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Heavy Quark Parameters and $|V_{cb}|$ from Spectral Moments in Semileptonic B Decays

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Abstract

We extract the heavy quark masses and non-perturbative parameters from the DELPHI preliminary measurements of the first three moments of the charged lepton energy and hadronic mass distributions in semileptonic B decays, using a multi-parameter fit. We adopt two formalisms, one of which does not rely on a $1/m_c$ expansion and makes use of running quark masses. The data are consistent and the level of accuracy of the experimental inputs largely determines the present sensitivity. The results allow to improve on the uncertainty in the extraction of $|V_{cb}|$.

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1 Introduction

The Operator Product Expansion (OPE) represents a foundation for extracting the $|V_{ub}|$ and $|V_{cb}|$ elements of the CKM mixing matrix from inclusive semileptonic (s.l.) B decays. In this framework, the decay width is expressed in terms of quark masses, and non-perturbative effects are described by expectation values of heavy quark operators, some of which are presently poorly known. The experimental accuracy already achieved, and that expected from the large data sets recorded by the B-factories, makes the ensuing theory uncertainty a serious limitation. Extracting the heavy quark masses and the non-perturbative parameters, arising from the $1/m_b^2$ and $1/m_b^3$ corrections, directly from the data has therefore become a key issue. There have already been $|V_{cb}|$ determinations from the first moment of distributions in s.l. and $b \to X_s \gamma$ decays, and the $1/m_b^3$ corrections, estimated from parameter ranges, have been found to represent an important source of uncertainty [1]. These ranges, based on dimensional arguments, are affected by a considerable degree of arbitrariness.

In order to circumvent these problems, we introduce in this Letter a multi-parameter fit to determine the relevant $1/m_b^2$ and $1/m_b^3$ parameters, together with the heavy quark masses, from the first three moments of the leptonic energy and hadronic mass spectra in s.l. B decays. Results are based on preliminary data obtained by the Delphi Collaboration. Moments are measured without cuts on the lepton energy in the B rest frame. We consider two formalisms, one of which is new and relies on fewer theoretical assumptions. The use of higher moments guarantees a sensitivity to these parameters and the simultaneous use of the hadronic and leptonic spectra ensures that a larger number of parameters can be kept free in the fit. We discuss the results both in terms of the extraction of the parameters and the implications for $|V_{cb}|$, and as a consistency check of the underlying theoretical assumptions.

2 Extracting Non-Perturbative Parameters

The moments of the hadronic and leptonic spectra in s.l. B decays have recently been measured by several experiments [1–5]. We consider here moments of the charged lepton energy distribution defined as

$$M_1(E_\ell) = \frac{1}{\Gamma} \int dE_\ell \, E_\ell \frac{d\Gamma}{dE_\ell}; \qquad M_n(E_\ell) = \frac{1}{\Gamma} \int dE_\ell \, (E_\ell - M_1(E_\ell))^n \frac{d\Gamma}{dE_\ell} \quad (n > 1), \tag{1}$$

and moments of the distribution of M_X, the invariant hadronic mass,

$$M_1(M_X) = \frac{1}{\Gamma} \int dM_X^2 \, (M_X^2 - \bar{M}_D^2) \frac{d\Gamma}{dM_X^2}; \qquad M_n(M_X) = \frac{1}{\Gamma} \int dM_X^2 \, (M_X^2 - \langle M_X^2 \rangle)^n \frac{d\Gamma}{dM_X^2} \quad (n > 1), \tag{2}$$

where $\bar{M}_D = 1.973$ GeV is the spin averaged D meson mass and no cut on the charged lepton energy is assumed.

The theoretical framework to interpret these data has long been known and it is based on the OPE. Different implementations exist, depending on the way the quark masses are treated. For instance, the m_b and m_c masses can be taken as independent parameters or subject to a constraint on $m_b - m_c$, imposed from the measured $B^{(*)}$ and $D^{(*)}$ meson masses. The second choice introduces a $1/m_c$ expansion. Another option concerns the normalization scheme used for quark masses and nonperturbative parameters. One approach is to use short-distance masses, such as the low-scale running masses. Alternatively, the pole mass scheme can be used.

