

BPS preons in M-theory and supergravity¹

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Abstract

After introducing the notion of BPS preons as the basic constituents of M-theory, we discuss the recent negative results in the search for solutions of the $D=10$ and $D=11$ supergravity equations preserving 31/32 supersymmetries *i.e.*, of preonic solutions. The absence of these supergravity preonic solutions may point out to a pure quantum nature of BPS preons, manifesting itself in the need of incorporating quantum (stringy/M-theoretic) corrections to the supergravity equations.

1 Introduction

BPS preons are BPS states preserving all but one supersymmetries. They were introduced six years ago [1], when the list of known supersymmetric solutions of $D=11$ and $D=10$ type II supergravities only contained solutions preserving 16 or less supersymmetries, or all the 32. These solutions described $k/32$ BPS states with $k \leq 16$ plus the fully (32/32)-supersymmetric vacua, the set of which contained then the Minkowski and some $AdS_d \otimes S^{D-d}$ spaces [2] as well as the pp -wave backgrounds [3].

Although it was already clear that BPS states with $k > 16$ were not forbidden by the M-theory algebra [4, 5], no supersymmetric solutions with $16 < k < 32$ were known in 2001. As a result, the existence of (31/32)-supersymmetric or *preonic* solutions of the supergravity equations was hardly expected at the time, and the possibility of a *preon conspiracy* by which only composites of some number of preons could be ‘observed’ as supergravity solutions, but not the single preons themselves, was already discussed in [1, 6]. The expectancy of finding preonic, 31/32 supersymmetric solutions increased when some examples preserving

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more than one-half, *i.e.* having more than 16 supersymmetries were found, mainly as plane wave solutions (see [7] and the references in [8, 6]). Nevertheless, the search for 31/32 supersymmetric solutions of the ‘free’ (no sources) supergravity equations, which we review here, has only produced negative results. Indeed, for type II supergravities [9, 10] and for simply connected solutions of $D=11$ supergravity [11], the existence of preonic solutions has now been ruled out. Nevertheless, these ‘no go’ conclusions may change if quantum M-theoretic or ‘stringy’ corrections are taken into account. Moreover, the preon conspiracy does not preclude the preon conjecture [1], since preons were introduced as M-theoretical objects and fundamental constituents of M-theory as a whole, rather than as specific solutions of supergravity, its low energy limit.

Let us begin by reviewing the definition of M-theoretic BPS preons [1, 12, 6].

2 BPS preons as M-theory constituents

A BPS preon [1] is an M-theory BPS state preserving all but one supersymmetries, namely 31 out of 32. It is equivalently labelled as $|BPS\ preon\rangle := |31/32\ susy\rangle := |31/32\ BPS\rangle$ (we note, however, that the preon concept is not restricted [1] to $D=11$). Denoting the 32 component supersymmetry generator by Q_α , the fact that a BPS state preserves k supersymmetries is expressed as

$$\boxed{\epsilon_I^\alpha Q_\alpha | \frac{k}{32} BPS \rangle = 0}, \quad \begin{matrix} \alpha=1,\dots,32 \\ I=1,\dots,k \end{matrix} \quad (a); \quad \boxed{|BPS\ preon\rangle = | \frac{31}{32} BPS \rangle} \quad (b), \quad (1)$$

where ϵ_I^α are the k bosonic spinors ($k = 31$ for a BPS preon) that characterize the k preserved supersymmetries. They will correspond to Killing spinors in the supergravity solutions case below.

The only supersymmetry broken by a BPS preon can be characterized by one bosonic spinor. This can be, for instance, a bosonic spinor u^α such that $u^\alpha Q_\alpha |BPS\ preon\rangle \neq 0$. Together with eq. (1), this implies that $\det\{u^\alpha, \epsilon_I^\alpha\} \neq 0$, which expresses that u^α is linearly independent of the 31 spinors ϵ_I^α . However, it is more convenient to characterize a BPS preon state by another spinor λ_α , which is orthogonal to all the bosonic spinors ϵ_I^α that determine the 31 supersymmetries preserved by the BPS preon,

$$\boxed{\epsilon_I^\alpha \lambda_\alpha = 0}, \quad \begin{matrix} \alpha=1,\dots,32 \\ I=1,\dots,31 \end{matrix} \quad \Rightarrow \quad |BPS\ preon\rangle = | \frac{31}{32} BPS \rangle = |\lambda\rangle. \quad (2)$$

Thus, the non-zero result of the action $Q_\alpha |BPS\ preon\rangle$ of Q_α on a preon state (1), has to be given in terms of the bosonic spinor λ_α that characterizes the preon. To express this explicitly, one has to introduce another state $|BPS\ preon^f\rangle = | \frac{31}{32}^f BPS \rangle = |\lambda^f\rangle$ with opposite Grassmann parity, fermionic if the 31/32 state is considered to be bosonic as it is usually (but not necessarily) the case. Then, the pair $\{|\lambda\rangle, |\lambda^f\rangle\}$ determines an ultrashort *BPS preonic multiplet* [13], on which the action of the supersymmetry generators reads

$$Q_\alpha |\lambda\rangle = \lambda_\alpha |\lambda^f\rangle \quad (a) \quad , \quad Q_\alpha |\lambda^f\rangle = \lambda_\alpha |\lambda\rangle \quad (b) \quad . \quad (3)$$

Notice that the bosonic and fermionic preonic states enter (3) in a completely symmetric manner, so that one might also think of identifying the BPS preon with a fermionic state [13]. This requires further study, and in this paper we shall concentrate on bosonic BPS preons as they might be described by purely bosonic 31/32 supersymmetric solutions of the supergravity equations and their M-theoretic generalizations *i.e.*, those incorporating quantum ‘stringy’ $((\alpha')^3)$ corrections.

