New Angle on the Strong CP and Chiral Symmetry Problems from a Rotating Mass Matrix

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Abstract

It is shown that when the mass matrix changes in orientation (rotates) in generation space for changing energy scale, then the masses of the lower generations are not given just by its eigenvalues. In particular, these masses need not be zero even when the eigenvalues are zero. In that case, the strong CP problem can be avoided by removing the unwanted θ term by a chiral transformation in no contradiction with the nonvanishing quark masses experimentally observed. Similarly, a rotating mass matrix may shed new light on the problem of chiral symmetry breaking. That the fermion mass matrix may so rotate with scale has been suggested before as a possible explanation for up-down fermion mixing and fermion mass hierarchy, giving results in good agreement with experiment.

A fermion mass matrix extracted directly from a Yukawa term in the action takes usually the following form:

$$m\frac{1}{2}(1+\gamma_5) + m^\dagger \frac{1}{2}(1-\gamma_5). \qquad (1)$$

However, by appropriately relabelling the right-handed fields in the singlet representation, which will not change the physics, the mass matrix can always be recast into a hermitian form independent of γ_5 [1]. It is this hermitian form of the fermion mass matrix that will exclusively be used in the present paper, which is henceforth taken as understood. Furthermore, to be specific, the analysis will be carried out explicitly only for the realistic case of 3 fermion generations, although it can readily be extended with only minor modifications to other numbers of fermion generations.

By a rotating fermion mass matrix then, we mean a fermion mass matrix which changes its orientation in generation space as the scale changes. Explicitly,

$$m(\mu) = U(\mu,\mu_0) m(\mu_0) U^{-1}(\mu,\mu_0), \qquad (2)$$

where $U(\mu,\mu_0)$ is unitary, whose explicit form will depend on the theory under consideration, but which we can leave unspecified for the present general discussion

That the mass matrix m can so rotate is expected. In much the same way as the familiar running of coupling strengths and mass values results as a consequence of renormalization, so will generally the rotation of the mass matrix. Indeed, even in the Standard Model, so long as the CKM or MNS mixing matrix is not diagonal, the fermion mass matrix will rotate with changing scale [2], although the rotation there is so slow as to be negligible for most practical purposes. Once beyond the Standard Model framework, however, it will not be difficult to imagine situations where rotation becomes more appreciable.

For the moment, we shall not address the question what dynamics will generate appreciable rotation or whether such dynamics is realistic, but concentrate first on the theoretical question of what physical consequences will result from a rotating fermion mass matrix whatever its origin, of which consequences, as we shall see, there are some of considerable interest. Only at the end of the paper shall we return to summarize some evidences for mass matrix rotation, both empirical and theoretical.

Not surprisingly, a rotating mass matrix will force on us some changes in notions we have grown used to in the situation when the mass matrix does not rotate. Indeed, one soon learns from experience when working with a rotating mass matrix that it would be unwise to take for granted any of these notions, no matter how familiar, without first checking whether it can be extended unchanged to the rotating case. One example of particular interest to the present discussion is the statement often made, based on experience gained from the nonrotating mass matrix, that chiral invariant interactions cannot generate nonzero physical masses from an initially chiral invariant mass matrix. We shall immediately see below that such a statement cannot in general be maintained without modifications for a rotating mass matrix.

Of course, whether the mass matrix rotates or not, any chiral invariant interactions will leave an initially chiral invariant mass matrix still chiral invariant. More precisely, one means that starting with a mass matrix of a certain rank, then under interactions of the same rank, the rank of the mass matrix will be maintained. For example, starting with a rank 1 mass matrix in 3 generations, the renormalized mass matrix will remain of rank 1, i.e.

$$m(\mu) = m_T(\mu)|\boldsymbol{\alpha}(\mu)\rangle\langle\boldsymbol{\alpha}(\mu)|, \tag{3}$$

or that it still has 2 zero eigenvalues. For a nonrotating mass matrix, i.e. when $|\boldsymbol{\alpha}(\mu)\rangle$ does not depend on μ, it then follows that 2 of the physical particles must still have zero mass, since the masses are just given by the eigenvalues. In case the mass matrix rotates, i.e. when $|\boldsymbol{\alpha}(\mu)\rangle$ does indeed depend on μ, however, this does *not* follow, since the masses of the physical particles are *not* all given just by the eigenvalues.

