Microwave Photon Detector in Circuit QED

G. Romero,[1] J. J. García-Ripoll,[2] and E. Solano[3]

[1]*Departamento de Física, Universidad de Santiago de Chile, USACH, Casilla 307, Santiago 2, Chile*
[2]*Instituto de Física Fundamental, CSIC, Serrano 113-bis, 28006 Madrid, Spain*
[3]*Departamento de Química Física, Universidad del País Vasco - Euskal Herriko Unibertsitatea,*
Apartado 644, 48080 Bilbao, Spain
(Dated: April 20, 2009)

In this work we design a metamaterial composed of discrete superconducting elements that implements a high-efficiency microwave photon detector. Our design consists of a microwave guide coupled to an array of metastable quantum circuits, whose internal states are irreversibly changed due to the absorption of photons. This proposal can be widely applied to different physical systems and can be generalized to implement a microwave photon counter.

Quantum optical photodetection [1] has occupied a central role in understanding radiation-matter interactions. It has also contributed to the development of atomic physics and quantum optics, with applications to metrology, spectroscopy, and quantum information processing [2]. The quantum microwave regime, originally explored using cavities and atoms [3, 4], is seeing a novel boost with the generation of *nonclassical propagating fields* [5] in circuit quantum electrodynamics (QED) [6–8]. In the last years we have witnessed a tremendous development of the field of quantum circuits [9–11]. These devices are built, among other things, from superconducting elements, Josephson junctions, Cooper-pair boxes [12], SQUID's, microwave guides and cavities [6–8], all of them cooled down to the quantum degenerate regime. Among numerous applications we may highlight the creation of artificial atoms or circuits with discrete quantum energy levels, and quantized charge [12], flux [13] or phase [14] degrees of freedom. These circuits find applications not only as quantum bits for quantum information processing as charge [15–17], flux [18, 19] or phase qubits [20–22], but also in the linear and nolinear manipulation of quantum microwave fields. In particular we remark the exchange of single photons between superconducting qubits and resonators [7, 23–25], the first theoretical efforts for detecting incoming photons [26], the generation of propagating single photons [5] and the nonlinear effects that arise from the presence of a qubit in a resonator [27, 28].

While the previous developments represent a successful marriage between quantum optics and mesoscopic physics, this promising field suffers from the absence of photodetectors. The existence of such devices in the optical regime allows a sophisticated analysis and manipulation of the radiation field which is crucial for quantum information processing and communication. This includes Bell inequality experiments, all optical and measurement-based quantum computing, quantum homodyne tomography, and most important quantum communication and cryptography [2].

There are several challenges to design a microwave photodetector in circuit QED: i) Available cryogenic linear amplifiers are unable to resolve the few photon regime. ii) Free-space cross-section between microwave fields and matter qubits are known to be small. iii) The use of cavities to enhance the coupling introduces additional problems, such as the frequency mode matching and the compromise between high-Q and high reflectivity. iv) The impossibility of performing continuous measurement without backaction [26], which leads to the problem of synchronizing the detection process with the arrival of the measured field. We observe that there are related theoretical and experimental proposals that may solve some but not all of these problems. Most of them are oriented towards unitary or coherent evolution of the photon-qubit system, aided by nonlinear dispersive [24, 27] or bifurcation effects [29]. But so far, to our knowledge, there has been no implementation of a microwave photon counter or even a simple microwave photon detector.

Our goal in this work is to design a metamaterial that performs single-shot microwave photodetection via irreversible absorption of photons [30]. It resembles a photographic film: when a photon enters the device, there is a large chance that it will be captured, leaving the system in a stable and mesoscopically distinguishable state that can be observed *a posteriori*. More precisely, we propose a general setup based on a one-dimensional waveguide that passes by a set of photon absorbers [Fig. 1]. We neither rely on any kind of cavity device nor aim at reaching a matter-field strong-coupling regime. The absorbers along the waveguide may be built with bistable quantum circuits similar to the ones used for implementing qubits. These circuits should be able to capture a photon and decay from an initial state $|0\rangle$ into a stable state $|g\rangle$. In analogy to the photographic film, these irreversible events constitute the measurement process itself. The final step consists on counting the number of activated absorbers which is related to the number of photons in the Fock state which was detected by the quantum measurement. The counting should be done af-
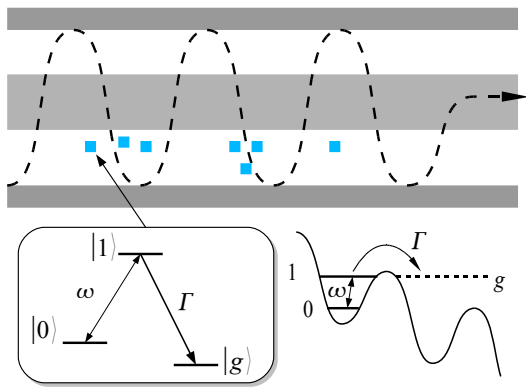
FIG. 1: (color online) Detector design. Sketch showing a one-dimensional waveguide (gray) coupled to a set of three-level absorbers (blue squares) in arbitrary positions. The microwave field (dashed line) excites coherently the state $|0\rangle$ to the upper state $|1\rangle$, which decays onto a long-lived stable state. An analogous setup uses current-biased Josephson Junctions (CBJJ), in which a washboard potential confines two metastable states that can decay into a continuum of current states.

