PHYSICAL REVIEW D **75,** 114011 (2007)

Vanishing chiral couplings in the large-N_C resonance theory

Jorge Portolés,[1] Ignasi Rosell,[1] and Pedro Ruiz-Femenía[2]

[1] *IFIC, Universitat de València–CSIC, Apartat de Correus 22085, E-46071 València, Spain*

[2] *Max-Planck-Institut für Physik (Werner-Heisenberg Institut), Föhringer Ring 6, 80805 München, Germany*

(Received 4 December 2006; published 25 June 2007)

The construction of a resonance theory involving hadrons requires implementing the information from higher scales into the couplings of the effective Lagrangian. We consider the large-N_C chiral resonance theory incorporating scalars and pseudoscalars, and we find that, by imposing LO short-distance constraints on form factors of QCD currents constructed within this theory, the chiral low-energy constants satisfy resonance saturation at NLO in the $1/N_C$ expansion.

DOI: 10.1103/PhysRevD.75.114011 PACS numbers: 12.39.Fe, 11.15.Pg, 12.38.−t

I. INTRODUCTION

Since the inception of chiral perturbation theory (χPT) [1] a lot of effort has been dedicated to the determination of the chiral low-energy constants (LECs), whether from hadronic observables or through spectral representations of Green functions that are order parameters of spontaneous chiral symmetry breaking. However it is well-known that the LECs of every effective field theory collect information from degrees of freedom that have been integrated out to obtain the low-energy Lagrangian. In consequence it has been put forward that chiral LECs would receive a contribution from the low-lying resonances that do not appear in χPT. This idea has been explored through the construction of a phenomenological Lagrangian (RχT) involving one multiplet of vector, axial-vector, scalar, and pseudoscalar resonances [2], and the conclusion that was achieved assesses the fact that the tree-level integration of the lightest resonance fields saturates the phenomenological values of $\mathcal{O}(p^4)$ chiral LECs. An extension of this result up to $\mathcal{O}(p^6)$ could be expected [3].

The resonance theory can be better understood within the framework of large-N_C QCD [4] where tree-level interactions between an infinite spectrum of narrow states implemented in a chiral invariant Lagrangian provide the LO ($N_C \to \infty$) contribution to Green functions of QCD currents. Thus the idea of matching the tree-level functions, evaluated within RχT, with those of QCD in the same limit [3,5–7] arises naturally, and it has been shown to broaden widely our knowledge on the construction of the theory by providing large-N_C estimates of the coupling constants in the Lagrangian that turn out to be in remarkable agreement with the phenomenology.

The $\mathcal{O}(p^4)$ couplings in χPT (L_i) and in the theory where the resonances are still active degrees of freedom ($\tilde{L}_i$) are related upon integration of the resonance fields:

$$L_i(\mu) = L_i^R(\mu) + \tilde{L}_i(\mu), \tag{1}$$

where $L_i^R(\mu)$ is the contribution stemming from the low-energy expansion of the resonance contributions. The statement of resonance saturation of the $\mathcal{O}(p^4)$ χPT couplings alleges then that $\tilde{L}_i(\mu) = 0$, i.e., that the values of the LECs are generated by the decoupling of the mesonic states which lie above the Goldstone particles. This assertion immediately raises the question of its validity for a determined value of μ or if the result is accomplished for any value (''extreme'' version of resonance saturation [8]). The latter possibility is specially interesting because of its simplicity and naturalness: the $L_i(\mu)$ couplings are then predicted as a function only of the resonance parameters, which can be extracted from the phenomenology or considering the matching procedure outlined before.

At LO in the $1/N_C$ expansion, the asymptotic behavior of QCD correlators require that $\tilde{L}_i = 0$ [5], in the RχT formulation where spin-1 mesons are described by antisymmetric tensor fields. In this limit, and considering the large-N_C resonance Lagrangian of Ref. [3], where only contributions from the lightest resonances are taken into account, Eq. (1) turns out to be