That this is the case may seem surprising at first sight, but it can be verified immediately as follows. To be specific, let us consider the charged leptons, assuming that the mass matrix rotates but remains of rank 1 at all scales, i.e. of the form (3). To identify the masses of the physical states, we need first to specify these physical states. The heaviest physical state, say $\mathbf{v}_\tau$, is easy; it is the single massive eigenstate $\boldsymbol{\alpha}(\mu)$ of the mass matrix (3) taken at the scale equal to its mass $\mu = m_\tau$, i.e. $\mathbf{v}_\tau = \boldsymbol{\alpha}(m_\tau)$. The other physical states, $\mathbf{v}_\mu$ and $\mathbf{v}_e$ have then to be orthogonal to $\mathbf{v}_\tau$ and to each other, for τ, μ, e are supposedly independent quantum states. Otherwise, if, say, $\mathbf{v}_\tau$ is allowed to have a nonzero component in $\mathbf{v}_\mu$ or $\mathbf{v}_e$, then τ can decay readily into $\mu\gamma$ or $e\gamma$ leading to blatant flavour-violations unseen in experiment. Hence, $\mathbf{v}_\mu$ and $\mathbf{v}_e$ must have eigenvalue zero at the scale $\mu = m_\tau$. But this zero eigenvalue at the scale $\mu = m_\tau$ is not the mass of the physical states μ and e, which has to be taken as the value(s) at the scale(s) equal to their mass(es). However, at any lower scale, $\mu < m_\tau$, the single massive eigenstate $\boldsymbol{\alpha}(\mu)$ of the rotating mass matrix will have rotated to a direction different to that of $\mathbf{v}_\tau$, its direction at $\mu = m_\tau$. It will then no longer be orthogonal to the plane spanned by $\mathbf{v}_\mu$ and $\mathbf{v}_e$. But at any scale, the plane spanned by the two zero eigenvectors is always orthogonal to the massive eigenvector, so that the state vectors $\mathbf{v}_\mu$ and $\mathbf{v}_e$ cannot both remain in the zero eigenspace at this lower scale. Hence, we conclude that $\mathbf{v}_\mu$ and $\mathbf{v}_e$ cannot both be eigenstates with zero eigenvalue of the mass matrix at any scale $\mu < m_\tau$, and confirm the assertion made at the end of the last paragraph.

At least one of the states $\mathbf{v}_\mu$ and $\mathbf{v}_e$ must have a nonvanishing component in the direction of the massive state $\boldsymbol{\alpha}(\mu)$ for any $\mu < m_\tau$ and acquire thereby a mass, thus contradicting the statement above that chiral interations cannot generate nonzero physical masses. That was just a notion gleaned from experience with nonrotating mass matrices which is now found inapplicable to rotating mass matrices. Notice that since the mass matrix m is still of rank 1, it has at every scale 2 linearly independent eigenvectors with eigenvalue zero. And any chiral transformation on these 2 states will leave physics invariant so that no chiral property of the mass matrix we started with has ever been lost. Only, by the above analysis, we find that, for the rotating mass matrix, in contrast to the nonrotating case, those states on which the chiral transformations leave physics invariant are *not* the physical states, the chiral transformation of which has thus no reason to keep the invariance.

The above example shows that for a rotating mass matrix, the physical masses are not in general given just by the eigenvalues of the rotating mass matrix, nor the physical states by the eigenvectors, so that zero eigenvalues do not necessarily imply zero physical masses. But it begs the question how the physical masses and states are then to be defined, to answer which further analysis would be required. Although such an analysis has already been given in the literature in the context of a specific model (DSM) [3, 4, 5, 6] for fermion mixing and mass hierarchy, it pays to review it here outside that context so as to exhibit its generality. Indeed, in doing so, the analysis gains also in lucidity, which is a help, for the analysis, though logically straightforward in principle, still needs a fair amount of care and patience to be carried out.