ter the absorption process has finished, thereby avoiding any disturbing backaction on the incoming signal. Remark that the detection process is in this sense passive and does not require any kind of control.

It is possible to develop a simple model for the detection based on the previous rather general requirements. We model the absorbing elements as three-level systems, with an internal frequency ω and a decay rate Γ from the excited state $|1\rangle$ to the final stable state $|g\rangle$ (see Fig. 1). Mathematically, this is described with a master equation for the density matrix ρ of the absorbers and the waveguide

$$\frac{d}{dt}\rho = -\frac{i}{\hbar}[H,\rho] + \mathcal{L}\rho. \tag{1}$$

The Hamiltonian contains terms for the absorbers and the radiation fields, ψ_l and ψ_r, propagating left and right with group velocity v_g, see Methods. The interaction between both is modeled using a delta-potential of strength V

$$\begin{aligned} H &= \sum_i \hbar\omega\,|1\rangle_i\,\langle 1| + i\hbar v_g \int dx\left[\psi_l^\dagger \partial_x \psi_l - \psi_r^\dagger \partial_x \psi_r\right] \\ &+ \sum_i \int dx V\delta(x-x_i)\left[(\psi_l + \psi_r)|1\rangle_i\,\langle 0| + \text{H.c.}\right], \end{aligned} \tag{2}$$

where x_i and $|0\rangle, |1\rangle$ denote the position and the states of the i-th absorber. The Liouvillian $\mathcal{L} = \sum_i \mathcal{L}_i$ has the standard decay terms for each of the absorbing qubits

$$\mathcal{L}_i\rho = \frac{\Gamma}{2}\left(2\,|g\rangle_i\,\langle 1|\,\rho\,|1\rangle_i\,\langle g| - |1\rangle_i\,\langle 1|_i\,\rho - \rho\,|1\rangle_i\,\langle 1|\right), \tag{3}$$

and it is proportional to the decay rate Γ. The solutions of this master equation can be found using an equivalent

non-Hermitian Hamiltonian $\bar{H} = H - i\sum_j \Gamma/2\,|1\rangle_j\,\langle 1|$, that rules the dynamics of the populations in $|0\rangle$ and $|1\rangle$. The norm of the wavefunction is not preserved by this equation, but precisely the decrease of the norm is the probability that one or more elements have absorbed a photon.

The simplest scenario that we can consider is a single absorber coupled to the microwave guide, a problem that has analytical solutions for any pulse shape. In the limit of long wavepackets it becomes more convenient to analyze the scattering states of $\bar{H}$. For a single absorbing element, these states are characterized by the intensity of the incoming beam, which we take as unity, and the intensity of the reflected and trasmitted beams, $|r|^2$ and $|t|^2$. The associated complex amplitudes are related by the scattering matrix T, as in $(t,0)^t = T(1,r)^t$. The single-absorber transfer matrix

$$T = \begin{pmatrix} 1 - 1/\gamma & -1/\gamma \\ 1/\gamma & 1 + 1/\gamma \end{pmatrix} \tag{4}$$

is a function of a single complex dimensionless parameter

$$\gamma = (\Gamma - i\delta)v_g/V^2, \tag{5}$$

which relates the properties of the circuit: the group velocity in the waveguide, v_g, the strength of the coupling between the absorbers and the waveguide, V, and the detuning of the photons from a characteristic frequency of the absorbers, $\delta = \omega - \omega_\mu$. The single-photon detection efficiency (absorption probability) is computed as the amount of radiation which is neither transmitted nor reflected. In terms of the elements of T, it is given by $\alpha = 1 - (1 + |T_{01}|^2)|T_{11}|^{-2} = 2\gamma(1+\gamma)^{-2}$.