$$\begin{aligned}
&L_1 = \frac{G_V^2}{8M_V^2}, \qquad L_2 = \frac{G_V^2}{4M_V^2}, \qquad L_3 = -\frac{3G_V^2}{4M_V^2} + \frac{c_d^2}{2M_S^2},\\
&L_5 = \frac{c_d c_m}{M_S^2}, \qquad L_8 = \frac{c_m^2}{2M_S^2} - \frac{d_m^2}{2M_P^2}, \qquad L_9 = \frac{F_V G_V}{2M_V^2},\\
&L_{10} = -\frac{F_V^2}{4M_V^2} + \frac{F_A^2}{4M_A^2}, \qquad L_4 = L_6 = L_7 = 0,
\end{aligned} \tag{2}$$

where F_V, F_A, G_V, c_d, and c_m are couplings of the RχT Lagrangian [2]. A μ dependence in the chiral couplings may appear through quantum corrections. Since the $1/N_C$ expansion is equivalent to a semiclassical approximation, there is a $1/N_C$ suppression for each loop; therefore NLO corrections in the large-N_C framework are given by one-loop diagrams generated by the RχT Lagrangian. Studies along this line of research have recently been carried out [8–12]. A proper question that follows is to find out if resonance saturation still holds at NLO in the $1/N_C$ expansion. The possible connection between resonance saturation and the implementation of short-distance constraints in the Lagrangian theory has been pointed out [8]. In Ref. [11], using the background field method, the full one-loop computation of the β function that renormalizes the resonance theory with scalar and pseudoscalar resonances was performed. Indeed one of the main conclusions of that work is that those $\tilde{L}_i$ related with the resonance content of the theory do not depend on μ when short-distance information is used to determine the LO resonance couplings.

In this article we provide an explanation of the latter fact concluding that resonance saturation of the $U(3)_L \otimes U(3)_R$ $\mathcal{O}(p^4)$ chiral LECs related with scalar and pseudoscalar resonances is satisfied at NLO in the $1/N_C$ expansion as a consequence of imposing the right high-energy behavior on form factors calculated within the theory. In particular, we show that $\tilde{L}_i(\mu) = 0$ for those couplings named as $\tilde{L}_4$, $\tilde{L}_5$, $\tilde{L}_8$ in the usual basis of $SU(3)_L \otimes SU(3)_R$ χPT [1], and for $\tilde{\alpha}_{18}$.[1] We also show the absence of running for the couplings $\tilde{L}_6$ and $\tilde{L}_7$, though for the latter we can only conclude that the NLO finite part of the combination $\tilde{L}_6 + \tilde{L}_7$ must vanish.

II. LARGE-N_C RESONANCE CHIRAL LAGRANGIAN

The $U(3)_L \otimes U(3)_R$ chiral Lagrangian with scalar and pseudoscalar resonance fields used in Ref. [11] (see also [3]) has, at leading order in $1/N_C$, the structure

$$\mathcal{L}_{\mathrm{R}\chi\mathrm{T}} = \mathcal{L}_2^{\chi\mathrm{PT}} + \mathcal{L}_{\mathrm{kin}}^R + \mathcal{L}_2^R + \mathcal{L}_2^{RR}, \tag{3}$$

where R stands for resonance nonets of scalars $S(0^{++})$ or pseudoscalars $P(0^{-+})$. $\mathcal{L}_2^{\chi\mathrm{PT}}$ is the $U(3)_L \otimes U(3)_R$ $\mathcal{O}(p^2)$ χPT Lagrangian [1]. The piece $\mathcal{L}_{\mathrm{kin}}^R$ contains the kinetic terms for the resonance fields, and $\mathcal{L}_2^R$ has the generic form $\langle R\chi^{(2)}\rangle$, with $\chi^{(2)}$ an $\mathcal{O}(p^2)$ chiral tensor. The second and third terms in Eq. (3) yield the most general Lagrangian that can give contributions to the chiral $\mathcal{O}(p^4)$ LECs after integrating out scalar and pseudoscalar resonances at tree-level [2,5]. Interaction terms among two resonances are included in $\mathcal{L}_2^{RR} \sim \langle RR\chi^{(2)}\rangle$, where $RR = SS$, PP, SP. Upon resonance integration double-resonance terms contribute first at $\mathcal{O}(p^6)$, but they can be required to satisfy the short-distance behavior of resonance form factors [12]. The truncation of the infinite tower of zero-width resonances of the large-N_C spectrum to the lowest-lying multiplet, as done in [3,11], is not essential in what follows, but can be assumed to ease the discussion. Likewise, the addition of interaction terms with three resonances [3] does not change the conclusions of this paper.

