Solution to Faddeev equations with two-body experimental amplitudes as input and application to $J^P = 1/2^+$, $S = 0$ baryon resonances

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Abstract

We solve the Faddeev equations for the two meson-one baryon system $\pi\pi N$ and coupled channels using the experimental two-body t-matrices for the πN interaction as input and unitary chiral dynamics to describe the interaction between the rest of coupled channels. In addition to the $N^*(1710)$ obtained before with the $\pi\pi N$ channel, we obtain, for $J^\pi = 1/2^+$ and total isospin of the three-body system $I = 1/2$, a resonance peak whose mass is around 2080 MeV and width of 54 MeV, while for $I = 3/2$ we find a peak around 2126 MeV and 42 MeV of width. These two resonances can be identified with the $N^*(2100)$ and the $\Delta(1910)$, respectively. We obtain another peak in the isospin 1/2 configuration, around 1920 MeV which can be interpreted as a resonance in the $Na_0(980)$ and $Nf_0(980)$ systems.

1 Introduction

Recent developments around three-body systems with two mesons and one baryon using chiral dynamics have brought new light into the nature of the $J^P = 1/2^+$ baryonic resonances. The study of such systems with strangeness $S = -1$ produced resonant states which could be identified with the existing low lying baryonic $J^P = 1/2^+$ resonances, two Λ and four Σ states [1, 2].

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Similarly, in the case of the $S = 0$ sector the $N^*(1710)$ appears neatly as a resonance of the $\pi\pi N$ system as well as including the channels coupled to $\pi\pi N$ within SU(3) [3]. Developments along the same direction produced a resonant state of $\phi K\bar{K}$ [4] which could be identified with the X(2175) resonance reported at BABAR [5, 6] and later on at BES [7]. The study of the three-body systems was done using Faddeev equations (FE) in the coupled channel approach. While most conventional studies of three-body systems use potentials in coordinate space, usually separable potentials to make the solution of the FE feasible, the approach of [1, 2] used two particle amplitudes generated within the unitary chiral approach in momentum space. Yet, the most novel finding in these works was the realization that, for s-waves and in the SU(3) limit, there was an exact cancellation between the off shell part of the two-body amplitudes and the three-body forces generated by the same chiral Lagrangians. To be more precise, the on shell amplitude means that the s-wave amplitude is calculated as a function of the Mandelstam variable s imposing $q^2 = m^2$ for the external momenta of the two body amplitudes. When these lines are inside the Faddeev diagrams where some line can be off shell, the full amplitude is separated into this "on shell" part plus and "off shell" part which goes as $q^2 - m^2$ for mesons and $q^0 - E(q)$ for baryons and vanishes when the external lines are on shell. This off shell part contains an inverse particle propagator and cancels one particle propagator rendering a Faddeev diagram with two two-body t-matrices into a three-body contact term, which has the same topology as genuine three-body interactions that stem from the chiral Lagrangians and cancel them exactly. As a consequence, one needs only the on shell two-body t-matrices and can ignore these three-body forces. This finding is novel for such studies and simplifies the work technically, although not much, since loops involve a changing s-variable, and consequently the s-dependent t-matrices must be inserted into the loop functions. This makes this approach different and technically more involved than the study of the two body interaction, where using arguments of the N/D method one can factorize on shell amplitudes outside the loop functions which involve only two hadron propagators [8, 9]. The strongest value of that finding in the three-body problem is that the results do not depend upon the off shell extrapolations of the amplitudes which is a source of uncertainty in the three-body calculations that rely upon a potential. Indeed, it is well known that given a certain physical amplitude, on shell by nature, one has an infinite number of potentials that give this amplitude upon solving the Schrödinger equation. The differences between the different potentials will only show in the off shell extrapolation of the amplitudes. However, this information enters the solution of the Faddeev equations and, hence, different potentials leading to the same on shell amplitude will provide different results upon solution of the Faddeev equations.

The problem stated above is most probably the main reason why recent works dealing with the $\bar{K}NN$ system lead to quite different results in the binding and the width. In this sense, we find a series of works based on Faddeev equations which lead to relatively large binding, of the order of $50 - 70$ MeV [10, 11, 12, 13], while other works based on variational methods lead to smaller bindings of the order of 20-30 MeV [14, 15, 16]. The widths also vary from $50 - 100$ MeV.

