf{y}_t$) to vectors and bold upper case ($\mathbf{W}$) to matrices. We denote temporal first difference by Δ and spatial difference by $\underline{\Delta} = \mathbf{I} - \mathbf{W}$, where $\mathbf{I}$ is the identity matrix and $\mathbf{W}$ is a spatial (or network) weights matrix. Then, $\mathbf{Wy}$ denotes the spatial lag of $\mathbf{y}$. The remainder of the paper is organised as follows. In Section 2, we develop a spatio-temporal Engle-Granger representation and the corresponding error correction model, and then take this to estimation in a spatio-temporal setting in Section 3. We develop an application to UK house prices in Section 4 and Section 5 concludes.

2 Spatio-Temporal Engle-Granger representations and ECM

In this section we describe a spatio-temporal Engle-Granger representation and error correction model. We will first define the spatial, then the temporal equilibrium and finally the combination of both. We start with a model with one lag of the dependent variable and a contemporaneous and lag of the explanatory variable. In addition to the above temporal lags, the model includes a spatial lag of the dependent and independent variables:

$$y_{i,t} = \beta_{i,0} + \beta_{i,1}x_{i,t} + \beta_{i,2}x_{i,t-1} + \alpha_i y_{i,t-1} + \pi_i \sum_{j=1,i\neq j}^{N} w_{ij}x_{j,t} + \rho_i \sum_{j=1,i\neq j}^{N} w_{ij}y_{j,t} + e_{i,t} \tag{1}$$

$$e_{i,t} = \gamma_i' f_t + \epsilon_{i,t} \tag{2}$$

for $i = 1, ..., N$ and $t = 1, ..., T$. The error component $e_{i,t}$ contains common factors f_t and their loadings γ_i which introduces potential strong cross-sectional dependence and temporal nonstationarity.[2] The idiosyncratic random component $\epsilon_{i,t}$ is an error term with finitely summable autocovariances.

This is a first order spatio-temporal autoregressive distributed lag model. All coefficients are assumed to be heterogeneous across cross-sectional units. Similar to the temporal lags $x_{i,t-1}$ and $y_{i,t-1}$, $\sum_{j=1,j\neq i}^{N} w_{ij}y_{j,t}$ and $\sum_{j=1,j\neq i}^{N} w_{ij}x_{j,t}$ are the spatial lags of $\mathbf{y}$ and $\mathbf{x}$ based on a spatial weights matrix $\mathbf{W}$. For the moment, we are not explicit about the spatial weights, and since they can represent both spatial weak and strong dependence, we do not require this to satisfy the spatial granularity condition of Pesaran (2006).

Both the variables $\mathbf{x}$ and $\mathbf{y}$ are potentially cointegrated across time and space. Therefore an error correction model which takes the time and the spatial or network dimension into account can be used to represent the short and long run relationships. For convenience we re-write the model with the spatial interactions in matrix form:

$$\mathbf{y}_t = (y_{1,t}, ..., y_{N,t})' \quad ; \quad \mathbf{x}_t = (x_{1,t}, ..., x_{N,t})' \tag{3}$$

$$\mathbf{w}_i = (w_{i,1}, ..., w_{i,N}) \tag{4}$$

with

$$w_{i,i} = 0, \text{ for } i = 1, ..., N \tag{5}$$

and we assume that:

$$\sum_{j=1}^{N} w_{i,j} = 1 \tag{6}$$

The above assumption implies that the spatial weights matrix $\mathbf{W}$ is row-normalised. Together with fixed spatial weights inherent in (1), this assumption ensures that the spatial weights matrix $\mathbf{W} = ((w_{i,j}))_{N\times N}$ has bounded row and column norms as $N \to \infty$. This is analogous to weak cross-sectional dependence. Note that we allow for potentially negative spatial weights, which is important

[2]Without loss of generality we assume only a single common factor. The model can be extended to multiple factors, see for example Pesaran (2006); Chudik and Pesaran (2015).

in many application contexts (Bhattacharjee and Holly, 2013; Bailey, Holly and Pesaran, 2016). An analogous relationship holds for $\mathbf{x}$. Then the model in (1) can be written as:

$$y_{i,t} = \beta_{i,0} + \beta_{i,1}x_{i,t} + \beta_{i,2}x_{i,t-1} + \alpha_i y_{i,t-1} + \pi_i \mathbf{w}_i \mathbf{x}_t + \rho_i \mathbf{w}_i \mathbf{y}_t + e_{i,t}. \tag{7}$$

In the spatial equilibrium all cross-sectional units (suitably normalized) have the same values for each successive time period:

$$\begin{aligned} x_{i,t} &= x_{j,t} = x_t^* \\ y_{i,t} &= y_{j,t} = y_t^* \end{aligned} \tag{8}$$

and since $\sum_{j=1,i\neq j}^{N} w_{ij} = 1$, then

$$\begin{aligned} \sum_{j=1,i\neq j}^{N} w_{ij}x_t^* &= x_t^* \\ \sum_{j=1,i\neq j}^{N} w_{ij}y_t^* &= y_t^*. \end{aligned} \tag{9}$$

Two important comments are in order. First, in the above, for simplicity of exposition, and without loss of generality, we made the assumption that the spatial long run relationships are:

$$\begin{aligned} \sum_{i=1}^{N} x_{i,t} &= c_x \\ \sum_{i=1}^{N} y_{i,t} &= c_y, \end{aligned}$$

which implies that the long run weights matrix is

$$\mathbf{W} = \frac{1}{N}11' - \mathbf{I},$$

where $\mathbf{1}$ is a $T \times 1$ vector of ones. This is consistent with the use of cross-section averages to model strong dependence (Pesaran, 2006). This simplification is without loss of generality, because if the long run $\mathbf{W}$ has some other form, one can scale $\mathbf{y}$ and $\mathbf{x}$ to set the value at each location equal to its spatial lag. One can also model spatial strong dependence using other methods, for example statistical factors, principal components or interactive fixed effects (Bai and Ng, 2007; Bai, 2009). However, following Pesaran (2006), cross-section averages are adequate in large (N, T) settings.

Second, the discussion also highlights that there can be a distinction between the long run and short run weights matrices. We use cross-section averages to represent the long run relationships for simplicity of exposition, retaining the notation $\mathbf{W}$ for short run dynamics. However, in our application, we fully exploit the flexibility of specifying weights matrices for the spatial short and long run dynamics.

Coming back to the derivation of the spatial error correction model, substituting the equilibrium values from (9) into equation (7) yields:

$$\begin{aligned} y_t^* =& \beta_{i,0} + \beta_{i,1} x_t^* + \beta_{i,2} x_{t-1}^* + \alpha_i y_{t-1}^* + \pi_i W_i x_t^* + \rho_i W_i y_{t-1}^* \quad (10) \\ =& \beta_{i,0} + (\beta_{i,1} + \pi_i) x_t^* + \beta_{i,2} x_{t-1}^* + \alpha_i y_{t-1}^* + \rho_i y_t^* \\ \Rightarrow y_t^* =& \frac{\beta_{i,0}}{1-\rho_i} + \frac{\beta_{i,1}+\pi_i}{1-\rho_i} x_t^* + \frac{\beta_{i,2}}{1-\rho_i} x_{t-1}^* + \frac{\alpha_i}{1-\rho_i} y_{t-1}^* \quad (11) \\ =& \frac{\beta_{i,0}}{\lambda_i} + \gamma_i x_t^* + \eta_i x_{t-1}^* + \delta_i y_{t-1}^* \end{aligned}$$

The final step defines the parameter values in the spatial equilibrium. Denoting by λ_i as the spatial equilibrium effect of $\mathbf{y}$, γ_i the spatial equilibrium effect of $\mathbf{x}$, δ_i the temporal equilibrium effect of $\mathbf{y}$ and η_i of $\mathbf{x}$:

$$\lambda_i = 1 - \rho_i \qquad \delta_i = \frac{\alpha_i}{\lambda_i} \quad (12)$$

$$\gamma_i = \frac{\beta_{i,1} + \pi_i}{\lambda_i} \qquad \eta_i = \frac{\beta_{i,2}}{\lambda_i} \quad (13)$$

