Large neutrino mixing and fermion mass hierarchy in $SO(10)$ GUT

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We present a common explanation of the fermion mass hierarchy and the large lepton mixing angles in the context of a grand unified flavor and gauge theory (GUTF). Our starting point is a $(SU(3)\times U(1))^F$ flavor symmetry and a $SO(10)$ GUT, a basic ingredient of our theory which plays a major role is that two different breaking pattern of the flavor symmetry are at work. On one side, the dynamical breaking of $(SU(3)\times U(1))^F$ flavor symmetry into $(U(2)\times Z_3)^F$ explains why one family is much heavier than the others. On the other side, an explicit symmetry breaking of $SU(3)^F$ into a discrete flavor symmetry leads to the observed tribimaximal mixing for the leptons. We write an explicit model where this discrete symmetry group is A_4. Naturalness of the charged fermion mass hierarchy appears as a consequence of the continuous $SU(3)^F$ symmetry. Moreover, the same discrete A_4-GUT invariant operators are the root of the large lepton mixing, small Cabibbo angle, and neutrino masses.

I. INTRODUCTION

Grand Unified Theory (GUT) [1, 2] are natural extensions of the Standard Model (SM) Indications toward GUT are the tendency to unify for the gauge couplings, and the possibility to explain charge quantization and anomaly cancellation. One of the main features of GUT is its potentiality to unify the particle representations and the fundamental parameters in a hopefully predictive framework. $SO(10)$ is the smallest simple Lie group for which a single anomaly-free irreducible representation (namely the spinor 16 representation) can accommodate the entire SM fermion content of each generation.

Flavor physics appears as new extra horizontal symmetries. After the recent experimental evidences about neutrino physics [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14], within the experimental errors, the neutrino mixing matrix is compatible with the so called tri-bimaximal matrix [15]

$$U_{TB} = \begin{pmatrix} -2/\sqrt{6} & 1/\sqrt{3} & 0 \\ 1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\ 1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \end{pmatrix} . \qquad (1)$$

At this stage the parameters both the quark [16] and lepton [17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28] sectors are known to a comparable level.

To explain at the same moment the charged fermion mass hierarchy and the lepton-quark mixing angle hierarchy is an unsolved problem, this is the flavor puzzle. The problem of the mass hierarchy is often addressed by introducing continuous flavor symmetries [29, 30]. On the other hand, discrete flavor symmetry such as 2-3 [31, 32, 33], $S3$ [34, 35, 36, 37], A_4 [38, 39, 40, 41], or other symmetries [42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53], where introduced to explain large lepton mixing angles, but in that case mass hierarchy remains unexplained.

A milestone in these studies has been the discovery that mass hierarchies and mixing angles can be not directly correlated among them in the flavor symmetry breaking [36, 54]. Fundamental steps in the realization of these ideas are given in [38, 39]. These new ingredients allow us to escape from the no-go theorem [55] that seems to indicate that a maximal mixing angle θ_{23} can never arise in the symmetric limit of whatever flavor symmetry (global or local, continuous or discrete), provided that such a symmetry also explains the hierarchy among the fermion masses and is only broken by small effects, as we expect for a meaningful symmetry.

In fact, in our theory, the mass hierarchy and large mixing angle are not originated at the same step in the symmetry breaking pattern.

Our final aim would be the construction of a grand unified $SO(10)$-like model where masses and mixing angles are generated by the flavor and gauge symmetry breaking.

We presented a viable $SO(10)$ model with discrete flavor symmetry in [38]. There we generated the observed lepton mixing but we fitted the fermion masses by assuming the group A_4 as flavor symmetry and the "constrain" of assigning right and left-handed fermion fields to the same representations. Indeed, we showed in [38] that the assignment of both left-handed and right-handed SM fields to triplets of A_4, that is therefore compatible with $SO(10)$, can lead to the charged fermion textures proposed in [56] and given by

$$M_f = \begin{pmatrix} h_0^f & h_1^f & h_2^f \\ h_2^f & h_0^f & h_1^f \\ h_1^f & h_2^f & h_0^f \end{pmatrix} , \tag{2}$$

with h_0^f, h_1^f and h_2^f distinct parameters. In [38], in order to obtain a mass matrix of the form of M_f in eq. (2) without spoiling the predictions of the neutrino sector, we introduced higher order operators containing simultaneously a set of $SO(10)$ representations **45**. The lepton mixing was naturally generated by the breaking pattern of A_4, while the fermion masses were obtained with a possible tuning in the flavor parameters not constrained by the symmetries.

We addressed the problem of the fine tuning in [39] where the A_4^F flavor discrete symmetry is embedded into $(SO(3)_L \times SO(3)_R)^F$. In that way we explicitly disentangled the mixing problem from the hierarchy one. We broke the continuous flavor $(SO(3)_L \times SO(3)_R)^F$ symmetry both dynamically and explicitly. The two breaking terms produced the charged fermion hierarchies on one hand and solved the leptonic mixing problem on the other hand. In this way not only a tribimaximal neutrino mixing was naturally generated but also the charged fermion hierarchies by dynamically breaking of the continuous left-right flavor symmetry. Finally the Cabibbo angle was obtained by taking into account higher order operators. However the left-right flavor group symmetry $(SO(3)_L \times SO(3)_R)^F$ of [39] is not compatible with a grand unified gauge group, like $SO(10)$, with all the fermions of one family in the same representation, because in left-right flavor symmetries the fermions of one family belong to different representations of the flavor group.