The OPE expresses lepton moments through quark masses as a double expansion in α_s and $1/m_b$:

$$M_n(E_\ell) = \left(\frac{m_b}{2}\right)^n \left[\varphi_n(r) + \bar{a}_n(r)\,\frac{\alpha_s}{\pi} + \bar{b}_n(r)\,\frac{\mu_\pi^2}{m_b^2} + \bar{c}_n(r)\,\frac{\mu_G^2}{m_b^2} + \bar{d}_n(r)\,\frac{\rho_D^3}{m_b^3} + \bar{s}_n(r)\,\frac{\rho_{LS}^3}{m_b^3} + ...\right], \tag{3}$$

where $r = (m_c/m_b)^2$. The higher coefficient functions $\bar{b}(r)$, $\bar{c}(r)$, ... are also perturbative series in α_s. The expectation values of only two operators contribute to the $1/m_b^3$ corrections: the Darwin term ρ_D^3 and the spin-orbital term ρ_{LS}^3. Due to the kinematic definition of the hadronic invariant mass M_X^2, the general expression for the hadronic moments includes M_B explicitly:

$$\begin{aligned} M_n(M_X) \quad = \quad & m_b^{2n} \sum_{l=0} \left[\frac{M_B - m_b}{m_b}\right]^l \Bigg\{ E_{nl}(r) + a_{nl}(r)\frac{\alpha_s}{\pi} + b_{nl}(r)\frac{\mu_\pi^2}{m_b^2} + c_{nl}(r)\frac{\mu_G^2}{m_b^2} \\ & + d_{nl}(r)\frac{\rho_D^3}{m_b^3} + s_{nl}(r)\frac{\rho_{LS}^3}{m_b^3} + ... \Bigg\}. \end{aligned} \tag{4}$$

It is possible to re-express the heavy quark masses, m_Q, in the above equations, in terms of the meson masses, M_{H_Q}, through the relation [6]:

$$M_{H_Q} = m_Q + \bar{\Lambda} + \frac{\mu_\pi^2 - \mu_G^2}{2m_Q} + \frac{\rho_D^3 + \rho_{LS}^3 - \rho_{nl}^3}{4m_Q^2} + \mathcal{O}\left(\frac{1}{m_Q^3}\right). \tag{5}$$

The use of these expressions introduces an explicit dependence of the non-local correlators contributing to ρ_{nl}^3. In the notation of [7], ρ_{nl}^3 corresponds to linear combinations of $\mathcal{T}_{1-4}$.

Here, we employ the following two formalisms. The first one is based on the kinetic running masses, $m_Q(\mu)$, and non-perturbative parameters, introduced in [8]. No charm mass expansion is assumed. The second formalism employs quark pole masses and the $B^{(*)}$ and $D^{(*)}$ meson mass relations. It represents a useful reference, as it has been already adopted in several studies.

Contributions through $O(\alpha_s^2\beta_0)$ [9, 10] and $O(1/m_b^3)$ [11–14] to the moments are available. Depending on the formulation adopted, the number of parameters involved at this order ranges from six to nine. Some of these parameters, like m_b and $\lambda_2 \simeq \mu_G^2/3$, are relatively well known. Others, notably those which appear at $O(1/m_b^3)$, are virtually unknown.

2.1 The $m_b(\mu)$, $m_c(\mu)$ and $\mu_\pi^2(\mu)$ Formalism

The running kinetic quark masses $m_b(\mu)$ and $m_c(\mu)$ are considered here as two independent parameters. Apart from $\mu_\pi^2(\mu)$ and $\mu_G^2(\mu)$, defined as expectation values in the actual B meson, there are two $1/m_b^3$ parameters, ρ_D^3 and ρ_{LS}^3. The effect of ρ_{LS}^3 turns out to be numerically small. In Eqs. (3) and (4) the mass ratio r is given by $(m_c(\mu)/m_b(\mu))^2$, and the b quark mass is understood as $m_b(\mu)$. The

Table 1: *Numerical values of the coefficients in Eq.(3) evaluated at r=0.06 and* $m_b(1\,\mathrm{GeV}) = 4.6\,\mathrm{GeV}$.

	φ_n	$\bar{a}_n$	b_n	$\bar{c}_n$	d_n	$\bar{s}_n$
$M_1(E_\ell)$	0.6173	0.015	0.31	-0.73	-3.7	0.2
$M_2(E_\ell)\,(\times 10)$	0.3476	0.026	1.7	-1.0	-10.2	-0.9
$M_3(E_\ell)\,(\times 10^2)$	-0.3410	0.066	3.4	1.3	-23	-4.2

Table 2: *Numerical values of the coefficients in Eq.(4) evaluated at r=0.06 and* $m_b(1\,\mathrm{GeV}) = 4.6\,\mathrm{GeV}$.

