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Study of the noise of micromechanical oscillators under quality factor enhancement via driving force control

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The performance of devices based on micro- and nanomechanical oscillators depends critically on the quality factor (Q). The quality factor can be externally increased about two orders of magnitude by coherent amplification of the oscillation at resonance with a fast feedback amplifier. Here, theory and experiments performed with microcantilevers are presented to study the oscillation noise under external Q enhancement and how it differs from the noise when the Q is naturally enhanced by decreasing the mechanical energy loss. The application of the feedback amplifier produces a significant increase of the thermal noise and the noise that arises from the cantilever-displacement sensor. The main consequence is that the signal-to-noise ratio (S/N) remains constant and independent of the Q enhancement when measuring the amplitude and phase of the oscillation in the slope detection technique. This behavior is opposite to the enhancement of the S/N when the Q naturally increases, which is proportional to $Q^{1/2}$, ignoring instrumental sources of noise. More important, by taking into account the maximum driving force provided by the actuator, it is concluded that external Q enhancement does not enhance the sensitivity of devices based on micro- and nanomechanical oscillators, using the slope detection technique. The lack of sensitivity enhancement is attributed to the fact that thermal forces are not altered by the increase of the quality factor via the fast feedback amplifier. Finally, it is proposed to use the fast feedback amplifier in a different measurement mode to obtain high sensitivity. This consists in the self-excitation of the cantilever without application of a reference driving force, and the measurement of the frequency of the oscillation. Self-excitation of the cantilever produces amplification of the noise and its squeezing around the resonant frequency, hence the oscillation resembles Brownian motion of the cantilever with a superior quality factor.
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I. INTRODUCTION

Micro- and nanomechanical oscillators are playing an increasing role in a wide range of fields such as nanotechnology, medicine, and communications.[1–3] Microcantilevers are employed in scanning probe microscopy for imaging at nanometer scale and probing single molecules.[4–6] Microarrays of cantilevers are used for writing and reading of ultrahigh data storage with terabit capacity.[7] Also cantilevered structures that are previously sensitized with either polymer films or biological receptors allow the specific detection of minute amounts of gases and biological substances.[8–11] Other applications include sensitive mechanical charge detectors[12,13] and high frequency signal processing.[14] For simplicity, hereafter micro- and nanomechanical oscillators will be referred to as cantilevers.

The operating principle of many devices based on cantilevers is the measurement of the oscillation and its variation when an external stimulus (that we wish to detect) interacts with the cantilever. The interaction produces a change of the resonant frequency and mechanical quality factor (Q) of the cantilever that are translated into a variation of the amplitude, phase, and frequency of the oscillation. The slope detection technique is commonly applied to monitor the oscillation changes, in which the cantilever is driven at a constant frequency near resonance, and the external interactions are detected as variations in amplitude or phase.[4]

Thermodynamics sets the ultimate sensitivity of cantilever-based devices. The cantilever fluctuates with respect to the rest position due to the random impacts of the surrounding molecules. In the same way, the cantilever dissipates the stored mechanical energy through its interaction with the surrounding thermal bath. This relationship between the thermal forces and the dissipation of mechanical energy is described by the fluctuation-dissipation theorem, which is usually applied to determine the electrical noise across a resistor.[15,16]

The fluctuation and dissipation are linked through the quality factor (Q) that is determined by the amount of energy loss with respect to the vibrational mechanical energy. The higher the energy dissipation, the lower the Q. The main sources of energy dissipation are the internal friction of the cantilever material, energy loss through the coupling of the cantilever to the support structure, and the viscous damping.[17,18] Since energy dissipation implies thermal coupling between the cantilever and the surrounding environment, the magnitude of the random thermal forces depends on Q, being inversely proportional to $Q^{1/2}$. On the other

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hand, the quality factor determines the linewidth of the cantilever resonance. The higher Q, the narrower is the resonant peak.

The performance of sensor devices based on cantilevers depends critically on the quality factor; the higher Q, the higher the signal-to-noise ratio. However the energy dissipation cannot be simply tailored during the fabrication process as can be done with other fundamental parameters such as the resonant frequency and spring constant, which exhibit simple relationships with the dimensions and the material. For instance, nanomechanical oscillators permit high operational frequencies with potential applications for high-Q filters in communications. However surface loss at this size scale gives a low Q.[16,18,19] In other applications, the cantilevers cannot be encapsulated in vacuum, and air and liquid damping degrade Q by orders of magnitude.[20,21]

Many approaches have been proposed to push further the thermomechanical limit. These include classical squeezing of the thermal noise via parametric amplification, in which a parameter of the cantilever (usually the spring constant) is periodically modulated.[22,23] Actively controlled external dissipative forces have been used to reduce the thermal noise amplitude to achieve an apparent temperature of 25 K at room temperature.[24] Another conceptually similar approach is the use of linear[19,25–31] and nonlinear[32] position-based fast feedbacks to change the effective response of the cantilever. Based on this concept, it is possible to change the quality factor by coherent (higher Q) or anticoherent (lower Q) amplification of the oscillation at resonance. Thus, Q has been artificially decreased to reduce the transient oscillation and increase the response speed of cantilever.[25–27]

Conversely, Q has been enhanced via a fast feedback amplifier to increase the sensitivity of cantilevers in a liquid that showed low Q.[19,28–31] In this case it has been widely reported that the signal increases with this technique, but attention has not been paid to the noise behavior. To elucidate if the external increase of Q can circumvent the thermomechanical limits, theoretical and experimental studies have been undertaken to determine the signal-to-noise ratio as function of the Q enhancement. This study has been performed using the slope detection technique. Alternatively, it is proposed to use the fast feedback amplifier in a different measurement mode, in which a constant driving force is not applied, and the noise is coherently amplified by the fast feedback amplifier for frequencies close to resonance. In this technique, the self-excited cantilever serves as the frequency-determining element.[28] The frequency is modulated by external interactions as in the frequency modulation technique.[33,34]

The paper is organized as follows. First, the theory of the signal and noise of the cantilever oscillation (amplitude and phase) is introduced in Sec. III. The aim of this section is to obtain the relationship between the signal and noise of cantilever-based devices and the intrinsic quality factor. Second, the effect of the increase of the Q via a fast feedback amplifier on the signal and noise of the amplitude and phase of the cantilever oscillation is analyzed in Sec. IV. This is studied theoretically following the formalism described in Sec. III, and it is compared with experiments performed with microcantilevers in air. In these experiments, the cantilever deflection is measured by the optical beam deflection technique, and the noise is predominantly of thermo-mechanical nature. Finally, in Sec. V, theory and experiments of the self-excitation technique are presented.