In this paper we merge all these ingredients together and we are able to construct a non renormalizable model with grand unified gauge group $SO(10)$ and with an extended flavor symmetry $(SU(3) \times U(1))^F$. In this new model both the tribimaximal lepton mixing matrix and the hierarchy among the mass of the 3rd and the other fermion families naturally appear from the symmetry breaking pattern. Our model is non renormalizable, however a renormalizable version of it can be easily constructed because the particular structure of the operators introduced here. For this purpose viable methods are well known, i.e. by integrated out given heavy extra fields [57].

Our effective $SO(10)$ invariant Lagrangian is

$$\mathcal{L} \;=\; \mathcal{L}_{SU(3)^F} + \delta\mathcal{L}_{A_4} , \tag{3}$$

where $\mathcal{L}_{SU(3)^F}$ is $SO(10) \times (SU(3) \times U(1))^F$ invariant and $\delta\mathcal{L}_{A_4}$ is the explicit breaking term of the $SU(3)^F$ symmetry that, at this level, leaves $SO(10)$ unbroken. The charge assignment of the fields is such that the $SU(3)^F$ invariant operator with lowest mass dimensions is only [38]

$$\mathcal{L}_{SU(3)^F} \;=\; h_0\, \mathbf{16}\, \mathbf{10}\, \mathbf{45}_A\, \mathbf{45}_B\, \mathbf{16}\, \Phi , \tag{4}$$

where Φ, singlet of $SO(10)$, transforms as $\overline{\mathbf{6}}$ with respect to $SU(3)^F$. The scalar fields **10**, $\mathbf{45}_A$ and $\mathbf{45}_B$ are singlets of $SU(3)^F$. As noticed in [38], thanks to the two **45**s scalar fields, the operator in eq. (4) can give no contribution to the neutrino sector for some set of the **45**s vev (i.e. in the explicit model of [38] one vev is proportional to the right-handed isospin T_{3_R} and the other one to the hypercharge Y, here the directions A and B are different but still with the same property).

When the scalar field Φ develops a vev in the direction $\langle \Phi^{ij} \rangle = 1\,(\forall\, i\,, j)$ we obtain a democratic mass matrices for all the charged fermions, that gives a massive 3rd family and two massless families. The democratic structure of the charged fermion mass matrices avoids the fine tuning needed to explain the mass hierarchy between the 3rd and the other two families that is usually needed in presence of charged fermion mass matrices of the form of eq. (2). The democratic mass matrices preserves the $(U(2) \times Z_3)^F$ subgroup of $(SU(3) \times U(1))^F$ that leaves invariant the $(1, 1, 1)$ vector in the flavor space. Therefore at this stage only one mixing angle can be generated.

The neutrino mass matrix and the first and second families masses arise when we switch on the explicitly breaking terms of $SU(3)^F$ into A_4. If we neglect the ordering problem of the **45**s and the possibility to have more than one flavon for each operator, the most general Lagrangian invariant under the flavor structure of the theory is

$$\begin{aligned} \delta\mathcal{L}_{A_4} &= h_{ijk}\, \phi^k\, \mathbf{16}^i\, \mathbf{10}\, \mathbf{45}_A\, \mathbf{45}_B\, \mathbf{45}_C\, \mathbf{45}_D\, \mathbf{16}^j + h'_{ijk}\, \phi^k\, \mathbf{16}^i\, \mathbf{45}_C\, \mathbf{45}_D\, \mathbf{10}\, \mathbf{45}_A\, \mathbf{45}_B\, \mathbf{16}^j + \\ & \quad h''_{ijk}\, \tilde{\phi}^k\, \mathbf{16}^i\, \mathbf{10}\, \mathbf{45}_C\, \mathbf{45}_D\, \mathbf{16}^j + g\, \mathbf{16}^i\, \overline{\mathbf{126}}\, \mathbf{16}^i\, \zeta_S + g'_{ijk}\, \mathbf{16}^i\, \overline{\mathbf{126}}\, \mathbf{16}^j\, \zeta_T^k \end{aligned} \tag{5}$$

where the indices $\{i, j, k, l\}$ are A_4, subgroup of $SU(3)$, indeces and the sum over the gauge indices is understood. The scalar field $\overline{\mathbf{126}}$ is a singlet $\mathbf{1}'$ of A_4, while the $\mathbf{45}_C$, and $\mathbf{45}_D$ are other scalars that transform as $\mathbf{45}$ of $SO(10)$, and are singlets of A_4. The flavon fields ϕ, $\tilde{\phi}$, ζ_T are triplets under A_4, while φ and ζ_S are singlets.

As found in [38] the terms in the second line of $\delta\mathcal{L}_{A_4}$ generates the light neutrino mass matrix. The terms in the first two lines in $\delta\mathcal{L}_{A_4}$ gives a contribution to the mass matrices that has the nice properties to commute with the leading order term obtained from eq. (3).