Next, we define the spatial difference as $\underline{\Delta} x_{i,t} = x_{i,t} - \mathbf{w}_i \mathbf{x}_t$. The spatial first difference is analogous to the temporal first difference with time series data. However, whereas in the time dimension the (causal) ordering is evident from the time index, the ordering in a spatial context is less clearly defined. The spatial weight matrix specifies a partial ordering since it assigns non zero values only to those cross-sectional units which are related to each other. Then, we use the long run coefficients to derive the spatial error correction model.

$$\begin{aligned} y_{i,t} =& \beta_{i,0} + \beta_{i,1} x_{i,t} + \eta_i \lambda_i x_{i,t-1} + (\gamma_i \lambda_i - \beta_{i,1}) \mathbf{w}_i \mathbf{x}_t + \delta_i \lambda_i y_{i,t-1} \quad (14) \\ &+ (1-\lambda_i) \mathbf{w}_i \mathbf{y}_t + e_{i,t} \\ y_{i,t} - \mathbf{w}_i \mathbf{y}_t =& \beta_{i,0} + \beta_{i,1} (x_{i,t} - \mathbf{w}_i \mathbf{x}_t) + \lambda_i \eta_i x_{i,t-1} + \gamma \lambda_i \mathbf{w}_i \mathbf{x}_t \quad (15) \\ &- \lambda_i \mathbf{w}_i \mathbf{y}_t + \delta_i \lambda_i y_{i,t-1} + e_{i,t} \\ \underline{\Delta} y_{i,t} =& \beta_{i,0} + \beta_{i,1} \underline{\Delta} x_{i,t} - \lambda_i (\mathbf{w}_i \mathbf{y}_t - \gamma \mathbf{w}_i \mathbf{x}_t) \quad (16) \\ &+ \lambda_i \eta_i (x_{i,t-1} + \delta_i / \eta_i y_{i,t-1}) + e_{i,t} \end{aligned}$$

Equation (16) is an ECM in a combined spatial and temporal dimension. It has one cointegrating relationship between the spatial lags of $\mathbf{x}$ and $\mathbf{y}$. Analogously to a temporal ECM, λ_i defines the spatial equilibrium effect, or the spatial cointegration vector. The second term encompassed by $\lambda_i \eta_i$, that is $(x_{i,t-1} + \delta_i / \eta_i y_{i,t-1})$ refers to the temporal cointegration relationship. Next, we derive the conditions for such a pair of time and space equilibria to exist.

In the temporal equilibrium the value of the variables x and y is constant across the time dimension, such that:

$$y_{i,t} = y_{i,t-1} = y_i^* \qquad x_{i,t} = x_{i,t-1} = x_i^* \tag{17}$$

$$\mathbf{w}_i\mathbf{y}_t = \mathbf{w}_i\mathbf{y}_{t-1} = \mathbf{w}_i\mathbf{y}^* \qquad \mathbf{w}_i\mathbf{x}_t = \mathbf{w}_i\mathbf{x}_{t-1} = \mathbf{w}_i\mathbf{x}^* \tag{18}$$

Using the temporal equilibrium conditions and equation (16) yields:

$$\begin{aligned} y_i^* =& \beta_{i,0} + \beta_{i,1}x_i^* - \beta_{i,1}\mathbf{w}_i\mathbf{y}^*, +\lambda_i\mathbf{w}_i\mathbf{y}^*, -\lambda_i\mathbf{w}_i\mathbf{y}^* \\ &+ \lambda_i\eta_i x_i^* + \lambda_i\gamma y_i^* + \mathbf{w}_i\mathbf{y}^* \end{aligned} \tag{19}$$

$$y_i^* = \frac{\beta_{i,0}}{1-\lambda_i\delta_i} + \frac{\beta_{i,1}+\lambda_i\eta_i}{1-\lambda_i\delta_i}x_i^* + \frac{\lambda_i\gamma_i - \beta_{i,1}}{1-\lambda_i\delta_i}\mathbf{w}_i\mathbf{x}^* + \frac{(1-\lambda_i)}{1-\lambda_i\delta_i}\mathbf{w}_i\mathbf{y}^* \tag{20}$$

$$= \frac{\beta_{i,0}}{\phi_i} + \kappa_i x_i^* + \omega_i\mathbf{w}_i\mathbf{x}^* + \mu_i\mathbf{w}_i\mathbf{y}^* \tag{21}$$

Analogous to the spatial equilibrium, we have the following coefficients under spatio-temporal equilibrium:

$$\phi_i = 1 - \lambda_i\delta_i \qquad \kappa_i = \frac{\beta_{i,1}+\lambda_i\eta_i}{\phi_i} \tag{22}$$

$$\mu_i = \frac{1-\lambda_i}{\phi_i} \qquad \omega_i = \frac{\lambda_i\gamma_i - \beta_{i,1}}{\phi_i} \tag{23}$$

Here, μ_i and ω_i capture the effect of the spatial lag and κ_i the effects of the explanatory variable in the spatio-temporal equilibrium. Equations (22) and (23) imply:

$$\kappa_i\phi_i - \lambda_i\eta_i = \omega_i\phi_i - \lambda_i\delta_i \tag{24}$$

$$\phi_i\left(\kappa_i + \omega_i\right) = \lambda_i\left(\eta_i + \gamma_i\right) \tag{25}$$

Plugging the equilibrium coefficients into equation (16) gives us the spatio-temporal ECM:

$$
\begin{aligned}
\underline{\Delta} y_{i,t} =& \beta_{i,0} + \beta_{i,1}\underline{\Delta} x_{i,t} + \lambda_i \left(\gamma_i \mathbf{w}_i \mathbf{x}_t - \mathbf{w}_i \mathbf{y}_t\right) \\
& + \kappa_i \phi_i x_{i,t-1} - \beta_{i,1} y_{i,t-1} + \left(1 - \phi_i\right) y_{i,t-1} + e_{i,t}
\end{aligned} \tag{26}
$$

$$
\begin{aligned}
=& \beta_{i,0} + \beta_{i,1}\underline{\Delta} x_{i,t} + \lambda_i \left(\gamma_i \mathbf{w}_i \mathbf{x}_t - \mathbf{w}_i \mathbf{y}_t\right) \\
& + \phi_i \left(\kappa x_{i,t-1} - y_{i,t-1}\right) \\
& + y_{i,t-1} - \beta_{i,1} x_{i,t-1} + e_{i,t}
\end{aligned} \tag{27}
$$

$$
\begin{aligned}
=& \beta_{i,0} + \beta_{i,1}\underline{\Delta} x_{i,t} - \lambda_i \left(\mathbf{w}_i \mathbf{y}_t - \gamma_i \mathbf{w}_i \mathbf{x}_t\right) \\
& + \beta_{i,1} \Delta x_{i,t} - \phi_i \left(y_{i,t-1} - \kappa_i x_{i,t-1}\right) \\
& + y_{i,t-1} - \beta_{i,1} x_{i,t-1} + e_{i,t}
\end{aligned} \tag{28}
$$

$$
\begin{aligned}
y_{i,t} - \mathbf{w}_i \mathbf{y}_t - y_{i,t-1} =& \beta_{i,0} + \beta_{i,1} \left(x_{i,t} - \mathbf{w}_i \mathbf{x}_t - x_{i,t-1}\right) \\
& - \lambda_i \left(\mathbf{w}_i \mathbf{y}_t - \gamma_i \mathbf{w}_i \mathbf{x}_t\right) \\
& - \phi_i \left(y_{i,t-1} - \kappa x_{i,t-1}\right) + e_{i,t}
\end{aligned} \tag{29}
$$

Equation (29), the spatio-temporal ECM, is a central contribution of this paper. It is new to the literature and expresses precisely the nature of spatio-temporal short run dynamics and partial adjustment to the spatial and temporal long run equilibria. The short run effect is $\beta_{i,1}$, ϕ_i is the speed of error correction or the partial adjustment to the temporal long run equilibrium, and λ_i is the partial adjustment to the spatial long run equilibrium.