Let us then go back to the beginning and ask in general terms how physical masses and states are to be extracted from a given mass matrix. At tree-level, where the concept of a mass matrix originates and where the mass matrix is independent of scale, the answer is easy; the measured masses are given just by the eigenvalues and the state vectors by the corresponding eigenvectors for the various mass states. On renormalization, when the mass matrix depends on scale, however, some care is needed, since the eigenvalues and eigenvectors can now be scale-dependent, and one needs to specify at what scale(s), if at any, these are to be identified as the physical masses and state vectors.

Suppose first that the scale-dependent renormalized mass matrix is, for some reason, nonrotating, as is the case to a fair approximation at least for quarks for the Standard Model, then the answer remains relatively simple, since, once diagonalized at some scale, the mass matrix will remain diagonal at any other scale, namely of the form: $m = \mathrm{diag}[\lambda_1(\mu), \lambda_2(\mu), \lambda_3(\mu)]$. Following then the usual convention that the physical masses are to be measured each at the scale equal to the mass itself, we can then identify the physical masses m_i of the 3 mass states as respectively the solutions to the equations $\lambda_i(\mu) = \mu$. For leptons, for example, we would have $m_\tau = \lambda_1(m_\tau)$, $m_\mu = \lambda_2(m_\mu)$, and $m_e = \lambda_3(m_e)$, while the state vectors are given by the corresponding eigenvectors which are scale-independent by the initial nonrotation ansatz. Notice, however, that even in this case, the measured masses are not just the eigenvalues of the same mass matrix but of three different matrices representing the mass matrix taken at three different scales, and so have departed already from the familiar simple notion valid at the tree-level.

What happens next for a rotating mass matrix? One can still of course diagonalize the mass matrix at every scale, but now both the eigenvalues λ_i and their corresponding eigenvectors, say $\boldsymbol{\beta}_i$, will depend on scale. This then raises immediately the question what states are to be identified as the physical particle states. It does not seem to make sense to identify just the eigenvectors at some scale as the state vectors of the physical particles at that scale, in other words, entertaining the concept of scale-dependent physical state vectors. Take again the charged leptons as example. If we were to identify the physical state of the τ at scale μ, say, as the highest eigenvector $\boldsymbol{\beta}_1(\mu)$, that of the μ as the next highest $\boldsymbol{\beta}_2(\mu)$, and that of the e as the last $\boldsymbol{\beta}_3(\mu)$, all taken at the same scale μ, then since the eigenvectors rotate, what appears as the τ vector at this scale μ will appear as a mixture of all 3 states at a different scale μ'. In other words, what we thought was the τ at the scale μ will start decaying into $\mu\gamma$ and $e\gamma$ at the other scale μ', giving thus copious flavour violation. This seems inadmissible. We ought to give the physical states a scale-independent meaning.

In the special case of a nonrotating mass matrix considered above where only the eigenvalues but not the eigenvectors depend on scale, it is conventional to define, as we did, the masses of the physical particles as the eigenvalues taken each respectively at the scale equal to its mass. So one may be tempted similarly to define for the rotating mass matrix the state vectors of the physical particles as the respective eigenvectors taken each at the scale equal to its mass. But this also will not work. For if we were to follow this prescription for the charged leptons, then one would define the physical τ state as the eigenstate of the mass matrix at scale $\mu = m_\tau$, with the eigenvalue $\lambda_1(m_\tau)$ as its mass and the corresponding eigenvector $\boldsymbol{\beta}_1(m_\tau)$ as its state vector $\mathbf{v}_\tau$. Similarly, for the μ, we would have $\lambda_2(m_\mu)$ as its mass and the vector $\boldsymbol{\beta}_2(m_\mu)$ as its state vector $\mathbf{v}_\mu$. Now two eigenvectors belonging to two different eigenvalues of the same hermitian matrix are necessarily orthogonal; hence $\boldsymbol{\beta}_1(\mu) \perp \boldsymbol{\beta}_2(\mu)$. But the state vector $\mathbf{v}_\tau = \boldsymbol{\beta}_1(m_\tau)$ of τ has no reason to be orthogonal to the state vector $\mathbf{v}_\mu = \boldsymbol{\beta}_2(m_\mu)$ of μ, being eigenvectors of the mass matrix $m(\mu)$ taken at different values of the scale μ. Indeed, if the mass matrix rotates, then the state vectors so defined for τ and μ would not be orthogonal to each other, which is physically untenable, since τ and μ are supposed to be independent quantum states. It would give rise again to unwanted flavour-violations.