After the breaking of A_4, it generates the first and second family masses and fix the mixing matrix in the lepton sector to be tribimaximal.

The plan of the paper is as follow. First, in sec. **II** we introduce the basic ingredient of the model, i.e. the general structure of the symmetry breaking, all the involved fields and how they transform under the gauge and flavor symmetries. Then in sec. **III** we show how the 3rd family masses are generated via the breaking $(SU(3) \times U(1))^F$ into $(U(2) \times Z_3)^F$, how the 1st and 2nd family masses are generated together with maximal mixings in the lepton sector, and how the neutrino masses are generated with a resulting tribimaximal mixing matrix in the lepton sector. Finally we show how the Cabibbo angle is naturally generated without the introduction of new operators. Finally in sec. **V** we report our conclusions.

II. BASIC INGREDIENTS

Let us first investigate the field content of the theory and the flavor charges. We report the field content of our model in Table (I). With our charge assignment, the only allowed operators of lower mass dimensions are given in eqs. (4-5), if we neglect the ordering problem of the **45** and the possibility to have more than one flavon for each operator. In this sense our Lagrangian is the most general one invariant under the flavor structure of the theory. Moreover, independently from the fact that nature prefer a dominant seesaw of type I (i.e. heavy Majorana right-handed neutrino mass and intermediate Dirac neutrino mass) or of type II (i.e. light Majorana left-handed neutrino mass) or a mixed scenario, the transformation properties of the ζ_S must be assumed to be $\mathbf{1}'$, as we will explain in sec. **IV B**.

In our opinion, the ordering problem can be related to a deeper structure of the theory, for example its version as a renormalizable model, and we will not investigate further it here. However the fact that will not be possible to express the directions A and B as rational combinations of C and D, together with the fact that **45** appears only as couples

$(\mathbf{45}_A$,$\mathbf{45}_B)$ and $(\mathbf{45}_C$,$\mathbf{45}_D)$ seems to us to indicate that the right representations to introduce are the irreducible part of the **2025** that can get a vev diagonal over the **16** matter fields with charges AB and CD. If this is the case we are really including all the allowed operator and there is not any more an ordering problem.

After symmetry breaking, once the Higgs acquire vevs, the quadratic part for the fermions of the Lagrangian in eqs. (4-5) can be rewritten in a compact form, i.e. with an abuse of notation in the $SO(10)$ contractions, as

$$
\begin{aligned}
L_{\text{Dirac}} \;=\;& h_0\ (\mathbf{16}_1\mathbf{16}'_1+\mathbf{16}_2\mathbf{16}'_2+\mathbf{16}_3\mathbf{16}'_3)\ v_{\mathbf{10}}\,+ && \text{(6a)}\\
&+\ \Big[h_1\ (\mathbf{16}_1\mathbf{16}'''_2+\mathbf{16}_2\mathbf{16}'''_3+\mathbf{16}_3\mathbf{16}'''_1)+h_2\ (\mathbf{16}_1\mathbf{16}'''_3+\mathbf{16}_2\mathbf{16}'''_1+\mathbf{16}_3\mathbf{16}'''_2)\Big]\ v_{\mathbf{10}}\ v_\phi && \text{(6b)}\\
&+\ \Big[h'_1\ (\mathbf{16}''_1\mathbf{16}'_2+\mathbf{16}''_2\mathbf{16}'_3+\mathbf{16}''_3\mathbf{16}'_1)+h'_2\ (\mathbf{16}''_1\mathbf{16}'_3+\mathbf{16}''_2\mathbf{16}'_1+\mathbf{16}''_3\mathbf{16}'_2)\Big]\ v_{\mathbf{10}}\ v_\phi && \text{(6c)}\\
&+\ \Big[h''_1\ (\mathbf{16}_1\mathbf{16}''_2+\mathbf{16}_2\mathbf{16}''_3+\mathbf{16}_3\mathbf{16}''_1)+h''_2\ (\mathbf{16}_1\mathbf{16}''_3+\mathbf{16}_2\mathbf{16}''_1+\mathbf{16}_3\mathbf{16}''_2)\Big]\ v_{\mathbf{10}}\ v_{\tilde\phi} && \text{(6d)}\\
&+\ g(\mathbf{16}_1\mathbf{16}_1+\mathbf{16}_2\mathbf{16}_2+\mathbf{16}_3\mathbf{16}_3)\ v_{\overline{\mathbf{126}}}\ v_{\zeta_S}+\Big[g'_1\mathbf{16}_1\mathbf{16}_2+g'_2\mathbf{16}_2\mathbf{16}_1\Big]\ v_{\overline{\mathbf{126}}}\ v_{\zeta_T} && \text{(6e)}
\end{aligned}
$$

where we have assumed that the two A_4-**3**plets ϕ and $\tilde\phi$ acquire vev in the $(1,1,1)$ direction of A_4, while the ζ_T vev is in the direction $(0,0,1)$. In eqs. (6) we introduced

$$
\begin{aligned}
\mathbf{16}'''_i &\equiv v_{\mathbf{45}_A}\ v_{\mathbf{45}_B}\ v_{\mathbf{45}_C}\ v_{\mathbf{45}_D}\ \mathbf{16}_i\,, \qquad && \mathbf{16}''_i \equiv v_{\mathbf{45}_C}\ v_{\mathbf{45}_D}\ \mathbf{16}_i\,,\\
\mathbf{16}'_i &\equiv v_{\mathbf{45}_A}\ v_{\mathbf{45}_B}\ \mathbf{16}_i\,, && \text{with}\quad i=1,2,3\,.
\end{aligned} \tag{7}
$$