The arbitrariness of the off shell amplitude is also well known in field theory, where the implementation of unitary transformations of the fields in the Lagrangian maintains the same on shell amplitudes but changes their off shell extrapolation. In this sense it is interesting to note that, although the off shell versus three-body cancellation discussed here is not explicitly shown in other three-body works using also chiral dynamics [17, 18], the approach is invariant upon these transformations, indicating that the mentioned cancellations apparently occur in the full calculation [19]. A similar independence on the off shell extrapolation has been shown in different reactions like the $\pi N \to \pi\pi N$ reaction [20] and the study of the interacting two pion exchange in the NN interaction [21]. However, the explicit realization of the off shell versus three-body forces indicates that one can neglect the three-body forces from the beginning, certainly simplifying the approach, and use only the two-body on shell amplitudes. Even more, these on shell amplitudes can be obtained from experiment and one can omit having to do a theory for the two-body interaction. There is a small caveat there, since sometimes in the loops one will need "on shell" amplitudes below threshold. This looks like a contradiction, but we made it clear the meaning on the on shell amplitude needed in the Faddeev equations, which is the one where $q^2 = m^2$ for the external momenta. Provided one has a suitable parametrization of the amplitude, the extrapolation below threshold fulfilling this condition is not a difficult task to accomplish. In many cases, like in the present one that we shall discuss here, one needs the information well within the physical range and the extrapolation is not even needed. This picture presented here is rather novel and the purpose of the present paper is to show how it works and how it can help whenever the theoretical models are not accurate enough.

With the perspective given above we shall tackle here the investigation of three-body systems with two mesons and a baryon with strangeness $S = 0$. The problem was already discussed in [3], where the $N^*(1710)$ was found as a resonant state of $\pi\pi N$. It was also found there that the implementation of other coupled channels barely changed the results obtained with the base of the $\pi\pi N$ states alone. Yet, there are other $J^P = 1/2^+$ states, like the $N^*(2100)$ and the $\Delta(1910)$, which do not appear with that base and the use of the amplitudes obtained with the lowest order chiral lagrangians. From the work of [22] we know that the chiral unitary approach using the lowest order chiral Lagrangian provides a fair amplitude up to $\sqrt{s} = 1600\ MeV$ but fails beyond this energy. For instance, the $N^*(1650)$ does not appear in the approach. As a consequence, any three-body states which would choose to cluster a πN subsystem into this resonance would not be obtained in the approach of [3]. In the present work we shall give the step to use experimental πN amplitudes and will show that in this case we reproduce the $N^*(1710)$ resonance without practically any modification with respect to [3], but the use of a more realistic πN interaction at higher energies leads also to the generation of the $N^*(2100)$ and the $\Delta(1910)$ resonances as three-body systems of two mesons and one baryon in coupled channels.

2 Formalism and Results

We follow the method developed in [1, 2, 3, 4] to calculate the three-body T-matrix and search for resonances. In [1, 2, 3, 4] a coupled channel Bethe-Salpeter equation is solved to calculate the required two-body t-matrices with the potentials obtained from chiral Lagrangians. These t-matrices, which contain the information of the two-body resonances, are then used as an input to solve the Faddeev equations in a coupled channel approach. The Faddeev equations, in our formalism, have been reformulated into algebraic ones, which are written in terms of two-body t-matrices depending on the invariant mass of the two interacting particles and loop functions, named G, which incorporate the off-shell dependence of the three-body diagrams,

$$T_R^{ij} = t^i g^{ij} t^j + t^i [G^{ijk} T_R^{jk} + G^{iji} T_R^{ji}] \quad i \neq j \neq k = 1, 2, 3. \tag{1}$$

We solve Eq. (1) for the $\pi\pi N$ system and coupled channels. The present work benefits from the previous study of the $\pi\pi N$ system and coupled channels [3], where the dynamical generation of the $N^*(1710)$ was found. This state was found when the total isospin 1/2 three-body T-matrix was evaluated by adding a nucleon to the two pions interacting in isospin zero in the energy range of the σ-resonance. The $N^*(1710)$ was thus interpreted as a resonance in the $\pi\pi N$ system, where the two pions rearrange to form the σ-resonance. The total energy range studied in [3] corresponded to a variation of the invariant mass of one of the πN pairs up to $\sim$ 1550 MeV. The calculations in [3] were limited to these energy range because the input πN t-matrix used in that work was taken from [22] which reproduces the πN scattering data well up to about 1600 MeV.