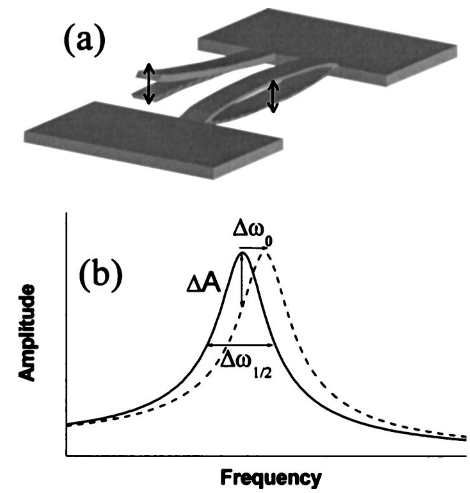

FIG. 1. (a) A schematic of a cantilever-based device with a single and a doubly clamped beam. (b) Hypothetical amplitude vs driving frequency curve of a cantilever before (solid line) and after (dashed line) the actuation of an external force. The external force produces a shift of the resonant frequency, $\Delta\omega_0$, to higher frequencies. ΔA is the corresponding change of the amplitude when the cantilever is driven at resonance before the external force. The smaller the resonant peak width ($\Delta\omega_{1/2}$), the higher ΔA.

II. MATERIALS AND METHODS

The experiments were performed with a home-built microcantilever sensor device, in which the cantilever deflection was measured with a resolution of 0.1 nm, approximately, by using the optical deflection technique. Commercially available cantilevers from Olympus were used, 200 μm long, 40 μm wide, and 0.8 μm thick. The cantilevers were coated with a 25-nm-thick layer of cobalt on both sides to allow magnetic excitation. The experiments were performed in air. Active control of the cantilever motion for Q enhancement was performed by using the commercially available electronics ARC from Infinitesima. The signal and noise of the cantilever vibration were measured by using home-made software programmed in LABVIEW (National Instruments).

III. THEORY OF THE SIGNAL AND NOISE OF CANTILEVER-BASED DEVICES

A. The slope detection technique

In a very good approximation, micro- and nanomechanical oscillators, such as singly and doubly clamped cantilevers [Fig. 1(a)], can be modeled as damped harmonic oscillators. The one-dimensional motion is governed by the differential equation,

$$m\frac{d^2z}{dt^2} + \gamma\frac{dz}{dt} + kz = F_0\cos(\omega_d t) + F_{\text{ext}}(z,t), \tag{1}$$

where m is the effective mass of the cantilever, γ is the damping constant, k is the spring constant, $F_0\cos(\omega_d t)$ is the driving force, and $\omega_d = 2\pi f$ is the angular driving frequency. The resonant frequency, the mass, and the spring constant are related to each other through $\omega_0 = (k/m)^{1/2}$. F_{ext} represents the external forces exerted on the cantilever. The quality factor and γ are related each other by $Q = m\omega_0/\gamma$.

If there are no external forces acting on the cantilever, the solution of Eq. (1) is

$$z = A_t e^{-t/\tau} \cos(\sqrt{1 - 1/4Q^2}\omega_0 t + \theta) + A \cos(\omega_d t - \varphi). \quad (2)$$

.

The first and second terms are the transient and steady oscillations, respectively; where A_t and A are the amplitudes of the transient and steady oscillations and θ and φ are the phase lags of the transient and steady oscillations with respect to the driving force. The transient oscillation exponentially decays with a time-constant $\tau = 2Q/\omega_0$. For times longer than τ, the transient oscillation can be neglected, and the steady oscillation is the signal carrying the information about external interactions. The amplitude and the phase lag of the steady oscillation with respect to the driving force is determined by the transfer function of the cantilever response X_R,[4,29] where $Ae^{-i\varphi} = X_R(\omega_{\text{d}})F_0$,

$$X_R(\omega_d) = \frac{1}{m\left(\omega_0^2 - \omega_d^2\right) + i\gamma\omega_d}. \quad (3)$$

In a first-order approximation, the effect of the external forces can be accounted for by including a small variation of the mass (δm), spring constant (δk), or damping constant ($\delta\gamma$) in the transfer function of the cantilever response [Eq. (3)], where $\delta m \ll m$, $\delta k \ll k$, and $\delta\gamma \ll \gamma$. For instance, in biochemical sensors based on cantilevers, adsorption of molecules on the cantilever surface produces a small increase of the mass and a small change of the spring constant.[8–11] In scanning probe microscopy, the cantilever is used to measure small forces between the tip and the sample. For small oscillation amplitudes where the force (F) acting on the cantilever varies linearly with the distance, the effect of the force can be accounted for by a change of the spring constant $\delta k = -\partial F/\partial z$.[4] External forces that produce a change of the mass and spring constant of the cantilever can be measured via variations of the resonant frequency where $\delta\omega_0/\omega_0 \cong 1/2(\delta k/k - \delta m/m)$. Another kind of external interaction involves energy dissipation and can be written as $F_{\text{ext}} = \delta\gamma\, dz/dt$. These interactions can arise from changes of the local viscosity and changes of the mechanical energy loss of the cantilever material. Dissipative interactions are detected through Q variations.

Commonly, the slope detection is used to measure the external forces [Fig. 1(b)]. The cantilever is driven at a constant frequency near resonance, and the external forces are measured as variation in amplitude or phase of the steady oscillation.[4] The sensitivity of the measurement is proportional to the slope of the amplitude and phase curves near resonance. Therefore, high sensitivity is obtained by using cantilevers with small linewidth. The linewidth of the resonant peak is determined by the quality factor by[29]

$$Q = \sqrt{3}\frac{\omega_0}{\Delta\omega_{1/2}}, \quad (4)$$

where $\Delta\omega_{1/2}$ is the frequency width at half of the maximum amplitude. The quality factor can also be written as $2\pi W_o/W_{\text{dis}}$, where W_o and W_{dis} are the stored vibrational energy and the energy lost per oscillation cycle, respectively. Since the energy loss is due to several dissipation mechanisms, Q can be expressed as $1/Q = \Sigma_i 1/Q_i$.[16–18] Dissipation mechanisms include internal friction of the cantilever, energy loss via clamping to the support structure, and viscous damping.

