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Yukawa Structure from $U(1)$ Fluxes in F-theory Grand Unification

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Abstract

In F-theory GUT constructions Yukawa couplings necessarily take place at the intersection of three matter curves. For generic geometric configurations this gives rise to problematic Yukawa couplings unable to reproduce the observed hierarchies. We point out that if the $U(1)_{B-L}/U(1)_Y$ flux breaking the $SO(10)/SU(5)$ GUT symmetry is allowed to go through pairs of matter curves with the same GUT representation, the quark/lepton content is redistributed in such a way that all quark and leptons are allowed to have hierarchical Yukawas. This reshuffling of fermions is quite unique and is particularly elegant for the case of three generations and $SO(10)$. Specific local F-theory models with $SO(10)$ or $SU(5)$ living on a del Pezzo surface with appropriate bundles and just the massless content of the MSSM are described. We point out that the smallness of the 3rd generation quark mixing predicted by this scheme (together with gauge coupling unification) could constitute a first hint of an underlying F-theory grand unification.

1 Introduction

In the search for string compactifications with low-energy physics as close as possible to observations two approaches seem feasible. In a top-bottom approach one starts from some string compactification which is fully consistent globally (e.g. with global RR-tadpole cancellation in the Type II case) and after a process of symmetry breaking one obtains a low-energy spectrum close to the SM (or the MSSM). On the other hand, in a bottom-up approach [1] one considers local configurations of lower dimensional Dp-branes, $p \leq 7$, which are localized on some region of the compact dimensions and reproduce SM physics. In this second case one does not care about the global aspects of the compactification and assumes that eventually the configuration may be embedded inside a fully consistent global model. This second bottom-up approach is not available in heterotic or Type I string compactifications since the SM fields would then live in the full six extra dimensions.

In principle one would say that having a globally consistent compactification would be more satisfactory. However, local configurations of branes may be more efficient in trying to identify promising classes of string vacua, independently of the details of the global theory. Eventually, if one successful bottom-up configuration is found one can then look for ways to embed it in a globally consistent model. In a stronger version of a local construction, the local SM or GUT physics is required to decouple from the gravitational sector in the infinite volume limit. This is the case of models derived from D3-branes at singularities [1, 2, 3, 4, 5] in which the SM physics only depends on the local geometry around the singularity [1]. In this situation one can cleanly disconnect the SM physics from the gravitational sector in such a way that local model building has an specific sense, since SM physics may indeed be separated from the rest of the compactification.

Another realization of the bottom-up philosophy has been recently considered in the context of local F-theory [6] grand unified models [7, 8, 9]. As particularly emphasized in [8, 9], if the F-theory 7-branes containing a GUT group wrap certain classes of complex two-dimensional surfaces S whose volume is contractible to zero (del Pezzo surfaces), one can take the overall infinite volume limit ($M_p \to \infty$) and get again the SM/GUT physics decoupled from the gravitational sector. The main advantage of a F-theory GUT version of the bottom-up approach is that gauge coupling unification is naturally guaranteed.

In the context of the MSSM the gauge coupling unification prediction [10] is in very good agreement with experiment and it seems sensible trying to incorporate it when searching for a realistic string vacuum. In an independent development it has been shown [11] that if MSSM matter is localized on 7-brane intersections and the source of SUSY breaking is modulus mediation, the emerging pattern of SUSY-breaking soft terms at tree level is consistent with all current experimental constraints. It was also shown that in order to obtain these successful results it was important that in the physical Yukawa couplings driving electroweak symmetry breaking all the fields involved should come from intersecting 7-branes. Interestingly enough this is the structure that appears in the F-theory GUT's constructed in [9] in which indeed non-vanishing Yukawa couplings appear only among fields living at intersecting 7-branes.

In the class of local F-theory GUT's constructed in [8, 9] (see also [12, 13, 14, 15, 16, 17, 18, 19, 20] for more recent work) the MSSM fields reside at certain matter curves corresponding to Riemann surfaces on the complex 2-fold S where the GUT symmetry lives. At those curves the singularity associated to the GUT symmetry ($SU(5)$ or $SO(10)$) is enhanced and at points at which three matter curves intersect there is a further enhancing of the singularity. These triple intersections of two quark/lepton matter curves with a Higgs curve enable Yukawa couplings to appear. However, there is a feature of the Yukawa couplings which is quite unattractive. For a Yukawa coupling to exist, the two matter curves associated to quarks/leptons must be different. Thus, for example, in an $SU(5)$ F-theory GUT of this class a coupling $\mathbf{10}_i \times \mathbf{10}_j \times \mathbf{5}_H$ is only non-vanishing when $i \neq j$. As emphasized in [9] such kind of U-quark mass matrix is unable to accommodate the hierarchical structure observed experimentally. In [9] a solution to this *F-theory GUT's Yukawa problem* was proposed in which it is assumed that the $\mathbf{10}$s (or the $\mathbf{16}$'s in $SO(10)$ GUT's) have self-pinching, a self-intersecting geometry in which the corresponding matter curve intersects itself on the surface S.

In this paper we point out that the F-theory Yukawa problem is naturally solved by slightly generalizing the conditions on the $U(1)$ fluxes breaking the GUT symmetry down to the SM. Indeed, in order to avoid that different MSSM particles could have different multiplicities it is assumed in [9] that the $U(1)$ flux through all matter curves Σ_i containing quark and leptons vanish,

$$\int_{\Sigma_i} F_{U(1)} \ = \ 0 \ . \tag{1.1}$$

Our main point is that this condition is too strong. It is enough to ask that if we have two matter curves Σ_1 and Σ_2 corresponding to the same GUT representation, in order to keep equal the number of generations for all quarks and leptons it is enough to request that this is true on average, i.e.

$$\int_{\Sigma_1} F_{U(1)} \ + \ \int_{\Sigma_2} F_{U(1)} \ = \ 0 \ . \tag{1.2}$$

If this is the case, one finds that the left-handed and right-handed quarks and leptons are redistributed unequally among the two matter curves Σ_1 and Σ_2 in such a way that all quarks and leptons are allowed to get non-vanishing Yukawa couplings in a way which allows for a hierarchical structure. Moreover, as a direct byproduct it follows that the mixing of the third generation quarks with the first two generations is suppressed. On the contrary, mixing among leptons is unconstrained. These two facts are in good qualitative agreement with experiment. We find this fact quite encouraging since we were only looking for a way to get Yukawa couplings for all quarks and leptons and not looking for any particular texture. This works both for $SO(10)$ and $SU(5)$ F-theory GUT's, although for $SO(10)$ this reshuffling of quarks and leptons is particularly simple and symmetric. In the $SO(10)$ case it is the flux of the $U(1)_{B-L}$ in the bulk which breaks the symmetry down to the left-right symmetric model and hence a further step of symmetry breaking down to the SM is needed. In the case of $SU(5)$ it is the hypercharge flux which breaks the bulk symmetry down to the SM. However the matter curves will feel $U(1)_{B-L}$ flux which again reshuffles the fermion spectrum in the matter curves associated to **10**s and $\overline{\mathbf{5}}$s. Although the number of quark/lepton generations is not fixed in these local F-theory constructions, it is intriguing that three generations is somewhat special. Indeed one can see that three is the minimum number of generations consistent with having a purely chiral quark/lepton spectrum (i.e. without vector-like fermion copies).

The structure of this paper is as follows. After reviewing the F-theory GUT constructions of [8, 9], we discuss in section 3 the *F-theory GUT Yukawa problem* and its solution in terms of the reshuffling of fermions induced by $U(1)$ fluxes in general terms. In section 4 we construct an specific local F-theory $SO(10)$ GUT with the symmetry broken by

$U(1)_{B-L}$ flux. A brief discussion of some phenomenological aspects is given. An $SU(5)$ local GUT is presented in section 5. Some final comments are left for section 6. A brief compendium on del Pezzo surfaces and other mathematical results are collected in an appendix.

2 F-theory Grand Unification

In this section we briefly review the basic formalism. We mostly follow [8, 9] where the reader can find a more complete discussion (see also [7, 19]). Some details about del Pezzo surfaces and other useful results are summarized in the appendix.

In F-theory GUT's the gauge theory arises from 7-branes wrapping a compact surface S of complex codimension one in the base of an elliptically-fibered Calabi-Yau fourfold. The gauge group G_S depends on the singularity type of the elliptic fiber. This gauge group can be broken by a background in a subgroup $H_S \subset G_S$. We will consider $H_S = U(1)$. $\mathcal{N}$=1 supersymmetry requires this $U(1)$ line bundle L to satisfy certain conditions stated in [8].

We will take S to be a del Pezzo surface. As stressed in [8, 9], in this case the resulting gauge theory on the 7-branes decouples from gravity in the bulk. In consequence the resulting GUT is more constrained. In particular, the allowed supersymmetric line bundles are completely characterized as reviewed in the appendix. In this framework it is possible to extract definite physical features of F-theory GUT's.