The motivation of this work is to extend the calculations made in [3] to higher energies by including the $N^*(1535)$ and $N^*(1650)$ in the input πN t-matrix and look for the other three-body isospin 1/2 and 3/2 states with $J^P = 1/2^+$ in the $\pi\pi N$ system and coupled channels. In order to do this, we use the experimental $L = 0$ phase shifts (δ) and inelasticities (η) [23] for the πN system in isospin 1/2 and 3/2 (Fig. 1, 2) and calculate from them the πN amplitudes in the isospin base (Fig. 3) using the relation

$$t^I = -\frac{4\pi E}{M} f^I, \quad \text{I=1/2, 3/2} \tag{2}$$

with

$$f^I = \frac{\eta^I e^{2i\delta^I} - 1}{2iq} \tag{3}$$

where η^I is the inelasticity, δ^I the phase shift, M is the nucleon mass, E is the πN center of mass energy and q is the corresponding momentum.

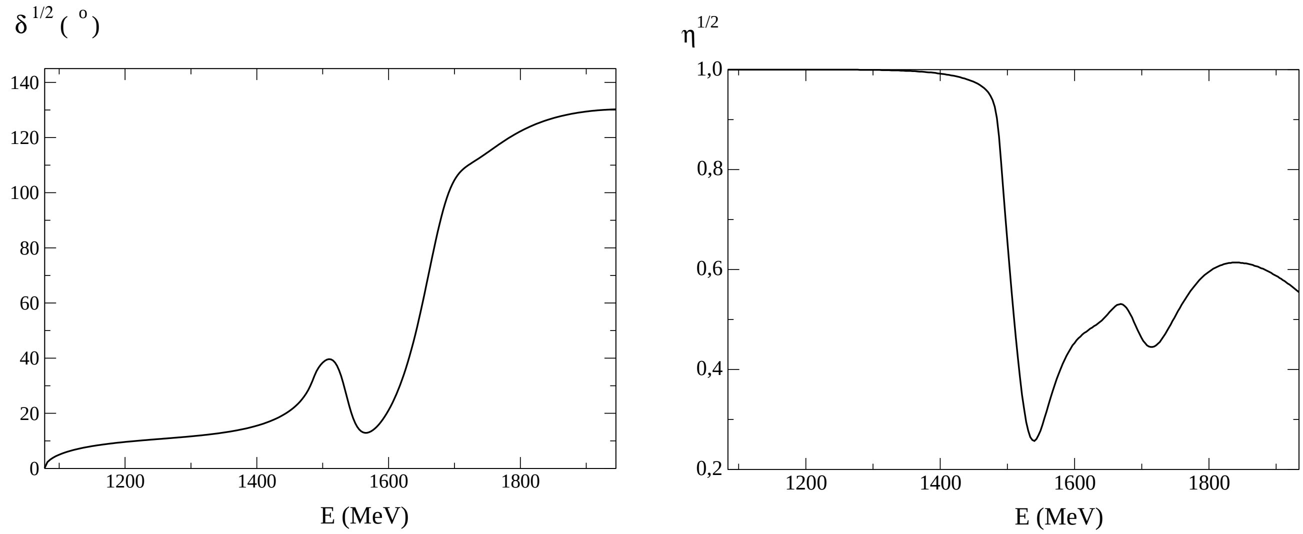

Figure 1: Experimental phase shifts and inelasticity for the πN interaction in isospin 1/2.

