Asymmetric and delayed activation of side-modes in multimode semiconductor lasers with optical feedback

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We study experimentally and numerically a multimode semiconductor laser subject to optical feedback and operating in the low-frequency fluctuation regime. We show that the low-frequency dropouts in the main modes are accompanied by sudden, asymetric, activations of dormant longitudinal side-modes. These activations are delayed with respect to the dropouts of the active modes. We compare experimental time traces of this phenomenon with results obtained from a multimode extension of the Lang-Kobayashi model that includes a parabolic gain profile. This new model satisfactorily reproduces both the asymmetric activation of the side-modes and their delay with respect to the dropouts.

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The use of an external cavity to reduce the spectral linewidth of semiconductor lasers is a well-established technique. However, under these conditions the interaction between the resulting delayed feedback and the laser nonlinearities leads, in a wide region of parameter space, to complex dynamical behavior. One of the most intriguing dynamical phenomena routinely found in semiconductor lasers with optical feedback is the recurrent appearance of sudden drops in the temporal evolution of the light intensity emitted by the laser under constant current driving [1]. Such dropouts, which arise for injection currents close to the laser threshold and for moderate feedback levels, occur at average time intervals much longer than the characteristic time scales of the laser, and are therefore known as low-frequency fluctuations (LFFs). The physical mechanism producing the power dropouts in the LFF regime is still subject to debate. Much work has been devoted to modelling this phenomenology by means of the well-known Lang-Kobayashi model [2], a delay-differential equation system which takes into account only one longitudinal mode of the laser and ignores multiple reflections from the external mirror. Within this model, the dropouts have been interpreted to be induced by crises, i.e. collisions of the system trajectory in phase space with saddle-type points of the dynamics, each crisis being preceded by a chaotic itinerancy between unstable spiral points [3]. Other studies have shown that the dropout process is to some extent a stochastically driven decay from the maximum gain mode due to spontaneous-emission noise [4].

The studies mentioned above assume single-mode operation of the semiconductor laser. But most of the low-cost semiconductor lasers available commercially operate in several longitudinal modes. In that sense, recent experiments have shown the importance of multimode dynamics in the LFF regime [5, 6]. Following these investigations, several multimode extensions of the LK model [7] were used to model the results obtained, both from dynamical [8, 9] and statistical [10] perspectives. In the framework of these investigations, it was observed that when a frequency-selective element (e.g. a diffraction grating) was introduced in the external cavity, each power dropout in the mode being fed back was accompanied by a sudden activation of several longitudinal side-modes of the laser (i.e., modes not subject to feedback) [11]. Different multimode extensions of the LK model have been proved to reproduce this phenomenon [12–14]. More recently, side-mode activations have been experimentally observed in a multimode laser subject to a *non-selective* optical feedback, demonstrating that the activation of the side-modes is a general feature of the LFF regime [15]. Moreover, it was shown that the activation of the side-modes is delayed with respect to the main mode dropouts and occurs in an asymmetric way. In this paper, we compare these experimental observations with results obtained from the numerical study of a multimode version of the LK model that assumes a parabolic profile of the gain and takes into account the frequency shift of the gain curve with the carrier population. Both measured and calculated values of the activation delay confirm that the side-mode activation is a consequence of the loss of power in the main modes. Furthermore, the model reproduces the asymmetry of the modal activation.

Our experimental setup consists of an index-guided AlGaInP semiconductor laser (Roithner RLT6505G), with a nominal wavelength of 658 nm. Its threshold current is $I_{\rm th} = 20.1\ mA$ for a temperature of $24.00 \pm 0.01\,^{\circ}$C. The injection current is set to $21.9 \pm 0.1\ mA$ all through the experiment. An antireflection-coated laser-diode objective is used to collimate the emitted light. An external mirror is placed 60 cm away from the front facet of the laser, introducing a delay time of $\tau = 4$ ns. The feedback strength is such that the threshold reduction due to it is 9.4%. Part of the total output intensity is received by a fast photodetector and sent to a HP 54720D 4 Gigasamples/s digital oscilloscope. The rest passes through a $1/8m$ CVI monochromator with a resolution better than 0.2 nm, which is used to filter different wavelengths in the laser output. The filtered radiation is detected by a Hamamatsu PS325 photomultiplier.