What then has gone wrong? For the heaviest generation fermion, such as τ, the definitions above have no apparent problem; its mass can be indeed taken as the highest eigenvalue of the mass matrix and its state vector as the corresponding eigenvector, both at the scale $\mu = m_\tau$. However, for the next heaviest generation such as μ, a problem begins to emerge. To extract the mass and state vector of μ, the mass matrix m has to be taken at an energy scale $\mu < m_\tau$, and at these energies, the τ state becomes unphysical.

To appreciate what this implies, let us recall the familiar parallel case of the analytic multi-channel S-matrix [7], e.g.:

$$S = \begin{pmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{pmatrix}. \tag{4}$$

This 3×3 matrix exists as a mathematical entity at all energies, but at energies below the physical threshold of the heaviest channel 1 where the state 1 becomes unphysical, what represents the physical S-matrix is just the 2×2 submatrix at the bottom right corner labelled by 2 and 3, namely

$$\hat{S} = \begin{pmatrix} S_{22} & S_{23} \\ S_{32} & S_{33} \end{pmatrix}. \tag{5}$$

Similarly, at energies below the second heaviest channel 2, the physical S-matrix is given just by the element S_{33}. In each case, the elements of the matrix referring to the higher channels continue to exist at the lower energies and represent there just the analytic continuations of the physical quantities above the appropriate thresholds but have no immediate physical meaning beneath those thresholds. They cannot, for example, contribute to unitarity sums, for below those thresholds, the higher states do not exist as physical states.

Like the 3×3 analytic S-matrix, the 3×3 mass matrix $m(\mu)$ is a mathematical construct which exists at all energy scales, but at energies μ less than the mass of the heaviest state m_1 where this state becomes unphysical, the physical mass matrix is given only by the 2×2 submatrix labelled by the remaining states. In case the mass matrix is nonrotating, or when the rotation is considered negligible as in most appliations of the Standard Model, we see that this makes no difference to our usual assertions about the physical masses. For when the matrix is diagonalized, the truncation of the 3×3 matrix gives for the physical 2×2 mass matrix for $\mu < m_1$ just $\hat{m} = \mathrm{diag}[\lambda_2(\mu), \lambda_3(\mu)]$. This will give for the physical mass of the second heaviest state m_2 again as just the solution to the equation $\lambda_2(\mu) = \mu$, as before, and similarly also for m_3.