However, the term on the left hand side and the term capturing the short run dynamics are not very informative. To provide better interpretation, we define joint spatio-temporal differencing as $y_{i,t} - y_{i,t-1} - \mathbf{w}_i \mathbf{y}_t + \mathbf{w}_i \mathbf{y}_{t-1} = \Delta\underline{\Delta} y_{i,t}$ and the equivalent for $\Delta\underline{\Delta} x_{i,t}$. The $\Delta\underline{\Delta}$ notation takes out first order nonstationarity across the two dimensions, space and time. It is equivalent in time series to transforming an I(1) process into a stationary I(0) by taking first differences across time. Here, the joint differencing is interpreted as temporal first difference of spatial difference, or vice versa.

Using this notation and adding on both sides $\mathbf{w}_i \mathbf{y}_t$ and $\mathbf{w}_i \mathbf{x}_t$ transforms equation (29) into:

$$
\begin{aligned}
\Delta\underline{\Delta} y_{i,t} = & \beta_{i,0} + \beta_{i,1} \Delta\underline{\Delta} x_{i,t} + \left(\mathbf{w}_i \mathbf{y}_{t-1} - \beta_{i,1} \mathbf{w}_i \mathbf{x}_{t-1}\right) \\
& - \lambda_i \left(\mathbf{w}_i \mathbf{y}_t - \gamma_i \mathbf{w}_i \mathbf{x}_t\right) - \phi_i \left(y_{i,t-1} - \kappa_i x_{i,t-1}\right) + e_{i,t},
\end{aligned} \tag{30}
$$

$\lambda_i \left(\mathbf{w}_i \mathbf{y}_t - \gamma_i \mathbf{w}_i \mathbf{x}_t\right)$ represents the spatial and $\phi_i \left(y_{i,t-1} - \kappa_i x_{i,t-1}\right)$ the temporal error correction term. However both terms are still potentially nonstationary with respect to the other dimension. We can rewrite the temporal long run relationship as

$$
y_{i,t-1} - \kappa_i x_{i,t-1} = \underline{\Delta} y_{i,t-1} - \kappa_i \underline{\Delta} x_{i,t-1} + \mathbf{w}_i \mathbf{y}_{t-1} - \kappa_i \mathbf{w}_i \mathbf{x}_{t-1} \tag{31}
$$

and the spatial long run relationship as

$$
\mathbf{w}_i \mathbf{y}_t - \gamma_i \mathbf{w}_i \mathbf{x}_t = \mathbf{w}_i \Delta \mathbf{y}_t - \gamma_i \mathbf{w}_i \Delta \mathbf{x}_t + \mathbf{w}_i \mathbf{y}_{t-1} - \gamma_i \mathbf{w}_i \mathbf{x}_{t-1} \tag{32}
$$

Then (30) is transformed as:

$$\begin{aligned}\Delta\underline{\Delta}y_{i,t} =&\beta_{i,0} + \beta_{i,1}\Delta\underline{\Delta}x_{i,t} + (\mathbf{w}_i\mathbf{y}_{t-1} - \beta_{i,1}\mathbf{w}_i\mathbf{x}_{t-1}) \\ &- \lambda_i(\mathbf{w}_i\Delta\mathbf{y}_t - \gamma_i\mathbf{w}_i\Delta\mathbf{x}_t + \mathbf{w}_i\mathbf{y}_{t-1} - \gamma_i\mathbf{w}_i\mathbf{x}_{t-1}) \\ &- \phi_i(\underline{\Delta}y_{i,t-1} - \kappa_i\underline{\Delta}x_{i,t-1} + \mathbf{w}_i\mathbf{y}_{t-1} - \kappa_i\mathbf{w}_i\mathbf{x}_{t-1}) + e_{i,t} \\ =&\beta_{i,0} + \beta_{i,1}\Delta\underline{\Delta}x_{i,t} - \phi_i(\underline{\Delta}y_{i,t-1} - \kappa_i\underline{\Delta}x_{i,t-1}) \\ &- \lambda_i(\mathbf{w}_i\Delta\mathbf{y}_t - \gamma_i\mathbf{w}_i\Delta\mathbf{x}_t) \\ &+ \mathbf{w}_i\mathbf{y}_{t-1}(1 - \lambda_i - \phi_i) - \mathbf{w}_i\mathbf{x}_{t-1}(\beta_{1,i} - \lambda_i\gamma_i - \phi_i\kappa_i) + e_{i,t} \\ =&\beta_{i,0} + \beta_{i,1}\Delta\underline{\Delta}x_{i,t} - \phi_i(\underline{\Delta}y_{i,t-1} - \kappa_i\underline{\Delta}x_{i,t-1}) \\ &- \lambda_i(\mathbf{w}_i\Delta\mathbf{y}_t - \gamma_i\mathbf{w}_i\Delta\mathbf{x}_t) \\ &+ \alpha_i\rho_i\mathbf{w}_i\mathbf{y}_{t-1} + (\pi_i - \beta_{1,i} - \beta_{2,i})\mathbf{w}_i\mathbf{x}_{t-1} + e_{i,t}\end{aligned} \quad (33),(34)$$

The advantage of equation (34) is that there are two distinct error correction terms: one capturing the temporal error correction $\phi_i(\underline{\Delta}y_{i,t-1} - \kappa_i\underline{\Delta}x_{i,t-1})$ and the other one capturing the spatial error correction $\lambda_i(\mathbf{w}_i\Delta\mathbf{y}_t - \gamma_i\mathbf{w}_i\Delta\mathbf{x}_t)$. Note that both these terms are stationary across one dimension and lagged (or spatially lagged) along the other. Then, if there is cointegration, then both error correction terms are stationary across the two dimensions.

The final two terms $\alpha_i\rho_i\mathbf{w}_i\mathbf{y}_{t-1} + (\pi_i - \beta_{1,i} - \beta_{2,i})\mathbf{w}_i\mathbf{x}_{t-1}$ have a strong dependence interpretation. Specifically, these are encompassed in common correlated effects of $\mathbf{y_{t-1}}$ and $\mathbf{x_{t-1}}$. This motivates cross-section averages to model spatial strong dependence, which in turn is justified in large (N,T) samples by common correlated effects (Pesaran, 2006). Hence, in some contexts, it may be appropriate to replace $(\mathbf{w}_i\Delta\mathbf{y}_t - \gamma_i\mathbf{w}_i\Delta\mathbf{x}_t)$ with $(\Delta\overline{\mathbf{y}}_t - \gamma_i\Delta\overline{\mathbf{x}}_t)$. Then, these terms can be interpreted as common correlated effects adjusted for strong spatial dependence. However, choice of the appropriate long run weights is typically context specific. We will discuss this issue in our application to UK house prices.

3 Spatio-Temporal ECM, Common Correlated Effects and Weak Dependence

The Pesaran (2006) common correlated effects (CCE) estimator approximates factors underlying strong cross-sectional dependence by cross-sectional averages, $\bar{y}_t = \frac{1}{N}\sum_{i=1}^{N} y_{i,t}$ and $\bar{x}_t = \frac{1}{N}\sum_{i=1}^{N} x_{i,t}$. In a spatial equilibrium with each cross-sectional unit having the same influence as any other, the spatial weights become $w_{i,j} = \frac{1}{N} \ \forall i,j \in N$. Therefore the spatial lags can be rewritten as $\mathbf{w}_i\mathbf{y}_t + 1/Ny_{i,t} = 1/N\sum_{i=1}^{N} y_{i,t} = \bar{y}_t$, respectively in their first difference as $\Delta\bar{y}_t = 1/N\sum_{i=1}^{N}\Delta y_{i,t}$. $\bar{y}_t$ is a scalar and the same for all cross-sectional units. Equation (34) can then be rewritten as

$$\Delta\underline{\Delta}y_{i,t} = \beta_{i,0} + \beta_{i,1}\Delta\underline{\Delta}x_{i,t} - \lambda_i\left(\Delta\bar{y}_t - \frac{1}{N}\Delta y_{i,t} - \tilde{\omega}_i\left[\Delta\bar{x}_t - \frac{1}{N}\Delta x_{i,t}\right]\right) \quad (35)$$
$$- \phi_i\left(\underline{\Delta}y_{i,t-1} - \kappa_i\underline{\Delta}x_{i,t-1}\right)$$
$$+ \alpha_i\rho_i(\bar{\mathbf{y}}_{t-1} - \frac{1}{N}y_{i,t-1}) + (\pi_i - \beta_{1,i} - \beta_{2,i})\left(\bar{\mathbf{x}}_{t-1} - \frac{1}{N}x_{i,t-1}\right) + e_{i,t}$$