Elastic external interactions that shift the resonant frequency can be detected by either an amplitude or phase change. For detection of amplitude variations, the driving frequency should preferably be in the region of highest slope of the resonant amplitude peak,[4] which occurs for $\omega_d - \omega_0 = \pm\omega_0/(8^{1/2}Q)$ where $dA/d\omega_d \sim F_0 Q^2/(k\omega_0)$. The maximum variation in the phase is achieved at resonance where $d\varphi/d\omega_d = 2Q/\omega_0$.

Dissipative external interactions can be detected via variations of the amplitude at resonance, A_0,

$$\frac{\partial A_0}{\partial \varepsilon_{\text{dis}}} = -\frac{1}{\pi}\frac{F_0}{k}Q^2, \quad (5)$$

where ε_{dis} is the ratio between the energy loss (W_{dis}) and the stored vibrational energy (W_o) per oscillation cycle.

B. Amplitude and phase noise

There are at least four sources of noise, which limit the sensitivity of devices based on cantilevers: (i) thermal vibrations of the cantilever, (ii) noise from the sensor of the cantilever displacement, (iii) noise from the actuator that provides the driving force, and (iv) noise due to the processing of the output signal of the displacement sensor (amplification, filtering, analog-to-digital conversion). However, thermal forces are generally dominant with respect to the force noise from the actuator due to electrical noise and mechanical vibrations between the actuator and the cantilever. Similarly, the noise of the output signal of cantilever-based devices is usually dominated by the displacement sensor. Therefore, the dominant sources of noise, thermal vibrations and the displacement sensor, will be considered for the derivation of the sensitivity in the following [Fig. 2(a)].

The driving force from the actuator can be written as $F_A(t) = c_A V_D e^{i\omega_d t}$, where c_A is the actuator sensitivity relating voltage to the generated force, V_D is the amplitude of the voltage applied to the actuator, and ω_d is the driving frequency [note that $c_A V_D$ is F_0 in Eq. (1)]. The cantilever motion is given by $z = Ae^{i(\omega_d t - \varphi)} + \delta z_{\text{th}}(t)$, where the first term is the steady oscillation in response to the driving force F_A whose amplitude A is given by $|X_R(\omega_d)| c_A V_D$, and δz_{th} is the Brownian motion due to the thermal forces. The output voltage of the displacement sensor is given by $V_0 = c_s z + \delta V_s$, where c_s is the proportionality constant that relates displacement to voltage, and δV_s is the noise due to the detection system. The output signal of the oscillator motion can be written as

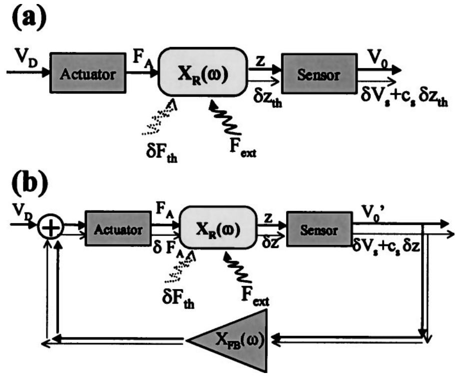

FIG. 2. Schematic of the flow of the signal and noise in cantilever-based devices without (a) and with (b) active control of the quality factor. Solid arrows represent signals, whereas dotted arrows represent noise.

$$V_0(t) = c_A c_s |X_R(\omega_d)| V_D e^{i(\omega_d t - \varphi)} + c_s \delta z_{\text{th}}(t) + \delta V_s(t). \quad (6)$$

Thermal noise arises from the thermal coupling between the oscillator and the medium, which is provided through the collision of the surrounding molecules. Thermal forces (F_{th}) are uncorrelated for time scales much longer than the mean collision time, thus the autocorrelation function is $\langle F_{\text{th}}(t)F_{\text{th}}(t')\rangle = 2m\omega_0 k_B T/Q \times \delta(t-t')$.[15,16] Therefore the thermal force has a Gaussian distribution with zero mean and a white spectral density whose value is

$$\psi_{\text{th}}(\omega) = \frac{2}{\pi}\frac{k_B T k}{Q\omega_0}. \quad (7)$$

For a small bandwidth $\Delta\omega$, the modulus of the thermal force is $|\delta F_{\text{th}}| = (\psi_{\text{th}}\Delta\omega)^{1/2}$, and the resulting Brownian motion amplitude is $|X_R(\omega)||\delta F_{\text{th}}|$. Since the measured magnitudes are the amplitude and phase of the oscillation at the driving frequency, we consider only the frequency component of the noise around ω_d. Then, thermal forces can be written as $\delta F_{\text{th}}(t) = (\psi_{\text{th}}\Delta\omega)^{1/2}\exp(i[\omega_d t + \alpha_{\text{th}}(t)])$, where $\alpha_{\text{th}}(t)$ is the uncorrelated phase that varies slowly with the time at an average frequency smaller than the measurement bandwidth. Similarly, the sensor noise can be written as $\delta V_s(t) = (S_s\Delta\omega)^{1/2}\exp(i[\omega_d t + \alpha_s(t)])$, where S_s is the spectral density of the displacement sensor noise and $\alpha_s(t)$ is the uncorrelated phase. In the slope detection technique, the bandwidth is limited by the quality factor due to the transient oscillation as shown earlier. Since the transient time constant is $\tau = 2Q/\omega_0$, the bandwidth is limited to values $\Delta\omega/\omega_0 \ll \pi/Q$. This allows one to write the output signal as

$$V_0(t) = \left[c_A c_s |X_R(\omega)| V_D + c_s |X_R(\omega)| \sqrt{\psi_{\text{th}}\Delta\omega}\, e^{i\alpha_{\text{th}}(t)} + \sqrt{S_s\Delta\omega}\, e^{i\alpha_s(t)}\right] e^{i(\omega_d t - \varphi)}. \quad (8)$$