We obtain the following expression by absorbing the vevs of the **45**s into the coupling constants

$$
\begin{aligned}
\mathbf{16}' &= \left(x'_Q\ Q,\ x'_U\ U^c,\ x'_D\ D^c,\ x'_L\ L,\ x'_E\ E^c,\ x'_N\ N^c\right), && \text{(8a)}\\
\mathbf{16}'' &= \left(x''_Q\ Q,\ x''_U\ U^c,\ x''_D\ D^c,\ x''_L\ L,\ x''_E\ E^c,\ x''_N\ N^c\right), && \text{(8b)}\\
\mathbf{16}''' &= \left(x'''_Q\ Q,\ x'''_U\ U^c,\ x'''_D\ D^c,\ x'''_L\ L,\ x'''_E\ E^c,\ x'''_N\ N^c\right) && \text{(8c)}
\end{aligned}
$$

where x'_f, x''_f, and x'''_f are the quantum numbers respectively of the product of the charges A and B, of the product of the charge C and D, and of the product of the charges A, B, C, and D. In particular we notice that

$$x'''_f = x''_f x'_f\,. \tag{9}$$

We report the charges of each fermion in Table (II).

III. DYNAMICAL BREAKING

A. $(SU(3)\times U(1))^F \to (U(2)\times Z_3)^F$ gives the charged fermion 3rd family masses

We assume that the Φ $SU(3)^F$-$\overline{\mathbf{6}}$plet field acquire a vev $\langle\Phi^{ij}\rangle = v_\Phi$ $(\forall\, i\,,j)$. In this case the charged fermion mass matrix obtained is the so-called democratic mass matrix [58] given by

$$M_{0f} = \frac{m_3^f}{3}\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix}, \tag{10}$$

where

$$
\begin{aligned}
m_3^U &= v_\Phi(x'_U+x'_Q)v^U_{\mathbf{10}}h_0\,, && \text{(11a)}\\
m_3^D &= v_\Phi(x'_D+x'_Q)v^D_{\mathbf{10}}h_0\,, && \text{(11b)}\\
m_3^N &= v_\Phi(x'_N+x'_L)v^U_{\mathbf{10}}h_0\,, && \text{(11c)}\\
m_3^E &= v_\Phi(x'_E+x'_L)v^D_{\mathbf{10}}h_0\,. && \text{(11d)}
\end{aligned}
$$

This matrix has only one eigenvalue different from zero, m_3^f, and can be assumed to be the mass of the 3rd family. To avoid any non diagonal contribution to the Dirac neutrino mass matrix we impose

$$x'_N + x'_L \;=\; 0\,. \tag{12a}$$

To have the bottom-tau unification, we must impose also

$$x'_L + x'_E \;=\; x'_Q + x'_D\,. \tag{12b}$$

The unitary matrix U that diagonalizes the symmetric matrix M_{0f} has one angle and the three phases undeterminated. One possible parametrization is given by [39]

$$U = \frac{1}{\sqrt{3}} \begin{pmatrix} \sqrt{2}\cos\theta\, e^{i\alpha} & \sqrt{2}\sin\theta e^{i(\beta+\gamma)} & 1 \\ -e^{i\alpha}(\frac{\cos\theta}{\sqrt{2}} + \sqrt{\frac{3}{2}}\sin\theta e^{-i\gamma}) & e^{i\beta}(\sqrt{\frac{3}{2}}\cos\theta - \frac{1}{\sqrt{2}}\sin\theta e^{i\gamma}) & 1 \\ -e^{i\alpha}(\frac{\cos\theta}{\sqrt{2}} - \sqrt{\frac{3}{2}}\sin\theta e^{-i\gamma}) & -e^{i\beta}(\sqrt{\frac{3}{2}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta e^{i\gamma}) & 1 \end{pmatrix}. \tag{13}$$

The freedom in the U matrix shows the remaining flavor symmetry $U(2)^F$. The unknow angle and phases are fixed only after breaking the democratic structure of M_{0f}, i.e. the $U(2)^F$ flavor symmetry, with a small perturbation δM_f, i.e.

$$M_f = M_{0f} + \delta M_f\,.$$

The effect of δM_f is to give a small mass to the first and second family and to fix the mixing angles. To have that the mixing matrix diagonalizing the full M_f belongs to the families of matrix of eq. (13), we must require that δM_f commute with M_{0f}. This has the nice consequence that we have automatically the selection of the breaking pattern of A_4 into Z_3.

IV. EXPLICITLY BREAKING $SU(3)^F \to A_4$

We will assume the presence of an hidden scalar sector that breaks spontaneously the continuous $SU(3)^F$ into the discrete A_4. Under this hypothesis it is quite natural to assume that the explicit breaking terms to be added to the Lagrangian are small.