For a sufficiently large number of cross sectional units $\bar{y}_t \pm \frac{1}{N}y_{i,t} \approx \bar{y}_t$ and $\mathbf{w}_i\mathbf{y}_t \approx \bar{y}_t$ because $\lim_{N\to\infty}\frac{1}{N}y_{i,t} = 0$. The same holds for the time difference $\Delta\bar{y}_t = \bar{y}_t - \bar{y}_{t-1}$. Equation (35) can be then simplified to:

$$\Delta\underline{\Delta}y_{i,t} = \beta_{i,0} + \beta_{i,1}\Delta\underline{\Delta}x_{i,t} - \lambda_i\left(\Delta\bar{y}_t - \tilde{\omega}_i\Delta\bar{x}_t\right) \quad (36)$$
$$- \phi_i\left(\underline{\Delta}y_{i,t-1} - \kappa_i\underline{\Delta}x_{i,t-1}\right) + \alpha_i\rho_i\bar{\mathbf{y}}_{t-1} + \left(\pi_i - \beta_{1,i} - \beta_{2,i}\right)\bar{\mathbf{x}}_{t-1} + e_{i,t}$$

Following Pesaran (2006), it is now standard to include cross-section averages in panel data models to account for potential spatial strong dependence. Then, our spatio-temporal ECM Equation (36) can be estimated as:

$$\Delta\underline{\Delta}y_{i,t} = \beta_{i,0} + \beta_{i,1}\Delta\underline{\Delta}x_{i,t} - \phi_i\left(\underline{\Delta}y_{i,t-1} - \kappa_i\underline{\Delta}x_{i,t-1}\right) \quad (37)$$
$$+ \gamma_{i,y}\Delta\bar{y}_t + \gamma_{i,x}\Delta\bar{x}_t + \delta_{i,y}\bar{\mathbf{y}}_{t-1} + \delta_{i,x}\bar{\mathbf{x}}_{t-1} + e_{i,t}$$

where $\gamma_{i,y} = -\lambda_i$ and $\gamma_{i,x} = \lambda_i\tilde{\omega}_i$. In the literature the nuisance coefficient estimates on the cross-sectional averages are not interpreted. However, as shown here, they can provide valuable insights into the spatial cointegration relationship and partial adjustment. Furthermore the last equation shows how the CCE estimator (Pesaran, 2006) can account for spatially integrated processes and implicitly model cointegration in a fashion similar to a temporal ECM. Alternatively, the spatial or network cross-section equilibrium can be modelled using principal components (Bai and Ng, 2007) or interactive fixed effects (Bai, 2009) which also incorporates strong network dependence. Likewise, in some other contexts, spatial weights matrices implied by geography or observed social networks can also be useful.

Thus, cross-section average weights capture, for large N and T, the spatial long run relationship and partial adjustment to it. Often interest also rests upon spatial modelling of the weak dependence part, included in the short run dynamics, that is, in spatial weights for modelling $\underline{\Delta}y_{i,t}$ and $\underline{\Delta}x_{i,t}$. Here, we can draw upon the recent spatial econometrics literature, which shows that an unrestricted weak dependence $\mathbf{W}$ is not identified in general (Bhattacharjee and Jensen-Butler, 2013). Then, weak dependence can be modelled using one of several estimators under alternate identifying assumptions: (a) symmetry (Bhattacharjee and Jensen-Butler, 2013); (b) sparsity (Ahrens and Bhattacharjee, 2015; Lam and Souza, 2019); (c) symmetry and sparsity (Bailey, Holly and Pesaran, 2016); (d) asymmetric hub-and-spokes network (Bhattacharjee and

Holly, 2013; Bailey and Holly, 2017); and (e) recursive ordering (Basak et al., 2018).

In the application here, we explore two alternatives for modelling the long run relationship: common correlated effects (Pesaran, 2006) and cross-section correlations (Bailey, Holly and Pesaran, 2016). For weak dependence, we employ the estimator proposed in Bailey, Holly and Pesaran (2016) based on multiple testing of cross-section correlations, under assumptions of sparsity and symmetry.

Hence, we propose estimation in two steps. In the first step, we estimate a simple model regressing y on x using standard panel models for potentially nonstationary data by including common correlated effects to account for strong cross-section dependence. After including sufficient temporal lags in this model to ensure weak dependence of the residuals, as evidenced using the Pesaran (2015) CD test, we estimate the weak dependence spatial weights by multiple testing of residual cross-correlations. We then construct $\Delta\underline{\Delta} y_{i,t}$ and $\Delta\underline{\Delta} x_{i,t}$. Then, in the second step, we estimate the spatio-temporal ECM Equation (36) using the mean group estimator (Pesaran and Smith, 1995). Under homogeneity assumptions on the cointegrating relationships, one can also use the pooled mean group estimator of Shin et al. (1999).

4 An application to house prices in the UK

The interest that many social scientists have in housing reflects, among other things, the importance it has in household budgets, in the design of social policy and even in the behaviour of the macro economy. Big differences in the way in which housing and financial markets function around the world have profound effects on how output and inflation in the different countries respond to changes in short-term interest rates, as well as to external shocks to asset markets. An important aspect of the interaction between the housing market and the macro economy arises from the link to the labour market as, for example, differences in the level of house prices between regions within countries lowers labour mobility.

4.1 Economic model

There is an extensive literature on the economics of housing and on the determination of house prices, yet many studies of house prices place more emphasis on demand compared to supply factors (Olsen, 1987). One reason for this is that fluctuations in house prices observed in many countries over time have the most immediate consequences for macroeconomic performance, reflecting factors on the demand side that trigger shifts along a very inelastic short run supply curve. However, if there is also an interest in the lower frequency movement of house prices, an analysis of how forces on the supply side impact upon house prices could be useful.

This is not to suggest that the theoretical literature has neglected supply side factors. The best known, and most elegant, models of the housing market derive the demand for housing from a well articulated utility maximising framework and allows the stock of housing to evolve in a similar manner to the practice in the modern literature on economic growth (Muth, 1976; Brueckner, 1981; Arnott et al., 1983, 1999; Glaeser et al., 2008; Glaeser and Gottlieb, 2009). Nevertheless, the housing stock is subject to a different process of construction and then refurbishment over the (extended) lifetime of the house. Here we do not focus on supply side factors.

On the demand side it is now standard to see the determination of house prices as the outcome of a market for the services of the housing stock and as an asset. A standard model of the demand for housing services includes permanent income, the real price of housing services and a set of other influences affecting changes in household formation such as demographic shifts. In equilibrium the real price of houses, p^h/p, is equal to the real price of household services, s, divided by the user cost of housing, c :

$$p^h/p = s/c.$$

Here, p is a general price index. Assume that alternative assets are taxed at the rate τ. c is then equal to the expected real after-tax rate of return on other assets with a similar degree of risk:

$$c = (r + \pi)(1 - \tau) - \pi^e,$$

where r is the risk-equivalent real interest rate on alternative assets and π^e is the expected rate of price inflation. Feldstein et al. (1978) assume that the alternative asset is some aggregate capital which can be financed by the issue of equity or the sale of bonds. The bonds are of an equivalent degree of risk to house ownership. Equity is riskier, so there is a market determined risk premium, ρ, on the holding of equity. In equilibrium the risk adjusted return on equity, ε, is equal to the return on bonds:

$$(1 - \tau)\varepsilon - \tau_c\pi - \rho = (r + \pi)(1 - \tau) - \pi^e$$

The return to equity is expressed as the dividend payout per unit of equity.

Another way of deriving the user cost of housing is to use the full intertemporal model of consumption in which in equilibrium the marginal rate of substitution between housing services and the flow of utility from consumption is:

$$\frac{u^h}{u^c} = \frac{p^h}{p}\{(1 - \tau)(r + \pi) - \pi^e - \Delta(ph/p)^e$$

where $\Delta(ph/p)^e$ is the expected appreciation in the real price of houses.

