

Superconductivity from gauge/gravity duality with flavor

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Abstract

We consider thermal strongly-coupled $\mathcal{N} = 2$ SYM theory with fundamental matter at finite isospin chemical potential. Using gauge/gravity duality, i. e. a probe of two flavor D7-branes embedded in the AdS black hole background, we find a critical temperature at which the system undergoes a second order phase transition. The critical exponent of this transition is one half and coincides with the result from mean field theory. In the thermodynamically favored phase, a flavor current acquires a vev and breaks an Abelian symmetry spontaneously. This new phase shows signatures known from superconductivity, such as an infinite dc conductivity and a gap in the frequency-dependent conductivity. The gravity setup allows for an explicit identification of the degrees of freedom in the dual field theory, as well as for a dual string picture of the condensation process.

Key words: Gauge/gravity correspondence, D-branes, Black Holes.

PACS: 11.25.Tq, 11.25.Uv, 04.70.Bw, 74.20.-z

Gauge/gravity duality is a powerful tool to compute observables of field theories at strong coupling. The original duality for superconformal field theories has been extended to field theories at finite temperature [1], using an AdS black hole as gravity dual. Moreover, the original correspondence has been extended to field theories with flavor degrees of freedom in the fundamental representation of the gauge group and to finite densities. In [2], a finite $SU(2)$ isospin density \tilde{d} was considered. At high isospin densities $\tilde{d} > \tilde{d}_c$, this configuration becomes unstable against flavor current fluctuations, corresponding to vector mesons [3].

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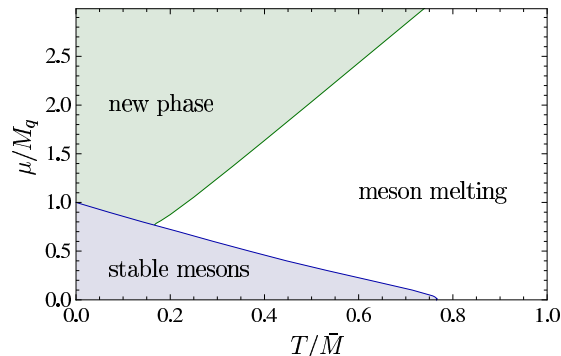


Figure 1: Phase diagram for fundamental matter in thermal strongly-coupled $\mathcal{N} = 2$ SYM theory [2], with μ the isospin chemical potential, M_q the bare quark mass, $\bar{M} = 2M_q\lambda^{-1/2}$, λ the 't Hooft coupling and T the temperature: In the blue shaded region, mesons are stable. In the white and green regions, the mesons melt. Here the new phase is stabilized while it was unstable in [2]. In this phase we find some features known from superfluids.

The central result of the present letter is that there is a new stable phase characterized by a ϱ -meson superfluid. We show explicitly that the new phase shown in fig. 1 is thermodynamically preferred. Moreover, we check that the phase is stable (at least against the fluctuations which caused the instability beforehand).

Moreover we find that this phase has properties of a superconductor, such as a second order phase transition with a critical exponent of $1/2$. The flavor current, which is analogous to the electromagnetic current, displays infinite dc conductivity and a gap in the frequency-dependent conductivity. Furthermore, we give a dual string-theoretical picture of the Cooper pairs and find a dynamical generation of meson masses similar to the Higgs mechanism. The stabilization mechanism is motivated by the results of [4, 5]. In the context of a phenomenological AdS/QCD model, a similar stabilization mechanism was first used in [6] to obtain a p-wave superconductor.

Our results provide an important link between gauge/gravity with flavor and the recent gauge/gravity realizations of superconducting states e. g. in [6, 7, 8], as well as a realization of a superconducting current from a top-down (super-) gravity model, rather than in a phenomenological bottom-up AdS/QCD model. Moreover, the degrees of freedom in the dual field theory are identified explicitly.

1. FIELD THEORY INTERPRETATION

In this letter we consider 3+1-dimensional $\mathcal{N} = 2$ supersymmetric Yang-Mills theory, consisting of a $\mathcal{N} = 4$ gauge multiplet as well as two $\mathcal{N} = 2$ supersymmetric hypermultiplets, called (ϕ_u, ψ_u) and (ϕ_d, ψ_d) . If the masses are degenerate, the theory has a global $U(2)$ flavor symmetry.