The curve shown in Fig. 2 reveals two regimes. If $\gamma \ll 1$, the decay channel $|1\rangle \rightarrow |g\rangle$ is very slow compared to the time required for a photon to excite the $|0\rangle \rightarrow |1\rangle$ transition, and only a small fraction of the photons is absorbed. If, on the other hand, the metastable state $|1\rangle$ decays too fast, $\gamma \gg 1$, there is a Zeno suppression of the absorption. From previous formula, the maximal achievable detection efficiency is 50%, a limit reached by tuning the single absorber on resonance with the microwave field. We conjecture that this may be a fundamental limit for any setup involving a single point-like absorber and no time-dependent external control.

A natural expectation would be that clustering many absorbers inside the waveguide increases the detection efficiency. As shown in Fig. 2, this is not true. If we have a cluster of N identical absorbers close together, we can compute the detection efficiency using the same formula but with the scattering matrix $T_{\text{cluster}} = T^N$. As far as the cluster size is smaller than a wavelength, the setup will be limited to a 50% maximum efficiency. There is a simple explanation for this. Since the cluster size is small, the photon sees the group of absorbers as a single element with a larger decay rate, $N\Gamma$. This renormalization just
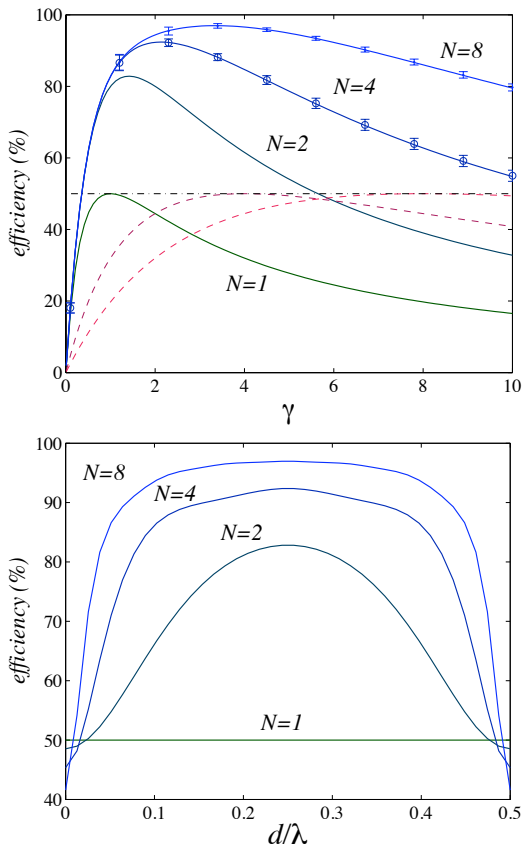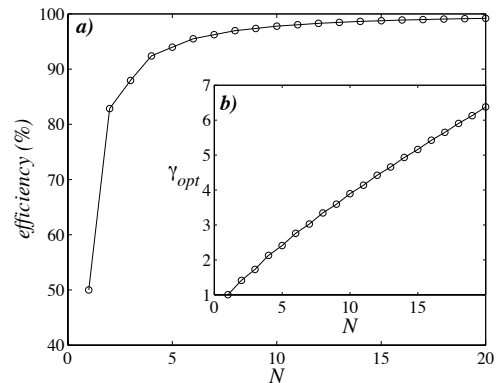
FIG. 3: (a) Maximal achievable single-photon detection efficiency as a function of the number of qubits along the microwave guide. (b) Optimal working parameters vs. number of qubits.

FIG. 2: (Color online) Detection efficiency when absorbers are on resonance (real γ). (Top) Absorption probability vs. effective decay rate $\gamma = \Gamma v_g / V^2$ in dimensionless units for a setup with $N = 1, 2, 4$ and 8 qubits (black, green, blue, red) either in cluster (dashed) or array (solid) regime. The error bars account for random deviations in the individual absorber properties, γ_i, of up to 40%. When absorbers are close together, the efficiency is limited to 50%, while in the other case there is no upper limit. (Bottom) Detection efficiency vs. separation d in a periodically distributed array of absorbers.

shifts the location of the optimal working point, leaving the maximum efficiency untouched.