The price of houses that satisfies the market for housing services and the asset market arbitrage condition is:

$$p^h/p = s/\{(1-\tau)(r+\pi) - \pi^e - \Delta(p/p)^e\} \tag{38}$$

The empirical model that can be derived from this form of analysis employs the device of proxying the unobservable real rental price of the flow of housing services, s, by the determinants of the demand for housing services, such as income and the housing stock. We take the above flexible model to data on US house prices.

4.2 Data

We use quarterly panel data, from 1997q1 to 2016q4, across local authorities in England. House prices at the United Kingdom Land Registry – which records all UK house transactions – are available monthly at the local authority level for England and Wales (from January 1995), Scotland (from January 2004) and Northern Ireland (from January 2005). The average of the 3 months is used to construct the quarterly estimates (GOV.UK, 2020).

Data on gross disposable household income are from the Office of National Statistics Office of National Statistics (2020*c*). Quarterly estimates are obtained by quadratic interpolation from annual figures. Annual population figures are obtained from Office of National Statistics (2020*a*). Quarterly estimates are obtained by quadratic interpolation from annual figures. The implicit deflator for consumer prices is the ratio of current price consumer expenditure to constant price consumer expenditure (Office of National Statistics, 2020*b*).

We focus on England only to obtain a balanced dataset. There are a total of 326 local authorities in our data set, made up of county councils, district councils, unitary authorities, metropolitan districts and London boroughs. We dropped the small local authority of Rutland which has a population of only about 30,000, so the sample for housing sales is very thin. The Isles of Scilly are included within Cornwall. Although there is an aggregate UK price index calculated from Land Registry data which is an hedonic, mix adjusted index, at the local authority level the price index is calculated from repeat sales. Thus we have 324 cross section units in total.

4.3 Discussion of Results

In this section we turn to a detailed discussion of the results. The steps we perform are outlined in more detail in the Appendix 6. First step estimation together with CCE cross-section averages produces residuals that satisfy the weak dependence condition based on the Pesaran (2015) CD test. We use these residuals to compute cross-section correlations, which are then used to conduct multiple testing to estimate the weak dependence weights matrix following the methodology in Bailey, Holly and Pesaran (2016). This provides 238 non-zero (1 or -1) elements in the spatial weights matrix, which is then row normalized using row sums of absolute values. The reported estimates in Table 1 are mean group panel estimates of a demand equation for real house prices across 324 local authorities. Estimation and inference is conducted in Stata using the `xtdcce2` command (Ditzen, 2018, 2019).

Column (1) is a standard panel data error correction model accounting for nonstationarity and possible cointegration in the temporal dimension; see, for example, Pesaran and Smith (1995). The evidence of cointegration is statistically significant but partial adjustment is weak and there is also substantial strong cross section dependence as evident from the CD test (Pesaran, 2015). Moreover, the long run relationship between real house prices and real incomes is rather high at 2.881. To correct for the strong cross section dependence we used the common correlated effects mean group estimator in column (2) (Pesaran, 2006). We have now eliminated the strong cross section dependence, and the coefficient of cross sectional dependence, α falls to 0.565. There is significant cointegration and strong partial adjustment to a long run relationship between real house prices and real personal income of about 0.75. These two columns represent what the current literature takes as best practice.

However, it is worthwhile examining the residuals of the model in more detail. At the moment the model ignores any possible overlap between local authority areas. A shock to Manchester has no consequences for the behaviour of house prices in contiguous areas so it ignores all spatial effects, but most critically potential nonstationarity and cointegration across the spatial dimension (Holly et al., 2011). In Figures 1 - 4 we plot on a map of England various features of the significant correlation coefficients after multiple testing (Bailey, Holly and Pesaran, 2016). Figure 1 plots the sum of the significant correlations for each local authority area. It does appear that there is a cluster of significant correlations around London and other large cities.[3] In Figures 2 and 3 we plot the negative and positive correlations and in Figure 4 the absolute sum of significant correlations. The plots suggest that there is a significant degree of spatial correlation that the results in columns (1) and (2) in Table 1 do not address.

We now use the results of sections 2 and 3 to integrate an explicit treatment of spatial as well as temporal effects into the model. This is the novel contribution of this paper. Column (3) shows estimates of a spatial error correction model which is an exact counterpart of the the temporal error correction model in panel data settings. Here, the spatial difference of y, $\underline{\Delta}\mathbf{y}$, is a linear function of $\underline{\Delta}\mathbf{x}$ – the spatial short run dynamics – together with partial adjustment to a long run spatial equilibrium captured by the spatial weights of y and x, that is, $\mathbf{Wy}$ and $\mathbf{Wx}$. Remarkably, there is evidence of spatial cointegration as well, which justifies the subject of this paper. Parallel to temporal cointegration, spatial cointegration here is interpreted as a (spatial) long run relationship between house prices and income whereby, for an index spatial local authority, if there is any disequilibrium between prices and income for its neighbours (as given by the spatial weights matrix), the prices in the index spatial unit adjust

[3]These correlations may be picking up the commuting patterns around London and other major labour market centres.

to partially mitigate against this disequilibrium.

Here, the role of the chosen spatial weights matrix is critical, because partial adjustment is with regard to disequilibrium amongst the neighbours of the index unit. Since our cross correlation weights matrix captures largely commuting patterns within local labour markets (Figures 1 to 4), spatial cointegration here can be interpreted as housing market forces negating opportunities for local arbitrage. However, if the weights matrix were given by cross section averages, this would imply price adjustments to a single house price index for England.

In addition to spatial cointegration, and as expected, the model has strong spatial dependence. This strong spatial dependence is not fully addressed by including common correlated effects in Column (4). This is because nonstationary temporal dynamics have not yet been modelled. In both models (3) and (4), the spatial long run effect is modelled using the estimated cross-correlation weights matrix (Bailey, Holly and Pesaran, 2016). Despite some residual strong spatial dependence, simultaneous evidence of spatial and temporal cointegration justifies our spatio-temporal ECM, to which we turn next.

Column (5) reports estimates of our basic spatio-temporal ECM model, which includes partial adjustment to a temporal equilibrium and a spatial equilibrium, but no common correlated effects. As expected from columns (1) through (4), we find strong evidence of cointegration in both dimensions. However, spatial strong dependence is present as evident from the CD test (Pesaran, 2015). Also, the final two common correlated effects terms in Equation (36) are not included, so this model is not entirely consistent with our theory. Hence, in column (6), we also include in our model cross-section averages of y_t, x_t, y_{t-1} and x_{t-1}. The evidence of cointegration across both the temporal and spatial dimensions persists. The Pesaran CD test (Pesaran, 2015) statistic is much reduced, but it still rejects the null hypothesis of weak cross section dependence at the 5% significance level.

The question is: why does strong cross section dependence still persist in model (6)? One reason may be that stationarity is not adequately achieved by taking spatial differences using the cross-correlation spatial weights matrix (Bailey, Holly and Pesaran, 2016). This line of thinking is also supported by the evidence from columns (3) and (4) where a pure spatial ECM does not remove in itself strong cross section dependence. An alternative that can be considered here is spatial weights implied by cross section averages, as discussed in Sections 2 and 3. The main place to apply these common correlated effects weights would be in the temporal partial adjustment term which relies critically on the spatial first differences being stationary (weakly dependent). Hence, in the final column (7), we apply common correlated effects weights only to the temporal long run, retaining the cross correlation spatial weights elsewhere, that is in the spatio-temporal short run dynamics and spatial error correction term.

To do so, we redefine the first spatial difference as $\underline{\underline{\Delta}} y_{i,t} = y_{i,t} - \bar{y}_t$, which then gives us a reformulation of Equation (36) as:

$$\begin{aligned}\Delta\underline{\Delta}y_{i,t} &= \beta_{i,0} + \beta_{i,1}\Delta\underline{\Delta}x_{i,t} - \lambda_i\left(\Delta\bar{y}_t - \tilde{\omega}_i\Delta\bar{x}_t\right) \\ &\quad - \phi_i\left(\underline{\underline{\Delta}}y_{i,t-1} - \kappa_i\underline{\underline{\Delta}}x_{i,t-1}\right) + \alpha_i\rho_i\overline{\mathbf{y}}_{t-1} + \left(\pi_i - \beta_{1,i} - \beta_{2,i}\right)\overline{\mathbf{x}}_{t-1} + e_{i,t}\end{aligned} \tag{39}$$

Taking teh above model to the data, in column (7), weak cross section dependence can no longer be rejected at the 5% level. This is supported by an estimated exponent of cross-sectional dependence $\hat{\alpha}$ of 0.505 with a lower 95% confidence bound of 0.497. When using the multiple testing approach on the residuals to obtain significant cross-correlations, we find that only a single cross-correlation remains significant. This suggests that we have adequately modelled strong and weak spatial dependence.