Using the monochromator we infer that 10-11 longitudinal modes are emitted by the laser in the absence of optical feedback. When feedback is added to the system, the maximum gain mode moves towards a higher wavelength and the optical spectrum broadens, mainly due to the activation of new modes in the long wavelength side of the spectrum. For the feedback parameters chosen, the laser exhibits low frequency fluctuations. In this situation, we can analyze the dynamical behavior of the different longitudinal modes. Figure 1 shows intensity time traces measured at eight wavelengths around the maximum gain mode of the laser with feedback, estimated to be located at ∼658.4 nm. We observe that the modes close to that maximum wavelength exhibit sudden dropouts in power, whereas inactive modes located at lower wavelengths undergo sudden activations in power simultaneously (in principle) to the dropouts. Note also that these side-mode activations do not occur, or are barely visible, in the other side of the spectrum. We remind at this point that the optical feedback acting upon our laser is not frequency selective.

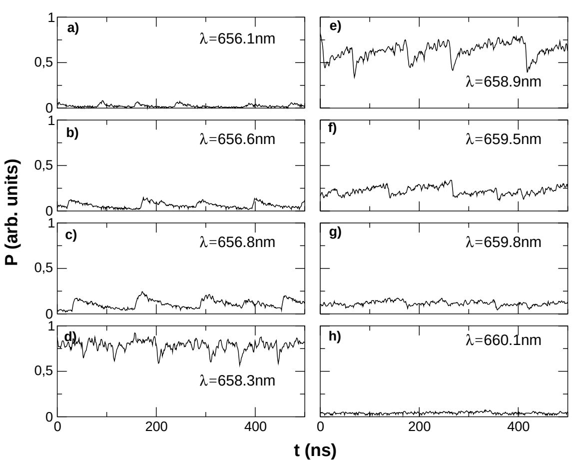

FIG. 1: Experimental time traces showing the temporal evolution of the emitted intensity, filtered at different wavelengths.

In order to reproduce the experimental results described above, we introduce at this point a multimode extension of the LK rate equations. The standard LK model consists of two equations describing the evolution of the electric field and the excess carrier number, respectively [2]. There are different ways of generalizing that standard single-mode model to describe the behavior of the different longitudinal modes of the laser. While all of them represent separately the complex envelopes of the electric fields corresponding to the different modes, some of them also distinguish between different carrier densities for each mode [8] (in a spirit close to the Tang-Statz-deMars model [16]), whereas many others consider that carriers are shared by all modes [7, 9, 10]. Within this latter type of models, there are some that consider mode interaction through self- and cross-saturation processes [7, 10], whereas others introduce a mode-dependent gain [9]. In what follows, we will make use of the latter type of approach, and consider a set of equations for the individual complex amplitudes of the slowly varying electric fields $E_m(t)$ of each mode m, and a single equation for the total excess carrier number $N(t)$ of the laser:

$$
\begin{aligned}
\frac{dE_m}{dt} &= \frac{1}{2}(1+i\alpha)(G_m(N)-\gamma_m)E_m(t) \\
&\quad + \frac{\kappa}{\tau_L}E_m(t-\tau)\exp(-i\omega_{0m}\tau) + F_m(t) \\
\frac{dN}{dt} &= \frac{I}{e} - \frac{N}{\tau_s} - \sum_{j=-M}^{M} G_m(N)|E_m|^2
\end{aligned}
\tag{1}
$$

where $m = -M \dots M$, and $m = 0$ corresponds to the mode located at the maximum of the gain curve of the solitary laser. The electric field amplitudes $E_m(t)$ are normalized so that $P_m(t) = |E_m(t)|^2$ measures the photon number in the m-th mode. The intrinsic laser parameters are the linewidth enhancement factor α, the mode-dependent cavity loss γ_m, and the internal round-trip time τ_L, all of which are assumed equal for all modes. Spontaneous emission is represented by the Langevin noise force $F_m(t)$, which is assumed to be Gaussian and white, with a correlation given by $\langle F_m^*(t)F_n(t')\rangle = R_{sp}\delta_{mn}\delta(t-t')$, where R_{sp} is the spontaneous emission rate. In the equation for the carrier density, τ_s is the lifetime of the electron-hole pairs, I is the injection current, and e is the magnitude of the electron charge.

The feedback parameters, namely the feedback level κ and the round-trip time of the external cavity τ, are also considered equal for all modes (in the case of κ, this assumption corresponds to a non-selective feedback). The phase shift $\omega_{0m}\tau$ appearing in the feedback term is due to the external-cavity roundtrip, with ω_{0m} representing the nominal frequency of the m-th mode, i.e. $\omega_{0m} = \omega_c + m\Delta\omega_L$, where ω_c is the frequency of the gain peak of the solitary laser and $\Delta\omega_L$ is the longitudinal mode spacing.