Quantum effects can be computed in this large-N_C framework and yield NLO corrections to tree-level results. Dimensional analysis tells us that one-loop diagrams are of $\mathcal{O}(p^4, p^2M_R^2)$, so it is obvious that additional operators are needed to renormalize $\mathcal{L}_{\mathrm{R}\chi\mathrm{T}}$ above [11]. Among those, we shall be interested in the counterterms from the Goldstone boson Lagrangian of order p^4, $\mathcal{L}_4^{\mathrm{GB}} = \sum_i \tilde{\alpha}_i \mathcal{O}_i$, which should be distinguished from the usual χPT Lagrangian expansion, $\mathcal{L}_4^{\chi\mathrm{PT}}$, as the couplings of both theories carry information about physics at different scales (notice that we write $\tilde{\alpha}_i$ as short for all the $\mathcal{O}(p^4)$ chiral couplings, including $\tilde{L}_i$). Resonance saturation at LO translates into the fact that $\tilde{\alpha}_i = 0$ and then $\mathcal{L}_4^{\mathrm{GB}}$ vanishes. At NLO, the absorbed divergences provide a scale dependence in the renormalized couplings $\tilde{\alpha}_i(\mu)$, as dictated by the renormalization group equations:

$$\mu \frac{d}{d\mu}\tilde{\alpha}_i = -\frac{\gamma_i}{16\pi^2}. \tag{4}$$

The γ_i are the divergent coefficients of the counterterms in $\mathcal{L}_4^{\mathrm{GB}}$ and have an explicit dependence with the couplings of $\mathcal{L}_{\mathrm{R}\chi\mathrm{T}}$. The leading logarithm in the evolution of the $\tilde{\alpha}_i$ constant can thus be obtained by plugging the LO values for the $\mathcal{L}_{\mathrm{R}\chi\mathrm{T}}$ couplings inside γ_i, i.e. ignoring the μ dependence on the right-hand side of the renormalization group equations. Consequently, a zero value for the divergent part of the $\tilde{\alpha}_i$ constant automatically implies that it does not run at one-loop in the large-N_C framework.

By explicit computation we found [11] that the divergent part of 6 out of the 16 $\mathcal{L}_4^{\mathrm{GB}}$ couplings vanishes after LO predictions for the constants in $\mathcal{L}_{\mathrm{R}\chi\mathrm{T}}$ are used.[2] The couplings that share this feature are the ones accompanying operators with a $\chi_\pm$ tensor, that are relevant for the renormalization of the two-point correlator functions of two scalar or pseudoscalar currents ($\tilde{L}_6$, $\tilde{L}_7$, and $\tilde{L}_8$), and for the scalar form factors to two Goldstone bosons ($\tilde{L}_4$, $\tilde{L}_5$, and $\tilde{\alpha}_{18}$). Next we show that the absence of running for both sets of couplings is a consequence of enforcing the correct high-energy behavior in the tree-level scalar and pseudoscalar form factors.

III. $\langle SS\rangle$ AND $\langle PP\rangle$ CORRELATORS

Let us consider the two-point functions built from two scalar (SS) or two pseudoscalar (PP) currents. Their tree-level expressions are given by one-particle exchanges, so they are booked as $\mathcal{O}(q^{-2})$ at large energies, q being the momentum flowing into the current vertex. The topologies that arise at one-loop from the Lagrangian in Eq. (3) thus yield the $\mathcal{O}(q^0)$ contributions that are shown in Fig. 1. The $\mathcal{L}_4^{\mathrm{GB}}$ operators $\langle\chi_+^2\rangle$, $\langle\chi_+\rangle^2$ and $\langle\chi_-^2\rangle$, $\langle\chi_-\rangle^2$ also contribute through local counterterm diagrams (see Fig. 1), and their divergent parts are fixed uniquely by the renormalization of the SS and PP correlators, respectively. Other diagrams with counterterms connected to the external currents with one or two propagators may also be required in order to

[1] $\tilde{\alpha}_{18}$ is the coupling of the operator $\mathcal{O}_{18} = \langle u_\mu\rangle\langle u^\mu\chi_+\rangle$, as defined in Ref. [11], which vanishes in the $SU(3)$ case.