Evidence of significant short run dynamics and cointegration in the spatial and temporal dimensions is retained. Then, column (7) represents our preferred model estimates. In the temporal domain, there is about 20% partial adjustment, per quarter, to a long run relationship between house prices and income with about unit long run elasticity. The spatial long run house price elasticity of income is about 1.5, but taking standard errors into consideration, this is only slightly above the temporal case. The strong partial adjustment is very notable, about 1, meaning that prices adjust fully within the immediate (spatial first order) neighbourhood as modelled by the cross correlation weights. The data does not suggest long distance adjustment, either to neighbours of neighbours (second order neighbourhood) or a broader spatial scale, such as a central price index across the whole of England. The short run elasticity is about 0.34, and this is very much in line with expectations.

Finally, we also explored a couple of other plausible model specifications. First, we attempted to model the spatial long run relationship using the cross section averages. However, as would be expected from the above discussion, this model does not fit equally well, implying that the price-income relationship at the local authority level do not adjust to the national average. Second, we also explored a traditional and popular weights matrix based on geographic contiguity between local authorities. This model also does not fit as well as our preferred model in column (7), with substantial spillovers beyond first order contiguous neighbours, implying that the spatial organisation is more nuanced. Local labour market dynamics and commuting for work explain spatial dynamics better than simple geography.

We would hope that if the model of column (7) is an adequate model, then the large amount of spatial patterns that we observe from the residuals of column (2) (weak cross section dependence) would be eliminated with the residuals of the model of column (7). When, indeed, we re-generated the residuals for (7) we found that we had eliminated all but one significant weak correlation coefficient (after multiple testing). This suggests that our temporal and spatial modelling has addressed successfully both strong and weak cross sectional dependence in the behaviour of house prices in 324 local authorities in England.

5 Conclusion

We develop spatial and spatio-temporal Engle-Granger representation results that provide corresponding error correction models (ECMs) that are new to the literature. The spatio-temporal ECM includes partial adjustment to two equilibrium relationships, one temporal and the other spatial, together with short run dynamics based on a spatio-temporal difference. The above ECM clearly clarifies the distinct role of spatial strong and weak dependence in non-stationary dynamic models. In addition, the crucial role of strong dependence is highlighted, and it can be modelled using the CCE estimator of Pesaran (2006). Weak dependence can be estimated using the various estimators of the spatial weights matrix available in the literature, and these estimated weights matrices can also be useful in understanding spatial cointegration.

Applied to data on house prices and personal incomes across local authorities in England, our model and estimation provides new evidence and interpretation of nonstationary spatio-temporal dynamics and partial adjustment to multiple equilibria. Importantly, there is evidence of spatial cointegration where there is a (spatial) long run relationship between house prices and income. The partial adjustment to this spatial equilibrium is very local, and can be well explained by local labour markets and commuting for work. Partial adjustment at a broader spatial scale, such as price adjustments to a single house price index for England, is not supported by the data.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Time	Time	Spatial	Spatial	Spatio-Temporal	Spatio-Temporal	Spatio-Temporal
Dep. Var.	$\Delta p_{i,t}$	$\Delta p_{i,t}$	$\underline{\Delta} p_{i,t}$	$\underline{\Delta} p_{i,t}$	$\Delta\underline{\Delta} p_{i,t}$	$\Delta\underline{\Delta} p_{i,t}$	$\Delta\underline{\underline{\Delta}} p_{i,t}$
$\hat{\beta}_{MG,1}$	0.666***	0.401***	1.042***	0.147***	0.531***	0.288***	0.338***
	(0.020)	(0.037)	(0.096)	(0.048)	(0.027)	(0.038)	(0.039)
$\hat{\phi}_{MG}$	-0.079***	-0.195***			-0.090***	-0.227***	-0.199***
	(0.002)	(0.007)			(0.005)	(0.008)	(0.008)
$\hat{\gamma}_{MG}$	2.881***	0.751***			1.756***	0.138	1.304***
	(0.110)	(0.169)			(0.214)	(0.121)	(0.514)
$\hat{\lambda}_{MG}$			-1.158***	-1.063***	-1.101***	-0.912***	-0.999***
			(0.114)	(0.055)	(0.043)	(0.040)	(0.040)
$\hat{\kappa}_{MG}$			0.447***	0.647***	0.018	0.763***	1.418***
			(0.116)	(0.233)	(0.266)	(0.248)	(0.289)
$\hat{\beta}_{MG,0}$	-7.278***	0.246	-0.406	2.522	-0.022	1.098***	0.271
	(0.564)	(0.695)	(0.445)	(2.285)	(0.056)	(0.372)	(0.293)
CSA time lags ($L_0 - L_1$)	none	0 - 1	none	0 - 4	none	1	0 - 2
R^2_{MG}	0.155	0.616	0.881	0.989	0.285	0.647	0.652
CD	1009.226	0.421	655.818	31.173	619.434	1.924	1.485
p-val	0.000	0.674	0.000	0.000	0.000	0.054	0.138
CI up	0.934	0.572	0.804	0.690	0.681	0.514	0.514
$\hat{\alpha}$	0.928	0.565	0.793	0.680	0.667	0.506	0.505
CI low	0.923	0.557	0.783	0.670	0.653	0.497	0.497
N	324	324	324	324	324	324	324
T	78	78	79	75	78	78	77

Table 1: Mean Group Estimation results with $\beta_{MG,1} = \frac{1}{N}\sum_{i=1}^{N}\beta_{1,i}$. CSA time lags is the structure of the lags of the cross-sectional averages, L_0 and L_1 defines the lower and upper bound of the spatial lags. 0 stands for contemporaneous cross sectional averages. R^2_{MG} is the mean group adjusted R^2 from Holly et al. (2010). $\hat{\alpha}$ is an estimate of the exponent of cross-sectional dependence (Bailey et al., 2019). Confidence intervals were calculated using 100 bootstrap repetitions. The estimated equation is Eq. (36): $\Delta\underline{\Delta} y_{i,t} = \beta_{0,i} + \beta_{1,i}\Delta\underline{\Delta} x_{i,t} - \phi\left(\underline{\Delta} y_{i,t-1} + \omega_i \underline{\Delta} x_{i,t-1}\right) - \lambda_i\left(\mathbf{w}_i \Delta \mathbf{y}_t + \kappa_i \mathbf{w}_i \Delta \mathbf{x}_t\right) + \sum_{l=L_0}^{L_1}\left(\gamma_{i,l,y}\bar{y}_{t-l} + \gamma_{i,l,x}\bar{x}_{t-l}\right) + \epsilon_{i,t}$, where $\bar{y}_t$ and $\bar{x}_t$ are the cross-sectional averages, $\underline{\Delta}$ indicates the first difference in the spatial dimension using a spatial lag, $\underline{\underline{\Delta}}$ indicates the first difference in the spatial dimension using cross-sectional averages Δ is the first lag in the time dimension. Column (7) is Equation (39), in which the spatial difference $\underline{\Delta}$ is replaced by the difference using cross-sectional averages $\underline{\underline{\Delta}}$. For the individual steps of the estimation see Section 3 and Appendix 6. $^{***}p < 0.01, ^{**}p < 0.05, ^{*}p < 0.1$