The $U(1)$ background breaks the gauge group G_S to the commutant $\Gamma_S \times U(1)$. The adjoint representation of G_S decomposes into a direct sum of irreducible representations of $\Gamma_S \times U(1)$, each generically labelled as $(\mathbf{r}, q)$. When S is a del Pezzo surface, the number of multiplets from the adjoint with $U(1)$ charge q, and transforming under $\mathbf{r}$ of Γ_S, is given by

$$N(\mathbf{r}, q) = h^1(S, L^q) = -\big[1 + \frac{1}{2} c_1(L^q) \cdot c_1(L^q)\big] \ , \tag{2.1}$$

where $h^1(S, L^q)$ is the dimension of the corresponding cohomology group and $c_1(L^q)$ is the first Chern class of the bundle L^q. Here we have already used that the admissible line bundles on a del Pezzo surface must satisfy $c_1(S) \cdot c_1(L) = 0$. Observe that the multiplicity does not depend on the sign of q. Thus, $N(\mathbf{r}, q) = N(\bar{\mathbf{r}}, -q)$, and only vector-like matter will result.

Charged multiplets with definite chirality will never originate from the adjoint in the breaking of G_S when S is a del Pezzo surface. Fortunately, in general there is another way to obtain charged matter in F-theory GUTs. Charged multiplets also arise by introducing non-compact surfaces S_i' asociated to extra 7-branes, each intersecting S at a Riemann surface Σ_i where the fields are localized. At each matter curve Σ_i the singularity type is enhanced to group G_{Σ_i} of rank at least one higher. The gauge group G_S is further enhanced to G_p of rank at least two higher at points in S where three matter curves intersect. For instance, when G_p has rank two higher in practice $G_p \supset G_S \times U(1)_a \times U(1)_b$, and from the decomposition of the adjoint of G_p one can identify the intersecting matter curves. The multiplets that materialize at each intersection descend from the adjoint of G_{Σ_i} and their degeneracy can be computed as reviewed shortly.

To be more concrete, we consider an example with $G_S = SO(10)$ and $G_p = E_7$. We need the decomposition of the adjoint $\mathbf{133}$ given by

$$\begin{aligned} E_7 &\supset SO(10) \times U(1)_a \times U(1)_b \\ \mathbf{133} &= \text{Adjoints} + [(\mathbf{10}, 2, 0) + (\mathbf{16}, -1, 1) + (\mathbf{16}, -1, -1) + (\mathbf{1}, 0, 2) + \text{c.c.}] \ . \end{aligned} \tag{2.2}$$

The last entries are the $U(1)$ charges. The $U(1)$ generators will be denoted Q_a and Q_b. The main point is that for each distinct charged representation that shows up in the adjoint branching there is a matter curve. In this example there are two distinct $\mathbf{16}$'s and a Higgs $\mathbf{10}$. Thus, there will be quark-lepton curves Σ_1 and Σ_2 associated each to a $\mathbf{16}_i$, and a Higgs curve Σ_H that hosts the $\mathbf{10}_H$. Since the curves are required to intersect, there will be a coupling $\mathbf{16}_1 \times \mathbf{16}_2 \times \mathbf{10}_H$, which is indeed allowed by gauge invariance.

On each matter curve G_S is enhanced to a group of rank at least one higher. In the $G_S = SO(10)$ and $G_p = E_7$ example, the singularity is enhanced to $G_{\Sigma_i} = E_6$ on Σ_i, $i = 1, 2$. Decomposing the adjoint of E_6 under $SO(10) \times U(1)_i'$ includes a $\mathbf{16}_i$. To determine the $U(1)_i'$ generators, notice that (2.2) shows that along the directions $(Q_a - Q_b) = 0$ and $(Q_a + Q_b) = 0$ there appear 32 additional neutral states and $SO(10)$ is indeed enhanced to E_6. Choosing a convenient normalization we then have

$$Q_1' = -\frac{1}{2}(Q_a - Q_b) \quad ; \quad Q_2' = -\frac{1}{2}(Q_a + Q_b) \ . \tag{2.3}$$

In this way there will be a $\mathbf{16}_1$ with charge $q'_1 = 1$, and a $\mathbf{16}_2$ with $q'_2 = 1$. Concerning the Higgs curve, the singularity must be enhanced to $G_{\Sigma_H} = SO(12)$. Decomposing the adjoint of $SO(12)$ under $SO(10) \times U(1)'_H$ yields the desired $\mathbf{10}_H$. From (2.2) we see that $SO(10) \to SO(12)$ along $Q_a = 0$. We choose $Q'_H = Q_a/2$ and $q'_H = 1$.

Following [8, 9] we now discuss how to compute the degeneracy of the multiplets living on a matter curve Σ at the intersection of S and some S'. On Σ the GUT group G_S is enhanced to $G_\Sigma \supset G_S \times G_{S'}$. We consider $G_{S'} = U(1)'$. In turn G_S is broken to $\Gamma_S \times U(1)$ by a background in $H_S = U(1)$. Similarly, $G_{S'}$ is broken by a background $H_{S'}$ that is taken to be $U(1)'$. The two line bundles are denoted L and L' respectively. The adjoint of G_Σ decomposes under $\Gamma_S \times U(1) \times U(1)'$ into a direct sum of irreducible representations that can be labelled $(\mathbf{r}, q, q')$. The degeneracy of a multiplet transforming into such a representation is given by

$$N(\mathbf{r}, q, q') = h^0(\Sigma, K_\Sigma^{1/2} \otimes L_\Sigma^q \otimes L_\Sigma'^{q'}) \ , \tag{2.4}$$

where $L_\Sigma \equiv L|_\Sigma$ and $L'_\Sigma \equiv L'|_\Sigma$ are the restrictions of the line bundles L and L' to Σ. Here K_Σ is the canonical line bundle of Σ. We will mostly consider the cases where Σ has genus zero and $K_\Sigma^{1/2} = \mathcal{O}_\Sigma(-1)$, or Σ has genus one and K_Σ is trivial. For the complex conjugate it follows that

$$N(\overline{\mathbf{r}}, -q, -q') = h^1(\Sigma, K_\Sigma^{1/2} \otimes L_\Sigma^q \otimes L_\Sigma'^{q'}) \ . \tag{2.5}$$

Furthermore, taking into account that L and L' are line bundles, the net multiplicity turns out to be

$$N(\mathbf{r}, q, q') - N(\overline{\mathbf{r}}, -q, -q') = (1-g) + c_1(K_\Sigma^{1/2} \otimes L_\Sigma^q \otimes L_\Sigma'^{q'}) \ , \tag{2.6}$$

where g is the genus of Σ. Finally, we remark that the $U(1)'$ decouples because the non-compact surface S' has formally infinite volume. Then, after computing the degeneracies we can drop the label q'.

In the finite volume case the extra $U(1)$ symmetries are anomalous and their gauge bosons get generically massive by combining with RR-fields of the full compactification. Note that, as pointed out in [21, 1], anomaly free $U(1)$'s like hypercharge or $U(1)_{B-L}$ may also become massive by combining with RR fields. There are conditions under which this may be avoided [21, 1] (see also [22]) and in the F-theory GUT's this can also be implemented [9]. This is not the case in heterotic compactifications in which setting flux along some $U(1)$ direction necessarily gives mass to the corresponding gauge boson [9] (see also [12]).

In the appendix we will further discuss the quantities involved in computing the multiplicities and will collect some useful results to this purpose.

3 $U(1)$ Fluxes and fermion splitting

As mentioned before, there is a generic problem for F-theory GUT's from the Yukawa coupling structure. Indeed, in these models Yukawa couplings come from the intersection of three curves inside the surface S. These curves generically correspond to different matter fields and the coupling constants have the structure [9]

$$Y_{ij} = \sum_p \Psi_R(p)^i \Psi_L(p)^j \Psi_H(p) \tag{3.1}$$

where $\Psi_{R,L,H}(p)$ are the internal wave functions of the left-handed, right-handed and Higgs matter fields evaluated at the intersection points p, which may be generically multiple. In $SO(10)$ GUT's the Yukawa couplings are of the form $\mathbf{16}_i \times \mathbf{16}_j \times \mathbf{10}_H$ and the fact that the curves involved must be different means that for $i = j$ those Yukawa couplings vanish, i.e. the structure of Yukawa couplings is of the form

$$Y_{q,l} \simeq \begin{pmatrix} 0 & x & x \\ x & 0 & x \\ x & x & 0 \end{pmatrix} \tag{3.2}$$

with x representing generically non-vanishing entries. This is illustrated in figure 1 for a particular $SO(10)$ example with two matter curves Σ_1 and Σ_2 having two and one generations respectively. From (3.2) one can then easily see that there is no regime in which one of the eigenvalues of the matrix is large with the other two small, as required to explain the data. Rather if one of the eigenvalues is small, the heavier ones are necessarily of the same order of magnitude. Something analogous happens in $SU(5)$ for the U-quark masses which come from couplings with the structure $\mathbf{10}_i \times \mathbf{10}_j \times \mathbf{5}_H$. Here again it is not possible to obtain one large eigenvalue (top quark) with the other two much smaller.