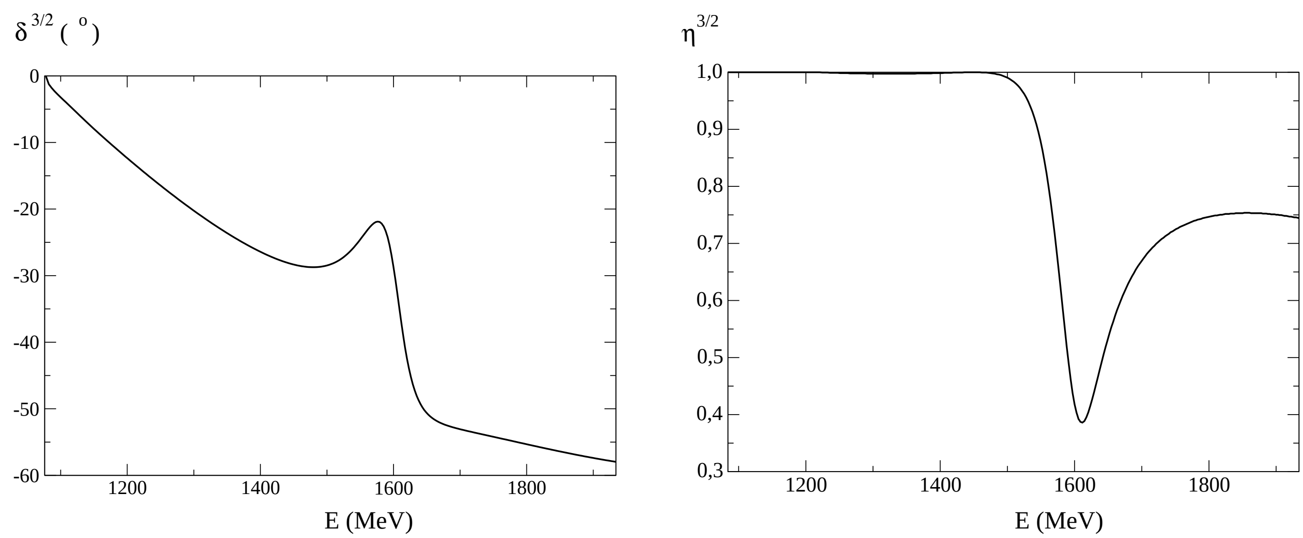

Figure 2: Experimental phase shifts and inelasticity for the πN interaction in isospin 3/2.

We require the input two-body t-matrices in the charge base to solve the Faddeev equations in our model. For this we use the relations

$$t_{\pi^0 n \to \pi^0 n} \;=\; \frac{2}{3}t^{3/2} + \frac{1}{3}t^{1/2}, \quad t_{\pi^0 n \to \pi^- p} = \frac{\sqrt{2}}{3}t^{3/2} - \frac{\sqrt{2}}{3}t^{1/2},$$

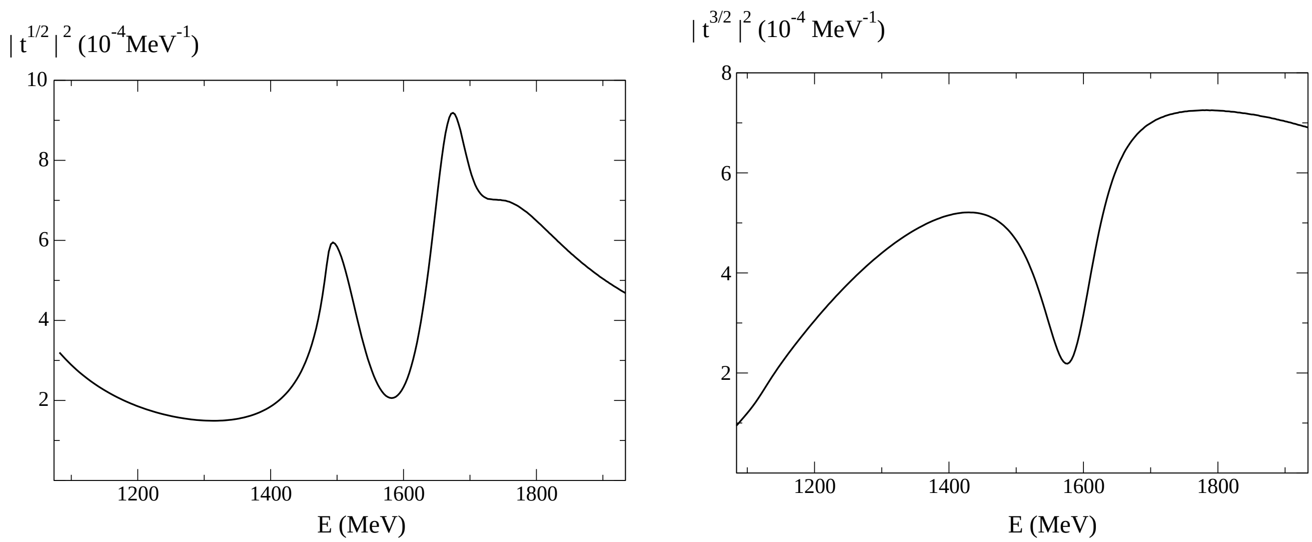

Figure 3: Experimental t-matrices for the πN interaction in isospin 1/2 and 3/2.

$$
\begin{aligned}
t_{\pi^- p\to\pi^- p} &= \frac{1}{3}t^{3/2} + \frac{2}{3}t^{1/2}, \quad t_{\pi^- n\to\pi^- n} = t^{3/2},\\
t_{\pi^+ n\to\pi^+ n} &= t_{\pi^- p\to\pi^- p}, \quad t_{\pi^0 p\to\pi^0 p} = t_{\pi^0 n\to\pi^0 n},\\
t_{\pi^0 p\to\pi^+ n} &= -t_{\pi^0 n\to\pi^- p}.
\end{aligned}
$$

In this way, we can extend the model for the $\pi\pi N$ interaction of [3] to higher energies where the invariant masses of the πN subsystems can be varied around 1650 MeV.