When the mass matrix rotates with changing scale, however, more care is needed, for in that case, we recall, we have not yet even identified the physical states 2 and 3. But we do know at least that these states are independent quantum states to the heaviest state, so their state vectors have to be orthogonal to that of the heaviest state. Hence, it follows that for a scale less than the mass of the heaviest state, $\mu < m_1$, the physical mass matrix, $\hat{m}(\mu)$, has to be the 2×2 submatrix in the 2-dimensional subspace orthogonal to the state vector $\mathbf{v}_1$ of the heaviest state. In particular, we may choose as the basis vectors of this orthogonal subspace the vectors $\boldsymbol{\beta}_2$ and $\boldsymbol{\beta}_3$ at $\mu = m_1$, which being eigenvectors of the mass matrix m at $\mu = m_1$ are automatically orthogonal to $\mathbf{v}_1$. In this basis, of course, the matrix $\hat{m}(\mu)$ at $\mu = m_1$ is diagonal, but because of rotation, it will not remain diagonal at lower values of μ. But $\hat{m}(\mu)$ can be digonalized afresh at each value of μ giving eigenvalues, say $\hat{\lambda}_2(\mu), \hat{\lambda}_3(\mu)$ and their corresponding eigenvectors, say $\hat{\boldsymbol{\beta}}_2(\mu), \hat{\boldsymbol{\beta}}_3(\mu)$. By the same logic as before for the heaviest state, we can now define the mass m_2 of the second heaviest state as the solution to the equation $\hat{\lambda}_2(\mu) = \mu$ and the corresponding eigenvector $\hat{\boldsymbol{\beta}}_2(m_2)$ as its state vector $\mathbf{v}_2$. We notice that the vector $\mathbf{v}_2$ so defined, being a vector in the orthogonal subspace spanned by the chosen basis vectors $\boldsymbol{\beta}_2(m_1), \boldsymbol{\beta}_3(m_1)$, is automatically orthogonal to $\mathbf{v}_1$, the state vector of the heaviest state. In other words, we have now guaranteed that the state vector of μ, for example, will be orthogonal to that of τ and avoided the pitfall met with before. Of course, the identification of $\mathbf{v}_2$ as the state vector of the second heaviest state also determines the state vector $\mathbf{v}_3$ of the lightest state as the vector orthogonal to both $\mathbf{v}_1$ and $\mathbf{v}_2$.

The procedure detailed in the preceding paragraph for identifying the masses and state vectors of the physical states applies to any mass matrix rotating with changing scales. Let us now specialize to the case of a rank 1 mass matrix of the form (3) above which is of some special interest, as will be seen later. This is easily diagonalized, having only one nonzero eigenvalue $\lambda_1(\mu) = m_T(\mu)$ with corresponding eigenvector $\boldsymbol{\beta}_1(\mu) = \boldsymbol{\alpha}(\mu)$. The other eigenvectors with degenerate eigenvalue zero can be taken as any two vectors orthogonal to $\boldsymbol{\alpha}(\mu)$, say $\boldsymbol{\beta}_2(\mu), \boldsymbol{\beta}_3(\mu)$. Following the procedure given above, we then identify the mass of the heaviest physical state m_1 as the solution to the equation $m_T(\mu) = \mu$ and its state vector as $\boldsymbol{\alpha}(m_1)$. For values of $\mu < m_1$, the physical mass matrix according to the above conclusion is the truncation of (3) to the subspace spanned by $\boldsymbol{\beta}_2(m_1), \boldsymbol{\beta}_3(m_1)$, i.e.

$$\hat{m}(\mu) = \hat{m}_T(\mu)|\hat{\boldsymbol{\alpha}}(\mu)\rangle\langle\hat{\boldsymbol{\alpha}}(\mu)|, \tag{6}$$

with

$$\hat{m}_T(\mu) = m_T(\mu)\sqrt{|\langle\boldsymbol{\alpha}(\mu)|\boldsymbol{\beta}_2(m_1)\rangle|^2 + |\langle\boldsymbol{\alpha}(\mu)|\boldsymbol{\beta}_3(m_1)\rangle|^2}, \tag{7}$$

and $\hat{\boldsymbol{\alpha}}(\mu)$ the normalized 2-vector defined as:

$$|\hat{\boldsymbol{\alpha}}(\mu)\rangle = (m_T(\mu)/\hat{m}_T(\mu)) \begin{pmatrix} \langle\boldsymbol{\alpha}(\mu)|\boldsymbol{\beta}_2(m_1)\rangle^* \\ \langle\boldsymbol{\alpha}(\mu)|\boldsymbol{\beta}_3(m_1)\rangle^* \end{pmatrix}. \tag{8}$$

This matrix $\hat{m}(\mu)$ vanishes of course when $\mu = m_1$ where the vectors $\boldsymbol{\beta}_2(m_1)$ and $\boldsymbol{\beta}_3(m_1)$ are by definition orthogonal to $\mathbf{v}_1 = \boldsymbol{\alpha}(m_1)$. For scales $\mu < m_1$, however, the vector $\boldsymbol{\alpha}(\mu)$ would have rotated to another direction giving thus a nonzero value to $\hat{m}_T(\mu)$. The 2×2 matrix $\hat{m}$ which, as we recall, is the physical mass matrix for $\mu < m_1$, is of rank 1 as is the original 3×3 mass matrix m. So the process gone through before of identifying mass values and state vectors of physical states can be repeated, only now in one less dimension. One can thus immediately conclude that the second heaviest state has a mass m_2 given by the solution to the equation $\hat{m}_T(\mu) = \mu$, and a state vector $\mathbf{v}_2 = \hat{\boldsymbol{\alpha}}(m_2)$. The process can be repeated again to deduce the mass of the lightest state m_3.