i	E_{i1}	E_{i2}	E_{i3}	a_{i0}	a_{i1}	b_{i0}	b_{i1}	c_{i0}	c_{i1}	d_{i0}	s_{i0}
1	0.839	1	0	0.029	0.013	-0.58	-0.58	0.31	0.87	3.2	-0.4
2	0	0.021	0	- 0.001	-0.002	0.16	0.34	0	-0.05	-0.8	0.05
3	0	0	-0.0011	0.0018	0.0013	0	0.034	0	0	0.15	0

perturbative coefficients additionally depend on μ/m_b and the mass normalization scale μ is set at $\mu = 1\,\mathrm{GeV}$. The functions φ_n in Eq. (3) are well-known parton expressions. The relevant coefficients are given in Table 1, for the central values of $m_b(1\ \mathrm{GeV}) = 4.6$ GeV and $r \simeq 0.06$ obtained in our fit. Although we quote only the leading-order perturbative coefficients, we also include second-order BLM corrections in the analysis. Detailed expressions for the coefficients will be presented elsewhere.

In the case of hadronic moments, we discard in Eq. (4) coefficients b_{nl}, c_{nl} with $l>1$, and d_{nl}, s_{nl} with $l>0$. The only non-vanishing E_{i0} coefficient is $E_{10} = r - \bar{M}_D^2/m_b^2$. The value of the other coefficients, at $r = 0.06$, are listed in Table 2. Here we consider only $\mathcal{O}(\alpha_s)$ corrections and evaluate them using $\alpha_s = 0.3$.

2.2 The $\bar{\Lambda}$ and λ_1 Formalism

This widely used scheme results from the combination of the OPE with the HQET. Following the notation of Ref. [14], the moments are expressed in the following general form:

$$M_n = M_B^k \Bigg[a_0 + a_1 \frac{\alpha_s(\bar{M}_B)}{\pi} + a_2\beta_0 \frac{\alpha_s^2}{\pi^2} + b_1 \frac{\bar{\Lambda}}{\bar{M}_B} + b_2 \frac{\alpha_s}{\pi}\frac{\bar{\Lambda}}{\bar{M}_B} + \frac{c_1\,\lambda_1 + c_2\,\lambda_2 + c_3\,\bar{\Lambda}^2}{\bar{M}_B^2} + \frac{1}{\bar{M}_B^3}\left(d_1\,\lambda_1\bar{\Lambda} + d_2\,\lambda_2\bar{\Lambda} + d_3\,\bar{\Lambda}^3 + d_4\,\rho_1 + d_5\,\rho_2 + \sum_{i=1,4} d_{5+i}\mathcal{T}_i \right) + O\left(\frac{\Lambda_{QCD}^4}{m_Q^4}\right)\Bigg], \qquad (6)$$

where $k = n$ and $k = 2n$ for leptonic and hadronic moments, respectively, while $a_0 = 0$ for hadronic moments. $\bar{M}_B = 5.3135$ GeV is the spin-averaged B meson mass. The second order BLM corrections[1] are expressed in terms of $\beta_0 = 11 - 2/3n_f$, where we take $n_f = 3$. The coefficients a_i, b_i, c_i, d_i are given in Table 3 for the first three leptonic, $M_{1,2,3}(E_\ell)$, and hadronic moments, $M_{1,2,3}(M_X)$. In the

[1] The terms $O(\alpha_s^2\beta_0)$ and $O(\alpha_s\bar{\Lambda})$ are not available for the third hadronic moment. In our analysis we employ an estimate and the related uncertainty is included in the fit.

Table 3: *Numerical values of the coefficients in Eq.(6).*

	a_0	a_1	a_2	b_1	b_2	c_1	c_2	c_3	
	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9
$M_1(E_\ell)$ $(\times 10)$	2.708	-0.004	-0.10	-0.548	-0.15	-3.99	-9.77	-0.77	
	-10.1	-15.3	-1.2	-9.7	3.1	-3.9	4.1	-2.0	9.8
$M_2(E_\ell)$ $(\times 10^2)$	0.710	-0.096	-0.18	-0.535	-0.10	-4.32	-5.75	-0.35	
	-7.2	-8.1	-0.2	-19.7	-5.4	1.3	10.2	-0.2	5.7
$M_3(E_\ell)$ $(\times 10^3)$	-0.257	-0.014	0.03	-0.017	-0.01	-2.14	2.88	0.20	
	0.5	5.6	0.4	-28.3	-11.4	5.2	9.6	1.0	-2.9
$M_1(M_X)$	0	0.052	0.096	0.225	0.10	1.04	-0.31	0.28	
	2.2	2.4	0.3	2.3	-1.2	1.6	0.8	1.5	0.4
$M_2(M_X)$ $(\times 10)$	0	0.054	0.078	0	0.14	-1.40	0	0.11	
	-1.6	-1.6	0.2	-8.7	2.4	-1.4	-4.2	0	0
$M_3(M_X)$ $(\times 10^2)$	0	0.106	–	0	–	0	0	0	
	-2.05	0	-0.03	14.45	0	0	0	0	0

leptonic case the a_1 coefficients agree with ref. [12], while the coefficients of the first two hadronic moments agree with [13, 14]. Details of the derivation will be presented elsewhere.