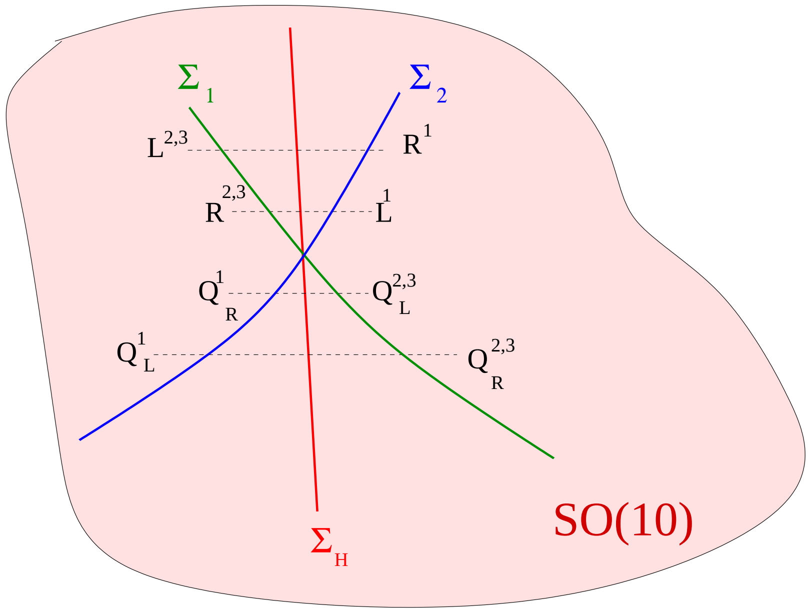

Figure 1: Triple intersection of two curves Σ_1 and Σ_2, with two and one generations of $SO(10)$ without fluxes going through them. Dashed lines represent fermions linked by a Yukawa coupling. Most Yukawa couplings are forbidden.

A possible solution to this problem advocated in [8, 9] consists of assuming that the matter curves where the fermions in problematic representations reside (**16**s in $SO(10)$, **10**s in $SU(5)$) have a *self-intersecting* geometry, the corresponding curve intersects itself inside S. This in general provides for non-vanishing diagonal entries in eq.(3.2) which could then be compatible with a hierarchical structure. Although this could certainly be a solution to the F-theory GUT Yukawa problem, we would like to argue in the present article that there is quite a natural and economical solution which does not require the assumption of any intricate geometry for the matter curves but rather relies on the gauge structure of the GUT.

Given that Yukawa terms couple right-handed and left-handed fermions to a Higgs field, the idea of the natural solution is to have a matter curve Σ_R with all right-handed fermions, another one Σ_L with all left-handed fermions and both intersecting a Higgs matter curve Σ_H. In this way all Yukawa couplings would be in principle allowed, there would be no self-intersections required. This would be for example the case in perturbative Type IIB orientifolds with e.g. a $SU(4) \times SU(2)_L \times SU(2)_R$ structure in which left- and right-handed quarks may be localized at different D7-brane intersections and transform as $(\mathbf{4},\mathbf{2}_L,\mathbf{1})$ and $(\overline{\mathbf{4}},\mathbf{1},\mathbf{2}_R)$ respectively. However our philosophy is to build GUT models which incorporate gauge coupling unification in a natural way. Unfortunately in GUT's like $SO(10)$ or $SU(5)$ this structure is not available since left- and right-handed fermions sit in the *same* GUT representations. However, there is a logical way out in F-theory GUTs. Imagine we start with two matter curves Σ_1 and Σ_2, each containing matter in the *same* GUT representation $\mathbf{R}$, e.g. $\mathbf{16}$ in $SO(10)$, or $\mathbf{10}$ in $SU(5)$. The same $U(1)$ flux which breaks the GUT symmetry down to the SM (or some extension) could perhaps be allowed to go through both matter curves and split the fields so that one curve contains right-handed fields and the other left-handed fields. Then again all Yukawa couplings would be allowed. Indeed we will see that $U(1)_{B-L}$ fluxes in $SO(10)$ or hypercharge flux in $SU(5)$ can under certain conditions reshuffle the matter content in curves Σ_1 and Σ_2 in such a way that Yukawa couplings for all quarks and leptons may appear. In fact there will not be purely right-handed Σ_R or left-handed Σ_L curves but the net effect will be quite analogous to that structure with an appropriate *flipping* of left by right in the curves.

In the models constructed in [8, 9] the net flux through each matter curve containing quarks and leptons is assumed to be zero. Indeed, threading flux through the quark-lepton curves seems risky. Take for example the $SU(5)$ case. If hypercharge flux is added, since the SM fields have different hypercharges, one would expect different multiplicity for each SM field, nothing we would like to have. However this is not true in general. As we will see, if we have two matter curves, Σ_1 and Σ_2, giving rise to the same GUT group representation, there are simple conditions under which a $U(1)$ flux may be allowed to go through the two curves in such a way that the net multiplicity for each multiplet is equal, despite all of them having different charges.

Let us be more specific. Consider two matter curves Σ_1 and Σ_2 which we will take for definiteness to have genus $g = 0$. Each curve contains matter transforming in the same GUT representation $\mathbf{R}_1 = \mathbf{R}_2$. We are adding fluxes through both curves along a $U(1) \subset G_S$, with G_S the GUT group living on the surface S. In addition these representations have charges q'_1 and q'_2 under groups $U(1)'_1$ and $U(1)'_2$ corresponding to the intersecting surfaces S'_1 and S'_2. Once the $U(1)$ flux is added the GUT symmetry is broken to $\Gamma_S \times U(1)$ and the two multiplets further decompose as

$$\mathbf{R}_1 \;=\; \bigoplus_\alpha (\mathbf{r}_\alpha, q_\alpha, q_1') \quad ; \quad \mathbf{R}_2 \;=\; \bigoplus_\alpha (\mathbf{r}_\alpha, q_\alpha, q_2') \ , \tag{3.3}$$

where the $\mathbf{r}_\alpha$ are irreducible representations of Γ_S.

We now assume that the restrictions of the $U(1)$ bundle L to the two curves have the forms $L_{\Sigma_1} = \mathcal{O}_{\Sigma_1}(1)^{1/n}$ and $L_{\Sigma_2} = \mathcal{O}_{\Sigma_2}(-1)^{1/n}$ respectively. We have allowed for possible fractional bundles, i.e. n= 5 for $SU(5)$, and $n = 4, 2$ for $SO(10)$ [9]. The main point here is that L_{Σ_1} and L_{Σ_2} have opposite instanton numbers. Instead of insisting that the fluxes through each individual curve vanish, we rather impose that the flux vanishes on average, namely

$$\int_{\Sigma_1} F_{U(1)} \ + \ \int_{\Sigma_2} F_{U(1)} \ = \ 0 \ . \tag{3.4}$$

There will also be the restrictions of the gauge bundles on S_1' and S_2' which we take to be of the general form $L'_{\Sigma_1} = \mathcal{O}_{\Sigma_1}(n_1)^{1/n}$ and $L'_{\Sigma_2} = \mathcal{O}_{\Sigma_2}(n_2)^{1/n}$ with n_1 and n_2 appropriately chosen integers. Under these conditions the multiplicity of each representation $(\mathbf{r}_\alpha, q_\alpha)$ due to each curve will be (we are assuming $g = 0$ for both curves)

$$\begin{aligned}
\Sigma_1 \ &: \ N_1(\mathbf{r}_\alpha, q_\alpha) \ = \ h^0(\Sigma_1, \mathcal{O}_{\Sigma_1}(-1 + \frac{1}{n}[q_\alpha + n_1 q_1']) \ = \ \frac{1}{n}(q_\alpha + n_1 q_1') \qquad (3.5)\\
\Sigma_2 \ &: \ N_2(\mathbf{r}_\alpha, q_\alpha) \ = \ h^0(\Sigma_2, \mathcal{O}_{\Sigma_2}(-1 + \frac{1}{n}[-q_\alpha + n_2 q_2']) \ = \ \frac{1}{n}(-q_\alpha + n_2 q_2')
\end{aligned}$$

so that the net multiplicity of each representation $(\mathbf{r}_\alpha, q_\alpha)$ is given by

$$N(\mathbf{r}_\alpha, q_\alpha) \ = \ \frac{1}{n}\ (n_1 q_1' \ + \ n_2 q_2') \tag{3.6}$$

Note that this multiplicity is *independent of the value of the charge* q_α of each of the multiplets $\mathbf{r}_\alpha$ inside the GUT representation $\mathbf{R}$. Thus, the same multiplicity for all different components can result irrespective of their charges. Note also that if $|n_1 q_1'|$ or $|n_2 q_2'|$ are smaller than some $|q_\alpha|$ the spectrum will then include additional vector-like multiplets of charge q_α. The crucial conclusion is that even though the net number of each $(\mathbf{r}_\alpha, q_\alpha)$ representation is the same, they will be distributed unequally between the two matter curves Σ_1 and Σ_2.

Note that this may also be understood as a single reducible matter curve such that the net flux through it is zero. Furthermore it may trivially extended to the case of more than two matter curves in which the net flux vanishes.

In the following we apply this simple idea to specific GUT models with two matter curves Σ_1 and Σ_2 and see how the Yukawa problem is then generically solved.

4 A $B-L$ fluxed $SO(10)$ model

Let us first consider the case of an $SO(10)$ GUT which is broken by $U(1)_{B-L}$ flux down to a left-right symmetric extension of the SM. A possible choice of curves and bundles is shown in table 1 (see the appendix for notation). A sketch of the matter curves involved is depicted in figure 2.