A finite isospin chemical potential μ is introduced as the source of the oper-

ator

$$J_0^3 \propto \bar{\psi} \tau^3 \gamma_0 \psi + \phi \tau^3 \partial_0 \phi = n_u - n_d, \quad (1)$$

where $n_{u/d}$ is the charge density of the isospin fields, $(\phi_u, \phi_d) = \phi$ and $(\psi_u, \psi_d) = \psi$. τ^i are the usual Pauli matrices. A non-zero vev $\langle J_0^3 \rangle$ introduces an isospin density as discussed in [2]. The isospin chemical potential μ explicitly breaks the $U(2) \simeq U(1)_B \times SU(2)_I$ flavor symmetry down to $U(1)_B \times U(1)_3$, where $U(1)_3$ is generated by the unbroken generator τ^3 of the $SU(2)_I$. Under the $U(1)_3$ symmetry the fields with index u and d have positive and negative charge, respectively.

In this letter we show that above a critical value for the isospin density the system is stabilized by a ρ -meson superfluid, i.e. a state with a non-vanishing vev of the flavor current component

$$\begin{aligned} J_3^1 &\propto \bar{\psi} \tau^1 \gamma_3 \psi + \phi \tau^1 \partial_3 \phi \\ &= \bar{\psi}_u \gamma_3 \psi_d + \bar{\psi}_d \gamma_3 \psi_u + \text{bosons}. \end{aligned} \quad (2)$$

The vev has p-wave symmetry and breaks both the $SO(3)$ rotational symmetry as well as the remaining Abelian $U(1)_3$ flavor symmetry. The rotational $SO(3)$ is broken down to $SO(2)_3$, which is generated by rotations around the x^3 axis. These symmetries are spontaneously broken. However only the $U(1)_3$, which acts on the gauge field on the D7 brane probe, is a dynamical symmetry in our approach, since we do not consider the backreaction of the D7 brane gauge field on the metric. This implies that we hold the metric background and thus also its symmetries fixed. Consequently only one Nambu-Goldstone boson is visible in our approach, namely the one due to spontaneous breaking of the dynamical $U(1)_3$.

Later we will compare our results to properties of superconductors: the condensate $\langle J_3^1 \rangle$ is the analog to the Cooper pairs of the BCS theory. Further, the $U(1)_3$ is the analog to the $U(1)_{\text{em}}$ and the current J^3 to the electric current J_{em} .

2. HOLOGRAPHIC SETUP

We consider asymptotically $AdS_5 \times S^5$ spacetime which is holographically dual to $\mathcal{N} = 4$ super-Yang-Mills theory with gauge group $SU(N_c)$. The dual description of a finite temperature theory is an AdS black hole [1]. We use the coordinates of [9],

$$ds^2 = \frac{\rho^2}{2R^2} \left(-\frac{f^2}{\tilde{f}} dt^2 + \tilde{f} d\vec{x}^2 \right) + \frac{R^2}{\rho^2} (d\rho^2 + \rho^2 d\Omega_5^2), \quad (3)$$

with

$$f = 1 - \frac{\rho_H^4}{\rho^4}, \quad \tilde{f} = 1 + \frac{\rho_H^4}{\rho^4}, \quad (4)$$

where R is the AdS radius and $d\Omega_5$ the metric of the unit 5-sphere.

Flavor degrees are added by embedding N_f coincident probe D7 branes with $N_f \ll N_c$ into the AdS black hole background [10, 11, 12]. Here we restrict ourselves to the case of zero quark mass and only comment on our massive results. In the massive case the physical interpretation does not change. For zero quark mass the induced metric G on the D7-branes is

$$ds^2(G) = \frac{\varrho^2}{2R^2} \left(-\frac{f^2}{\tilde{f}} dt^2 + \tilde{f} d\vec{x}^2 \right) + \frac{R^2}{\varrho^2} d\varrho^2 + R^2 d\Omega_3^2. \quad (5)$$