The mode-dependent gain coefficient G_m appearing in the electric field equation of (1) is assumed to have a parabolic frequency profile

$$G_m(N) = G_c(N-N_0)\left[1-\left(\frac{\omega_m-\omega_{\text{peak}}(N)}{\Delta\omega_g}\right)^2\right], \quad (2)$$

where G_c is the differential gain coefficient at the peak gain of the solitary laser, N_0 is the carrier number at transparency, $\Delta\omega_g$ is the gain width of the laser material, and ω_m is the instantaneous frequency of the m-th mode, given by

$$\omega_m(t) = \omega_{0m} + \frac{d\phi_m(t)}{dt}. \quad (3)$$

In this expression, $\phi_m(t)$ is the phase of the slowly varying complex electric field of the m-th mode. On the other hand, the center of the parabolic profile (2) occurs at a peak frequency ω_{peak} that shifts with the carrier population as [17]

$$\omega_{\text{peak}}(N) = \omega_c + \omega_N(N-N_{th}) \quad (4)$$

where ω_N is a constant and N_{th} is the carrier number at the laser threshold.

In our calculations, we assume nine active optical modes (i.e. M=4), and consider that γ_m is mode independent. In this approximation, the spacing between the modes of the solitary laser is given by $\Delta\omega_L = 2\pi/\tau_L$. We use typical values for the diode laser parameters: $\alpha = 4$, $\tau_s = 2$ ns, $\tau_L = 8.3$ ps, $\gamma_m = 5\times10^{11}$ s^{-1}, $G_c = 4\times10^3$ s^{-1}, $N_0 = 1.1\times10^8$, $R_{sp} = 5\times10^{11}$ s^{-1}, and $\Delta\omega_g = 2\pi\times2.82$ THz. Finally, we choose $\omega_c\tau = 0 \bmod 2\pi$, so that the feedback phase is $\omega_{0m}\tau = m\Delta\omega_L\tau$ $(\bmod 2\pi)$, i.e. different for every mode. The LFF regime can be observed in a wide range of feedback parameters when the laser is pumped close to its solitary threshold. In the following, we choose $\kappa = 7.5\times10^{-2}$, $\tau = 5$ ns, and $I = 1.015\times I_{th}$. Figure 2 shows the temporal evolution of eight modes of the laser for these parameters. The time traces have been averaged over 4 ns, in order to compare the numerical results with those obtained by a photodetector, which is bandwidth limited.

As shown in Fig. 2, low-frequency fluctuations are observed in every active longitudinal mode. Just after a dropout in the main mode of the laser with feedback ($m = -2$), all modal intensities start to rise proportionally to their relative gain. The mode of the solitary laser with largest modal gain ($m = 0$) increases faster than the others [18]. However, simultaneously to the recovery of the total intensity, the carrier number decreases and the gain peak shifts rapidly towards lower frequencies, as can be observed in Fig. 3(c). This figure displays the total output of the laser, the carrier number, and the shift of the gain peak frequency with respect to its value at the solitary laser threshold. For the parameter values chosen here, the total shift is approximately 190 GHz, which corresponds to 1.6 times the modal frequency spacing. As a result of this shift, mode 0 is no longer the dominant one. After a short time interval, the frequency of the

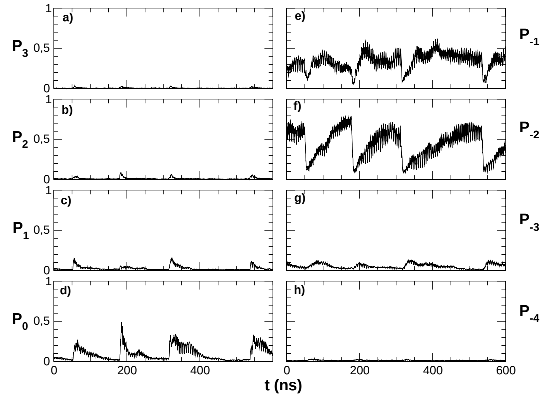

FIG. 2: Numerical time series of the power of different modes (in arb. units) emitted by the laser, from $m = -4$ to $m = 3$, as computed from the multimode LK model (1). Model parameters are given in the text.