The main result is that we can indeed increase the absorption efficiency by separating the absorbing elements a fixed distance d longitudinally along the waveguide. The total scattering matrix for the array is given by $T_{\text{array}} = \prod_{j=1}^{N} exp(-i2\pi\sigma^z d/\lambda)T_j$, where σ^z is a Pauli matrix and the scattering of each absorber may be different. In this case the microwave pulse does no longer see the the detection array as a big particle, and we obtain an collective enhacement of the absorption probability. Remarkably, an arbitrarily high detection efficiency can be reached by increasing the number of absorbers and tuning their separation d. Already with two and three qubits we can achieve 80% and 90% detection efficiency, see Figs. 2 and 3. Furthermore, as we have seen numerically, the more qubits we have, the less sensitive the

whole setup becomes to the experimental parameters, see Fig. 2. This shows that the proposed setup is both robust and scalable.

The previous analysis is of a general kind. It only requires a coupling between a waveguide and absorbers that may capture a photon and irreversibly decay to one or more stable states. A practical implementation of our setup, which does not require strong coupling or cavities, consists on a coplanar coaxial waveguide coupled to a number of current-biased Josephson junctions (CBJJ) [14, 20]. We will now sketch the microscopic derivation of Eq. (2) for this setup and relate the efficiency to the parameters of the circuit.

First of all, since the CBJJs are described by a washboard potential for the phase degree of freedom, we can identify $|0\rangle$ and $|1\rangle$ with the two lowest metastable levels in a local minimum, see Fig. 1. The energy levels around such a minimum are well described by a harmonic oscillator with a frequency ω that depends on the bias current. Furthermore, these levels have finite lifetimes before they decay into the continuum of current states, but since that the decay rate of state $|0\rangle$, Γ_0, can be made 1000 times smaller [14] than that of $|1\rangle$, Γ_1, we will approximate $\Gamma_0 \simeq 0, \Gamma_1 \equiv \Gamma$.

The microwave guide is described by a Lagrangian [6]

$$L = \int dx \left[\frac{l}{2}(\partial_t Q)^2 - \frac{1}{2c}(\partial_x Q)^2 \right], \qquad (6)$$

where l and c are the inductance and capacitances per unit length. The quantization of the charge field Q introduces Fock operators a_p associated to the normal modes of the line. If we assume periodic boundary conditions, then $w_p(x,t) = \exp[i(px - \omega_p t)]/L^{-1/2}$, where L is the length of the waveguide and the dispersion relation is $\omega_p = v_g|p| = |p|/\sqrt{cl}$. When the relevant modes of the electromagnetic field are concentrated in a small interval $\mathcal{B}$ around a central momentum p_0, we can introduce

right and left moving fields $\psi_r(x,t) = \sum_{p\in\mathcal{B}} a_p w_p(x,t)$, and $\psi_l(x,t) = \sum_{p\in\mathcal{B}} a_{-p} w_{-p}(x,t)$, and approximate the waveguide Hamiltonian as $H = \sum_p \omega_p a_p^\dagger a_p$, which corresponds to the first line in Eq. (2).

Finally, for the interaction between the absorbers and the guide we use a capacitive coupling in the dipole approximation [6]. The corresponding Hamiltonian has the form

$$H_{\text{int}} = \frac{C_g}{C_g + C_J} \sqrt{\frac{\hbar\omega_\mu}{c}} (\psi_r + \psi_l + \text{H.c.}) \frac{e}{\alpha}(a + a^\dagger), \quad (7)$$

The first fraction depends on the capacitances of the junction and of the gate between the junction and the microwave, C_J and C_g, respectively. The second term gives the strength of the electric potential inside the waveguide and it is proportional to the fields. Finally, the third term is just the charge operator for the CBJJ expressed using harmonic approximation around a minimum of the washboard potential. In particular, $a \simeq |0\rangle\langle 1|$ and $\alpha^2 = E_C/\hbar\omega$ is the dimensionless parameter of this oscillator, expressed in terms of the junction capacitance, $E_C = (2e)^2/C_J$, and the plasma frequency. Note that when we combine all constants to form the interaction strength V there is not explicit dependence on the length of the microwave guide. Qualitatively, while in cavity experiment the qubit only sees a small fraction of the standing waves with which it interacts, in our setup each absorber gets to see the whole of the photon wavepacket after a long enough time.

In terms of the microscopic model, it is possible to write the parameter that determines the detector efficiency as follows

$$\gamma = \frac{\alpha^2}{c_{12}} \frac{\hbar}{eZ_0} \frac{\Gamma_1 - i(\omega - \omega_\mu)}{\omega_\mu}, \quad (8)$$

where we have introduced the dimensionless constant $c_{12} = C_g/(C_g + C_J)$. It is evident from Eq. (9) that, in order to optimize the efficiency, we have several experimental knobs to play with. In particular we have considered the following values, close to current experiments[20] a junction capacitance of $C_J = 4.8$pF, $c_{12} = 0.13$ and $\omega = 5$ GHz. Putting the numbers together, and letting the waveguide impedance oscillate between 10 and 100 Ω, the optimal operation point for a single junction gives a necessary decay rate $\Gamma \simeq 10 - 100$ MHz. Increasing C_g by a factor 2 triples the optimal decay rate, $\Gamma \simeq 30 - 300$ MHz.