The Dirac-Born-Infeld (DBI) action determines the embedding of D7-branes as well as the gauge field on these branes,

$$S_{\text{DBI}} = -T_{D7} \int d^8 \xi \text{Str} \sqrt{|\det(G + 2\pi\alpha' F)|}, \quad (6)$$

with non-Abelian gauge field $F = dA + c\lambda^{-1/2} [A, A]$ on the D7-branes. The symmetrized trace prescription in this DBI action is only valid to fourth order in α' . However the corrections to the higher order terms are suppressed by N_f^{-1} . Here we use two different approaches to evaluate (6). First, we expand the DBI action to fourth order. Second, to make the calculation of the full action (6) feasible, we modify the symmetrized trace prescription by omitting the commutators of the generators τ^i and setting $(\tau^i)^2 = 1$. The numerics presented are obtained by the second prescription. However for both approaches we find properties of a superconductor, the critical exponent of one half up to an error of ten percent and a second order phase transition. The numerical values of thermodynamical quantities differ in the two approaches due to the modification of the effective action for the meson interaction. Still, the qualitative behavior remains the same.

In the following we restrict to $N_f = 2$ flavors. An isospin chemical potential μ may be introduced by a non-vanishing time component of the non-Abelian gauge field $A_0 = A_0^3(\varrho) \tau^3$ on the D7-brane probe [2]. The operator J_3^1 (see (2)) is dual to the non-vanishing gauge field component $A_3^1(\varrho)$ on the D7 branes.

The equations of motion for $A_0^3(\varrho)$ and $A_3^1(\varrho)$ obtained from the DBI action (6) are only satisfied if the asymptotic expansion near the boundary is of the form

$$A_0^3 = \mu - \frac{\tilde{d}_0^3}{2\pi\alpha'} \frac{\varrho_H}{\rho^2} + \dots, \quad A_3^1 = -\frac{\tilde{d}_3^1}{2\pi\alpha'} \frac{\varrho_H}{\rho^2} + \dots, \quad (7)$$

where $\rho = \varrho/\varrho_H$ is the dimensionless AdS radial coordinate. ϱ_H is the radius of the horizon. According to the standard AdS/CFT correspondence, the absence of a constant term in the expression for A_3^1 above ensures that the corresponding symmetry $U(1)_3$ is broken spontaneously and not explicitly. Moreover, according to AdS/CFT, μ is the isospin chemical potential and the parameters \tilde{d} are related to the vev of the currents J by

$$\tilde{d}_0^3 = \frac{2^{\frac{5}{2}} \langle J_0^3 \rangle}{N_f N_c \sqrt{\lambda T^3}}, \quad \tilde{d}_3^1 = \frac{2^{\frac{5}{2}} \langle J_3^1 \rangle}{N_f N_c \sqrt{\lambda T^3}}. \quad (8)$$

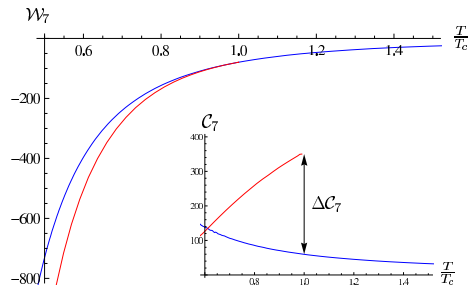


Figure 2: Dimensionless grand potential \mathcal{W}_7 (large plot) and specific heat \mathcal{C}_7 (small plot) versus temperature T/T_c . Blue: the unstable phase with $A_3^1 \equiv 0$. Red: the new stable phase with $A_3^1 \neq 0$. Note that the dimensionful specific heat $C_7 \propto T^3 \mathcal{C}_7$ is rising above T_c .

2.1. Legendre transformation

We Legendre-transform the action which gives rise to first order equations of motion which are easier to handle numerically. The conjugate momenta are defined by

$$p_0^3 = \frac{\delta S_{\text{DBI}}}{\delta(\partial_\rho A_0^3)}, \quad p_3^1 = \frac{\delta S_{\text{DBI}}}{\delta(\partial_\rho A_3^1)}. \quad (9)$$

These conjugate momenta are functions of the AdS radial coordinate ρ — and not constant in contrast to [2, 9, 13] — due to the non-Abelian term $A_3^1 A_0^3$ in the DBI action.