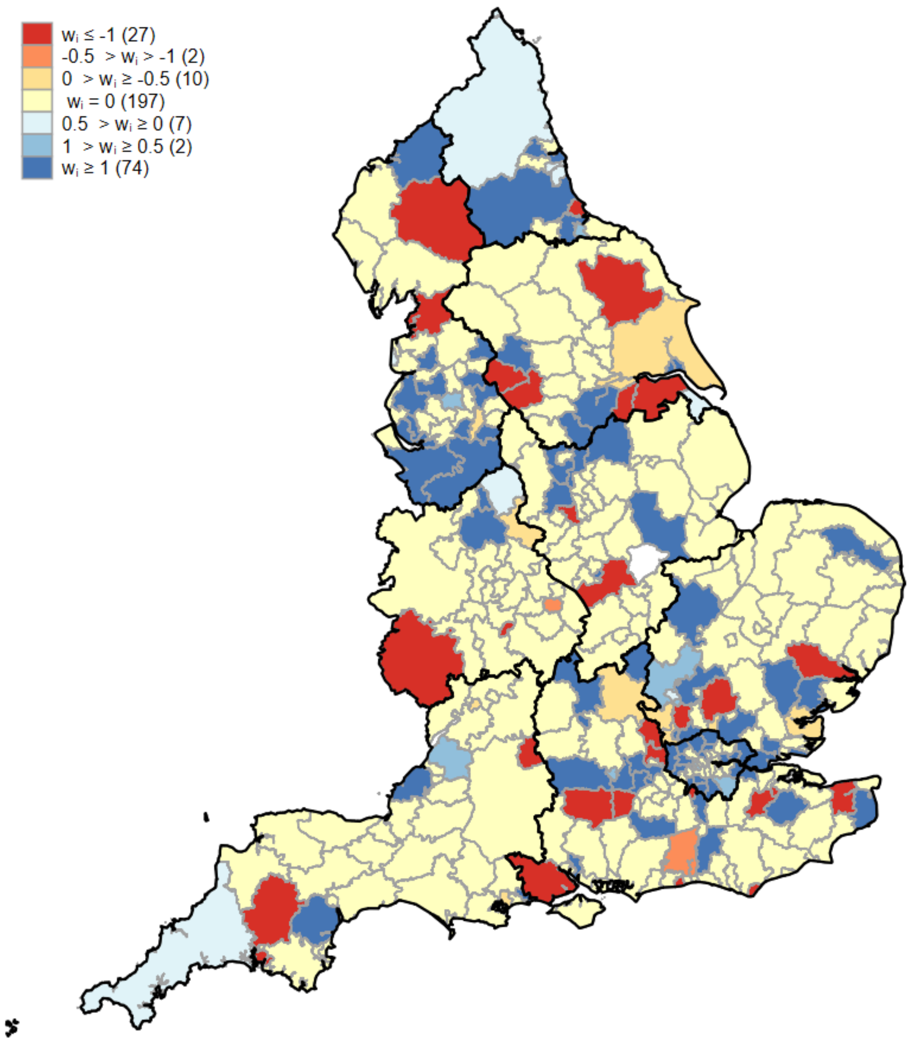

Figure 1: Sum of spatial weights of 324 local authorities in England ($w_i = \sum_{j=1}^{N} w_{i,j}$). Spatial weights are generated from significant cross-correlations.

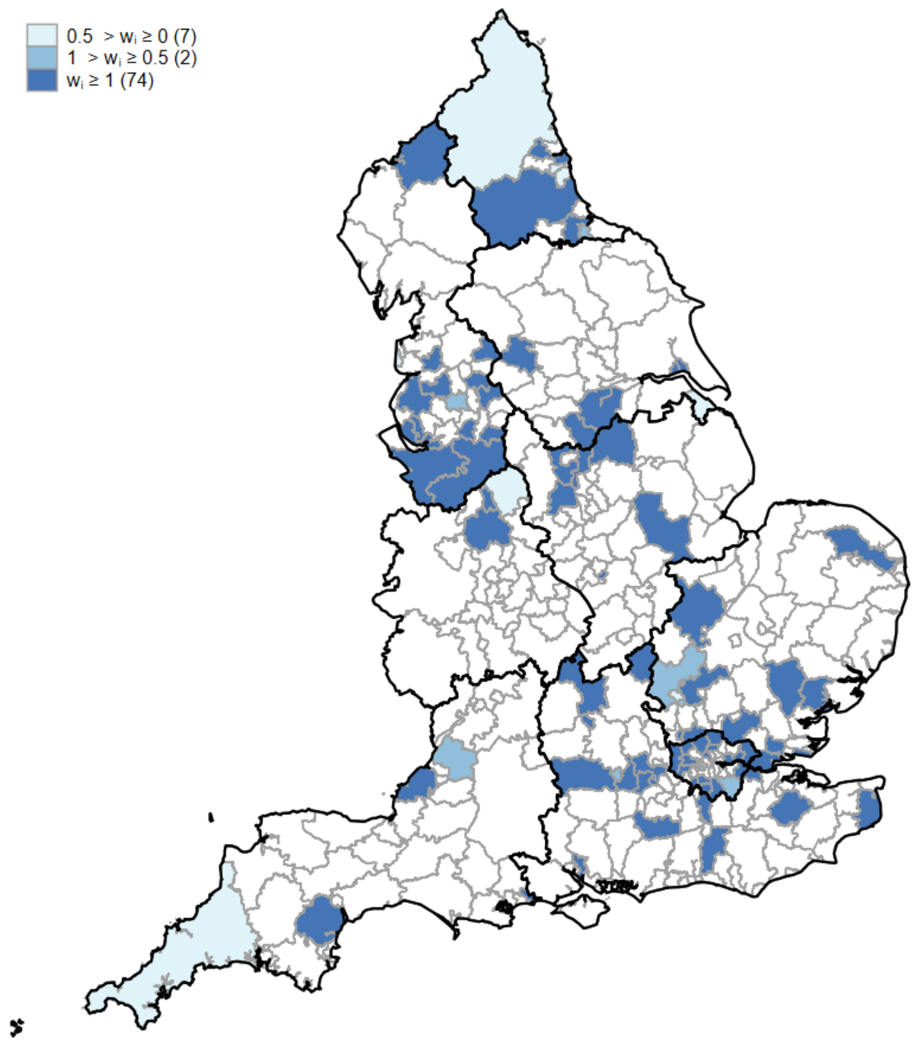

Figure 2: Sum of positive spatial weights of 324 local authorities in England ($w_i = \sum_{j=1}^{N} w_{i,j}, w_{i,j} > 0$). Spatial weights are generated from significant cross-correlations.