Multiplet	Curve	Class	g_Σ	L_Σ	L'_Σ
$\mathbf{16}_1$	Σ_1	$H - E_1 - E_3$	0	$\mathcal{O}_{\Sigma_1}(1)^{1/2}$	$\mathcal{O}_{\Sigma_1}(3)^{1/2}$
$\mathbf{16}_2$	Σ_2	$H - E_2 - E_4$	0	$\mathcal{O}_{\Sigma_2}(-1)^{1/2}$	$\mathcal{O}_{\Sigma_2}(3)^{1/2}$
$\mathbf{10}$	Σ_H	$-K_S$	1	$\mathcal{O}_{\Sigma_H}(p_1 - p_2)^{1/2}$	$\mathcal{O}_{\Sigma_H}$
$(\mathbf{16} + \overline{\mathbf{16}})$	Σ_ϕ	$3H - E_1 - E_2$	1	$\mathcal{O}_{\Sigma_\phi}(p_3 - p_4)^{1/2}$	$\mathcal{O}_{\Sigma_\phi}(p_3 - p_4)^{-3/2}$

Table 1: Curves and bundles of the $SO(10)$ model with $U(1)_{B-L}$ flux and $L = \mathcal{O}_S(E_1 - E_2)^{1/2}$.

We have F-theory 7-branes wrapping a del Pezzo surface S with a singularity corresponding to a $SO(10)$ gauge symmetry. Adding $U(1)_{B-L}$ flux breaks the symmetry and the adjoint decomposes as

$$SO(10) \supset SU(3) \times SU(2)_R \times SU(2)_L \times U(1)_{B-L} \tag{4.1}$$

$$\mathbf{45} = \text{Adjoints} + (\overline{\mathbf{3}}, \mathbf{1}, \mathbf{1}, -4) + (\mathbf{3}, \mathbf{1}, \mathbf{1}, 4) + (\overline{\mathbf{3}}, \mathbf{2}, \mathbf{2}, 2) + (\mathbf{3}, \mathbf{2}, \mathbf{2}, -2)$$

where the last entry is the $B-L$ charge. Upon symmetry breaking the $SO(10)$ adjoint gives rise to fields transforming under the unbroken gauge group according to this decomposition.

In general in the presence of flux there may be massless exotics from the adjoint transforming as $(\overline{\mathbf{3}}, \mathbf{2}, \mathbf{2}, 2)$, $(\mathbf{3}, \mathbf{1}, \mathbf{1}, 4)$ and/or their conjugates. As discussed in [9], in the case of $SU(5)$, exotics disappear if some appropriate powers of the $U(1)$ bundle correspond to a dP_N divisor associated to E_8 roots. In $SO(10)$ models one cannot get rid of all exotics

[9], but one can still remove most of them. To this end we make the choice

$$L = \mathcal{O}_S(E_1 - E_2)^{1/2} \ , \tag{4.2}$$

where the E_i are exceptional divisors. We can now compute the multiplicity of the states descending from the adjoint using (2.1). We obtain

$$N(\overline{\mathbf{3}},\mathbf{2},\mathbf{2},2) \ = \ h^1(S,L^2) \ = \ 0 \quad ; \quad N(\mathbf{3},\mathbf{2},\mathbf{2},-2) \ = \ h^1(S,L^{-2}) \ = \ 0 \tag{4.3}$$

because $c_1(L^2)\cdot c_1(L^2) = -2$, reflecting that L^2 indeed corresponds to an E_8 root. On the other hand,

$$N(\overline{\mathbf{3}},\mathbf{1},\mathbf{1},-4) \ = \ h^1(S,L^{-4}) = \ -\big[1+\frac{1}{2}c_1(L^{-4})^2\big] \ = \ 3 \tag{4.4}$$

and the same result for $N(\mathbf{3},\mathbf{1},\mathbf{1},4) = h^1(S,L^4)$. So we will have three sets of vector-like chiral multiplets

$$3[(\overline{\mathbf{3}},\mathbf{1},\mathbf{1},-4)+(\mathbf{3},\mathbf{1},\mathbf{1},4)] \tag{4.5}$$

In fact, after further symmetry breaking down to the SM these fields have the quantum numbers of vector-like right-handed D-quarks. We will argue later that these states are expected to become massive after the symmetry breaking process from $SU(3)\times SU(2)_R\times SU(2)_L\times U(1)_{B-L}$ down to the SM.

We have an $SO(10)$ gauge group in our S surface. At some complex curves Σ_i the rank of the singularity is enhanced so that matter transforming as both **16** and **10** does appear. In particular, **16**s arise from curves in which the singularity is enhanced to E_6 whereas **10**s appear at curves where the symmetry is enhanced to $SO(12)$. These curves will intersect at points of double enhancing up to E_7. Recalling the decomposition of the adjoint of E_7 under $SO(10)\times U(1)_a\times U(1)_b$ displayed in (2.2) we see that the coupling $(\mathbf{16},-1,1)\times(\mathbf{16},-1,-1)\times(\mathbf{10},2,0)$ is allowed so that in principle one can engineer three curves Σ_i , $i=1,2$, and Σ_H, associated to each representation and giving rise to Yukawa couplings. In particular, we have two types of matter curves Σ_1 and Σ_2 supporting **16**s that we can use for the fermion reshuffling mechanism described in the previous section.

We have chosen the matter and Higgs curves as indicated in table 1. They verify the condition that Σ_1 and Σ_2 have non-trivial (and positive) intersection among themselves as well as with the Higgs curve Σ_H, so that Yukawa couplings are possible. The matter curves

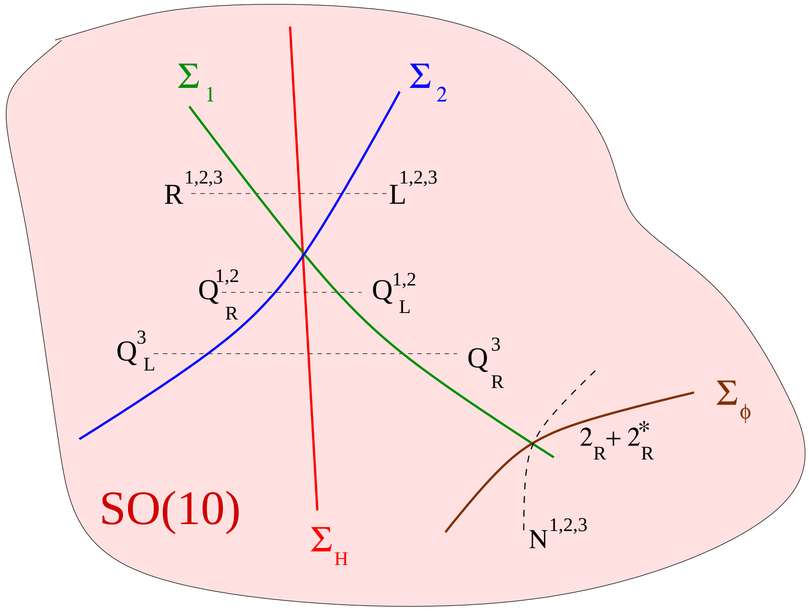

Figure 2: Sketch of the structure of the $B-L$ fluxed $SO(10)$ model. Dashed lines represent fermions linked by a Yukawa coupling. Yukawa couplings for all fermions are allowed.

are also chosen so that the restrictions of the $U(1)_{B-L}$ line bundle $L=\mathcal{O}_S(E_1-E_2)^{1/2}$ to the Σ_i have opposite instanton numbers. Since $(E_1-E_2)\cdot\Sigma_1=1$ and $(E_1-E_2)\cdot\Sigma_2=-1$ we find that

$$\begin{aligned} L_{\Sigma_1} &= \mathcal{O}_{\Sigma_1}(1)^{1/2} \ , \\ L_{\Sigma_2} &= \mathcal{O}_{\Sigma_2}(-1)^{1/2} \ , \end{aligned} \tag{4.6}$$

as required in order to have $\int_{\Sigma_1} F_{U(1)_{B-L}} + \int_{\Sigma_2} F_{U(1)_{B-L}} = 0$.

The curve Σ_H supporting the Higgses that couple to matter fermions is selected to have genus one. Doublet-triplet splitting can be realized in this setting as we now explain. Since the Higgs curve is such that $\Sigma_H\cdot(E_1-E_2)=0$, the restriction of the $U(1)_{B-L}$ line bundle L to Σ_H has degree zero. We can then choose

$$L_{\Sigma_H} = \mathcal{O}_{\Sigma_H}(p_1-p_2)^{1/2} \ , \tag{4.7}$$

where p_1 and p_2 are two independent degree one divisors on Σ_H (see the appendix). We also know that under $SO(10)\supset SU(3)\times SU(2)_R\times SU(2)_L\times U(1)_{B-L}$ the **10** has a branching

$$\mathbf{10} = (\mathbf{1},\mathbf{2},\mathbf{2},0) \ + \ (\overline{\mathbf{3}},\mathbf{1},\mathbf{1},2) \ + \ (\mathbf{3},\mathbf{1},\mathbf{1},-2) \tag{4.8}$$

so that the $B-L$ flux will not affect the doublets that are neutral. To compute the multiplicities according to (2.4) we still need to specify the bundle L'. A simple option is to choose a trivial line bundle. In this way we obtain

$$N(\mathbf{1},\mathbf{2},\mathbf{2},0) \;=\; h^0(\Sigma_H,\mathcal{O}_{\Sigma_H}) \;=\; 1\;, \tag{4.9}$$

because on a curve of genus one the only holomorphic sections of the trivial bundle are the constant functions. The upshot is that there is a massless Higgs doublet. Recalling that Σ_H has trivial canonical line bundle, for the triplet we instead find

$$N(\overline{\mathbf{3}},\mathbf{1},\mathbf{1},2) = \; h^0(\Sigma_H,\mathcal{O}_{\Sigma_H}(p_1-p_2)) = 0\;, \tag{4.10}$$

because the divisor (p_1-p_2) is not effective. Using this result together with the index theorem (2.6), and the fact that $c_1(\mathcal{O}_{\Sigma_H}(p_1-p_2))=0$, it also follows that

$$N(\mathbf{3},\mathbf{1},\mathbf{1},-2) = h^0(\Sigma_H,\mathcal{O}_{\Sigma_H}(p_1-p_2)^{-1}) = 0\;. \tag{4.11}$$

We thus obtain a minimal set of massless Higgs doublets.