Hereafter, it will be assumed that the driving force oscillates at resonance, where $\omega_d = \omega_0$ and $\varphi = \pi/2$. As shown in Sec. III A, elastic and dissipative interactions can be measured at resonance with high sensitivity via phase and amplitude changes, respectively. The electrical output signal [Eq. (8)] can be written as

$$V_0 = a(\alpha_{\text{th}}, \alpha_s) e^{i\phi(\alpha_{\text{th}}, \alpha_s)} e^{i(\omega_0 t - \pi/2)}, \quad (9)$$

where a and ϕ are the instantaneous amplitude signal and the instantaneous phase fluctuation, respectively. Since the thermal and displacement sensor noises are not correlated with the time, the signal and noise corresponding to the amplitude and phase can be calculated from the mean and rms values of a and ϕ, integrating for α_{th} and α_s. The obtained mean values are $a_m = \langle a\rangle \cong c_s Q F_0/k$ and $\langle\phi\rangle = 0$. The amplitude of the driving force is $F_0 = c_A V_D$. The noise of the amplitude signal and phase are given by

$$\delta a \equiv \left\langle (a - \langle a\rangle)^2 \right\rangle^{1/2} \cong \frac{1}{\sqrt{2}}\sqrt{\beta Q + S_s}\,\Delta\omega^{1/2}, \quad (10)$$

where

$$\beta = \frac{2}{\pi} c_s^2 \frac{\langle z_{\text{th}}\rangle^2}{\omega_0}. \quad (11)$$

The phase noise is proportional to the ratio between the amplitude noise and mean amplitude,

$$\delta\phi = \langle\phi^2\rangle^{1/2} \cong \frac{\delta a}{\langle a\rangle} = \frac{1}{\sqrt{2}}\frac{k\Delta\omega^{1/2}}{c_s F_0}\frac{\sqrt{\beta Q + S_s}}{Q}. \quad (12)$$

The phase and amplitude noises depend on the ratio between the parameters β and S_s, which gives an estimation of the relative contribution of the Brownian motion and displacement sensor noise to the overall noise. High values of β/S_s occur for oscillators with large thermal vibrations such as those with low spring constants and sensitive displacement sensors. This is the case of soft microcantilevers detected by the optical beam deflection technique, as shown in the following. In this case, the amplitude noise is proportional to $Q^{1/2}$, and the signal-to-noise ratio is $a_m/\delta a \sim Q^{1/2}$. The phase noise $\sim 1/Q^{1/2}$. Small β/S_s occurs for devices where the noise owing to the displacement detection dominates over the thermal vibrations of the oscillator. This occurs in microcantilevers with integrated piezoresistors for readout of the displacement, where the fabrication technology gives high spring constants and the resistance measurement is noisier than with optical techniques.[35] In this case, the amplitude noise is approximately independent of Q with a signal-to-noise ratio $a_m/\delta a \sim Q$, whilst the phase noise is $\sim 1/Q$.

A parameter is proposed to define the sensitivity of the device. For elastic interactions that shift the resonant frequency, I define the sensitivity (σ_{elas}) as the ratio between the resonant frequency-derivative of the phase shift ($\partial\varphi/\partial\omega_0$) and the phase noise at resonance ($\delta\varphi$). The calculation of the elastic sensitivity gives

$$\sigma_{\text{elas}} = 2\sqrt{2}\frac{\gamma}{S_s^{1/2}}\frac{Q^2}{\sqrt{\beta/S_s Q + 1}}, \quad (13)$$

where

$$\gamma = \frac{c_s F_0}{k\omega_0\Delta\omega^{1/2}}. \quad (14)$$

For dissipative interactions, the sensitivity is defined as the ratio between the derivative of the amplitude signal at resonance with respect to the dissipated-stored energy ratio, $\partial a/\partial\varepsilon_{\mathrm{dis}}$, and the amplitude noise at resonance (δa). The dissipation sensitivity is then given by

$$\sigma_{\mathrm{dis}} = \frac{\sqrt{2}}{\pi}\frac{\gamma\omega_0}{S_s^{1/2}}\frac{Q^2}{\sqrt{\beta/S_sQ+1}}. \quad (15)$$

As can be noted in Eqs. (13) and (15), the sensitivity of cantilever-based devices follows the same Q dependence for dissipative and elastic interactions, $\sim Q^2/(\beta Q/S_s+1)^{1/2}$. Higher Q, higher sensitivity; however the enhancement depends on the relative contributions of the displacement sensor and Brownian motion to the overall noise. For devices where the noise is dominated by the Brownian motion ($\beta \gg S_s$), the sensitivity is $\sim Q^{3/2}$, approximately. The enhancement is more pronounced for $\beta \ll S_s$ where the noise is dominated by the displacement sensor, and the sensitivity increases as $\sim Q^2$, approximately.

IV. EFFECT OF THE Q ENHANCEMENT VIA A FEEDBACK AMPLIFIER ON THE SIGNAL AND NOISE OF CANTILEVERS

A. Theory

The quality factor of cantilevers can be externally increased by a fast feedback amplifier that amplifies and delays the signal corresponding to the vibration, V_0', and feeds this signal back to the actuator.[19,28–31] The delay time is adjusted to produce a shift of 90° at the resonant frequency. The output signal from the actuator, for a feedback amplifier gain g, can be written as

$$F_A'(t) = c_A V_D e^{i\omega_d t} + e^{i\pi/2} c_A g V_0'(t). \quad (16)$$

The steady-state oscillation is obtained by solving the closed-loop equation $V_0'(t) = c_s X_R(\omega_d) F_A'(t) + c_s \delta z_{\mathrm{th}}(t) + \delta V_s(t)$, in which $X_R(\omega_d) = |X_R(\omega_d)| e^{-i\varphi}$ [Fig. 2(b)],

$$V_0'(t) \cong \left[c_A c_s |X_R'(\omega_d)| V_D + c_s |X_R'(\omega_d)| \sqrt{\psi_{\mathrm{th}}\Delta\omega} e^{i\alpha_{\mathrm{th}}(t)} + \frac{|X_R'(\omega_d)|}{|X_R(\omega_d)|}\sqrt{S_s\Delta\omega} e^{i\alpha_s(t)} \right] e^{i(\omega_d t-\varphi)}, \quad (17)$$

where

$$X_R'(\omega) = \frac{X_R(\omega)}{1 - i c_s c_A g X_R(\omega)}. \quad (18)$$

X_R' is the transfer function of a harmonic oscillator that corresponds to the physical properties of the cantilever except for the quality factor that is increased to Q'. The relationship between the modified quality factor Q' and the feedback amplifier gain is

$$g = \frac{k}{c_s c_A}\left(\frac{1}{Q} - \frac{1}{Q'}\right). \quad (19)$$

Hereafter, Q and Q' will be referred to as the natural and effective quality factors, respectively. The natural quality factor is determined by the mechanical energy dissipation of the cantilever that depends mainly on the internal energy loss of the cantilever material and of the viscous damping. For instance, the natural quality factor can be increased by lowering the pressure of the environment. Effective quality factor is that obtained by applying the fast feedback amplifier, and is determined by the gain g.