The non-perturbative parameters in Eq.(6) are related to those in Section 2.1 by the following relations, valid up to $\mathcal{O}(\alpha_s)$:

$$\mu_\pi^2 = -\lambda_1 - \frac{\mathcal{T}_1 + 3\mathcal{T}_2}{m_b}; \quad \mu_G^2 = 3\lambda_2 + \frac{\mathcal{T}_3 + 3\mathcal{T}_4}{m_b}; \quad \rho_D^3 = \rho_1\,; \quad \rho_{LS}^3 = 3\rho_2\,. \tag{7}$$

Perturbative corrections introduce a significant numerical difference between the parameters in the two schemes. At $\mu = 1$ GeV:

$$\bar{\Lambda} \simeq M_B - m_b(1\ \text{GeV}) - \frac{\mu_\pi^2 - \mu_G^2}{2m_b} - 0.26\ \text{GeV}\ ; \qquad -\lambda_1 \simeq \mu_\pi^2(1\ \text{GeV}) - 0.17\ \text{GeV}^2\,. \tag{8}$$

A well known problem of this formalism is the instability of the perturbative series, due to the use of the pole quark masses. Large higher order corrections are however expected to cancel in the relation between physical observables, as long as all observables involved in the analysis are computed at the same order in α_s [6, 15]. We also note that, as a consequence of the HQET mass relations for the mesons, the intrinsic expansion parameter in Eq.(6) is $1/M_D$, rather than $1/M_B$. The convergence of this expansion has been questioned, in view of indications [16, 17] that the matrix elements $\mathcal{T}_i$ of some non-local operators could be larger than that expected from dimensional estimates.

3 Fits and Results

This analysis is based on the preliminary DELPHI measurements [3, 4] of the first three moments of the hadronic mass and charged lepton energy, summarised in Table 4. Owing to the large boost of B

Table 4: *Preliminary* DELPHI *results for the three leptonic and hadronic moments.*

Moment	Result	(stat)	(syst)	
$M_1(E_\ell)$	(1.383	$\pm$ 0.012	$\pm$0.009)	GeV
$M_2(E_\ell)$	(0.192	$\pm$ 0.005	$\pm$0.008)	GeV^2
$M_3(E_\ell)$	(-0.029	$\pm$ 0.005	$\pm$0.006)	GeV^3
$M_1(M_X)$	(0.534	$\pm$ 0.041	$\pm$ 0.074)	GeV^2
$M_2(M_X)$	(1.226	$\pm$ 0.158	$\pm$ 0.152)	GeV^4
$M_3(M_X)$	(2.970	$\pm$ 0.673	$\pm$ 0.478)	GeV^6

hadrons in $Z^0 \to b\bar{b}$ events, the acceptance of the analyses can be extended down to the start of the lepton energy spectrum, making their theoretical interpretation more direct. The results correspond to B_d^0 and B_u^- mesons decays only.

We perform a χ^2 fit to these six moments, using the two theoretical frameworks discussed above. In the fit we also impose some additional constraints derived from independent determinations.

In the kinetic mass scheme, we fit the full set of parameters: m_b(1 GeV), m_c(1 GeV), μ_π^2, ρ_D^3 and ρ_{LS}^3. We impose μ_G^2=(0.35$\pm$0.05) GeV^2 [16] and ρ_{LS}^3=(-0.15$\pm$0.15) GeV^3. Two mass constraints have also been applied: m_b(1 GeV)=(4.57$\pm$0.10) GeV [18], and, to be conservative, m_c(1 GeV)=(1.05$\pm$0.30) GeV. The most stringent is that on m_b(1 GeV). It must be noted that this constraint is largely equivalent to that derived from the first moment of the photon energy spectrum in $b \to s\gamma$ in other studies [2]. Results are obtained for $\alpha_s(m_b) = 0.22 \pm 0.01$ and are shown in Table 5. In order to study the effect of the bounds on $m_{b,c}$ introduced, the fit has been repeated unconstrained. Results are consistent, although the accuracy on the masses degrades. In particular we find m_b(1 GeV)=(4.61$\pm$0.15) GeV. It is interesting to observe that the mass constraints applied are of the scale of the fit sensitivity. Also, the central values of the heavy quark masses are in good agreement with independent determinations [18, 19].