2.1 Exploring the $\pi\pi N$ system

We first study the $\pi\pi N$ system with total charge zero considering $\pi^0\pi^0 n$, $\pi^0\pi^- p$, $\pi^+\pi^- n$, $\pi^-\pi^+ n$ and $\pi^-\pi^0 p$ as coupled channels. We calculate the three-body T_R matrix in total isospin 1/2 by keeping the $\pi\pi$ system in isospin zero and for total energy around 1700 MeV. The πN t-matrix above threshold has been calculated using Eq. (2) and the phase shifts and inelasticities shown in Figs. 1, 2 and for energies below threshold we have followed [22]. For the $\pi\pi$ interaction we use the t-matrix obtained and studied thoroughly in [24], where the dynamical generation of the $\sigma(600)$, $f_0(980)$ and $a_0(980)$ resonances was found and the theoretical results for physical observables coincided well with the experimental ones. We find exactly the same peak at 1704 MeV as obtained in [3]. In this way, we ensure that we reproduce our previous results even by using the experimental data for the πN interaction. With this assurance, we now look for resonances in the higher energy region.

In Fig. 4 we show the isospin 1/2 $T_R^* \equiv \sum_{ij}(T_R^{ij} - t^i g^{ij} t^j)$ matrix (see [1] for this definition)

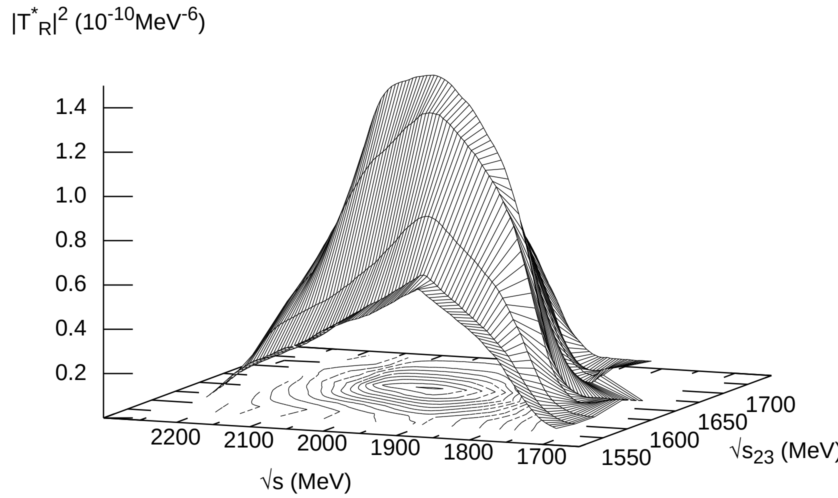

Figure 4: The $N^*(2100)$ in the $\pi\pi N$ system with five coupled channels.

versus the total energy of the three-body system, $\sqrt{s}$, and the invariant mass of the meson-baryon subsystem formed by the second and third particle, $\sqrt{s_{23}}$, which has been kept in isospin $I_{23} = 1/2$. A peak around an energy of 2100 MeV with a width of $\sim$ 250 MeV appears when $\sqrt{s_{23}}$ is close to 1670 MeV. These results are compatible with the findings of various partial wave analyses indicated by the PDG [25] about the $N^*(2100)$, for which the peak position is found in the range 1855 - 2200 MeV and the width in the range of 69-360 MeV.

Therefore, by studying only the $\pi\pi N$ channels, we find that the resonance $N^*(2100)$ has a large coupling to $\pi N^*(1650)$ and that the inclusion of the $N^*(1650)$ in the πN subsystem is essential to generate a resonance at 2100 MeV.

In this former study, we do not find evidence for any resonance in the isospin 3/2 configuration, but the situation is different when we introduce coupled channels, as we discuss below.