The masses m_2 and m_3 so obtained are seen clearly to have no reason to be, and will in general not be, zero when the mass matrix m rotates. It is thus shown that the two lower generations do naturally acquire nonzero masses simply as a result of the rotation of the mass matrix m, confirming thus the conclusion reached before, only now, as a result of the above analysis, one knows exactly what these nonzero masses are or at least how to compute them, and also the corresponding state vectors. And in deducing this conclusion, one has nowhere changed the rank 1 nature of m nor the chiral structure of the action. At every scale μ, the mass matrix m, being of rank 1, has always 2 eigenstates with the eigenvalue zero, namely $\boldsymbol{\beta}_2(\mu)$ and $\boldsymbol{\beta}_3(\mu)$ in the above notation, and a chiral transformation on these 2 states will leave the action invariant. Only these states, $\boldsymbol{\beta}_2(\mu)$ and $\boldsymbol{\beta}_3(\mu)$, on which a chiral transformation leaves physics invariant, are not to be identified with the physical states which are the states $\mathbf{v}_2$ and $\mathbf{v}_3$. These physical states $\mathbf{v}_2$ and $\mathbf{v}_3$ are linear combinations of the "chiral" massless states $\boldsymbol{\beta}_2(\mu)$ and $\boldsymbol{\beta}_3(\mu)$ at any scale μ but they contain in addition an admixture of the massive eigenstate $\boldsymbol{\beta}_1(\mu)$ at that scale. It is this admixture of the massive eigenstate in the physical states $\mathbf{v}_2$ and $\mathbf{v}_3$ which gives them each a mass and destroys at the same time the invariance under a chiral transformation on them. Their nonzero masses arise purely from the rotation, as the energy scale μ changes, of the vector $\boldsymbol{\alpha}(\mu)$ away from its direction at $\mu = m_1$, i.e. the direction of the state vector $\mathbf{v}_1$ for the heaviest state. It thus appears that simply by virtue of the rotation, the mass of the heaviest state has leaked a little into the lower generations to give them small but nonvanishing masses, hence the rather fanciful name of "leakage mechanism" we have coined earlier for it [4].

We notice that to actually evaluate the physical masses arising from a rotating mass matrix in accordance to the above procedure, we shall need to know the rotation matrix $U(\mu, \mu_0)$ as defined in (2) above, which will depend on the underlying theory. Besides, we need also to specify at exactly what scales the physical masses are to be measured, which in the above analysis we have chosen, following the usual convention, to be the scales equal to the masses themselves. This last is reasonable if we are dealing with freely or quasi-freely propagating particles like the charged leptons or the heavy quarks, but may not be the most convenient for confined objects like the light quarks u and d. However, for the general assertion that nonzero masses can result from a chiral invariant but rotating mass matrix, we shall not need to be specific about either the rotation matrix $U(\mu, \mu_0))$ nor the scales at which the physical masses are measured. The assertion would follow so long as the mass matrix does rotate and the physical masses of the different physical states are to be measured at different scales, as indicated already in the remarks made right at the beginning.

That nonzero masses can result from a rotating chiral invariant mass matrix is not just a theoretical curiosity but can lead to some quite interesting physical consequences. The reason is that there are many instances when from various theoretical considerations it may seem physically desirable to start with a chiral invariant mass matrix for fermions but is hindered by the experimental fact that the fermions we see seem all to possess nonzero masses, although their masses are in some cases rather small. However, it would now appear from above that if we allow the mass matrix to rotate then we can both keep the chiral invariance and have nonzero masses for our physical particles, thus bypassing the apparent contradiction.