The effect of the feedback amplifier on the cantilever oscillation can be deduced by analyzing Eq. (17). First, if the noise sources (the Brownian motion and the displacement sensor noise) are neglected, then the output signal is determined by the new cantilever response to the driving force [the first term of Eq. (17)]. Therefore, the cantilever behaves as having a quality factor Q' that is determined by the gain of the feedback amplifier g [Eq. (19)].

Now, let us add the thermal forces [the second term of Eq. (17)], which produce the superposition of the Brownian motion on the driven oscillation. The Brownian motion is the product of the transfer function modulus of the modified cantilever response with the quality factor Q', $|X_R'(\omega)|$, and the thermal force amplitude $(\psi_{\mathrm{th}}\Delta\omega)^{1/2}$ [Eq. (7)]. Therefore, the frequency distribution of the thermal noise is determined by $|X_R'(\omega)|$, consistent with a cantilever having a quality factor Q'. However, the magnitude of the thermal noise is higher than the noise of a cantilever having a natural quality factor Q'. This is because the feedback amplifier cannot alter the motion of the surrounding molecular impinging on the mechanical oscillator, and the thermal force is determined by the natural quality force, being $\sim 1/Q^{1/2}$ instead of $\sim 1/Q'^{1/2}$. Therefore, the amplified Brownian motion exhibits a frequency distribution consistent with a quality factor Q', but with a magnitude $(Q'/Q)^{1/2}$ times higher.[31]

Finally, the displacement sensor noise [the third term of Eq. (17)] is also altered by the feedback amplifier. When the feedback amplifier is not applied, this noise is independent of the quality factor, and it only depends on the characteristics of the sensor. However, external Q enhancement produces the feedback of the displacement sensor noise to the actuator through the feedback amplifier. This results in the amplification of this noise $|X_R'|/|X_R|$ times, that is approximately Q'/Q times for frequencies close to resonance.

As in Sec. III B, the output signal [Eq. (17)] is calculated at resonance, and it can be written as $V_0' = a'(\alpha_{\mathrm{th}},\alpha_s)\exp[i\phi'(\alpha_{\mathrm{th}},\alpha_s)]\exp[i(\omega_0 t-\pi/2)]$, where a' and ϕ' are the instantaneous amplitude signal and the instantaneous phase fluctuation around $\pi/2$, respectively. Thereby integrating for α_{th} and α_s, the mean and rms values of a' and ϕ' can be obtained. The mean amplitude signal and phase fluctuation are $a_m' \equiv \langle a'\rangle \cong c_s Q' F_0/k$ and $\langle\phi'\rangle = 0$. This indicates that the mean values of the amplitude and phase are determined by the new cantilever response, i.e., the signal corresponds to a cantilever with a quality factor Q'. The amplitude and phase noises are given by,

$$\delta a' \equiv \langle (a'^2 - \langle a'\rangle^2)\rangle^{1/2} \cong \frac{1}{\sqrt{2}}\Delta\omega^{1/2}\sqrt{\beta Q + S_s}\frac{Q'}{Q}, \quad (20)$$

$$\delta\phi' = \langle \phi'^2\rangle^{1/2} \cong \frac{1}{\sqrt{2}}\frac{k\Delta\omega^{1/2}}{c_s F_0}\frac{\sqrt{\beta Q + S_s}}{Q}. \quad (21)$$

As it was anticipated above, the noise is not consistent with a cantilever having a quality factor Q', it exhibits a Q'

dependence that differs with respect to the dependence with the natural quality factor [Eqs. (10) and (12)]. Independent of the origin of the noise, thermo-mechanical and/or from the displacement sensor, the amplitude noise is Q'/Q times the noise without Q enhancement, and it increases more quickly with the effective quality factor than with the natural quality factor. This leads to $a'_m/\delta a'$ amplitude signal-to-noise ratio being independent of the effective quality factor, whilst it increases with the natural quality factor between $\sim Q^{1/2}$ and $\sim Q$, depending on the nature of the noise (Sec. III B). The phase noise remains constant, not decreasing with Q' as it does when the natural quality factor increases.

The calculation of the sensitivity for dissipative (σ'_{elas}) and elastic (σ'_{dis}) interactions gives Q'/Q times the sensitivity without external Q enhancement [shown in Eqs. (13) and (15)]. Thus, the sensitivity for both interactions, elastic and dissipative, is proportional to the effective quality factor Q'. However, this enhancement is smaller than that obtained when the natural quality factor is increased, where the sensitivity is approximately proportional to $Q^{3/2}$ or Q^2, depending on the dominant source of noise (see Sec. III B).

B. Effect of actuator and sensor displacement limits on the *Q* enhancement

Although the sensitivity increases with Q', it must be taken into account that Q enhancement is limited by the amount of feedback tolerated by the cantilever-based device. This is mainly determined by the maximum force that can be provided by the actuator, F_{max}. During Q enhancement, the maximum actuator output is provided at resonance where coherence excitation is obtained. The amplitude of the actuator force, for a constant driving force amplitude F_0, is

$$|F'_A(t)| = \sqrt{\left(\frac{Q'}{Q}F_0\right)^2 + 2\left(\frac{Q'}{Q} - 1\right)^2 \delta F^2}, \tag{22}$$

where δF is referred to as the force noise given by

$$\delta F = \frac{k}{c_s Q}\delta a, \tag{23}$$

where δa is given by Eq. (10).