We now consider the effect of the $U(1)_{B-L}$ flux on the **16**s living on the two matter curves Σ_1 and Σ_2. To begin, we recall the well known branching of the **16** under $SO(10)\supset SU(3)\times SU(2)_R\times SU(2)_L\times U(1)_{B-L}$

$$\mathbf{16} = (\mathbf{1},\mathbf{2},\mathbf{1},3)+(\mathbf{3},\mathbf{1},\mathbf{2},1)+(\overline{\mathbf{3}},\mathbf{2},\mathbf{1},-1)+(\mathbf{1},\mathbf{1},\mathbf{2},-3)\;. \tag{4.12}$$

Substituting the bundle data of table 1 in (2.4) one finds for the matter fields from curve Σ_1 (with $q'=1$) the multiplicities

$$\begin{aligned}
N(L_R) &= N(\mathbf{1},\mathbf{2},\mathbf{1},3) &&= h^0(\Sigma_1,\mathcal{O}_{\Sigma_1}(-1+\tfrac{3}{2}+\tfrac{3}{2})) &&= 3\;,\\
N(Q_L) &= N(\mathbf{3},\mathbf{1},\mathbf{2},1) &&= h^0(\Sigma_1,\mathcal{O}_{\Sigma_1}(-1+\tfrac{1}{2}+\tfrac{3}{2})) &&= 2\;,\\
N(Q_R) &= N(\overline{\mathbf{3}},\mathbf{2},\mathbf{1},-1) &&= h^0(\Sigma_1,\mathcal{O}_{\Sigma_1}(-1-\tfrac{1}{2}+\tfrac{3}{2})) &&= 1\;,\\
N(L_L) &= N(\mathbf{1},\mathbf{1},\mathbf{2},-3) &&= h^0(\Sigma_1,\mathcal{O}_{\Sigma_1}(-1-\tfrac{3}{2}+\tfrac{3}{2})) &&= 0\;.
\end{aligned} \tag{4.13}$$

For the second curve Σ_2 one instead obtains

$$\begin{aligned}
N(L_R) &= N(\mathbf{1},\mathbf{2},\mathbf{1},3) &= h^0(\Sigma_2,\mathcal{O}_{\Sigma_2}(-1-\frac{3}{2}+\frac{3}{2})) &= 0\ , \\
N(Q_L) &= N(\mathbf{3},\mathbf{1},\mathbf{2},1) &= h^0(\Sigma_2,\mathcal{O}_{\Sigma_2}(-1-\frac{1}{2}+\frac{3}{2})) &= 1\ , \\
N(Q_R) &= N(\overline{\mathbf{3}},\mathbf{2},\mathbf{1},-1) &= h^0(\Sigma_2,\mathcal{O}_{\Sigma_2}(-1+\frac{1}{2}+\frac{3}{2})) &= 2\ , \\
N(L_L) &= N(\mathbf{1},\mathbf{1},\mathbf{2},-3) &= h^0(\Sigma_2,\mathcal{O}_{\Sigma_2}(-1+\frac{3}{2}+\frac{3}{2})) &= 3\ .
\end{aligned} \tag{4.14}$$

Therefore, the matter content from each curve is

$$\begin{aligned}
\Sigma_1 &: 3\times L_R + 2\times Q_L + 1\times Q_R\ , \\
\Sigma_2 &: 3\times L_L + 2\times Q_R + 1\times Q_L\ .
\end{aligned} \tag{4.15}$$

Note that altogether there are three net generations but the fields have been redistributed and there are no complete **16**s in any of the two curves.

The remarkable result is that now the rule that states that only fields coming from different curves (i.e. $\Sigma_1\times\Sigma_2\times\Sigma_H$) can have trilinear couplings leads to interesting textures. Indeed note that with the matter content summarized in (4.15) the rule implies the Yukawa structure:

$$h_Q \sim \begin{pmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & x \end{pmatrix} \ ;\ h_L \sim \begin{pmatrix} x & x & x \\ x & x & x \\ x & x & x \end{pmatrix} \tag{4.16}$$

where x means something non-vanishing. We see that all fermions may now get a mass without any self-interaction for the curves. This is important in itself but as a byproduct we obtain three qualitative predictions:

- The third generation quarks mix little with the first two generations.
- First and second generations may have substantial (Cabbibo) mixing.
- The mixing of leptons may generically be large.

These three points are in good qualitative agreement with experiment. (Actually, before the breaking of the left-right symmetry there is no mixing, since the U-and D-quark mass matrices are always proportional. The above statements apply once the L-R symmetry has been broken and this proportionality ceases to be exact, see comments at the end of the section). Additional fermion hierarchies may result once one computes the Yukawa couplings in terms of the values of internal wave functions at the intersection points as in eq.(3.1). For example, the 3rd generation quarks will have larger Yukawas if the wave functions of Q_L^3, U_R^3, and D_R^3 evaluated at some intersection point p are larger than those of the corresponding quarks of the first two generations. In particular, if the wave functions at this point satisfy $\Psi_L^\tau(p)\Psi_R^\tau(p) \simeq \Psi_L^b(p)\Psi_R^b(p) \simeq \Psi_L^t(p)\Psi_R^t(p)$, one would obtain relationships among the 3rd generation Yukawas of the form $h_\tau \simeq h_b \simeq h_t$.

The gauge symmetry $SU(3) \times SU(2)_R \times SU(2)_L \times U(1)_{B-L}$ has to be further broken down to the SM group. To this purpose we need to have vector-like right-handed doublets in our massless spectrum. To obtain these fields we introduce a curve Σ_ϕ giving rise to multiplets $(\mathbf{16} + \overline{\mathbf{16}})$ before flux is added. Then the flux must split them so that only a vector-like pair of right-handed doublets $[(\mathbf{1}, \mathbf{2}, \mathbf{1}, 3) + \text{c.c.}]$ remains. This can be achieved by considering a genus one curve where the restricted $U(1)_{B-L}$ bundle has degree zero again. Some $B-L$ flux must pierce the curve in order to split the representations and remove all fields except the right-handed doublets. A possible curve can have a class

$$\Sigma_\phi = 3H - E_1 - E_2 \tag{4.17}$$

which has $g = 1$ and $\Sigma_\phi \cdot (E_1 - E_2) = 0$. The restriction of the $B-L$ flux onto Σ_ϕ can then again be taken of the form

$$L_{\Sigma_\phi} \;=\; \mathcal{O}_{\Sigma_\phi}(p_3 - p_4)^{1/2} \ , \tag{4.18}$$

with p_3 and p_4 degree one divisors in Σ_ϕ. We now choose a non trivial restriction of the $U(1)'$ bundle given by

$$L'_{\Sigma_\phi} \;=\; \mathcal{O}_{\Sigma_\phi}(p_3 - p_4)^{-3/2} \ . \tag{4.19}$$

To compute the multiplicities we need the decomposition of the **16** shown in (4.12). Inserting into (2.4) we find

$$N(\mathbf{1}, \mathbf{2}, \mathbf{1}, 3) \;=\; h^0(\Sigma_\phi, \mathcal{O}_{\Sigma_\phi}) \;=\; 1 \ . \tag{4.20}$$

The rest of the multiplets have zero multiplicity since the $(p_3 - p_4)$ divisor is not effective and has degree zero. Thus, the only remaining fields are right-handed doublets transforming as $[(\mathbf{1}, \mathbf{2}, \mathbf{1}, 3) + (\mathbf{1}, \mathbf{2}, \mathbf{1}, -3)]$ whose vev would break the symmetry down to the SM.

The final spectrum is that of the MSSM. However there are still the three copies of exotics $[(\overline{\mathbf{3}}, \mathbf{1}, \mathbf{1}, -4)^{exot} + \text{c.c}]$ coming from the adjoint of $SO(10)$. In fact, after the symmetry is broken down to the SM those multiplets generically get massive. Indeed, note that in general there ia a coupling $\mathbf{45} \times \mathbf{16}_\phi \times \overline{\mathbf{16}}_\phi$ that gives rise to

$$(\overline{\mathbf{3}}, \mathbf{1}, \mathbf{1}, -4)^{exot} \times (\mathbf{1}, \mathbf{2}, \mathbf{1}, 3) \times (\mathbf{3}, \mathbf{2}, \mathbf{1}, 1) \tag{4.21}$$

The triplets $(\mathbf{3}, \mathbf{2}, \mathbf{1}, 1)$ inside Σ_ϕ are massive KK states and may be exchanged so that an effective operator of the form

$$(\mathbf{1}, \mathbf{2}, \mathbf{1}, 3) \times (\mathbf{1}, \mathbf{2}, \mathbf{1}, -3) \times (\overline{\mathbf{3}}, \mathbf{1}, \mathbf{1}, -4)^{exot} \times (\mathbf{3}, \mathbf{1}, \mathbf{1}, 4)^{exot} \tag{4.22}$$

is generated. When the right-handed doublets get a vev the three pairs of triplets from the adjoint will dissappear from the massless spectrum.