As an example of such instances where a chiral invariant mass matrix seems desirable, consider first the strong CP problem. This arises from the fact that colour gauge invariance in QCD admits in principle a CP-violating term in the action of the form;

$$\mathcal{L}_\theta = -\frac{\theta}{64\pi^2}\tilde{F}F, \tag{9}$$

associated with topologically non-trivial field configurations, where θ is a real but otherwise arbitrary parameter [8]. Such a term in the action would exhibit itself, for example, in a nonvanishing electric dipole moment of the neutron estimated to be of the order [8]:

$$d_n \sim |\theta| e m_\pi^2/m_N^3 \sim 10^{-16}|\theta| \text{ e cm}. \tag{10}$$

The present experimental limit for d_n has already been pushed down to less than 3×10^{-26} e cm, which means that this free parameter θ in the theory, if it really exists, will have to be assigned a value: $|\theta| < 10^{-10}$. It would appear therefore that nature has some hidden mechanism for suppressing this angle θ which has not yet been accounted for in the standard formulation of chromodynamics. The favourite mechanism suggested is to supplement colour symmetry by an additional $U(1)$ symmetry [9], the breaking of which, however, would give rise to a new particle called the axion [9, 10], which has been searched for experimentally but never yet observed. Besides, new experiments are being carried out which are expected to push down further the limit of the electric dipole moment of the neutron, which is already making it uncomfortable for most current suggestions for the suppression mechanism [11]. It would thus be of interest to explore other possibilities for suppressing the θ angle or for eliminating it altogether.

The reason why a rotating fermion mass matrix matters in the strong CP problem is that the θ angle term (9) in the action can be removed by a chiral transformation on the fermion fields which leaves the physics invariant provided that the quark mass matrix has at least one zero eigenvalue [8]. Unfortunately, as far as known, no quark can be assigned a zero mass in the current interpretation of the existing experimental data, and would thus be at variance with the above proviso, if chiral invariance necessarily implies a zero mass for a physical state. However, in the above analysis, one sees that, if the mass matrix rotates, then the physically measured masses of all physical fermion states can be nonzero even though the mass matrix appearing in the action has zero eigenvalues. That being the case, it would seem to offer a possible resolution to the above dilemma, at least in principle [12].

As a second example, consider the chiral symmetry breaking problem in QCD. As noted already, the QCD action is "almost" chiral invariant. Indeed, if the light quarks u and d actually have zero mass, then the QCD action would be invariant under chiral $su(2) \times su(2)$. It thus seem attractive to entertain the notion that the QCD action may in fact be invariant under chiral $su(2) \times su(2)$ to start with, but then undergoes a spontaneous breaking of this symmetry to give the u and d quarks each a mass. Although the actual mechanism for the spontaneous breaking of this chiral $su(2) \times su(2)$ has not been fully understood, the idea has generated a host of important results too numerous to be here enumerated [8]. Now, if it were true, as suggested by the above analysis, that the light quarks u and d can acquire each a physical mass different from zero without the action ever losing its chiral invariance so long as the mass matrix rotates, would it not then cast a new light on to the problem? The masses of u and d arising from rotation via the "leakage mechanism" would be naturally small. In other words, their smallness would appear as a consequence of chiral invariance instead of being a hindrance to it.

Of course, this new angle for looking at the strong CP and chiral symmetry breaking problems would be no more than exchanging these two mysteries for another, namely that of mass matrix rotation, unless one can find a viable theoretical reason why the fermion mass matrix should rotate, or else some evidence in nature that it does do so. It turns out that both such exist though both are as yet of a circumstantial nature. Nevertheless, they seem to us already to be a sufficient incentive for the rotation scenario to be seriously entertained.