[2] Actually, there is one more $\mathcal{L}_4^{\mathrm{GB}}$ coupling, $\tilde{H}_2$, whose divergent part also vanishes. However, the saturation of the couplings H_1 and H_2 by resonances has no physical significance, as these constants depend on the renormalization scheme used in QCD, and will not be included in our analysis.

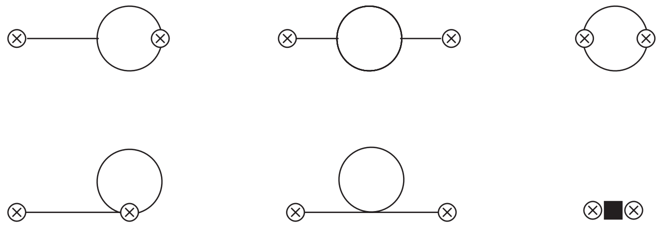

FIG. 1. Topologies in the one-loop scalar and pseudoscalar correlators. The lines can represent both Goldstone and resonance fields.

absorb all the divergences from the one-loop graphs. Among the latter, note that the divergences arising from tadpoles do not play any role in the determination of the local counterterms.

The relevant topologies involve loops with two propagators. After reduction to scalar integrals, all terms are proportional to the scalar two- and one-point functions $B_0(q^2, M^2, M'^2)$ and $A_0(M^2)$ [13], with M, M' any of the masses inside the loops. The divergences that have to be canceled by the local counterterms of $\mathcal{L}_4^{\mathrm{GB}}$ are the ones proportional to $\mathcal{O}(q^0)$. Because of the fact that the $1/\epsilon$ terms from the one-point scalar function are proportional to a mass squared, it is easy to convince oneself that the $\mathcal{O}(q^0)$ divergences in the SS and PP correlators come solely from the two-point functions B_0.

The spectral functions of the scalar and pseudoscalar correlators are generated from the discontinuities of the two-point functions. Using the optical theorem, the spectral function can be written as a sum over the form factors of all absorptive contributions:

$$\mathrm{Im}\,\Pi(q^2) = \sum_n \xi_n(q^2)|\mathcal{F}_n(q^2)|^2. \tag{5}$$

At one-loop, any of the possible absorptive contributions, n, comes from the two-particle cuts in the diagrams of Fig. 1. If we stick to the particle content in $\mathcal{L}_{\mathrm{R}\chi\mathrm{T}}$, the terms in the sum correspond to $n = \phi\phi$, $R\phi$, RR, where ϕ denotes a Goldstone boson and $R = S, P$ is a resonance field. The one-loop spectral function is thus entirely determined by the tree-level scalar and pseudoscalar form factors to these two-particle states. It is a commonly accepted statement that the individual form factors of QCD currents should vanish at infinite momentum transfer [14]. In RχT the appropriate high-energy behavior is guaranteed by the well-known relations among the resonance couplings at LO in the large-N_C limit. Since the kinematic factors $\xi_n(q^2)$ behave as $\mathcal{O}(1)$ in the $q^2 \to \infty$ limit for the allowed two-particle cuts from $\mathcal{L}_{\mathrm{R}\chi\mathrm{T}}$, the short-distance behavior of the form factor leads immediately to a vanishing $\mathcal{O}(q^0)$ term for the spectral functions. As the $\mathcal{O}(q^0)$ absorptive and divergent parts of the correlators come together in the B_0's, it follows that they are affected by the same suppression. We therefore reach the conclusion that the divergent $\mathcal{O}(q^0)$ piece of the SS and PP correlators, responsible for the running of $\tilde{L}_6$, $\tilde{L}_7$, and $\tilde{L}_8$, must vanish if the tree-level scalar and pseudoscalar form factors computed from the theory behave as $1/q^2$ at large q^2. In more physical terms, imposing the right short-distance properties at the Lagrangian level produces ultraviolet finite results for the $\mathcal{O}(q^0)$ correlators so that the renormalization of the local terms is not needed.