Our proposal has the following potential limitations and imperfections. First, the bandwidth of the detected photons has to be small compared to the time required to absorb a photon, roughly proportional to $1/\Gamma$. Second, the efficiency might be limited by errors in the discrimination of the state $|g\rangle$ but these effects are currently negligible [25]. Third, dark counts due to the decay of

the state $|0\rangle$ can be corrected by calibrating Γ_0 and postprocessing the measurement statistics. Fourth, fluctuations in the relative energies of states $|0\rangle$ and $|1\rangle$, also called dephasing, are mathematically equivalent to an enlargement of the incoming signal bandwidth by a few megahertz and should be taken into account in the choice of parameters. Finally and most important, unknown many-body effects cause the non-radiative decay process $1 \to 0$, which may manifest in the loss of photons while they are being absorbed. In current experiments[25] this happens with a rate of a few megahertzs, so that it would only affect very long wavepackets.

Our design can be naturally extended to implement a photon counter using a number of detectors large enough to capture all incoming photons. Furthermore, our proposal can be generalized to other level schemes and quantum circuits that can absorb photons and irreversible decay into long lived and easily detectable states.

[1] L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge U.Press, New York, 1995), chap. 9,14.
[2] D. Bouwmeester, A. Ekert, and A. Zeilinger, *The Physics of Quantum Information* (Springer Verlag, Berlin, 2008).
[3] H. Walther, B. T. H. Varcoe, B. G. Englert, and T. Becka, Rep. Prog. Phys. **69**, 1325 (2006).
[4] S. Haroche and J.-M. Raymond, *Exploring the Quantum* (Oxford Univ. Press Inc., New York, 2006).
[5] A. A. Houck *et al*, Nature **449**, 328 (2007).
[6] A. Blais *et al*, Phys. Rev. A **69**, 062320 (2004).
[7] A. Wallraff *et al*, Nature **431**, 162 (2004).
[8] I. Chiorescu *et al*, Nature **431**, 159 (2004).
[9] J. Q. You and F. Nori, Physics Today **58**, 110000 (2005).
[10] R. J. Schoelkopf and S. M. Girvin, Nature **451**, 664 (2008).
[11] J. Clarke and F. K. Wilhelm, Nature **453**, 1031 (2008).
[12] V. Bouchiat, D. Vion, P. Joyez, D. Esteve, and M. H. Devoret, Phys. Scr **T76**, 165 (1998).
[13] J. E. Mooij *et al*, Science **285**, 1036 (1999).
[14] J. M. Martinis, M. H. Devoret, and J. Clarke, Phys. Rev. Lett. **55**, 1543 (1985).
[15] D. Vion *et al*, Science **296**, 886 (2002).
[16] T. Yamamoto *et al*, Nature **425**, 941 (2003).
[17] J. Majer *et al*, Nature (London) **449**, 443 (2007).
[18] C. H. van der Wal *et al*, Science **290**, 773 (2000).
[19] I. Chiorescu, Y. Nakamura, C. Harmans, and J. Mooij,

Science **299**, 1869 (2003).

[20] A. J. Berkley *et al*, Science **6**, 1548 (2003).

[21] R. W. Simmonds *et al*, Phys. Rev. Lett. **93**, 077003 (2004).

[22] M. Steffen *et al*, Science **313**, 1423 (2006).

[23] K. B. Cooper *et al*, Phys. Rev. Lett. **93**, 180401 (2004).

[24] D. I. Schuster *et al*, Nature (London) **445**, 515 (2007).

[25] M. Hofheinz *et al*, Nature **454**, 310 (2008).

[26] F. Helmer, M. Mariantoni, E. Solano, and F. Marquardt, e-print arXiv.org:0712.1908.

[27] J. M. Fink *et al*, Nature **454**, 315 (2008).

[28] L. S. Bishop *et al*, e-print arXiv:0807.2882.

[29] I. Siddiqi *et al*, Phys. Rev. Lett. **93**, 207002 (2004).

[30] J. J. García-Ripoll, E. Solano, and G. Romero, Spanish Patent Application No. 200802933 (2008).