Given F_{max}, the maximum effective quality factor that can be achieved is

$$\frac{Q'_{\mathrm{max}}}{Q} = \frac{1}{2 + (F_0/\delta F)^2}\left\{2 + \sqrt{4 + \left[\left(\frac{F_{\mathrm{max}}}{\delta F}\right)^2 - 2\right]\left[\left(\frac{F_0}{\delta F}\right)^2 + 2\right]}\right\}. \tag{24}$$

Taking into account the limitation of the amount of feedback for Q enhancement, we compare the sensitivity of the cantilever without Q enhancement applying the maximum driving force to maximize the amplitude signal-to-noise ratio and minimize the phase noise [Eqs. (10) and (12)], to the sensitivity with Q enhancement with the maximum effective quality factor for a given driving force F_0. The ratio between both sensitivities ($\sigma'_{\mathrm{max}}/\sigma_{\mathrm{max}}$) with and without Q enhancement is plotted in Fig. 3. This calculation concludes that, independent of the noise sources and the type of measured interaction, elastic or dissipative, the sensitivity is not improved with the enhancement of the quality factor via a feedback amplifier, it can even be degraded for small values of the fixed driving force F_0. This is a consequence of (i) the amplification of the thermo-mechanical and displacement sensor noises by Q'/Q and (ii) the limit of the actuator.

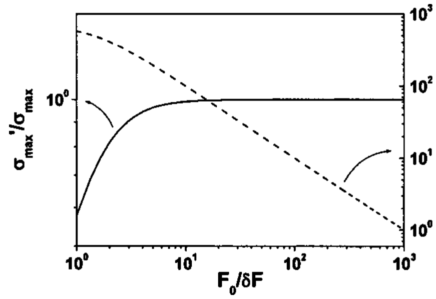

FIG. 3. Left axis corresponds to the ratio between the maximum sensitivities of a device based on micro-nanomechanical resonators with and without Q enhancement as a function of the amplitude of the constant driving force (F_0) normalized with respect to the force noise, δF. Right axis corresponds to the maximum effective quality factor achieved as a function of the constant driving force. The maximum excitation force is set to 1000 times the force noise. The maximum sensitivity without Q enhancement is achieved by applying the maximum driving force at resonance. With Q enhancement, the maximum sensitivity is obtained with the highest quality factor that depends on the driving force amplitude F_0.

C. Experimental characterization of the signal and noise with *Q* enhancement

Figure 4(a) shows the amplitude and its noise as a function of the driving frequency for a silicon nitride microcantilever with a nominal spring constant $k = 0.1$ N/m that has been previously coated with a thin layer of cobalt for magnetic actuation. The noise was measured by acquiring 200

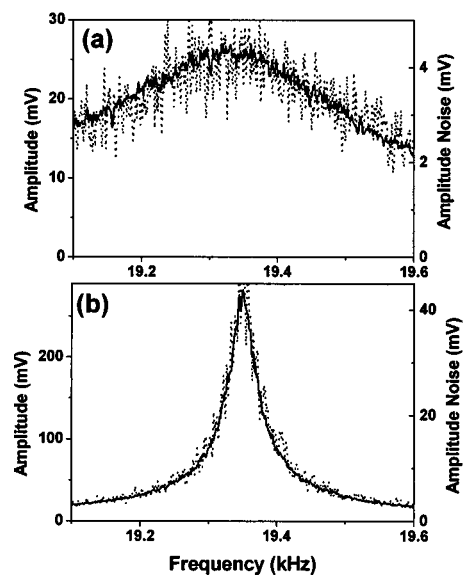

FIG. 4. Experimental frequency-dependence of the cantilever amplitude (solid line) and its noise (dotted line) without (a) and with Q enhancement (b). The natural Q is of about 55, and the enhanced Q' is of about 555. The experimental was performed in air at room temperature.

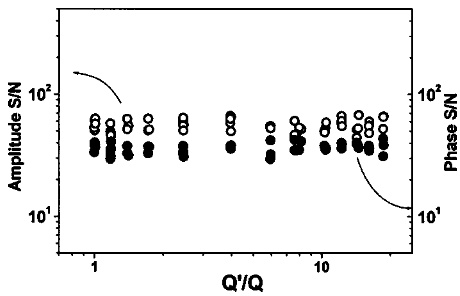

FIG. 5. Experimental signal-to-noise ratio of the amplitude (open symbols) and phase shift (closed symbols) of the cantilever oscillation, as a function of the effective quality factor that is manually controlled by the gain of the feedback amplifier. The signal-to-noise of the phase is calculated from the ratio between 180 and the measured phase noise in degrees. Natural resonant frequency and quality factor were 19.3 kHz and 55, respectively. Experiments were performed in air at room temperature.

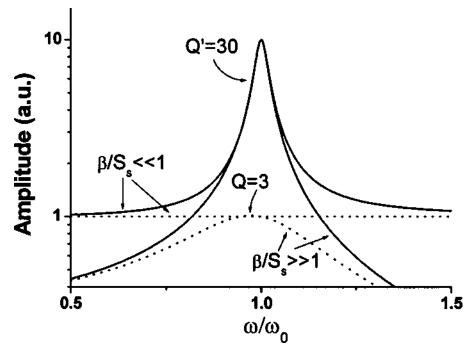

FIG. 6. Theoretical amplitude signal spectrum for a cantilever-based device without a reference driving frequency, with (solid line) and without (dotted line) the application of the fast feedback amplifier to increase the natural quality factor $Q=3$ to $Q'=30$. Two asymptotic cases are depicted, devices where the dominant source of noise arises from the displacement sensor ($\beta/S_s \ll 1$), and devices where the noise is of thermomechanical nature ($\beta/S_s \gg 1$).

values of the amplitude per frequency value, and calculating the rms value. A resonant frequency and a Q of 19.32 kHz and 55 were determined by fitting the amplitude curve with the harmonic oscillator model [Eq. (3)]. The frequency distribution of the amplitude noise follows the same frequency dependence of the amplitude, indicating that the dominant source is the Brownian cantilever motion [see Eq. (8)]. If the noise would be dominated by the displacement sensor, in this case the photodetector, the amplitude noise would exhibit a flat frequency distribution, approximately. Coherent amplification of the microcantilever oscillation at resonance by the feedback amplifier increases the quality factor to 555 [Fig. 4(b)]. The amplitude noise also exhibits the same frequency distribution, consistent with the thermal noise of a cantilever having the enhanced quality factor.