The whole argument above can be simplified as follows. If we expand the correlator in q^2, we realize that the $\mathcal{O}(q^0)$ terms from the different one-loop diagrams are either zero or proportional to a unique function, $B_0(q^2, 0, 0)$, so that

$$\Pi(q^2 \to \infty) = \lambda B_0(q^2, 0, 0) + \mathcal{O}(q^{-2}), \tag{6}$$

with λ a combination of resonance parameters. When we impose relations among the couplings so that the imaginary part of the correlators vanishes, we are indeed setting $\lambda = 0$. This cancels out the whole $\mathcal{O}(q^0)$ term, including the $1/\epsilon$ and the finite parts. The saturation of the couplings at NLO in the large-N_C counting is thus complete: the running is zero and a local NLO finite piece from the $\tilde{L}_6 + \tilde{L}_7$ and $\tilde{L}_8$ couplings is not allowed because of its wrong high-energy behavior, since for massless quarks the correlator $SS - PP$ vanishes as $1/q^4$ [15]. The absence of a NLO piece from $\tilde{L}_8$ in $\mathcal{L}_{\mathrm{R}\chi\mathrm{T}}$ is consistent with a recent determination of the χPT low-energy coupling $L_8(\mu)$ [10]. We would like to point out that the result in Eq. (6) is not modified if an arbitrary number of resonance multiplets is considered, provided their interactions follow the structure given by the Lagrangian in Eq. (3).

IV. SCALAR FORM FACTOR

Similarly, counterterms of the operators $\tilde{L}_4$, $\tilde{L}_5$, and $\tilde{\alpha}_{18}$ can be determined by the renormalization of the scalar form factor of 2 Goldstone bosons. At one-loop the form factor behaves as q^2 at large energies, and the allowed topologies are shown in Fig. 2. The last diagram represents the local counterterms of $\tilde{L}_4$, $\tilde{L}_5$, and $\tilde{\alpha}_{18}$ that absorb the $\mathcal{O}(q^2)$ divergences.

We shall prove first that the $\mathcal{O}(q^2)$ term of the one-loop calculation is only proportional to $q^2 B_0(q^2, 0, 0)$. For the bubble topologies (diagrams in the first line of Fig. 2) this is inferred from the discussion above. A new feature arises from the three-propagator integrals. After the reduction of the one-loop diagrams with three propagators is done, the

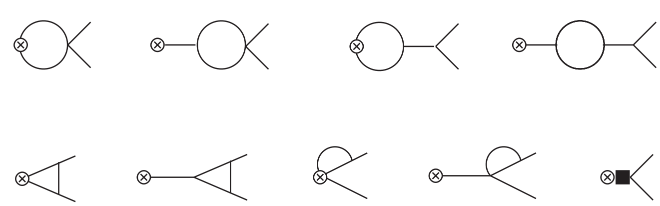

FIG. 2. Topologies in the one-loop scalar form factor to two Goldstone bosons. Tadpole diagrams have not been drawn.

leading term in the $q^2 \to \infty$ limit can only be proportional to $q^2 B_0(q^2, 0, 0)$ or, *a priori*, to $q^4 C_0(q^2, 0, 0, 0)$, based on pure dimensional grounds and on the fact that the scalar three-point function, C_0, behaves as $1/q^2$. However it is easy to show that no terms proportional to $q^4 C_0$ can arise from the triangle loops. Choose the routing of the loop momentum k such that it is assigned to the vertical line in the triangle. The $\mathcal{O}(p^2)$ vertices connected to the outgoing Goldstone bosons, with momenta p_1 and p_2, can thus yield $p_1 \cdot k$, $p_2 \cdot k$, or k^2, p_1^2, p_2^2 factors. Take, for example, the upper outgoing line to be p_1, and write the upper vertex factors as $p_1 \cdot k = 1/2[(k + p_1)^2 - k^2 - p_1^2]$ and $k^2 = (k^2 - M^2) + M^2$, with M the mass of the particle in the vertical propagator. These factors then give either one square mass term multiplying the three-propagator integral, or they have the structure of one of the propagators joining at the vertex. In the latter case one gets two-propagator integrals that yield B_0 or A_0 functions. In particular, only the two-point function which arises when the vertical propagator is canceled out can pick an additional q^2 from the other vertex and yield a $q^2 B_0$. On the other hand, a scalar three-point function only survives if we pick the mass squared term from each vertex. We thus conclude that C_0 enters the result with a M^4 factor in front. Possible one-point functions A_0 do not contribute to the leading order in q^2 either, since they are proportional to a square mass. The same is true for the last two one-loop diagrams in Fig. 2. Consequently, the behavior of the scalar form factor of 2 Goldstone bosons at large energies reads

$$\mathcal{F}(q^2 \to \infty) = \lambda' q^2 B_0(q^2, 0, 0) + \mathcal{O}(q^0), \qquad (7)$$

λ' being a combination of resonance parameters.