2.2 Inclusion of the $\pi K\Sigma$, $\pi K\Lambda$ and $\pi\eta N$ channels

Next, we solve the Faddeev equations with fourteen coupled channels: $\pi^0\pi^0 n$, $\pi^0\pi^- p$, $\pi^0 K^+\Sigma^-$, $\pi^0 K^0\Sigma^0$, $\pi^0 K^0\Lambda$, $\pi^0\eta n$, $\pi^+\pi^- n$, $\pi^+ K^0\Sigma^-$, $\pi^-\pi^+ n$, $\pi^-\pi^0 p$, $\pi^- K^+\Sigma^0$, $\pi^- K^0\Sigma^+$, $\pi^- K^+\Lambda$ and $\pi^-\eta p$. As there are no data for $K\Sigma \to K\Sigma$, $K\Lambda \to K\Lambda$, etc., we use the model of [22] to calculate

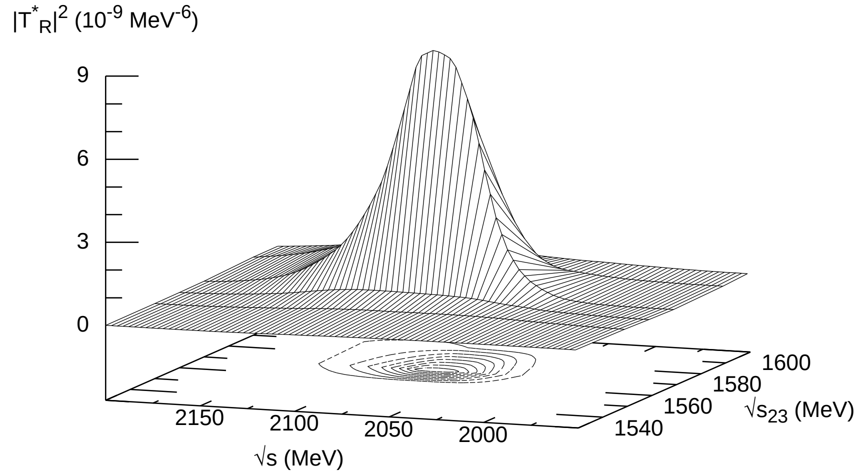

Figure 5: The $N^*(2100)$ in the $\pi\pi N$ system including 14 coupled channels.

the corresponding amplitudes. The πN interaction below threshold is determined using the same model as for $K\Sigma$ and $K\Lambda$ and above the threshold we use the experimental results.

In Fig. 5 we show the result obtained in $I = 1/2$ for this case by keeping the subsystem of particles 2 and 3 in $I_{23} = 1/2$. We obtain a peak at an energy of 2080 MeV with a width of 54 MeV for a $\sqrt{s_{23}}$ near 1570 MeV, which we identify with the $N^*(2100)$ listed in the *PDG* [25]. The inclusion of the $\pi K\Sigma$, $\pi K\Lambda$ and $\pi\eta N$ channels makes the resonance more pronounced (by an order of magnitude in the squared T^*_R-matrix) and much narrower. These changes in the results can be easily understood with respect to the previous ones obtained with only five coupled channels by noticing that now the wave function of the resonance contains extra components which have smaller phase space in the decay of the resonance. At the same time, the $\pi\pi N$ component becomes smaller due to the normalization of the wave function and, hence, the decay into $\pi\pi N$ is also reduced.

We plot now the T^*_R-matrix for total isospin $I = 3/2$ with $I_{23} = 1/2$ for the $\pi K\Lambda$ channel in Fig. 6. A peak is found at a total energy of ~ 2126 MeV with ~ 42 MeV of width. In this case, the invariant mass $\sqrt{s_{23}}$, at which the peak appears, is around 1590 MeV. This peak can be identified with the $\Delta(1910)$ listed in [25], whose position, given by different partial wave analyses, ranges up to 2070 MeV and the width varies from 190-500 MeV.

Thus the introduction of the $\pi K\Sigma$, $\pi K\Lambda$ and $\pi\eta N$ channels, together with the inclusion of

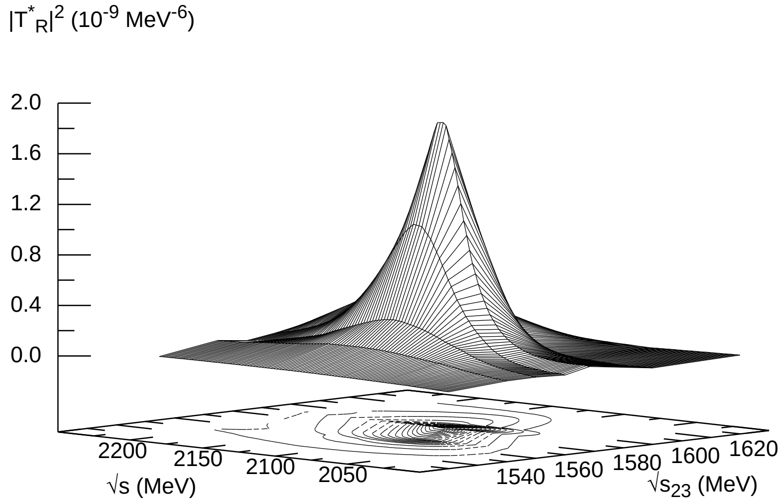

Figure 6: The $\Delta(1910)$ in the $\pi K \Lambda$ system including 14 coupled channels.

the $N^*(1650)$ in the πN t-matrix, is important to get this resonance. One should note that we get smaller widths than the experimental ones. The πN decay channels are not considered in our approach and they should contribute to increase the widths. Note that this can be done even with a small πN component, as implicitly assumed here, since there is more phase space for decay into the πN channel (see [1] for more discussion).