A. $A_4 \to Z_3$ generates the charged fermions 1st and 2nd family masses and mixing

When the ϕ and $\tilde{\phi}$ A_4-**3**plets take vev as $\langle\tilde{\phi}\rangle \propto \langle\phi\rangle = v_\phi\,(1,1,1)$ we have new contributions to the mass matrices. I.e., for the charged leptons we get the operator

$$\delta^E_{ijk}\,\epsilon_{\alpha\beta}\,H^\alpha_d\,\left(L^\beta_i\,E_j\,\phi_k\right) \to \epsilon_{\alpha\beta}\,H^\alpha_d\left[\delta^E_1\,(L^\beta_2 E_3 + L^\beta_3 E_1 + L^\beta_1 E_2) + \delta^E_2\,(L^\beta_3 E_2 + L^\beta_1 E_3 + L^\beta_2 E_1)\right] v_\phi\,, \tag{14}$$

where the two δ^E_i arise by the two different contractions of A_4. The value of δ^Es can be read from the Lagrangian in eq. (6) and is

$$\delta^E_1 \;=\; (h_1 x'''_E + h_2 x'''_L) + (h'_1 x''_L x'_E + h'_2 x'_L x''_E) + (h''_1 x''_E + h''_2 x''_L)\,, \tag{15}$$
$$\delta^E_2 \;=\; (h_2 x'''_E + h_1 x'''_L) + (h'_2 x''_L x'_E + h'_1 x'_L x''_E) + (h''_2 x''_E + h''_1 x''_L)\,. \tag{16}$$

Because the $SO(10)$ unification, the operators in the up, down and neutrino sectors have similar expressions. In particular for the contributions to the Dirac neutrino mass matrix we have

$$\delta^N_1 \;=\; (h_1 x'''_N + h_2 x'''_L) + (h'_1 x''_L x'_N + h'_2 x'_L x''_N) + (h''_1 x''_N + h''_2 x''_L)\,, \tag{17}$$
$$\delta^N_2 \;=\; (h_2 x'''_N + h_1 x'''_L) + (h'_2 x''_L x'_N + h'_1 x'_L x''_N) + (h''_2 x''_N + h''_1 x''_L)\,. \tag{18}$$

At this stage we do not want any contribution to the Dirac neutrino mass matrix, without any fine tuning in the coupling constants hs. For this reason we have to impose the conditions

$$x_L''' = 0 = x_N''' \,, \quad x_L'' x_N' = 0 = x_L' x_N'' \,, \quad x_L'' = 0 = x_N'' \,. \tag{19}$$

By reducing the set of conditions in eqs. (9), (12), and (19) the only non trivial solution is

$$\begin{aligned} x' &= A\,B \propto 3\,(Y^2) - 12\,(T_{3_R}^2) + 20\,T_{3_R} Y && \text{(20a)} \\ x'' &= C\,D \propto (Y\,T_{3_R}) && \text{(20b)} \\ x''' &= A\,B\,C\,D = x'' x' && \text{(20c)} \end{aligned}$$

In particular we notice that we have

$$x_Q'' = x_Q''' = x_L'' = x_L''' = x_N'' = x_N''' = 0 \,. \tag{21}$$

Finally, by taking into account the relations in eq. (21), and the relation (9), we get from eq. (15)

$$\begin{aligned} \delta_1^E &= x_E''(x_E' h_1 + x_L' h_2' + h_1'') \,, & \delta_2^E &= x_E''(x_E' h_2 + x_L' h_1' + h_2'') \,, && \text{(22a)} \\ \delta_1^U &= x_U''(x_U' h_1 + x_Q' h_2' + h_1'') \,, & \delta_2^U &= x_U''(x_U' h_2 + x_Q' h_1' + h_2'') \,, && \text{(22b)} \\ \delta_1^D &= x_D''(x_D' h_1 + x_Q' h_2' + h_1'') \,, & \delta_2^D &= x_D''(x_D' h_2 + x_Q' h_1' + h_2'') \,, && \text{(22c)} \\ \delta_1^N &= 0 \,, & \delta_2^N &= 0 \,. && \text{(22d)} \end{aligned}$$

We notice here the importance of having the three operators. In fact, for example, if we had only one than we should obtain the relations

$$\frac{m_e}{m_\mu} = \frac{m_d}{m_s} = \frac{m_u}{m_c} \,. \tag{23}$$

While the first relation can be assumed true at the unification scale, with the given uncertainty in the determination of the fermion masses at such scale, the second relation is surely false. The introduction of the other operators allows us to escape from this consequence. We notice that there could be a direct relation between the fact that $\frac{m_d}{m_s} \neq \frac{m_u}{m_c}$ and the presence of a non zero CP-violating phase. The effect of the other explicit breaking terms in the mass matrices is translated in a perturbation of the democratic mass matrices of eq. (10), that is

$$M^f = \frac{m_3^f}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \to \tilde{M}^f = \frac{m_3^f}{3} \begin{pmatrix} 1 & 1+\delta_1^f & 1+\delta_2^f \\ 1+\delta_2^f & 1 & 1+\delta_1^f \\ 1+\delta_1^f & 1+\delta_2^f & 1 \end{pmatrix} \tag{24}$$