In the alternative approach based on pole masses, the fit extracts $\bar{\Lambda}$, λ_1, λ_2, ρ_1 and ρ_2. We fix $\mathcal{T}_i = 0$ and impose two constraints from $M_{B^*} - M_B$ and $M_{D^*} - M_D$ which effectively reduce by two the number of free parameters. The results are given in Table 6.

Projections of the constraints from the six moments in the m_b-μ_π^2 and m_b-ρ_D^3 planes are shown in Fig. 1 and those in the $\bar{\Lambda}$-λ_1 and $\bar{\Lambda}$-ρ_1 planes in Fig. 2. The χ^2/n.d.f. of the fits is 0.96 and 0.35 in the two formulations. Since the contributions proportional to ρ_{LS}^3 in the moment expressions are numerically suppressed, the fit is only marginally sensitive to its size and the result is determined by the constraint applied. By removing this, the fit would give ρ_{LS}^3=(-1.0$\pm$0.7) GeV^3.

In contrast, the value of the leading $1/m_b^3$ correction (parameterised by ρ_D^3 or ρ_1) can be determined with satisfactory accuracy and its range agrees with theoretical expectations [16].

Systematic uncertainties due to ranges of residual parameters which have been fixed and missing terms in the expansions have been estimated. For the running mass formalism we propagate the

Table 5: *Results of fit for the $m_b(\mu)$, $m_c(\mu)$ and $\mu_\pi^2(\mu)$ formalism.*

Fit Parameter	Fit Values	Fit Uncertainty	Syst. Uncertainty	
m_b(1 GeV)	4.59	$\pm$ 0.08	$\pm$ 0.01	GeV
m_c(1 GeV)	1.13	$\pm$ 0.13	$\pm$ 0.03	GeV
μ_π^2(1 GeV)	0.31	$\pm$ 0.07	$\pm$ 0.02	GeV^2
ρ_D^3	0.05	$\pm$ 0.04	$\pm$ 0.01	GeV^3

Table 6: *Results of fit for the $\bar{\Lambda}$-λ_1 formalism.*

Fit Parameter	Fit Values	Fit Uncertainty	Syst. Uncertainty	
$\bar{\Lambda}$	0.40	$\pm$ 0.10	$\pm$ 0.02	GeV
λ_1	-0.15	$\pm$ 0.07	$\pm$ 0.03	GeV^2
λ_2	0.12	$\pm$ 0.01	$\pm$ 0.01	GeV^2
ρ_1	-0.01	$\pm$ 0.03	$\pm$ 0.03	GeV^3
ρ_2	0.03	$\pm$ 0.03	$\pm$ 0.01	GeV^3

uncertainty on α_s and evaluate the effect of removing the BLM corrections from the lepton moments. In this scheme that is a small effect and higher order perturbative corrections are expected to be under control. Dimensional estimates suggest that $1/m_b^4$ effects do not exceed the present experimental resolution. Other systematic uncertainties will be addressed in a dedicated publication.

For the $\bar{\Lambda}$-λ_1 formalism we take the effect of $\mathcal{T}_i = (0.0 \pm 0.50)^3$ GeV^3, $\alpha_s = 0.22 \pm 0.01$ and we also estimate the effect of the missing corrections to third moments as $M_B^6(0.001 \pm 0.0005)\,\beta_0\,(\alpha_s/\pi)^2$ and $M_B^6(0.003 \pm 0.003)\,\bar{\Lambda}/\bar{M}_B\,\alpha_s/\pi$.

The fit was also repeated using only the first two moments, leaving free m_b(1 GeV), μ_π^2(1 GeV) and $\bar{\Lambda}$, λ_1, respectively. The other parameters were fixed to the central values obtained in the full fit. Results agreed with those from the full fit. In particular, the values of $\bar{\Lambda} = 0.42 \pm 0.07$(stat.) GeV and $\lambda_1 = (-0.17 \pm 0.05(\text{stat.}))$ GeV^2 agree with the recent result reported by the Cleo Collaboration [1], which uses the first moments of the charged lepton energy to obtain $\bar{\Lambda} = 0.39 \pm 0.07$ GeV and $\lambda_1 = (-0.25 \pm 0.05)$ GeV^2.

There are several facets of these results to be looked at. One interesting piece of information comes from the correlation between m_c and m_b extracted from the fit. It corresponds to $m_c(1\text{ GeV}) = 1.63 \times (m_b(1\text{ GeV}) - 3.91)$. Therefore a competitive value of the charm mass can be extracted from a precise determination of m_b. Using, for instance, $m_b(1\text{ GeV}) = (4.60 \pm 0.05)$ GeV would give $m_c(1\text{ GeV}) = (1.13 \pm 0.09)$ GeV. This can be compared to the present typical lattice uncertainties which range between 50 and 120 MeV [20].