Using the asymptotics (7), the densities \tilde{d} may be calculated from the asymptotic value of the dimensionless conjugate momenta $\tilde{p} = p/(2\pi\alpha' N_f T_{D7} \rho_H^3)$, $\tilde{p}_0^3 \rightarrow \tilde{d}_0^3$ and $\tilde{p}_3^1 \rightarrow -\tilde{d}_3^1$. The Legendre-transformed action is given by

$$\tilde{S}_{\text{DBI}} = -N_f T_{D7} \int d^8 \xi \sqrt{-G} \left[\left(1 - \frac{2c^2 (\tilde{A}_0^3 \tilde{A}_3^1)^2}{\pi^2 \rho^4 f^2} \right) \left(1 + \frac{8(\tilde{p}_0^3)^2}{\rho^6 \tilde{f}^3} - \frac{8(\tilde{p}_3^1)^2}{\rho^6 \tilde{f} f^2} \right) \right]^{\frac{1}{2}}. \quad (10)$$

The action simplifies to the result of [2] if $A_3^1 \equiv 0$. A non-zero gauge field A_3^1 , which also implies a non-zero conjugate momentum p_3^1 , decreases the value of the Legendre-transformed action \tilde{S}_{DBI} . Since the contribution of the flavors to the free energy F_7 is given by

$$F_7 = T \tilde{S}_{\text{DBI}}, \quad (11)$$

the phase with non-zero gauge field A_3^1 is thermodynamically favored. Since the action \tilde{S}_{DBI} diverges on-shell due to the infinite AdS volume, the action must be renormalized. Details may be found in [14, 2].

3. PHASE TRANSITION

In this section we study the second order phase transition to the phase with non-zero gauge field A_3^1 . We work in the grand-canonical ensemble. The

contribution of the flavors to the grand potential is given by the on-shell action S_{DBI}

$$\Omega_7 = \frac{\lambda N_c N_f V_3 T^4}{32} \mathcal{W}_7 = T S_{\text{DBI}}, \quad (12)$$

where the dimensionless quantity \mathcal{W}_7 is plotted in fig. 2. In this figure we also present the contribution of the flavors to the specific heat C_7 ,

$$C_7 = \frac{\lambda N_c N_f V_3 T^3}{32} \mathcal{C}_7 = T \frac{\partial^2 \Omega_7}{\partial T^2}. \quad (13)$$

The temperature scale is defined by

$$\frac{T}{T_c} = \frac{\tilde{\mu}_c}{\tilde{\mu}} = \left(\frac{(\tilde{d}_0^3)_c}{\tilde{d}_0^3} \right)^{\frac{1}{3}}, \quad (14)$$

with the dimensionless critical chemical potential $\tilde{\mu}_c \approx 2.85$ and density $(\tilde{d}_0^3)_c \approx 20.7$. Fig. 2 shows a smooth transition in the grand potential and a discontinuous step in the specific heat at the phase transition. This implies that the transition is second order.

From a fit to the numerical result for the order parameter \tilde{d}_3^1 near the critical density, we obtain the critical exponent of the transition to be $1/2$ up to an error of 10%, in agreement with the mean field theory result. Since the order parameter \tilde{d}_3^1 and the density \tilde{d}_0^3 increase rapidly as $T \rightarrow 0$, we expect that the probe approximation ignoring the backreaction of the D branes to the geometry breaks down near $T = 0$.

Let us compare our results to QCD. In QCD, the pion condensate is of course the natural state in isospin asymmetric matter. The condensation of a particle sets in if the isospin chemical potential is larger than the mass of this particle. According to this rule, the pions condense first in QCD since they are the Nambu-Goldstone bosons of the spontaneous chiral symmetry breaking and therefore the lightest particles. However the dual field theory which we consider in this letter is supersymmetric at zero temperature and therefore chiral symmetry cannot be broken spontaneously. In this supersymmetric theory, the vector and scalar mesons have the same mass at zero temperature. Due to finite temperature effects, the mass of the vector and scalar mesons can become different as we increase the temperature. It is a priori unclear which particle will condense. In our model we checked that the vector mesons condense first such that the ρ -meson condensation state, which we consider in this letter, is the physical ground state of our system near the phase transition.

Since the system shares similarities with superconductors, it is interesting to investigate the remnant of the Meissner-Ochsenfeld effect. Switching on a magnetic field $H_3^3 = F_{12}^3$ along the lines of [15, 16], we find a critical line in the magnetic field and temperature plane. This line separates the normal phase from the superconducting one at finite external magnetic field. Similarly to the discussion of [8], this suggests the existence of a Meissner phase at small magnetic field and temperature.