with the obvious correspondences $v_E = v_D$. The mass matrices of eq. (24) are diagonalized by

$$\tilde{U}_\omega = \frac{1}{\sqrt{3}} \begin{pmatrix} \omega & \omega^2 & 1 \\ \omega^2 & \omega & 1 \\ 1 & 1 & 1 \end{pmatrix} \,, \tag{25}$$

corresponding to the U of eq. (13) with $\theta = \pi/4$, $\alpha = 2\pi/3$, $\beta = 5\pi/6$ and $\gamma = \pi/2$. The mass matrices $\tilde{M}^f$ of eq. (24) give an heavy 3rd family mass m_3^f and small 1st and 2nd family masses satisfying

$$\frac{m_1^f}{m_3^f} = \frac{\omega\,\delta_1^f + \omega^2\,\delta_2^f}{3 + \delta_1^f + \delta_2^f} \,, \qquad \frac{m_2^f}{m_3^f} = \frac{\omega^2\,\delta_1^f + \omega\,\delta_2^f}{3 + \delta_1^f + \delta_2^f} \,. \tag{26}$$

B. Neutrino masses and mixing

The Yukawa interactions for the neutrinos come from the coupling of the fermion field **16** with the $\overline{\mathbf{126}}$ Higgs and the (ζ_S, ζ_T) flavons. The components of the $\overline{\mathbf{126}}$ that can acquire a vev[1] are a triplet Δ, three singlets[2] $\tilde{\Delta}$, a doublet Γ and two other singlets[2] $\tilde{\Gamma}_\alpha$ of the weak $SU(2)_L$. When the A_4-triplet field ζ_T takes vev in the A_4 direction $\langle\zeta_T\rangle \sim (0,0,1)$ - notice that this alignment is different from the one used in many models as for example in [38, 46] -, the resulting neutrino mass matrices are given by

$$M_{\nu\nu} = \begin{pmatrix} a_{\nu\nu} & b_{\nu\nu} & 0 \\ b_{\nu\nu} & \omega\, a_{\nu\nu} & 0 \\ 0 & 0 & \omega^2\, a_{\nu\nu} \end{pmatrix}, \qquad M_{\nu N} = \begin{pmatrix} a_{\nu N} & b_{\nu N} & 0 \\ b_{\nu N} & \omega\, a_{\nu N} & 0 \\ 0 & 0 & \omega^2\, a_{\nu N} \end{pmatrix}, \qquad M_{NN} = \begin{pmatrix} a_{NN} & b_{NN} & 0 \\ b_{NN} & \omega\, a_{NN} & 0 \\ 0 & 0 & \omega^2\, a_{NN} \end{pmatrix} \tag{27}$$

where as and bs are the product of the vevs of the $\overline{\mathbf{126}}$ components with the coupling constants g and g'. All the mass matrices in eq. (27) are diagonalized by the same mixing matrix. I.e. we get

$$M_x = \begin{pmatrix} a_x & b_x & 0 \\ b_x & \omega\, a_x & 0 \\ 0 & 0 & \omega^2\, a_x \end{pmatrix} = \tilde{V}_\nu^\star \begin{pmatrix} \omega^2\, a_x + b_x & 0 & 0 \\ 0 & \omega^2\, a_x & 0 \\ 0 & 0 & -\omega^2\, a_x + b_X \end{pmatrix} \tilde{V}_\nu^\dagger \tag{28}$$

with $x \in \{\nu\nu, \nu N, NN\}$. The common mixing matrix $\tilde{V}_\nu$ is given by

$$\tilde{V}_\nu = \begin{pmatrix} \frac{\omega}{\sqrt{2}} & 0 & -i\frac{\omega}{\sqrt{2}} \\ \frac{\omega^2}{\sqrt{2}} & 0 & i\,\frac{\omega^2}{\sqrt{2}} \\ 0 & 1 & 0 \end{pmatrix}. \tag{29}$$

The fact that all the mass matrices are diagonalized by the same mixing matrix $\tilde{V}_\nu$, translates in the nice result that, independently on the seesaw mechanism acting to generate the low energy neutrino mass, the neutrino mixing matrix is given by $\tilde{V}_\nu$ itself. In fact we have

$$M_{low} \;=\; M_{\nu\nu} + M_{\nu N}\frac{1}{M_{NN}}M_{\nu N}^T \tag{30}$$

and consequently, by indicating with an index $^\Delta$ the corresponding diagonalized matrix, we get

$$\begin{aligned} \tilde{V}_\nu^T M_{low} \tilde{V}_\nu &= \tilde{V}_\nu^T M_{\nu\nu}\tilde{V}_\nu + \tilde{V}_\nu^T M_{\nu N}(\tilde{V}_\nu\tilde{V}_\nu^\dagger)\frac{1}{M_{NN}}(\tilde{V}_\nu^\star\tilde{V}_\nu^T)M_{\nu N}^T\tilde{V}_\nu \\ &= \tilde{V}_\nu^T M_{\nu\nu}\tilde{V}_\nu + \tilde{V}_\nu^T M_{\nu N}\tilde{V}_\nu\frac{1}{\tilde{V}_\nu^T M_{NN}\tilde{V}_\nu}\tilde{V}_\nu^T M_{\nu N}^T\tilde{V}_\nu \\ &= M_{\nu\nu}^\Delta + M_{\nu N}^\Delta\frac{1}{M_{NN}^\Delta}M_{\nu N}^\Delta \\ &= M_{low}^\Delta\,, \end{aligned} \tag{31}$$