Now consider the absorptive contributions of the one-loop diagrams. According to the discussion above, only the two-particle cuts in the s channel contribute to the $\mathcal{O}(q^2)$ imaginary part of the scalar form factor, proportional to $q^2 B_0$. The optical theorem states that the one-loop form factor into two Goldstone bosons is given by the sum of the tree-level form factors to all possible intermediate states times the conjugate tree-level scattering amplitude of the intermediate state to two Goldstone (which is of $\mathcal{O}(q^2)$ in the $q^2 \to \infty$ limit),

$$\mathrm{Im}\,\mathcal{F}(q^2) = \sum_n \xi_n(q^2) \mathcal{F}_n(q^2) A^*_{\mathrm{scatt}}(q^2). \qquad (8)$$

If the tree-level form factors $\mathcal{F}_n$ obey the $1/q^2$ suppression, we conclude that the $\mathcal{O}(q^2)$ term of $\mathrm{Im}\mathcal{F}$ must vanish, and therefore $\lambda' = 0$ in Eq. (7). Consequently there is neither an $\mathcal{O}(q^2)$ divergence to be absorbed by the local counterterms of $\tilde{L}_4$, $\tilde{L}_5$, and $\tilde{\alpha}_{18}$, nor any $\mathcal{O}(q^2)$ finite piece coming from the loops. A possible NLO finite piece from the $\tilde{L}_4$, $\tilde{L}_5$, and $\tilde{\alpha}_{18}$ operators cannot thus be canceled by possible loop contributions, and it is not allowed if the scalar form factor to two Goldstone bosons has to obey the $1/q^2$ behavior at NLO in the large-N_C counting.

V. OTHER $\mathcal{L}_4^{\mathrm{GB}}$ COUPLINGS

It is tempting to apply the preceding discussion to study the renormalization of the vector-vector and axial-vector–-axial-vector correlators, and to the vector form factor into two Goldstone bosons, since they are the key objects to determine the divergent piece of the couplings $\tilde{L}_9$ and $\tilde{L}_{10}$. This requires the introduction of vector and axial-vector meson fields in the large-N_C Lagrangian, which can be done systematically [2,3]. A problem, however, arises from the fact that the spin-1 field propagator behaves as $\mathcal{O}(q^0)$ at large q^2 and breaks the q^2 counting advocated before for scalar and pseudoscalar resonances. This fact can produce one-loop terms that are higher than $\mathcal{O}(q^2)$ enhanced with respect to the tree-level ones when spin-1 resonances flow inside the loops (see, e.g., the one-loop vector form factor computation in Ref. [9]). The proof given above applies only to the leading order divergence for large q^2 associated to each intermediate state. Thus from the loops which involve cuts with spin-1 resonances, we can only conclude that their contributions to the divergent part of certain $\mathcal{L}_{6,8}^{\mathrm{GB}}$ couplings vanish if the corresponding tree-level form factors have the right short-distance suppression. The cancellation of the subleading divergent term, relevant for the renormalization of the $\mathcal{L}_4^{\mathrm{GB}}$ operators, is more subtle for the loops which involve cuts with spin-one resonances, and very likely requires a detailed study of the allowed vertex structures [16]. For the rest of $\mathcal{L}_4^{\mathrm{GB}}$ couplings, namely, $\tilde{L}_1$, $\tilde{L}_2$, $\tilde{L}_3$ and α_3, α_4, α_{17} [11], that are relevant for the renormalization of the elastic Goldstone boson scattering amplitude at one-loop, we can expect that the analysis of the high-energy behavior of the tree-level scattering amplitude of Goldstone bosons to the possible intermediate states could yield constraints on the running of these couplings, but at the moment this is just a desirable conjecture.