In the running mass scheme, the expansions of Eq.(5), for the B and D mesons are not used in

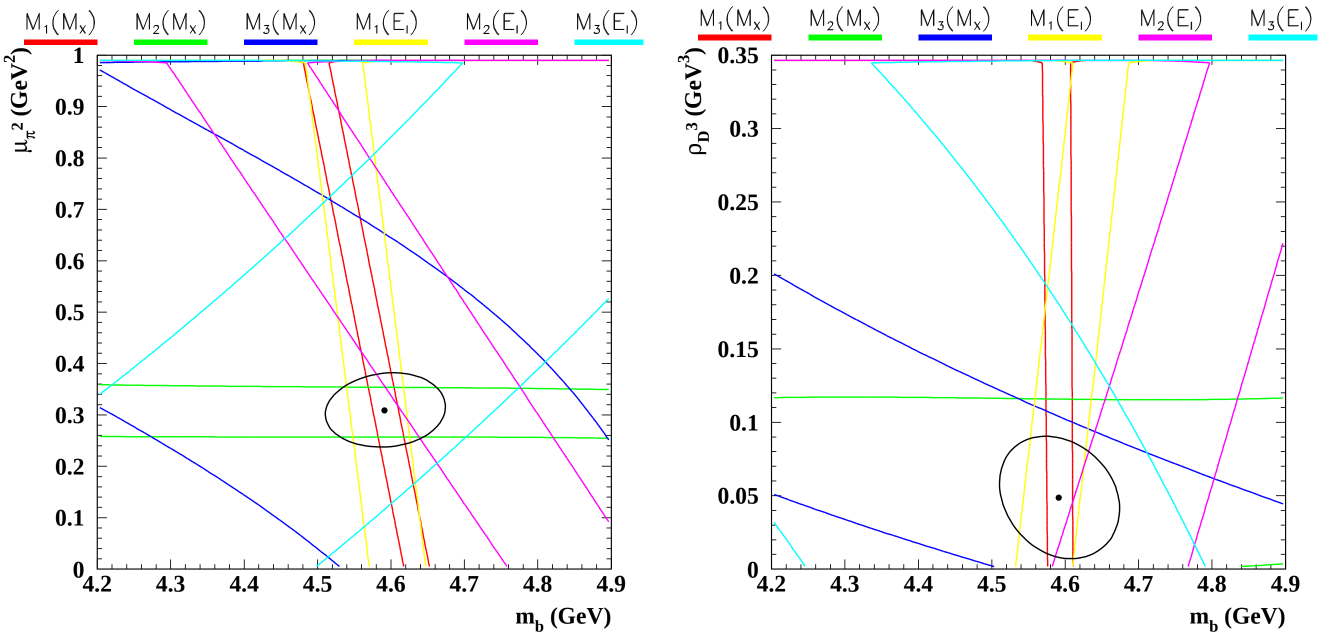

Figure 1: *The projection of the constraints of the six measured moments on the* $m_b(1\text{ GeV})$-$\mu_\pi^2(1\text{ GeV})$ *(left) and* $m_b(1\text{ GeV})$-ρ_D^3 *(right) planes. The bands correspond to the total measurement accuracy and are given by keeping all the other parameters at their central values. The ellipses represent the 1* σ *contours.*

the fit. It is therefore possible to test *a posteriori* the consistency of the meson mass expansion by comparing the $\bar{\Lambda}$ values obtained in the two cases. We find $\bar{\Lambda}(B) - \bar{\Lambda}(D) = -0.086 \pm 0.092$. This is also a test of the size of the non-local terms.

In both approaches, the OPE predictions for the six moments, computed with the available precision, have a common intersection in the multi-parameter space and the quality of the fit is good. Within the present experimental accuracy, we therefore do not see the need to introduce higher order terms to establish agreement with the data. In particular, the first leptonic and hadronic moments are highly correlated and depend on nearly the same combination of heavy quark masses. Fixing this from $M_1(M_X)$, one finds $M_1(E_\ell) = 1.377$ GeV which agrees well with the measured value of (1.383 ± 0.015) GeV. This provides a non-trivial consistency check of the OPE. The overall agreement represents both a test of the theory and suggests constraints on the size of the $1/m_b^4$ terms and of other missing corrections. Similarly, the observed agreement strongly supports the validity of quark-hadron duality in the B decay shape variables.