It is not the purpose of this paper to give a full phenomenological study of the models presented. Several comments are however in order:

- Dirac neutrino masses appear generically with the same size as the rest of quarks and leptons from the couplings $\mathbf{16}_1 \times \mathbf{16}_2 \times \mathbf{10}_H$. On the other hand, one possible way to implement the see-saw mechanism is the following. Large masses for the right-handed neutrinos (which live in the Σ_1 curve) may appear if there are couplings of the form $\mathbf{16}_1 \times \overline{\mathbf{16}}_\phi \times N_i$, with N_i three SM singlets (see figure 2). Once the right-handed doublets inside $\overline{\mathbf{16}}_\phi$ get a vev, the three right-handed neutrinos become massive and the standard sea-saw mechanism will be at work. In this connection note that Σ_ϕ has positive intersection with Σ_1 so that the required coupling is in principle allowed.

- In this model R-parity is automatic due to the $SO(10)$ structure which forbids $\mathbf{16}^3$ couplings. Hence there are no dim=4 baryon or lepton number violating couplings. On the other hand baryon number violating dim=5 operators (like e.g. $QQQL$)

are suppressed. Such operators appear from couplings of quarks and leptons to the massive color triplets inside the **10**. However, due to the split structure of the multiplets in the **16**s one can check that the baryon number dim=5 operators always involve at least one third-generation quark. Thus, the amplitude has additional Cabbibo suppression which makes the exchange of colored Higgsses harmless.

- The quark mass structure in (4.16) before the breaking of the $SU(2)_R \times SU(2)_L \times U(1)_{B-L}$ symmetry gives mass matrices for U- and D-quarks which are proportional so that at this level there is in fact no mixing. However, once further symmetry breaking takes place the insertion of vevs for the right-handed doublets in Σ_ϕ gives rise to mixings between the right-handed D-quarks from the **16**s and the color triplets in the **10** so that the massless eigenstates are mixed linear combinations. Then the U- and D-quark mass matrices will cease to be proportional and mixing will generically occur.

We finish with some comments about possible extensions/generalizations of the model. There is some freedom in the choice of line bundles in the $U(1)'_i$ associated to each matter curve. For example, instead of bundles $L'_{\Sigma_i} = \mathcal{O}_{\Sigma_i}(3)^{1/2}$ for both Σ_1 and Σ_2 we could have chosen $L'_{\Sigma_1} = \mathcal{O}_{\Sigma_1}(4)^{1/2}$ and $L'_{\Sigma_2} = \mathcal{O}_{\Sigma_2}(2)^{1/2}$, or even $L'_{\Sigma_1} = \mathcal{O}_{\Sigma_1}(5)^{1/2}$ and $L'_{\Sigma_2} = \mathcal{O}_{\Sigma_2}(1)^{1/2}$. With these alternative choices one also gets three net quark/lepton generations. However one can easily check that with asymmetric assignments the massless spectrum includes extra vector-like multiplets beyond the standard three generations. Also in these cases always some quark or lepton remains massless.

The case with *three generations is also somewhat special* in these $SO(10)$ GUT's. Indeed, although in principle one can get any number of generations, three is the minimal number for which the spectrum does not contain additional vector-like matter fields. Note also in this respect that the bigger the number of generations, the bigger the $U(1)'$ instanton numbers n_1 and n_2. A larger number of generations would make more difficult the eventual embedding of the local model into a complete global F-theory compactification. This is because $U(1)$ backgrounds induce D3-brane charge and the latter is bounded in a compact model.

One can also consider adding fluxes along the $U(1)$ in the branching of $SO(10)$ into

$SU(5) \times U(1)_5$. Flipped $SU(5)$ models with this kind of flux were considered in [9]. However, it is easy to check that allowing for this $U(1)$ flux to go through the matter curves does not add anything new to the problem of Yukawa couplings. This is obvious since the gauge group $SU(5)$ remains unbroken and its representations contain both left- and right-handed fermions unified.

5 A $SU(5)$ F-theory GUT model

We now discuss the case of $SU(5)$ GUT's. Consider then a bulk 7-brane with gauge group $SU(5)$. In the simplest situations **10**s are obtained at curves where the $SU(5)$ singularity is enhanced to $SO(10)$ whereas $\overline{\mathbf{5}}$s will appear at curves where the singularity in enhanced to $SU(6)$. At points at which these curves intersect the symmetry is further enhanced to E_6 or $SO(12)$. This is the case of the examples considered in [9]. In particular we have the branchings

$$
\begin{aligned}
E_6 &\supset SU(5) \times U(1)_1 \times U(1)_2 && (5.1)\\
\mathbf{78} &= \text{Adjoints} + \left[(\mathbf{10},-1,-3) + (\mathbf{10},4,0) + (\mathbf{5},-3,3) + (\mathbf{1},5,3) \ + \ \text{c.c.}\right] \\
SO(12) &\supset SU(5) \times U(1)_3 \times U(1)_4 && (5.2)\\
\mathbf{66} &= \text{Adjoints} + \left[(\mathbf{10},4,0) + (\overline{\mathbf{5}},-2,2) + (\overline{\mathbf{5}},-2,-2) \ + \ \text{c.c.}\right]
\end{aligned}
$$

The first branching shows that two **10**s from different Riemann curves can form a Yukawa coupling with some **5**. On the other hand, the second shows that one of the **10** curves (but not the other) may couple also to pairs of $\overline{\mathbf{5}}$'s to provide for D-quark and lepton Yukawas. One can now think of constructing a model with two **10** curves and one matter $\overline{\mathbf{5}}$ curve and repeating the reshuffling idea we used before but using now hypercharge flux through the two curves associated to the **10**s. However, it is easy to realize that this cannot possibly work. The problem is that after turning on the hyperflux, as we will see below, the three generations of quarks and leptons from the **10**s split into two sets with Σ_1 and Σ_2 containing respectively $(3 \times E_R + 2 \times Q_L + 1 \times U_R)$ and $(2 \times U_R + 1 \times Q_L)$. Now, according to our observation above, only one of the **10** curves (in particular Σ_2) may couple to $\overline{\mathbf{5}}$s to form Yukawas. That means that only one D-quark generation and no leptons would be allowed to get Yukawas. This shows that in an F-theory $SU(5)$ GUT in which matter resides on curves with minimal rank enhancing the presence of flux through the matter curves cannot possibly solve by itself the Yukawa coupling problem.

On the other hand, the fact that $SO(10)$ contains $SU(5)$ as a subgroup and that we were able to build a $SO(10)$ model with no Yukawa problem already tells us that it should be possible to construct $SU(5)$ GUT's with the appropriate properties to solve it. Since $SO(10)$ has rank higher in one unit this means that the adequate couplings should be possible if in the matter curves the $SU(5)$ singularity is enhanced in two units up to E_6. This double enhancing in the rank of the singularity is indeed possible, as remarked in [15]. In these matter curves both $\mathbf{10}$s and $\overline{\mathbf{5}}$s can be present. We know that a $\mathbf{16}$ of $SO(10)$ contains these $SU(5)$ multiplets, the relevant branching being

$$\begin{aligned} SO(10) &\supset SU(5) \times U(1)_5 \\ \mathbf{16} &= (\mathbf{10}, -1) + (\overline{\mathbf{5}}, 3) + (\mathbf{1}, -5) \ . \end{aligned} \tag{5.3}$$

Then, the E_7 branching (2.2) indicates that one can consider a pair of curves Σ_1 and Σ_2 each supporting a $\mathbf{10}$, a $\overline{\mathbf{5}}$, and a singlet. At the intersection of these curves the symmetry is enhanced to E_7, just like in the $SO(10)$ GUT of the previous section. As remarked in [15], in these doubly enhanced singularity curves there could be in principle additional matter fields from the further Higgssing of $SO(10)$ down to $SU(5)$. In what follows we will ignore this issue and simply examine what would be the effect on the Yukawas of non-vanishing fluxes through these matter curves. An specific local realization of curves and bundles is summarized in table 2. An sketch of the matter curves involved is depicted in figure 3. The structure is quite analogous to the previous $SO(10)$ model. The main difference is that we have an $SU(5)$ symmetry on S which is broken down to the SM by hypercharge fluxes. Furthermore, now there are simultaneous fluxes along $U(1)' \times U(1)_5$ that break $G_{S'}$. The corresponding line bundles are denoted L' and $\tilde{L}$.