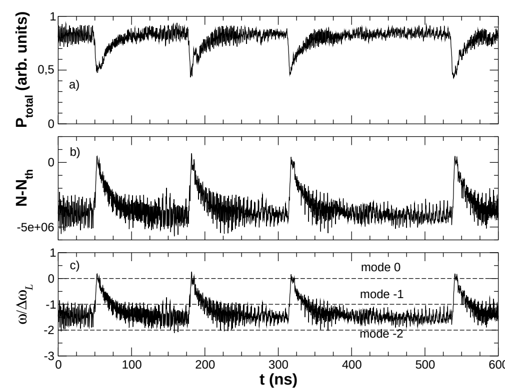

FIG. 3: Numerical time series corresponding to the situation of Fig. 2: (a) total output, (b) carrier number, (c) frequency shift.

gain peak oscillates around the frequency of mode -2, which is then the dominant one until the next dropout. Consequently, mode -2 continues to grow steadily [Fig. 2(f)], draining most of the electron-hole pairs while the photon number in the other modes saturates or begins to decrease until the next dropout event [Figs. 2(a)-(d) and (g)-(h)]. Most of the modes reach the spontaneous-emission level. These sudden activation of the side modes and their progressive extinction lead to the generation of bursts. The activation of the side-modes is not symmetric with respect to the dominant mode ($m = -2$). Indeed, the activations of modes -3 and -4 are much less pronounced than those of modes 0 and 1, although the corresponding modal gains are almost equal at the end of the recovery process. The asymmetry in the modal dynamics is the result of the shift of the gain peak towards lower frequencies. Just after a dropout, the modes located close to mode 0 increase faster than the other modes. When the total power recovers, modes -3 and -4 do not benefit from the shift of the gain curve since, at this time, the carrier population is low.

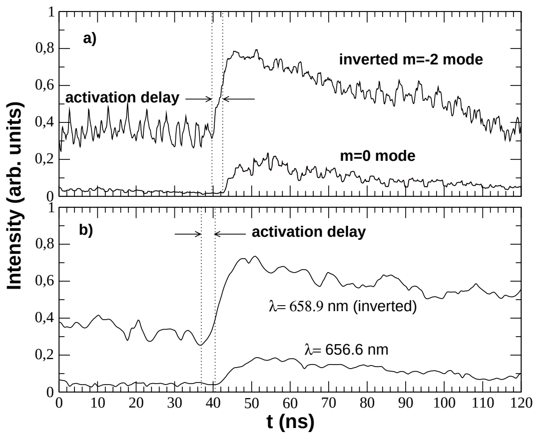

FIG. 4: Time series of the inverted dropout of the main mode and the activation of the side mode: (a) numerical model, (b) experiments.

When the dropout events are analyzed in short time scales, it can be seen that the side mode ($m = 0 \dots 3$) activations begin slightly after the dominant modes ($m = -1,\ -2$) drop out. This characteristic is shown in Fig. 4, for both the LK model and the experiment. In the two cases, an activation event is compared with the corresponding inverted time series of the main-mode dropout, and under this condition it can be seen that the dropout starts *earlier* than the activation. We can estimate the time delay between these two events by averaging the time series over several dropouts (in order to eliminate fluctuations before and after the events), and comparing the time instants when the time series corresponding to the two modes have maximum slope [15]. In the numerical case, the delay is measured with an statistics larger than 5000 dropout events, and the delay between the dropouts in mode $m = -2$ and the activations in mode $m = 0$ is estimated to be 3.2 ± 1.8 ns. This value is on the order of the one measured experimentally [15], and on the order of the carrier lifetime assumed in the model ($\tau_s = 2$ ns). This result supports our earlier conclusion that the activation of the side modes is a natural consequence of the loss of power of the dominant modes.

In summary, we have studied the side-mode activation of a multimode semiconductor laser in the low-frequency fluctuation regime. The intensity dropouts of the main modes are related with activations of side-modes at lower wavelengths. Numerical results obtained from a multimode Lang-Kobayashi model show, in agreement with experimental observations, that these activations appear after the main-mode dropouts and occur in an asymmetric way. Our model assumes a parabolic profile of the gain and takes into account the frequency shift of the gain curve with the carrier population. Other models previously reported in the literature [8, 10] do not exhibit this behavior.

Statistical analysis of the activation delay shows that its value is of the order of the carrier lifetime of the laser. Our experimental and numerical results thus demonstrate that the activation of the side-modes are a natural consequence of the loss of power of the dominant modes. As an additional conclusion, the qualitative agreement between the numerical results and the experimental observations give validity to the multimode extension of Lang-Kobayashi model proposed here.

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[18] Note that we are not taking into account the variation of the gain spectrum during turn-on. This approximation identifies the main mode of the solitary laser with the maximum gain mode during turn-on transients.