These arise as follows. Since, according to the above analysis, the lower generations can acquire each a mass by "leakage" from the generation above, it follows that only the heaviest generation needs be given a mass (i.e. starting with a mass matrix m of rank 1) for all generations to end up with nonzero physical masses. Now given the empirical fact that fermions of the heaviest generation are in every known case very much heavier than the others, a rank 1 mass matrix has long been taken as a good starting point for a phenomenological description [13]. The "leakage mechanism" from rotation now provides one with a concrete procedure for actually producing finite masses for the lower generations starting from a rank 1 mass matrix. Such a scenario is phenomenologically particularly attractive for the following reason. Since the masses of each lower generation arise only as consequences of "leakage" from those of the generation above, they are expected to have progressively smaller values, dropping by large factors from generation to generation. In other words, we have here an immediate qualitative explanation for the fermion mass hierarchy observed in experiment. Furthermore, since state vectors for different flavours are to be defined each at the scale equal to its mass, it follows that the state vectors of up-states will not be aligned to those of down-states, given their different masses, even if their mass matrices are always aligned at the same scale. For example, the state vector $\mathbf{v}_t$ of the t-quark is the first eigenvector of the mass matrix of U-type quarks evaluated at $\mu = m_t$, while the state vector $\mathbf{v}_b$ of the b-quark would be the first eigenvector of the mass matrix of D-type quarks but evaluated at $\mu = m_b$. Hence, even if the mass matrices of the 2 quark types are always aligned at the same μ, the 2 state vectors $\mathbf{v}_t$ for t and $\mathbf{v}_b$ for b will not be aligned, meaning that there will be nontrivial mixing between the t and b states. In other words, a single rotating rank 1 mass matrix has already the potential to explain not only the fermion mass hierarchy experimentally observed but also the intriguing mixing pattern between the U and D flavours.

The idea outlined in the preceding paragraph for explaining the fermion mass hierarchy and mixing pattern can be put to empirical test in two ways. First, starting with the experimental quark and lepton masses and mixing angles, and interpreting them as arising from a single rotating rank 1 mass matrix, one asks whether the result is consistent with all the data points lying on a smooth rotation curve. This was done and the answer is affirmative within experiemtnal errors [14]. Conversely, one can start by constructing a model for rotation giving a rotating rank 1 mass matrix depending on some paramters, and then proceed to fit the experimental data with the model. This was done with a model called the Dualized Standard Model (DSM) which was able to give a good fit to nearly all the mass ratios and mixing angles with only 3 adjustable real parameters [5, 6]. We find these tests rather compelling, given that the fermion mass hierarchy and mixing pattern have otherwise no generally accepted explanation, and can, we think, be taken as at least circumstantial evidence for mass matrix rotation.

As to theoretical justification for why the fermion mass matrix should rotate and at such speed as to produce the above phenomena, our judgment is bound to be a little subjective, given our past experience. We can say, however, that the model DSM cited above for fitting data was meant to be only phenomenological, having been constructed with the object in mind, and thus contains some ad hoc assumptions while satisfying no strict demand for internal consistency. Besides, it is seen [5] that its apparent success as outlined above does not depend so much on its details but largely just on the fact that the rotating fermion mass matrix it produced has 2 rotational fixed points, one at $\mu = 0$ and the other at $\mu = \infty$. However, a new self-consistent model has now been constructed on a firmer theoretical basis [15]. It has as its motivation an explanation of some of the Standard Model's basic features, and incorporates 't Hooft's confinement picture for symmetry-breaking [16, 17] while purporting to give a new geometrical meaning to Higgs fields. It has thus a very different structure from the previous model. Nevertheless, the new model leads logically also to a rotating fermion mass matrix of rank 1 with still the desired fixed points at $\mu = 0, \infty$. Besides, it has overcome some of DSM's shortcomings and gained some new good features such as the possibility of a CP-violating phase. It is thus hopeful that the fit to experimental data now being carried out may equal or perhaps even surpass that obtained before with the DSM. If this results, then the empirical observations made in the preceding paragraphs would have been put on a firmer theoretical footing.

Although the evidence for mass matrix rotation as outlined above, whether empirical or theoretical, is as yet only circumstantial, it appears to us already sufficient to suggest that this possibility be taken seriously. That being the case, it may in turn cast a new light on to the strong CP and chiral symmetry problems, as observed above.

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