We start from a surface S with 7-branes corresponding to a $SU(5)$ gauge group. Switching on magnetic flux through the hypercharge direction breaks the GUT symmetry. The resulting hypercharge values of the matter fields can be read off from the

Curve	Class	g_Σ	Multiplet	L_Σ	$L'^{q'}_\Sigma \otimes \tilde{L}^{q_5}_\Sigma$
Σ_1	$H - E_1 - E_3$	0	$\mathbf{10_1}$	$\mathcal{O}_{\Sigma_1}(1)^{1/5}$	$\mathcal{O}_{\Sigma_1}(9)^{1/5}$
			$\overline{\mathbf{5}}_\mathbf{1}$	$\mathcal{O}_{\Sigma_1}(1)^{1/5}$	$\mathcal{O}_{\Sigma_1}(3)^{1/5}$
			$\mathbf{1_1}$	$\mathcal{O}_{\Sigma_1}(1)^{1/5}$	$\mathcal{O}_{\Sigma_1}(3)$
Σ_2	$H - E_2 - E_4$	0	$\mathbf{10_2}$	$\mathcal{O}_{\Sigma_2}(-1)^{1/5}$	$\mathcal{O}_{\Sigma_2}(6)^{1/5}$
			$\overline{\mathbf{5}}_\mathbf{2}$	$\mathcal{O}_{\Sigma_2}(-1)^{1/5}$	$\mathcal{O}_{\Sigma_2}(12)^{1/5}$
			$\mathbf{1_2}$	$\mathcal{O}_{\Sigma_2}(-1)^{1/5}$	$\mathcal{O}_{\Sigma_2}$
Σ_H	$-K_S$	1	$\overline{\mathbf{5}}_\mathbf{H} + \mathbf{5_H}$	$\mathcal{O}_{\Sigma_H}(p_1 - p_2)^{1/5}$	$\mathcal{O}_{\Sigma_H}(p_1 - p_2)^{-3/5}$

Table 2: Curves and bundles of the $SU(5)$ model with hypercharge flux and $L = \mathcal{O}_S(E_1 - E_2)^{1/5}$.

decompositions

$$\begin{aligned} SU(5) &\supset SU(3) \times SU(2) \times U(1)_Y \\ \mathbf{24} &= \text{Adjoints} + (\mathbf{3}, \mathbf{2}, -5) + (\overline{\mathbf{3}}, \mathbf{2}, 5) \\ \mathbf{10} &= (\mathbf{3}, \mathbf{2}, 1) + (\overline{\mathbf{3}}, \mathbf{1}, -4) + (\mathbf{1}, \mathbf{1}, 6) \\ \overline{\mathbf{5}} &= (\mathbf{1}, \mathbf{2}, -3) + (\overline{\mathbf{3}}, \mathbf{1}, 2) \end{aligned} \tag{5.4}$$

where the last entry denotes the hypercharge. As remarked in [9], with the choice

$$L = \mathcal{O}_S(E_1 - E_2)^{1/5} \tag{5.5}$$

the exotics $[(\mathbf{3}, \mathbf{2}, -5) + (\overline{\mathbf{3}}, \mathbf{2}, 5)]$ are absent from the massless spectrum.

Let us now study the effect of the fluxes on the matter curves containing quarks and leptons. We will assume that there are two genus zero curves Σ_1 and Σ_2 in which the $SU(5)$ symmetry is doubly enhanced up to E_6. As we said, at each of these curves there are three types of multiplets, $\mathbf{10}$, $\overline{\mathbf{5}}$, and a singlet. In the intersecting 7-branes wrapping S'_1 and S'_2 there are bundles along $U(1)'$ and another $U(1)_5$. To make contact with the previous $SO(10)$ model it is important to realize that the $U(1)_{B-L}$ generator may be expressed in terms of hypercharge and $U(1)_5$ as follows

$$Q_{B-L} = \frac{1}{5}(2Y - 3Q_5) \; . \tag{5.6}$$

This relation helps to understand the data in table 2. Note that the first Chern classes of the restricted line bundles $L'^{q'}_{\Sigma_1} \otimes \tilde{L}^{q_5}_{\Sigma_1}$ felt by the various multiplets in Σ_1 are given

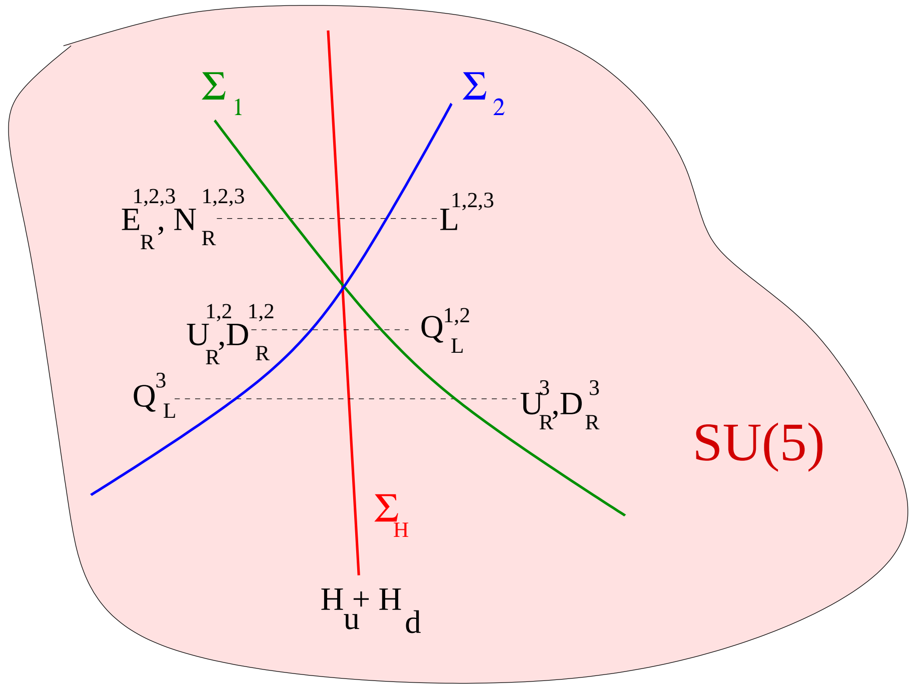

Figure 3: Sketch of the structure of the hypercharge fluxed $SU(5)$ model.

by $-\frac{3}{10}q_5 + \frac{3}{2}q'$, where $q' = 1$ and q_5 is the $U(1)_5$ charge in eq.(5.4). For Σ_2 the Chern classes are instead $\frac{3}{10}q_5 + \frac{3}{2}q'$. Now, by virtue of eq.(5.6), it follows that the total degree of $L^Y_{\Sigma_1} \otimes L'^{q'}_{\Sigma_1} \otimes \tilde{L}^{q_5}_{\Sigma_1}$ is equal to $\frac{1}{2}q_{B_L} + \frac{3}{2}q'$, which is precisely the total degree of $L^{q_{B-L}}_{\Sigma_1} \otimes L'^{q'}_{\Sigma_1}$ in the $SO(10)$ model. For Σ_2 there is also perfect matching with the $SO(10)$ bundles in table 1. The important point is that along the matter curves Σ_i there is actually $U(1)_{B-L}$ flux (rather than just hypercharge), with Σ_1 and Σ_2 getting opposite fluxes. Given this fact, everything is quite similar to the parent $SO(10)$ model. In particular, it is easy to check that the fermions are redistributed as in the $SO(10)$ case.

As an example let us see what happens with the fermions belonging to a **10**. As shown in the table, the restriction of the hypercharge bundle to Σ_1 and Σ_2 are

$$\begin{aligned} L_{\Sigma_1} &= \mathcal{O}_{\Sigma_1}(1)^{1/5} , \\ L_{\Sigma_2} &= \mathcal{O}_{\Sigma_2}(-1)^{1/5} , \end{aligned} \tag{5.7}$$

where we have used that $(E_1 - E_2) \cdot \Sigma_1 = 1$ and $(E_1 - E_2) \cdot \Sigma_2 = -1$. The spectrum from the **10** in Σ_1 can be deduced from the multiplicities computed using the data in the table. Substituting in (2.4) yields

$$\begin{array}{rclclcl} N(E_R) & = & N(\mathbf{1},\mathbf{1},6) & = & h^0(\Sigma_1,\mathcal{O}_{\Sigma_1}(-1+\frac{6}{5}+\frac{9}{5})) & = & 3\ , \\ N(Q_L) & = & N(\mathbf{3},\mathbf{2},1) & = & h^0(\Sigma_1,\mathcal{O}_{\Sigma_1}(-1+\frac{1}{5}+\frac{9}{5})) & = & 2\ , \\ N(U_R) & = & N(\overline{\mathbf{3}},\mathbf{1},-4) & = & h^0(\Sigma_1,\mathcal{O}_{\Sigma_1}(-1-\frac{4}{5}+\frac{9}{5})) & = & 1\ . \end{array} \tag{5.8}$$

Similarly, for the second curve Σ_2 one obtains

$$\begin{array}{rclclcl} N(E_R) & = & N(\mathbf{1},\mathbf{1},6) & = & h^0(\Sigma_2,\mathcal{O}_{\Sigma_2}(-1-\frac{6}{5}+\frac{6}{5})) & = & 0\ , \\ N(Q_L) & = & N(\mathbf{3},\mathbf{2},1) & = & h^0(\Sigma_2,\mathcal{O}_{\Sigma_2}(-1-\frac{1}{5}+\frac{6}{5})) & = & 1\ , \\ N(U_R) & = & N(\overline{\mathbf{3}},\mathbf{1},-4) & = & h^0(\Sigma_2,\mathcal{O}_{\Sigma_2}(-1+\frac{4}{5}+\frac{6}{5})) & = & 2\ . \end{array} \tag{5.9}$$

The multiplets from both curves are then

$$\begin{array}{rcl} \Sigma_1 & : & 3\times E_R + 2\times Q_L + 1\times U_R\ , \\ \Sigma_2 & : & 2\times U_R + 1\times Q_L\ . \end{array} \tag{5.10}$$

This is the content of three $\mathbf{10}$s of $SU(5)$ unequally distributed between the two curves. It is easy to check that the full content in both curves is

$$\begin{array}{rcl} \Sigma_1 & : & [3\times E_R\ +\ 2\times Q_L\ +\ 1\times U_R]_{10_1}\ +\ [1\times D_R]_{\overline{5}_1}\ +\ [3\times N_R]_{1_1} \\ \Sigma_2 & : & [1\times Q_L\ +\ 2\times U_R]_{10_2}\ +\ [2\times D_R\ +\ 3\times L_L]_{\overline{5}_2} \end{array} \tag{5.11}$$

which corresponds to the same distribution we found in the $SO(10)$ model.