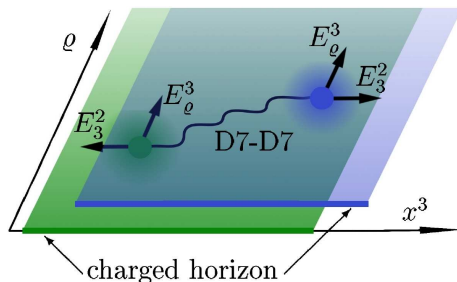


Figure 3: Sketch of our string setup: Strings spanned from the horizon of the AdS black hole to the D7-branes (green and blue plane) induce a charge at the horizon [2, 9, 17]. In the setup considered here, there are also D7-D7 strings present as shown in the figure. These D7-D7 strings are distributed along the AdS radial coordinate ρ , since they have to balance the flavorelectric and gravitational, i.e. tension forces. Thus these D7-D7 strings distribute the charges along the AdS radial coordinate, leading to a stable configuration of reduced energy. This corresponds to a superconducting condensate given by the Cooper pairs.

4. STRING THEORY PICTURE

In the string context, the non-zero fields A_0^3 and A_3^1 induce two non-zero flavorelectric $SU(2)$ fields $E_3^2 = F_{03}^2 = A_0^3 A_3^1$ and $E_\rho^3 = F_{0\rho}^3 = -\partial_\rho A_0^3$ as well as a non-zero flavormagnetic field $B_{3\rho}^1 = F_{3\rho}^1 = -\partial_\rho A_3^1$ on the D7-branes (we use the notation of [9, 2] with ρ being related to the radial AdS direction). These fields are generated by D7-D7 strings stretched between the two probe branes and by strings stretched from the D7-branes to the horizon. The D7-D7 strings are new to our setup. They move from the horizon into the bulk and thus distribute the isospin charge. This stabilizes the system.

Let us first describe the unstable configuration in absence of the field A_3^1 . As known from [17, 9, 2], the non-zero field A_0^3 is induced by fundamental strings which are stretched from the D7-brane to the horizon of the black hole. Since the tension of these strings would increase as they move to the boundary, they are localized at the horizon, i.e. the horizon is effectively charged. By increasing their density, the charge on the D7-brane at the horizon and therefore the energy of the system grows. In [2], the critical density was found beyond which this setup becomes unstable. In this case, the strings would prefer to move towards the boundary due to the repulsive force on their charged endpoints generated by the field E_ρ^3 .

The setup is now stabilized by the new non-zero field A_3^1 . This field is induced by D7-D7 strings moving in the x^3 direction. This may be interpreted as a current in x^3 direction which induces the magnetic field $B_{3\rho}^1 = -\partial_\rho A_3^1$.

Let us now explain the mechanism by which the D7-D7 strings may propagate into the bulk. Due to the non-Abelian structure, the field E_ρ^3 and the magnetic field $B_{3\rho}^1$ induce the field E_3^2 , corresponding to an interaction between the two string types. This field E_3^2 stretches the D7-D7 strings in the x^3 direction. The position of the string in the ρ coordinate is fixed such that the gravitational force induced by the change in tension balances the force induced

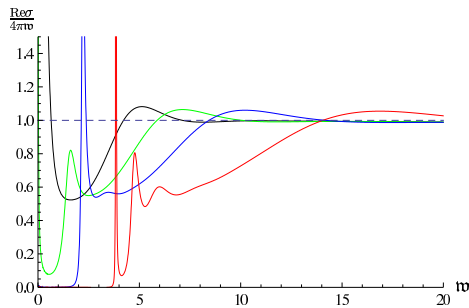


Figure 4: Real part of the conductivity $\text{Re } \sigma$ versus frequency $\omega = \omega/(2\pi T)$ at different temperatures: $T \approx 0.90T_c$ (black), $T \approx 0.66T_c$ (green), $T \approx 0.46T_c$ (blue), $T \approx 0.28T_c$ (red).

by the field E_2^3 . This means that the energy of the setup is minimized. Our numerical calculations show that this is the case.

The double importance of the D7-D7 strings is given by the fact that they are both responsible for stabilizing the new phase by lowering the charge density, as well as being the dual of the Cooper pairs since they break the $U(1)_3$ symmetry.