We do not find any evidence of the $\Delta(1750)$, which could indicate a different structure for this state that the one studied in this work.

3 Exploring the Nf_0 and Na_0 systems by taking $N\pi\pi$, $NK\bar{K}$ and $N\pi\eta$ as coupled channels

Until now, we have investigated possible resonant states in the $\pi\pi N$ system and its coupled channels which have been obtained by adding a pion to pseudoscalar-baryon systems which couple strongly in $J^\pi = 1/2^-$ and isospin 1/2 configuration, i.e., πN, $K\Sigma$, $K\Lambda$ and ηN. The invariant mass of this pseudoscalar-baryon subsystem has been varied around that of the $N^*(1535)$ and $N^*(1650)$, hence, treating the three-body system as a πN^* system with $1500 < M_{N^*} < 1760$ MeV, although within the three-body Faddeev equations. There are other configurations of this three-body system, like

$Na_0(980)$ and $Nf_0(980)$, which we have not discussed so far.

In order to study such a system, we must take $NK\bar{K}$, $N\pi\pi$ and $N\pi\eta$ as coupled channels, such that the $\pi\pi$ and $K\bar{K}$ subsystem dynamically generate the $f_0(980)$ and the $\pi\eta$ subsystem along with $K\bar{K}$ generates the $a_0(980)$ resonance. In this way, we can study the $Nf_0(980)$ and $Na_0(980)$ systems simultaneously by projecting the three-body channels in the isospin base while keeping the subsystem made of the particles two and three in isospin zero or one and by changing the corresponding invariant mass, s_{23}, around 980 MeV. Concretely, we take the following coupled channels into account: $n\pi^0\pi^0$, $p\pi^0\pi^-$, $n\pi^0\eta$, $n\pi^+\pi^-$, $n\pi^-\pi^+$, $p\pi^-\pi^0$, $p\pi^-\eta$, nK^+K^-, $nK^0\bar{K}^0$, pK^0K^-. With these channels we solve the few-body equations in the formalism developed in [1, 2, 3, 4]. For total isospin 1/2, with the 2-3 subsystem projected in isospin one, we obtain a peak around 2080 MeV, with a width of 51 MeV, which we relate with the peaks shown in Figs.4 and 5 as there $Na_0(980)$ partner. Interestingly, along with this $N^*(2100)$ state, we find another peak with even larger strength in the squared three-body amplitude, at $\sqrt{s} = 1924$ MeV, with a width of 20 MeV. We show this peak in Fig. 7 for the $NK\bar{K}$ channel.

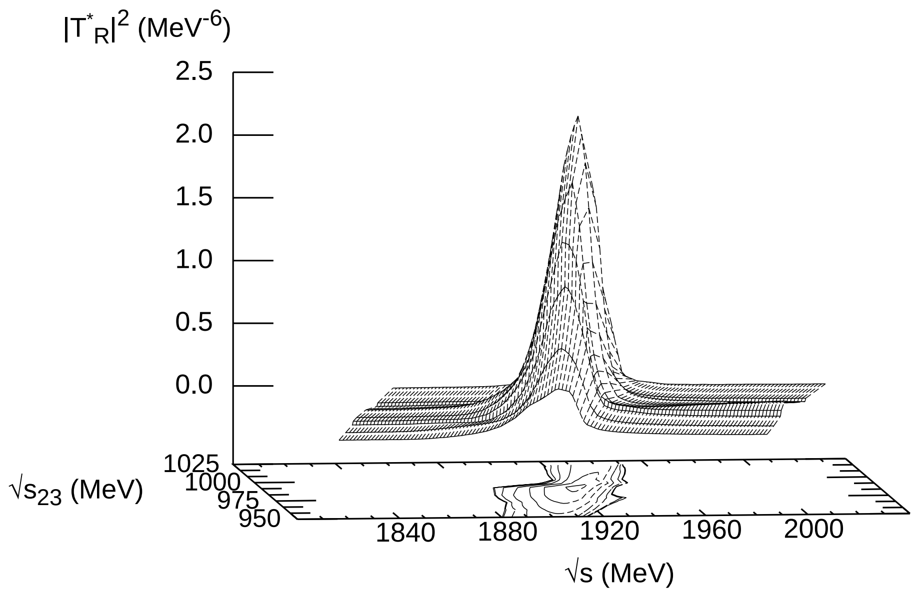

Figure 7: A possible $N^*(1910)$ in the $NK\bar{K}$ channels.