Figure 5 shows the experimental measurement of the signal-to-noise ratio as a function of the effective quality factor that is controlled by manually adjusting the feedback amplifier gain. Phase signal-to-noise ratio is defined as the ratio between 180 and the phase noise in degrees. The experimental data show that both the phase and amplitude signal-to-noise ratios are approximately independent of the effective quality factor. This confirms the theoretical result achieved in Sec. IV A. Notice that the increase of the natural quality factor would produce an increase of the signal-to-noise ratio $\sim Q^{1/2}$ as the noise is dominated by the thermal forces in these experimental conditions.

The experimental results shown in Fig. 5 lead to the conclusion that the same cantilever-sensor sensitivity can be achieved without Q enhancement by maximizing the driving force as demonstrated in Sec. IV B. In conclusion, theory and experiments indicate that Q enhancement by application of a fast feedback amplifier does not produce enhancement of the sensitivity of cantilever-based devices using the slope detection technique, the sensitivity can even be degraded for small values of the driving force (see Fig. 3 and related text).

V. THE SELF-EXCITATION TECHNIQUE

In the slope detection technique, the frequency of the constant vibrating force that drives the cantilever serves as reference to monitor the oscillation changes. Normally this frequency is near resonance and the amplitude and phase shift of the oscillation are measured. Thus, the spectral density of the motion consists of a δ-function peak at the driving frequency with a small contribution of noise at the sidebands superposed. However, if we ignore the displacement sensor noise, the driving force could be removed, and the cantilever would be driven by the thermal forces that exhibit a white spectral density. Hence the spectral density of the cantilever motion would be determined only by the modulus of the transfer function of the cantilever response, $|X_R(\omega)|$, that corresponds to a damped harmonic oscillator [Eq. (3)]. Measurement of the thermal oscillation would allow determination of the resonant frequency and its variation. However, a Brownian motion signal much higher than the noise from the displacement sensor is an experimental situation not always achieved. Here, this is circumvented by amplification of the noise through self-excitation of the cantilever via the feedback amplifier used for Q enhancement. This results in an output signal from the displacement sensor with the following spectral density:

$$N(\omega) = (c_s|X_R'(\omega_d)|)^2\psi_{\rm th} + \left(\frac{|X_R'(\omega_d)|}{|X_R(\omega_d)|}\right)^2 S_s. \quad (25)$$

The oscillation amplitude at a frequency ω is $(N(\omega)\Delta\omega)^{1/2}$, where $\Delta\omega$ is the measurement bandwidth. Figure 6 shows the theoretical amplitude as a function of the frequency without driving force, for a natural quality factor of 3 that is enhanced to an effective value of 30 by the feedback amplifier. When the dominant source of noise is the thermal force ($\beta/S_s \gg 1$), the frequency distribution of the noise corresponds to a damped harmonic oscillator with a quality factor of 30. In this situation, the spectral density is described by the first term of Eq. (25), hence the amplitude spectrum can be written as $|X_R'(\omega)|(\psi_{\rm th}\Delta\omega)^{1/2}$, where $X_R'(\omega)$ is the oscillator response with the effective quality factor $Q'=30$, and $(\psi_{\rm th}\Delta\omega)^{1/2}$ represents the thermal force for a

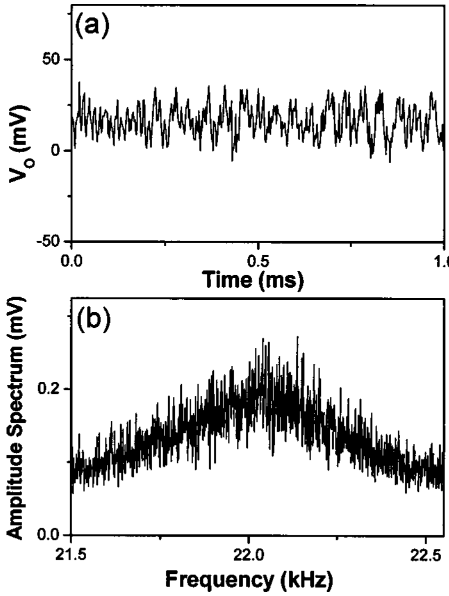

FIG. 7. Experimental measurement of the cantilever vibration noise (a) and its Fourier transform (b) in the absence of an external driving force. The resonant frequency and the nominal spring constant were 22 kHz and 0.10 N/m, respectively. A natural quality factor of 45 is determined by fitting curve (b) with the damped harmonic oscillator model. Experiments were performed in air at room temperature.

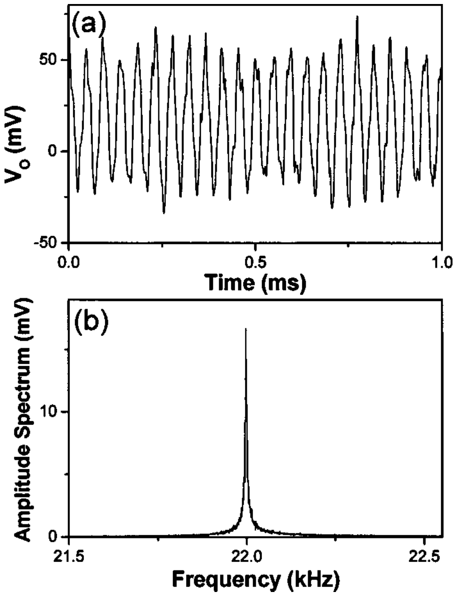

FIG. 8. Experimental measurement of the cantilever vibration noise (a) and its Fourier transform (b) in the absence of an external driving force and after application of the feedback amplifier. The natural quality factor of about 45 is enhanced to 7810. The experimental conditions and the cantilever are the same as those in Fig. 7.

bandwidth $\Delta\omega$. More interestingly, the noise narrowing around the resonant frequency can also be obtained for devices, in which the noise arises from the displacement sensor ($\beta/S_s \ll 1$). In these devices, the amplitude noise spectrum is flat prior to application of the feedback amplifier. However, the feedback of the white noise from the displacement sensor produces a filtering through the cantilever resonance, giving an amplitude spectrum proportional to the ratio between the effective and natural cantilever responses $|X_R'(\omega)|/|X_R(\omega)|$. For frequencies close to resonance, $X_R(\omega) \cong X_R(\omega_0)$, and the response of the cantilever is $\sim X_R'(\omega)$, approximately.