where we inserted twice the identity matrix $(\tilde{V}_\nu\tilde{V}_\nu^\dagger) = 1 = (\tilde{V}_\nu^\star\tilde{V}_\nu^T)$. The result is that the neutrino mass matrix is diagonalized by the mixing matrix $\tilde{V}_\nu$. On the other hand, the charged leptons are diagonalized by $L \to \tilde{U}_\omega\, L$, so we obtain a tribimaximal mixing for the lepton sector, that is

$$V_{leptons} = \tilde{U}^\dagger \cdot \tilde{V}_\nu = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{32}$$

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[1] We neglect here any contribution from the $(\mathbf{1},\mathbf{1},\mathbf{6})$-plet of the Pati-Salam subgroup of $SO(10)$.
[2] The three singlets $\tilde{\Delta}$ and the two singlets $\tilde{\Gamma}$ are respectively a triplet and a doublet under the $SU(2)_R$ of the Pati-Salam group.

The same operator that generates the neutrino masses and the lepton mixing matrix, generates also the Cabibbo angle in the quark sector. In fact we have that, with the inclusion of the contributions from the $\overline{\mathbf{126}}$, the charged fermion mass matrices of eq. (24) acquire a very small diagonal contribution and become[3]

$$M^f = \frac{m_3^f}{3} \begin{pmatrix} 1+\rho^f & 1+\delta_1^f & 1+\delta_2^f \\ 1+\delta_2^f & 1+\rho^f\omega^2 & 1+\delta_1^f \\ 1+\delta_1^f & 1+\delta_2^f & 1+\rho^f\omega \end{pmatrix} .$$

In the basis rotated by $\tilde{U}_\omega$ of eq. (25), namely $\tilde{M}^f \equiv \tilde{U}_\omega^\dagger \, M_{eff}^f \, \tilde{U}_\omega$, the charged fermion mass matrices are now given by

$$\tilde{M}^f = \begin{pmatrix} m_1^f & \tilde{\rho}^f\omega^2 & 0 \\ 0 & m_2^f & \tilde{\rho}^f\omega^2 \\ \tilde{\rho}^f\omega^2 & 0 & m_3^f \end{pmatrix} \qquad (33)$$

where $\tilde{\rho}^f = m_3^f/3\rho^f$. Let's assume that the $\tilde{\rho}^f$ are small arbitrary parameters of order $m_3^f O(\lambda^5)$, where λ is the Cabibbo angle. The crucial point is that this assumption has the consequences that our operators give negligible effects in the down and charged lepton sectors, since for the down and charged leptons we have $(m_1^{D,L}, m_2^{D,L}, m_3^{D,L}) \sim (\lambda^4, \lambda^2, 1)$ and $\tilde{M}^{D,L}$ may be considered diagonals. On the contrary for the up quarks we have that $(m_1^U, m_2^U, m_3^U) \sim (\lambda^7, \lambda^4, 1)$ and therefore the off-diagonal entry (1,2) cannot be neglected: the matrix $\tilde{M}^U$ is diagonalized by a rotation in the 12 plane with $\sin\theta_{12} \approx \lambda$. This rotation produces the Cabibbo angle in the CKM. In fact while M^D is still diagonalized by U_ω, we have that M^U is diagonalized by $V_L^{U\dagger} \, U_\omega^\dagger M^U U_\omega \, V_R^U$ where V_{LR}^U are unitary matrix, approximatively rotations in the 12 plane, and therefore the CKM mixing matrix is given by

$$V_{CKM} = (V_L^U)^\dagger \, U_\omega^\dagger \, U_\omega \equiv (V_L^U)^\dagger \, .$$

The charm and top quark masses are almost unaffected by the corrections and still are given by m_2^U and m_3^U respectively. The up quark mass is obtained by tuning the $\tilde{\rho}^U$.

V. CONCLUSIONS

In this work we addressed the two aspects of the flavor puzzle: the charged fermion mass and the mixing hierarchies. Following the idea that the mass hierarchy and large mixing angles are not originated at the same step in the symmetry breaking pattern, we introduced a GUTF $SO(10) \times (SU(3) \times U(1))^F$ model.

On one hand, a democratic structure for the charged fermion mass matrices arises from the vev of a scalar that transforms as a $\bar{\mathbf{6}}$ under the flavor group $SU(3)^F$. In this way the hierarchy between the 3rd family charged fermion masses and the others two is explained in a natural way. When the flavor group is dynamically broken, the CKM is given by an undetermined rotation in the $1-2$, while neutrino are massless and the lepton mixing is undetermined.