In conclusion we have established that those $U(3)_L \otimes U(3)_R$ chiral LECs of the large-N_C resonance theory related with scalar and pseudoscalar resonances do not run at NLO when the theory is properly devised, i.e. once the right high-energy behavior of form factors has been implemented by tuning the couplings of the resonance theory at LO. In between we also conclude that any NLO finite contribution to $\tilde{L}_{4,5,8}$ and $\tilde{L}_6 + \tilde{L}_7$ should also vanish. This outcome together with the LO result ($\tilde{L}_i = 0$) confirms the statement of resonance saturation of chiral LECs up to NLO.

ACKNOWLEDGMENTS

We wish to thank G. Amorós, G. Ecker, S. Peris, A. Pich, and J. J. Sanz-Cillero for their useful comments on the manuscript. I. R. is supported by a FPU contract (MEC). This work has been supported in part by the EU MRTN-CT-2006-035482 (FLAVIAnet), by MEC (Spain) under Grant No. FPA2004-00996 and by Generalitat Valenciana under Grants No. ACOMP06/098 and GV05/015.

[1] S. Weinberg, Physica (Amsterdam) **96A**, 327 (1979); J. Gasser and H. Leutwyler, Ann. Phys. (N.Y.) **158**, 142 (1984); Nucl. Phys. **B250**, 465 (1985); **B250**, 517 (1985); **B250**, 539 (1985).

[2] G. Ecker, J. Gasser, A. Pich, and E. de Rafael, Nucl. Phys. **B321**, 311 (1989); J.F. Donoghue, C. Ramirez, and G. Valencia, Phys. Rev. D **39**, 1947 (1989).

[3] V. Cirigliano, G. Ecker, M. Eidemüller, R. Kaiser, A. Pich, and J. Portolés, Nucl. Phys. **B753**, 139 (2006).

[4] G. 't Hooft, Nucl. Phys. **B72**, 461 (1974); **B75**, 461 (1974); E. Witten, Nucl. Phys. **B160**, 57 (1979).

[5] G. Ecker, J. Gasser, H. Leutwyler, A. Pich, and E. de Rafael, Phys. Lett. B **223**, 425 (1989).

[6] S. Peris, M. Perrottet, and E. de Rafael, J. High Energy Phys. 05 (1998) 011; M. Knecht and E. de Rafael, Phys. Lett. B **424**, 335 (1998); M. Knecht, S. Peris, and E. de Rafael, Phys. Lett. B **443**, 255 (1998); M. Golterman, S. Peris, B. Phily, and E. De Rafael, J. High Energy Phys. 01 (2002) 024.

[7] B. Moussallam, Phys. Rev. D **51**, 4939 (1995); Nucl. Phys. **B504**, 381 (1997); M. Knecht and A. Nyffeler, Eur. Phys. J. C **21**, 659 (2001); J. Bijnens, E. Gamiz, E. Lipartia, and J. Prades, J. High Energy Phys. 04 (2003) 055; P. D. Ruiz-Femenía, A. Pich, and J. Portolés, J. High Energy Phys. 07 (2003) 003; V. Cirigliano, G. Ecker, M. Eidemüller, A. Pich, and J. Portolés, Phys. Lett. B **596**, 96 (2004); V. Cirigliano, G. Ecker, M. Eidemüller, R. Kaiser, A. Pich, and J. Portolés, J. High Energy Phys. 04 (2005) 006.

[8] O. Catà and S. Peris, Phys. Rev. D **65**, 056014 (2002).

[9] I. Rosell, J. J. Sanz-Cillero, and A. Pich, J. High Energy Phys. 08 (2004) 042.

[10] I. Rosell, J. J. Sanz-Cillero, and A. Pich, J. High Energy Phys. 01 (2007) 039.

[11] I. Rosell, P. Ruiz-Femenía, and J. Portolés, J. High Energy Phys. 12 (2005) 020.

[12] I. Rosell, Ph.D. thesis, Universitat de València, 2007, arXiv:hep-ph/0701248.

[13] G. Passarino and M. J. G. Veltman, Nucl. Phys. **B160**, 151 (1979).

[14] G.P. Lepage and S.J. Brodsky, Phys. Lett. **87B**, 359 (1979); Phys. Rev. D **22**, 2157 (1980).

[15] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B147**, 385 (1979).

[16] I. Rosell, P. Ruiz-Femenía, and J. J. Sanz-Cillero (work in progress).