Assuming that there is a Higgs curve Σ_H with triple intersection with Σ_1 and Σ_2 and looking at the distribution of fermions one observes that there is a structure for quark masses of the form

$$h_U \sim \begin{pmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & x \end{pmatrix} \quad ; \quad h_D \sim \begin{pmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & x \end{pmatrix} . \tag{5.12}$$

For the lepton masses the pattern turns out to be

$$h_L \sim \begin{pmatrix} x & x & x \\ x & x & x \\ x & x & x \end{pmatrix} \quad ; \quad h_N \sim \begin{pmatrix} x & x & x \\ x & x & x \\ x & x & x \end{pmatrix} . \tag{5.13}$$

This is analogous to what we found for both U- and D-quarks in the $SO(10)$ example, the main difference being that the mass matrices for U- and D-quarks are now in general not strictly proportional since the wave function of U_R and D_R fields may now differ and hence there will be Cabbibo mixing. The same happens with charged leptons and Dirac neutrino masses which are no longer proportional. Again the lesson is that a small mixing for the third generation is predicted, whereas other mixings in the CKM (and neutrino mixing) are not generically small.

The $SU(5)$ model may be considered in some sense as a variant of the parent $SO(10)$ model. However it has the advantage that the symmetry breaking down to the SM does not require explicit field theoretical Higgssing. It would be interesting to find an F-theoretical 7-brane recombination process connecting both models. In any event it is clear that in order to obtain Yukawa couplings for all fermions the flux of a particular $U(1)$, that of $U(1)_{B-L}$, is required. In the $SO(10)$ model this flux acts in the bulk S surface. In $SU(5)$ the $U(1)_{B-L}$ flux goes through the matter curves, while in the bulk only hypercharge flux is felt.

Let us make some comments about the phenomenological properties of these $SU(5)$ models. As in the $SO(10)$ case, right-handed neutrinos appear generically in the spectrum. However, in the present case possible Majorana right-handed neutrino masses are not forbidden by the bulk gauge interactions but only by the $U(1)_5$ and $U(1)'$ symmetries whose gauge bosons are presumably massive via combination with RR-fields. Under these conditions Majorana masses for right-handed neutrinos could appear induced by stringy instanton effects [23]. In this model R-parity is automatic due to the absence of self-couplings of matter $\overline{\mathbf{5}}$-plets. In addition baryon number violating dim=5 operators are suppressed just like in its parent $SO(10)$ model.

At this level the third generation quarks have no mixing with the first two generations. Experimentally that mixing is of order 10^{-2}–10^{-3} so this is a good starting point. Eventually some corrections should give rise to this mixing. A natural possibility is again instanton induced couplings. Notice in this respect that such mixing is forbidden to leading order by the $U(1)'$ symmetries under which the fermion curves are charged. Those $U(1)$'s are generically anomalous and their gauge boson massive so that instanton effects of the type considered in [23] could give rise to the required non-vanishing but small third generation mixing. Instanton effects inducing non-perturbative corrections to Yukawa couplings in perturbative Type II orientifold models have been recently considered in [24].

6 Final comments and outlook

Yukawa couplings in F-theory GUT's come from the intersection of three different matter curves. At first sight this fact makes the structure of Yukawa couplings unrealistic, unless one assumes that the involved matter curves have a self-intersecting structure. In this paper we have remarked that a slight generalization of the conditions on the $U(1)_Y/U(1)_{B-L}$ fluxes breaking the GUT symmetry solves this problem in a natural and attractive way. When we have two matter curves Σ_1 and Σ_2 giving rise to the same GUT representation, instead of requesting that these fluxes vanish in each individual matter curve it is enough to impose that the fluxes, particularly $U(1)_{B-L}$ fluxes, cancel on average (i.e.$\int_{\Sigma_1} F_{U(1)_{B-L}} + \int_{\Sigma_2} F_{U(1)_{B-L}} = 0$). Under these conditions the SM fermions redistribute asymmetrically on the two matter curves so that Yukawa couplings for all quarks and leptons are allowed. This structure predicts suppressed CKM mixing for the third quark generation and unsuppressed mixing for the rest of the generations and also for leptons. This prediction is in good qualitative agreement with experiment.

We have constructed specific local F-theory $SO(10)$ and $SU(5)$ configurations with this structure. It would be interesting to further develop the phenomenology of these models. In particular it would be interesting to explore whether one can find configurations with good quantitative agreement with observed quark/lepton masses and mixings. These models have the low-energy spectrum of the MSSM. The structure of SUSY-breaking soft terms under the assumption of modulus dominance has recently been analysed in [11], where compatibility with low-energy experimental constraints and radiative electroweak symmetry breaking was shown. The possibility that SUSY is induced by a particular variety of gauge mediation has also been recently studied in [17, 15]. It would be interesting to see whether the splitting of matter fields on two different curves Σ_1 and Σ_2, as explored in the present paper, has a bearing on these and other phenomenological aspects of F-theory GUT's. Let us finally comment that the smallness of the mixing of the third quark generation with the first two families could perhaps be a first hint, together with gauge coupling unification, of an underlying F-theory grand unification.

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A Brief compendium on del Pezzo surfaces and other mathematical results

The first step to construct an F-theory GUT is to specify the surface S wrapped by the 7-branes. In this work we mostly consider $S = dP_8$. The del Pezzo surfaces dP_N, $N = 1, \cdots, 8$, are defined as the blowup of $\mathbb{P}^2$ at N points. The canonical class of dP_N is

$$K_S = -c_1(S) = -3H + \sum_{i=1}^{N} E_i \ , \tag{A.1}$$

where H is the hyperplane class and the E_i are exceptional divisors. We will also need the intersections

$$H \cdot H = 1 \quad ; \quad H \cdot E_i = 0 \quad ; \quad E_i \cdot E_j = -\delta_{ij} \ . \tag{A.2}$$

The generators $\alpha_i = E_i - E_{i+1}$, $i = 1, \cdots, N-1$, and $\alpha_N = H - E_1 - E_2 - E_3$, have intersection products equal to minus the Cartan matrix of the Lie algebra E_N and can be regarded as simple roots.

An important result is that the admissible line bundles on dP_N are in one to one correspondence with roots of the E_N algebra [8, 9]. For example, the line bundle of the $U(1)$ flux that breaks the GUT group can be chosen to be $L = \mathcal{O}_S(\alpha_i)^{1/n}$. Fractional bundles are allowed as long as the physically relevant powers are integers.

After selecting the surface S one has to define the matter curves Σ_i that must be divisors of S. Since S has complex dimension two, the Σ_i are Riemann surfaces and their genus is given by

$$2g_i - 2 = \Sigma_i \cdot (\Sigma_i + K_S) \ . \tag{A.3}$$

For instance, for $S = dP_N$, taking $\Sigma_i = E_i$ yields $g_i = 0$, as it should because each E_i is a $\mathbb{P}^1$. We will only consider curves of genus zero or one.

To evaluate the degeneracies of charged multiplets living on a matter curve Σ we need to know $h^0(\Sigma, K_\Sigma^{1/2} \otimes L_\Sigma^q \otimes L_\Sigma'^{q'}) = \dim_{\mathbb{C}} H^0_{\bar\partial}(\Sigma, K_\Sigma^{1/2} \otimes L_\Sigma^q \otimes L_\Sigma'^{q'})$. The product of line bundles is another line bundle of degree equal to the degree of its associated divisor. A useful result is that when Σ has genus zero $h^0(\Sigma, \mathcal{O}_\Sigma(d))$ vanishes when the degree d is negative and it is equal to $(d+1)$ for $d \geq 0$.

The restricted bundle L_Σ is determined by the line bundle L on S that is introduced from the start to break the GUT group G_S. Since L has a corresponding divisor D on S, L_Σ will be a line bundle of degree equal to $\Sigma \cdot D$. On the other hand, the form of the line bundle L' on the intersecting surface S' need not be given explicitly because the gauge theory on S' is completely decoupled. One only has to specify the restriction L'_Σ.

When Σ has genus zero the restricted line bundles can be characterized by their degrees. However, when Σ has genus one it is necessary to give the associated divisor that might be effective or not. A divisor $\mathcal{D}$ of Σ can be written as a formal linear combination of irreducible codimension one hypersurfaces. Since Σ is a curve, these hypersurfaces are points p_m and $\mathcal{D} = \sum_m a_m p_m$. The divisor is effective if $a_m > 0$, $\forall m$. The degree of $\mathcal{D}$ is equal to $\sum_m a_m$.

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