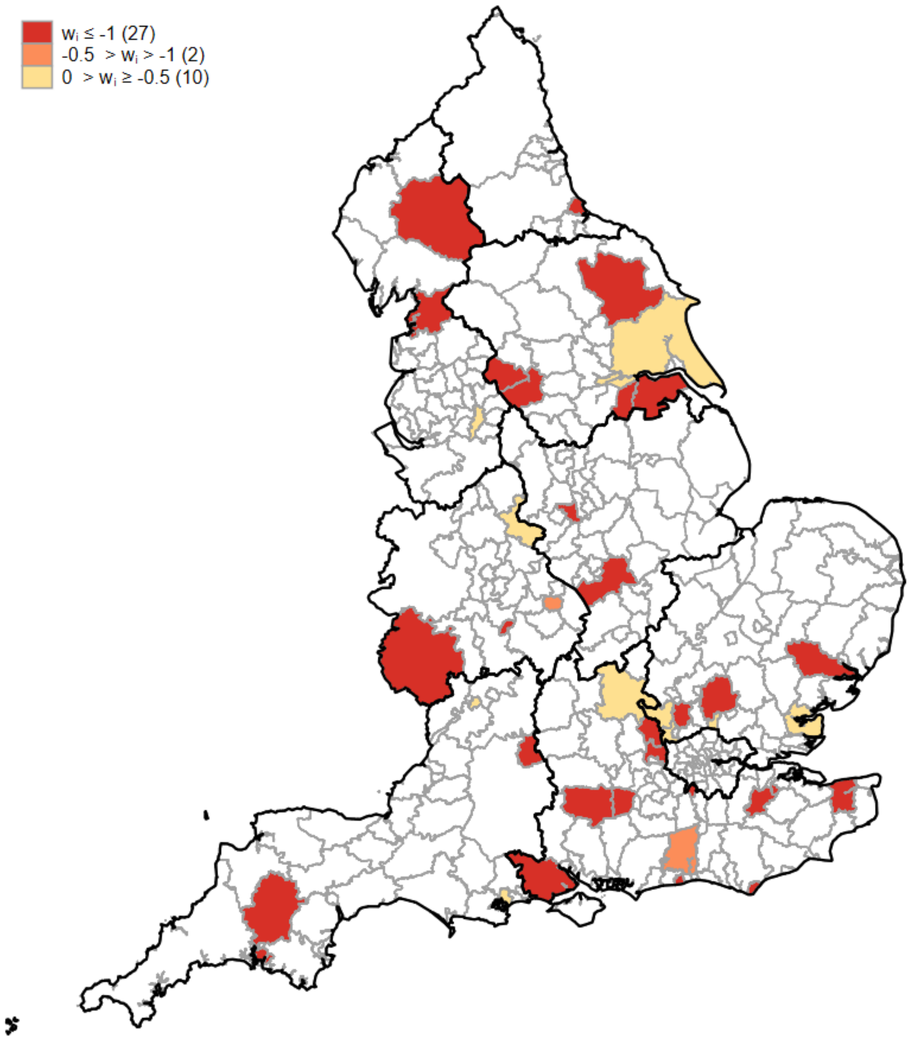

Figure 3: Sum of negative spatial weights of 324 local authorities in England ($w_i = \sum_{j=1}^{N} w_{i,j}, w_{i,j} < 0$). Spatial weights are generated from significant cross-correlations.

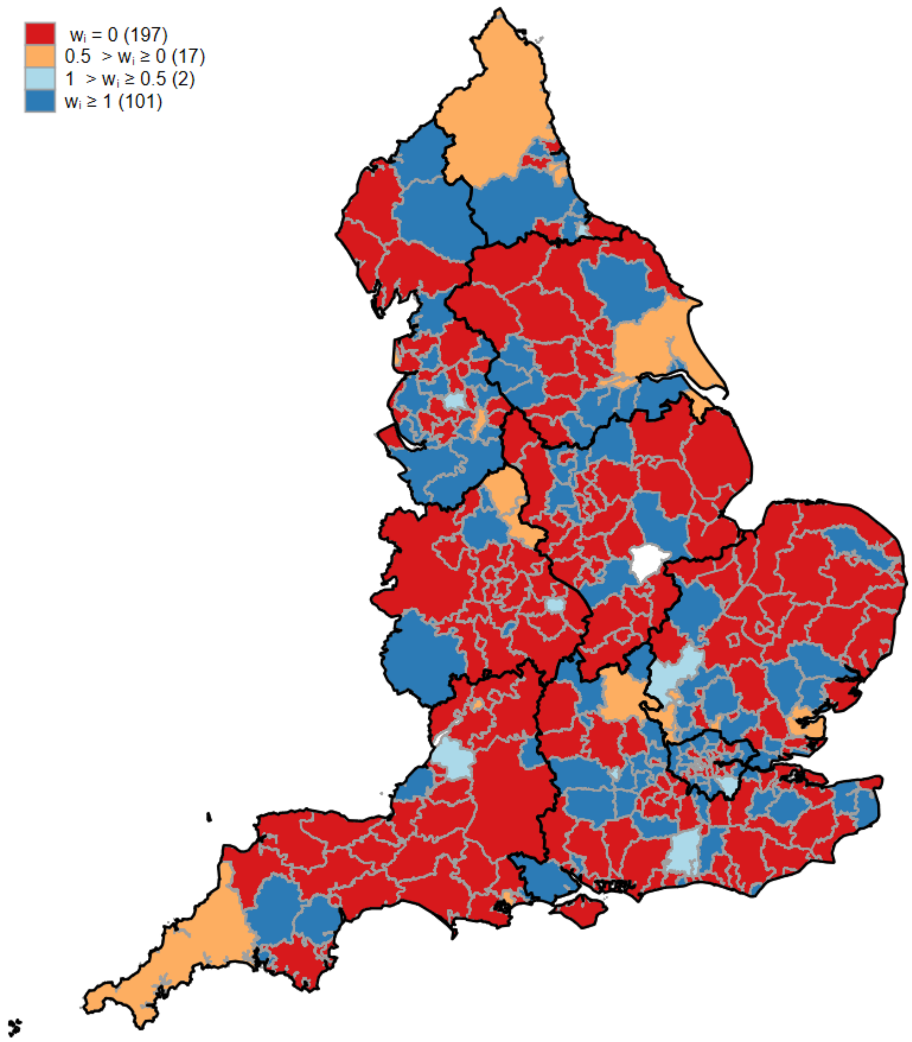

Figure 4: Sum of absolute spatial weights 324 local authorities in England ($w_i = \sum_{j=1}^{N} |w_{i,j}|$). Spatial weights are generated from significant cross-correlations.

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6 Appendix: Estimation Steps

1. Estimate a simple model to obtain the pair wise-correlations

$$\Delta y_{i,t} = \beta_{i,0} + \beta_{i,1} y_{i,t-1} + \beta_{i,2}\Delta x_{i,t} + \beta_{i,3} x_{i,t-1} + \sum_{l=1}^{p_x} \gamma_{x,i,l}\bar{x}_t + \sum_{l=1}^{p_y} \gamma_{y,i,l}\bar{y}_t + \epsilon_{i,t}$$

2. Obtain the pair wise correlation matrix from the residuals $\rho_{i,j} = \frac{1}{N}\sum_{t=1}^{T} \hat{\epsilon}_{i,t}\hat{\epsilon}_{j,t}$:

$$\tilde{W} = \begin{pmatrix} \hat{\rho}_{1,1} & \hat{\rho}_{1,2} & \dots & \hat{\rho}_{1,N} \\ \hat{\rho}_{2,1} & \hat{\rho}_{2,2} & \dots & \hat{\rho}_{2,N} \\ \vdots & & \ddots & \vdots \\ \hat{\rho}_{N,1} & \dots & \dots & \hat{\rho}_{N,N} \end{pmatrix}$$

3. Use multiple testing to obtain significant pair wise-correlations with $\rho_{i,j} > c_p = \phi^{-1}\left(1 - \frac{p/2}{n^\delta}\right)$ which then gives W and row standardise W.

4. Calculate spatial lags as $\sum_{s=1}^{N} w_{i,s} y_{i,t}$ and $\sum_{s=1}^{N} w_{i,s} x_{i,t}$.

5. Calculate $\Delta\underline{\Delta} y_{i,t} = y_{i,t} - y_{i,t-1} - \mathbf{w}_i \mathbf{y}_t + \mathbf{w}_i \mathbf{y}_{t-1}$

6. Estimate the following models:

$$\begin{aligned} \Delta\underline{\Delta} y_{i,t} = {} & \beta_{i,0} + \beta_{i,1}\Delta\underline{\Delta} x_{i,t} - \lambda_i \left(\mathbf{w}_i \Delta \mathbf{y}_t - \omega_i \mathbf{w}_i \Delta \mathbf{x}_t\right) \\ & - \phi_i \left(\underline{\Delta} y_{i,t-1} - \kappa_i \underline{\Delta} x_{i,t-1}\right) \\ & + \alpha_i \rho_i \overline{\mathbf{y}}_{t-1} + \left(\pi_i - \beta_{1,i} - \beta_{2,i}\right) \overline{\mathbf{x}}_{t-1} + e_{i,t} \end{aligned}$$

   respectively for large N:

$$\begin{aligned} \Delta\underline{\Delta} y_{i,t} = {} & \beta_{i,0} + \beta_{i,1}\Delta\underline{\Delta} x_{i,t} - \lambda_i \left(\Delta \bar{y}_t - \tilde{\omega}_i \Delta \bar{x}_t\right) \\ & - \phi_i \left(\underline{\Delta} y_{i,t-1} - \kappa_i \underline{\Delta} x_{i,t-1}\right) \\ & + \alpha_i \rho_i \overline{\mathbf{y}}_{t-1} + \left(\pi_i - \beta_{1,i} - \beta_{2,i}\right) \overline{\mathbf{x}}_{t-1} + e_{i,t} \end{aligned}$$

   Further cross-sectional averages can be added to both regressions.