This state is about 7 MeV below the $NK\bar{K}$ threshold (assuming an average mass for the kaons of 496 MeV and 939 MeV for the nucleon). Therefore, this result indicates that the $Na_0(980)$ system gets bound at around 1920 MeV. This possibility has been already suggested by the authors in [26], in which they study the $NK\bar{K}$ channel using effective two-body potentials to describe the $\bar{K}N$, $\bar{K}K$, KN interactions. They find that the $NK\bar{K}$ system can get bound while the $K\bar{K}$ subsystem acts like the a_0. Our result is, thus, in agreement with the suggestions in [26]. Interestingly, the existence of a $1/2^+$ N^* resonance around 1935 MeV has also been proposed earlier [27] on the basis of a study of the data on the $\gamma p \to K^+\Lambda$ reaction in an isobar model, although other theories [28] which include explicitly resonances up to 1855 MeV can reproduce these data (though further work along these lines to include higher mass resonances is under way [29]).

Since the peak found is below the three-body threshold and, in the two-body problem, the poles for the $f_0(980)$ and $a_0(980)$ appear below the $K\bar{K}$ threshold, the three particles in the system have associated complex momenta in the momentum representation. To avoid the use of unphysical complex momenta in the three-body system, which will lead to imaginary energies in the real plane, we give a minimum value, around 50 MeV, to the momentum of the particles. We have check the sensitivity of our results to the mentioned choice by changing the minimum momentum from 50 MeV to 100 MeV and we find the peak and width to remain almost unchanged.

We also projected the t-matrix in isopin 1/2 by keeping the $K\bar{K}$ system in isospin zero, i.e., looking at the $Nf_0(980)$ system. In this case too, we find a peak around 1923 MeV with a width of 30 MeV and another one around 2052 MeV with a width of 60 MeV. The strength of the first peak mentioned above is very similar to the corresponding one found in the $Na_0(980)$ system, but the strength of the second peak is bigger as compared to that in $Na_0(980)$.

From the whole study we would conclude that there are two N^*'s with $J^\pi = 1/2^+$ in the energy region $1800 < \sqrt{s} < 2200$ MeV.

4 Conclusions

To summarize, we have extended our previous study of the $\pi\pi N$ system and coupled channels [3], where the generation of the $N^*(1710)$ was found, to higher energies. In this work the new input is the experimental data on the πN interaction where the information on excitation of both the $N^*(1535)$ and the $N^*(1650)$ is present, the latter of which was absent in our previous work [3]. Here, apart from confirming the $N^*(1710)$, we find evidence for the other $1/2^+$ N^*, i.e., the $N^*(2100)$, and also for the $1/2^+$ $\Delta(1910)$ resonance. The findings reported here indicate that the inclusion of the $N^*(1650)$ in the interaction of the πN subsystem is essential to generate these higher mass $1/2^+$ resonances. We have made a first search including only the $\pi\pi N$ channels where a resonance having the properties of $N^*(2100)$ was found. Later we included the $\pi K\Sigma$, $\pi K\Lambda$ and $\pi\eta\Sigma$ channels where the same resonance is produced but with larger magnitude and narrower width, indicating the addition of more channels to which the resonance couples strongly. No isospin 3/2 resonances is found in the study of the $\pi\pi N$ channels alone. However, the $\Delta(1910)$ is found on inclusion of the $\pi K\Sigma$, $\pi K\Lambda$ and $\pi\eta\Sigma$ channels. Further, we have investigated the $NK\bar{K}$, $N\pi\pi$ and $N\pi\eta$ channels where the $K\bar{K}-\pi\pi$ subsystem rearranges itself as a $f_0(980)$ resonance, while $K\bar{K}-\pi\eta$ acts like the $a_0(980)$. We obtain a new peak at $\sim$ 1924 MeV, apart from the one corresponding to the $N^*(2100)$, with a strong coupling to $Na_0(980)$ and $Nf_0(980)$. Finally, we conclude this work by stating that the study of three-body systems, for the cases where a complete theoretical two-body input is not available, is also possible in our formalism using on shell experimental amplitudes.

Acknowledgments

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