Figures 7 and 8 show the experimental microcantilever "noise" motion and the corresponding amplitude spectrum before and after application of the feedback amplifier, respectively. Here, a reference driving force has not been applied. When the feedback amplifier is not applied, the frequency of the noise motion [Fig. 7(a)] can be hardly discerned, however the Fourier transform [Fig. 7(b)] clearly shows a relatively broad distribution around the resonance frequency that can be fitted with the damped harmonic oscillator model, indicating the thermal nature of the noise. The measured natural quality factor is about 45. Application of the feedback amplifier produces the self-excitation of the cantilever by its thermal fluctuations, narrowing the frequency distribution of the motion around resonance. In fact, the resonant frequency can be clearly distinguished in the resulting cantilever motion [Fig. 8(a)]. The amplitude spectrum corresponds to an effective quality factor of about 7810 [Fig. 8(b)]. Similar to the slope detection technique, the maximum effective quality factor is restricted by the limit of the actuator (Sec. IV B). However, higher Q enhancement can be obtained in the self-excitation technique than in the slope detection technique where a constant driving force of about 10 times the force noise is necessary, restricting the actuation room for the feedback amplifier.

In the slope detection technique, the bandwidth and the quality factor are not independent. The bandwidth is determined by the decay time-constant of the transient oscillation that is proportional to the effective quality factor Q'. In the self-excitation technique, as the dominant frequency of the noise is measured instead of the amplitude and phase, the bandwidth is not limited by either the natural or the externally controlled quality factor. The bandwidth is determined by the characteristics of the frequency measurement technique that depends also on the required frequency range and resolution.

The frequency noise arises from the vibration at the sidebands of the resonant frequency, hence smaller bandwidths as well as higher quality factors improve the frequency resolution. The mean and rms values of the frequency can be obtained by averaging with the spectral noise density, $N(\omega)$ [Eq. (25)], as the statistical weight function. Thus, the average frequency is given by

$$\langle\omega\rangle = \frac{\int_{\omega_0-\Delta\omega/2}^{\omega_0+\Delta\omega/2} \omega N(\omega) d\omega}{\int_{\omega_0-\Delta\omega/2}^{\omega_0+\Delta\omega/2} N(\omega) d\omega}. \tag{26}$$

For bandwidths $\Delta\omega \ll \omega_0$, the average and noise values of the frequency can be analytically calculated. This gives $\langle\omega\rangle \cong \omega_0$ and $\delta\omega = \langle(\omega-\omega_0)^2\rangle^{1/2}$ given by

$$\frac{\delta\omega}{\omega_0} = \frac{1}{2\sqrt{3}}\sqrt{\frac{\left[3\beta Q + 3S_s + \left(\frac{Q'}{Q}x\right)^2 S_s\right]x}{S_s Q^2 x + [\beta Q Q'^2 + (Q'^2 - Q^2)S_s]\arctan x} - \frac{3}{Q'^2}}, \quad (27)$$

where $x = Q\Delta\omega/\omega_0$. When the dominant source of noise is the thermal motion of the cantilever ($\beta \gg S_s$), Eq. (27) gives $(\delta\omega/\omega_0)^2 = 1/2\pi(\Delta\omega/\omega_0)1/Q$, which is similar to the frequency resolution deduced by Albrecth *et al.*[33]

The frequency noise was experimentally measured as function of the effective quality factor Q' for $\Delta\omega = 800$ Hz (Fig. 9). The natural quality factor Q was 45 and the resonant frequency was 22 kHz, approximately. A good agreement was found between the experimental data (symbols) and theory (red line) for the asymptotic case, in which the dominant source of noise is the Brownian motion of the cantilever ($\beta \gg S_s$). Here, we have obtained values of Q' of about 50 000, which would give a frequency resolution of about 0.1 Hz for a bandwidth of 1 Hz, approximately.

VI. CONCLUSIONS

It seems intuitive that the thermo-mechanical limits should not be beaten with external electronics. Coherent amplification of the cantilever oscillation produces the increase of the quality factor. As long as the cantilever-based device is noise-free, no difference is found between cantilevers with effective quality factors determined by external electronics and cantilevers with natural quality factors determined by the physical properties. However, thermal noise is always present, and its amplification by the positive feedback loop produces a contradiction between the quality factor determined from the cantilever response that depends on the gain of the feedback amplifier and the quality factor determined from the thermal forces that is not altered by the feedback amplifier, and it depends on the thermal coupling of the cantilever with the surrounding medium. The main consequence of this contradiction is that the sensitivity is not enhanced in the slope detection technique by externally increasing the quality factor. However, if the driving force is not applied and the feedback amplifier is used for self-excitation of the cantilever, the "apparent" Brownian motion is consistent with an enhanced quality factor. This allows the direct measurement of the resonant frequency with high sensitivity.

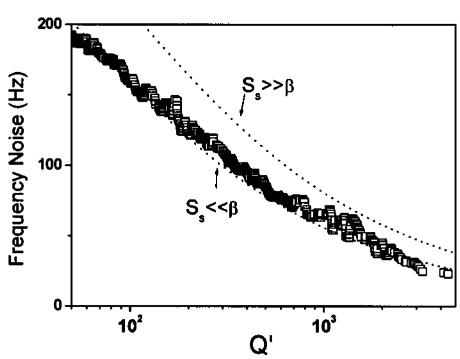

FIG. 9. Experimental measurement of the frequency noise (symbols) of a microcantilever in the self-excitation technique, as a function of the effective quality factor for a bandwidth of 800 Hz. The resonant frequency, spring constant, and natural quality factor are 22.0 kHz, 0.1 N/m, and 45, respectively. The theoretical frequency noise is also depicted (lines) for the asymptotic cases where the dominant source of noise is either the cantilever thermal motion ($\beta \gg S_s$) or the displacement sensor ($\beta \ll S_s$).

ACKNOWLEDGMENT

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