5. FLUCTUATIONS

The full gauge field \hat{A} on the branes consists of the field A and fluctuations a ,

$$\hat{A} = A_0^3 \tau^3 dt + A_3^1 \tau^1 dx_3 + a_\mu^a \tau^a dx^\mu, \quad (15)$$

where τ^a are the $SU(2)$ generators. The linearized equations of motion for the fluctuations a are obtained by expanding the DBI action in a to second order. We will analyze the fluctuations a_2^3 and $X = a_2^1 + ia_2^2$, $Y = a_2^1 - ia_2^2$.

5.1. Fluctuations in a_2^3

We calculate the frequency-dependent conductivity $\sigma(\omega)$ using the Kubo formula,

$$\sigma(\omega) = \frac{i}{\omega} G^R(\omega, q = 0), \quad (16)$$

where G^R is the retarded Green function of the current J_2^3 dual to the fluctuation a_2^3 , which we calculate using the method obtained in [18]. The current J_2^3 is the analog to the electric current since it is charged under the $U(1)_3$ symmetry. In real space it is transverse to the condensate. Since this fluctuation is the only one which transforms as a vector under the $SO(2)$ rotational symmetry, it decouples from the other fluctuations of the system. The real part of the frequency-dependent conductivity $\text{Re } \sigma(\omega)$ is presented in fig. 4. It shows the appearance and growth of a gap as we increase the condensate \tilde{d}_3^1 . Using the Kramers-Kronig relation, which connects the real and imaginary part of the complex conductivity, we find a delta peak at $\omega = 0$ in the real part of the conductivity, $\text{Re } \sigma(\omega) \sim \pi n_s \delta(\omega)$. As expected from Ginzburg-Landau theory, our

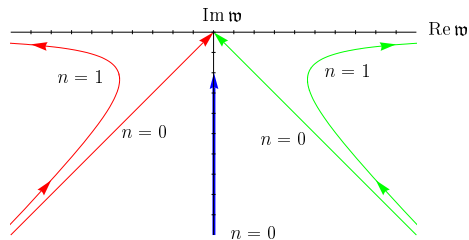


Figure 5: Movement of quasinormal modes at increasing temperature T : The different colors indicate the different fluctuations X , Y and a_2^3 . A Higgs mechanism is evident as explained in the text.

numerics show that the superconductive density n_s vanishes linearly at the critical temperature, $n_s \propto (1 - T/T_c)$ for $T \approx T_c$. As a second distinct effect fig. 4 shows prominent peaks which can be interpreted as mesonic excitations, as confirmed by our massive calculation. This is reminiscent of results for condensed matter systems where prominent quasiparticle peaks appear (e. g. [19]).

5.2. Fluctuations in $X = a_2^1 + ia_2^2$, $Y = a_2^1 - ia_2^2$

As shown in fig. 5, our setup is stable with respect to the fluctuations X and Y . Furthermore, fig. 5 shows that the quasinormal modes of higher excitations $n > 1$ move to larger frequencies and closer to the real axis. This corresponds to the formation of stable massive mesons. Such a behavior is known in gauge/gravity duality for mesons which are built from massive quarks (e. g. [3]). Thus we observe a dynamical mass generation for the mesons which we expect to be similar to the Higgs mechanism in the field theory. This is dual to the gravity gauge fields eating the Nambu-Goldstone bosons. This reasoning also applies to the massive mesonic excitations in the fluctuations a_2^3 . The explicit identification of the Nambu-Goldstone bosons and the massive vector mesons is postponed to future investigations.

6. DISCUSSION AND OUTLOOK

We found a stringy realization of holographic superconductivity for which the field theory action is known. It will be interesting to study the D7-D7 string condensate further, e. g. the drag force of strings pulled through this condensate. We expect that in the condensate this force vanishes in contrast to the original black hole background [20]. The methods presented in this letter can also be applied to D2/D6 or D3/D5 (see also [21]) systems.

ACKNOWLEDGEMENTS. We are grateful to T. Dahm, S. Gubser, C. Herzog, R. Meyer, A. O'Bannon, S. Pufu and F. Rust for discussions. This work was supported in part by *The Cluster of Excellence for Fundamental Physics - Origin and Structure of the Universe*.

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