On the other hand, the explicitly breaking of $SU(3)^F$ into A_4 generates automatically the first and second family charged fermion masses $m_{1,2} \ll m_3$. However, in order to fit the hierarchy between the masses of the first and second families, we require a tuning.

Finally, the same operators generate the neutrino masses, the large mixing lepton angle and the Cabibbo angle. In fact, assuming that the light neutrino Yukawa interactions come from the couplings with an A_4 singlet and an A_4

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[3] We notice that the ratio of the vev of the doublets in the $\overline{\mathbf{126}}$ can have a different value from the one in the $\mathbf{10}$, for this reason we put the index f to the ρ parameter, with the relation $\rho^D = \rho^E$.

triplet that acquires vev in the direction $(0, 0, 1)$, we have showed that the lepton mixing matrix is the tribimaximal one. In particular in our model these operators give corrections to the entry $(1, 2)$ of all charged fermion mass matrices. If the ratio between this correction and m_c is of the order of the Cabibbo angle λ, we obtain that a rotation of order λ in the 12 plane appears in the up mass matrix. However the down and charged lepton mass matrices are almost unaffected by such corrections. This mismatching gives up the Cabibbo angle in the quark sector as a net result.

Acknowledgments

One of us (M.P.) acknowledge the kind hospitality of the Murcia University and the IFIC in Valencia for kind hospitality. M.P., E.T-L, and S.M. thank the Milan University for kind hospitality during the final part of the work. We acknowledge a MEC-INFN grant, Fundacion Seneca (Comunidad Autonoma de Murcia) grant, a CYCIT-Ministerio of Educacion (Spain) grant. Work partially supported by MEC grants FPA2005-01269 and FPA2005-25348-E, by Generalitat Valenciana ACOMP06/154. This work has been supported by the Spanish Consolider-Ingenio 2010 Programme CPAN (CSD2007-00042).

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	L	Q	N	E	U	D	H_u	H_d	$\overline{126}$	45_A	45_B	45_C	45_D	Φ	ϕ	$\tilde{\phi}$	ζ_S	ζ_T
$SO(10)$	$\mathbf{16}$						$\mathbf{10}$		$\overline{\mathbf{126}}$	$\mathbf{45}$	$\mathbf{45}$	$\mathbf{45}$	$\mathbf{45}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$SU(3)^F$	$\mathbf{3}$						$\mathbf{1}$		$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\overline{\mathbf{6}}$				
$U(1)^F$	$-\frac{1}{2}(\delta+\chi+\omega)$						$\omega-\rho-\sigma$		$\omega+\chi$	$\sigma-\beta$	$\beta+\chi$	χ	δ	$\delta+\rho$	$\rho-\chi$	$\rho+\sigma$	δ	δ
A_4	$\mathbf{3}$						$\mathbf{1}$		$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$		$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}'$	$\mathbf{3}$

TABLE I: The field content of the model. With this charge assignment, all the allowed operators are the only ones in our Lagrangian of eqs. (4-5), as explained in the text.

	X	Y	$B-L$	T_{3R}
Q	1	1/3	1	0
U^c	1	-4/3	-1	1/2
D^c	-3	2/3	-1	-1/2
L	-3	-1	-3	0
E^c	1	2	3	-1/2
N^c	5	0	3	1/2

TABLE II: $U(1)$ gauge quantum numbers for the low energy matter fields [59].

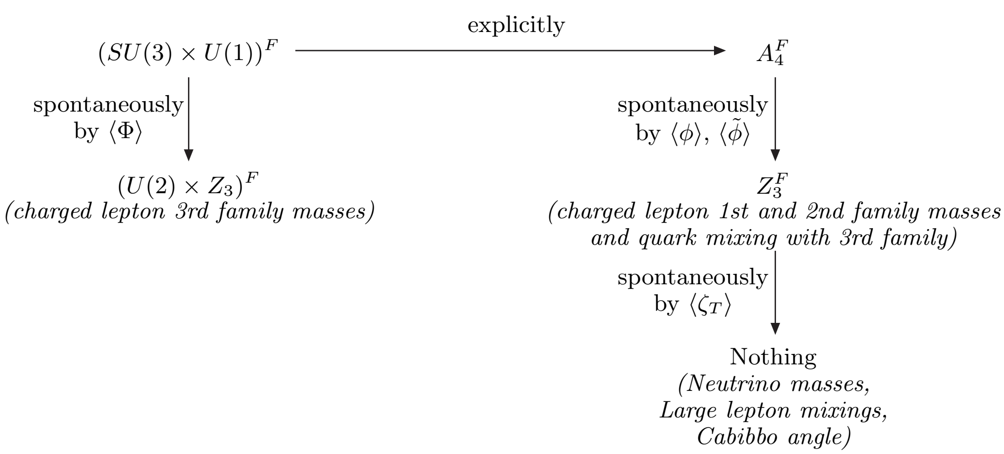

FIG. 1: Diagrammatic representation of the flavor symmetry structure of the model. The upper horizontal arrow indicates the explicit global symmetry breaking $SU(3)^F \to A_4^F$ due to the Yukawa terms induced by a hidden sector. The other arrows show the spontaneous breaking. The hierarchy among the masses is not directly related to the mixing angles.