At present the achieved experimental resolution matches the available theoretical accuracy. With more precise data soon becoming available, it is important to improve the latter, particularly for higher hadronic moments. One way to improve the convergence of the heavy quark expansion could be to employ different kinematic variables. We propose to consider $\mathcal{N}_X^2 = M_X^2 - 2\tilde{\Lambda} E_X$, where M_X and E_X are the hadronic mass and energy and $\tilde{\Lambda}$ a fixed mass parameter. Choosing $\tilde{\Lambda}$ near

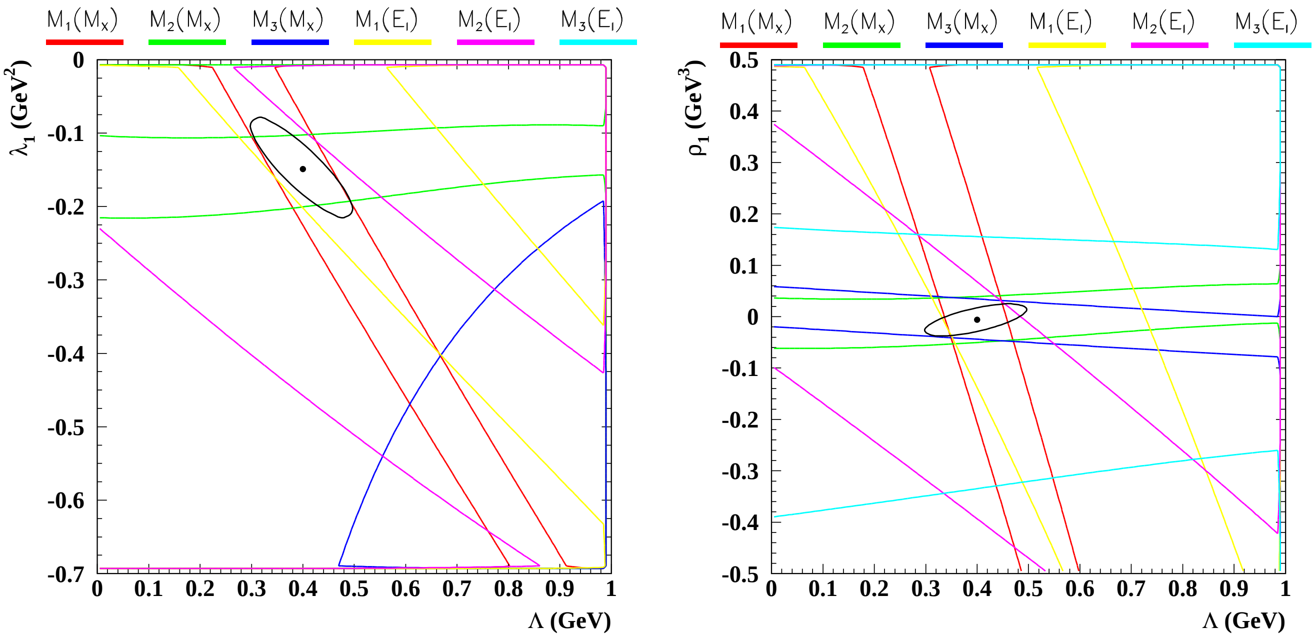

Figure 2: *The projection of the constraints of the six measured moments on the* $\bar{\Lambda}$-λ_1 *(left) and* $\bar{\Lambda}$-ρ_1 *(right) planes. The bands correspond to the total measurement accuracy and are given by keeping all the other parameters at their central values. The ellipses represent the 1* σ *contours.*

$M_B - m_b(1 \text{ GeV}) \simeq 0.65$ GeV, suppresses terms with $l \geq 1$ in Eq. (4) and results in a better convergence of higher moments [21]. The use of this variable should be feasible at B factories, where the kinematics allows an accurate reconstruction of both the mass and energy of the hadronic system in s.l. B decays.

3.1 Implications for $|V_{cb}|$

The value of $|\mathrm{V_{cb}}|$ obtained from the total s.l. decay width depends on the OPE parameters extracted above. We discuss now the implications of our results for $|\mathrm{V_{cb}}|$, using the input parameters given in Table 7. The uncertainties on the BR($b \to X\ell^-\nu$) have been increased compared to ref. [22] for not using the heavy quark forward-backward asymmetries in the LEP global electroweak fit and to account for the ±15% uncertainty on the equality of s.l. partial width of b baryons and mesons.

Table 7: Input values used for the determination of $|V_{cb}|$.

Measurement	Value [22]
b-hadron lifetime	(1.564 ± 0.014) ps
BR($b \to X\ell^-\nu$)	(10.59 ± 0.31) %
BR($b \to X_u\ell^-\nu$)	(0.17 ± 0.05) %

The inclusive s.l. decay width has been calculated through second order in perturbative QCD. Second order BLM corrections were obtained in [23], all-order BLM terms are available from [24], whereas second-order non-BLM corrections have been estimated in [25]. Non-perturbative corrections start at order $\mathcal{O}(1/m_b^2)$ [11] and $\mathcal{O}(1/m_b^3)$ corrections have also been calculated [7]. Electroweak corrections have also been taken into account [26].

The determination of $|\mathrm{V_{cb}}|$ and the contributions of the various parameters in the kinetic mass scheme is described in [27]. An approximate formula which displays the dependence on the different parameters is:

$$\begin{aligned} |\mathrm{V_{cb}}| = |\mathrm{V_{cb}}|_0 \quad & \Big[1 - 0.65\left(m_b(1) - 4.6\ \mathrm{GeV/c^2}\right) + 0.40\left(m_c(1) - 1.15\ \mathrm{GeV/c^2}\right) \\ & +0.01\left(\mu_\pi^2 - 0.4\ \mathrm{GeV^2}\right) + 0.10\left(\rho_D^3 - 0.12\ \mathrm{GeV^3}\right) \\ & +0.06\left(\mu_G^2 - 0.35\ \mathrm{GeV^2}\right) - 0.01\left(\rho_{LS}^3 + 0.17\ \mathrm{GeV^3}\right)\Big]. \end{aligned} \tag{9}$$

A detailed discussion of the theoretical uncertainties on $|V_{cb}|$ goes beyond the scope of this paper. Here we focus on the uncertainty arising from the heavy quark masses and non-perturbative parameters determined in the fit. It is evaluated using the full fit error matrix which leads to $\pm 1.5\%$. There is an additional uncertainty coming from the limited accuracy of the theoretical expressions which have been used. We take the range $m_b/2 < \mu' < m_b$ for the scale μ' at which α_s is evaluated and find a $\pm 1\%$ effect[2]. In summary, we obtain:

$$|\mathrm{V_{cb}}| = 0.0419 \times \left(1 \pm 0.016|_{meas} \pm 0.015|_{fit} \pm 0.010|_{pert}\right), \tag{10}$$

where the first uncertainty reflects the accuracy on the s.l. width determination.

The expression for the inclusive b s.l. width in the pole mass scheme is known to the same order. The fit results have been used to obtain:

$$|\mathrm{V_{cb}}| = 0.0413 \times \left(1 \pm 0.016|_{meas} \pm 0.017|_{fit} \pm 0.006|_{nl} \pm 0.021|_{pert}\right). \tag{11}$$

Again, the first two uncertainties correspond to the decay width measurement and to the fitted parameters, respectively. The third uncertainty refers to the $\mathcal{T}_{i=1,4}$ parameters which have been varied within the range $(0 \pm (0.5)^3)$ GeV^3. The uncertainty from the truncation of the perturbative QCD series is again estimated by varying the scale at which α_s is evaluated between $m_b/2$ and $2m_b$. Here the perturbative uncertainty is larger and reflects the slower convergence of the perturbative series when the pole mass scheme is employed.

4 Conclusions

The values of the heavy quark masses have been determined, together with the leading $1/m_b^2$ and $1/m_b^3$ parameters, from a fit to the first three moments of the charged lepton energy and hadronic

[2]Incorporating the third-order BLM correction suppresses this scale dependence. Combining Refs. [24] and [28], we find the third-order BLM correction to $\Gamma_{\rm sl}(b \to c)$ to be $\approx -50(\alpha_s/\pi)^3$ in this scheme. This increases $|V_{cb}|$ by 1% for $\mu' = m_b$, and leaves it nearly unchanged, compared to two loops, for $\mu' = m_b/2$.

mass spectra in s.l. decays, measured in a preliminary analysis of the DELPHI data. The absence of a charged lepton energy cut in the analysis makes the heavy quark expansion more reliable and allows us to include higher moments. We have adopted two different formalisms: one based on low-energy running quark masses, which does not rely on a $1/m_c$ expansion, and the other on pole quark masses. The constraints from the six moments agree well and the size of the dominant $1/m_b^3$ term has been found to be compatible with theoretical estimates. The fit is largely insensitive to non-local correlators and to the spin-orbital operator.

Propagating the ranges of the OPE parameters to the determination of $|V_{cb}|$ reduces the theoretical uncertainty due to the $1/m_b^3$ corrections below 2%. Furthermore, the use of a fit changes the nature of these uncertainties and partly removes the arbitrariness arising from estimates based on parameters ranges.

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Note

During the final stage of this work, a new analysis of s.l. moments has appeared [29]. There are several differences with our approach, but the results are qualitatively consistent with our findings.

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