Applying the supersymmetry generator to eq. (3a) and using (3b), one finds that $Q_\beta Q_\alpha |\lambda\rangle = \lambda_\beta \lambda_\alpha |\lambda\rangle$, which leads to another equivalent definition of a BPS preon state [1, 12, 13],

$$\boxed{P_{\alpha\beta} |BPS\ preon\rangle = \lambda_\alpha \lambda_\beta |BPS\ preon\rangle}. \quad (4)$$

The symmetric generalized momentum $P_{\alpha\beta}$ operator above is the bosonic generator in the superalgebra

$$\{Q_\alpha, Q_\beta\} = 2P_{\alpha\beta}, \quad [P_{\alpha\beta}, P_{\gamma\delta}] = 0, \quad [P_{\alpha\beta}, Q_\gamma] = 0, \quad \alpha = 1, \dots, 32, \quad (5)$$

which has a central extension structure and is associated to the maximally extended rigid tensorial superspace $\Sigma^{(n(n+1)/2|n)}$, where n is the dimension of the appropriate spinor (32 for $D=11$).

The second definition (4) of a BPS preon indicates that any BPS state $|k/32\ BPS\rangle$ preserving k supersymmetries may be considered [1] as a composite of $\tilde{n} = 32 - k$ BPS preons, $|BPS\ preon^l\rangle = |\lambda^l\rangle$, $l = 1, \dots, \tilde{n}$:

$$|\frac{k}{32}\ BPS\rangle = |\lambda^{(1)}\rangle \otimes \dots \otimes |\lambda^{(32-k)}\rangle \equiv \bigoplus_{r=1}^{\tilde{n}} |\lambda^{(r)}\rangle, \quad \tilde{n} = 32 - k. \quad (6)$$

To conclude this, we begin by noticing that the eigenvalue matrix $p_{\alpha\beta}^k$ of $P_{\alpha\beta}$ for a BPS state preserving k supersymmetries $|\frac{k}{32}\ BPS\rangle$ has rank $\tilde{n} = 32 - k$ (one for a preon). This is expressed by the equation

$$p_{\alpha\beta}^k = \lambda_\alpha^1 \lambda_\beta^1 + \dots + \lambda_\alpha^{\tilde{n}} \lambda_\beta^{\tilde{n}} = \sum_{r=1}^{\tilde{n}=32-k} \lambda_\alpha^r \lambda_\beta^r, \quad P_{\alpha\beta} |\frac{k}{32}\ BPS\rangle = p_{\alpha\beta}^k |\frac{k}{32}\ BPS\rangle, \quad (7)$$

where λ_α^r are the \tilde{n} bosonic spinors that characterize the supersymmetries broken by the $k/32$ state and that appear in the expression

$$Q_\alpha |\frac{k}{32}\ BPS\rangle = \lambda_\alpha^1 |f^1\rangle + \dots + \lambda_\alpha^{\tilde{n}} |f^{\tilde{n}}\rangle = \bigoplus_{r=1}^{\tilde{n}} \lambda_\alpha^r |f^r\rangle, \quad \tilde{n} = 32 - k, \quad (8)$$

where $|f^1\rangle, \dots, |f^{\tilde{n}}\rangle$ are some fermionic states. The additive structure of the generalized momentum eigenvalue matrix $p_{\alpha\beta}^k$ of a $|\frac{k}{32}\ BPS\rangle$ state, eq. (7), then shows that such a BPS state may be treated as a composite [1] of a $\tilde{n} = 32 - k$ independent BPS preons, each of them characterized by an independent spinor λ_α , eq. (6). Thus, the number \tilde{n} of *broken* supersymmetries $\tilde{n} = 32 - k$ is the number of preons that compose the BPS state; each preon breaks one supersymmetry (see [12, 6] for further discussion), and a fully supersymmetric vacuum contains no preons.

The $\tilde{n} = 32 - k$ *preonic spinors* λ_α^r that characterize the preons making the $|\frac{k}{32} BPS \rangle$ state are orthogonal to the k (Killing) spinors ϵ_I^α that characterize the k supersymmetries preserved by it, eq. (1a),

$$\boxed{\epsilon_I^\alpha \lambda_\alpha^r = 0}, \quad \alpha = 1, \dots, 32, \quad I = 1, \dots, k, \quad r = 1, \dots, \tilde{n} \quad (\tilde{n} = 32 - k). \quad (9)$$

To conclude this section we note that the fermionic states $|f \rangle$ in (8) differ from the original bosonic $k/32$ -supersymmetric BPS state $|\frac{k}{32} BPS \rangle$ (6) by replacing one of the bosonic preons $|\lambda \rangle$ in (6) by its fermionic superpartner $|\lambda^f \rangle$, namely $|f^r \rangle := |\lambda^{(1)} \rangle \otimes |\lambda^{(2)} \rangle \otimes \dots \otimes |\lambda^{(r)f} \rangle \otimes \dots \otimes |\lambda^{(32-k)} \rangle$.

3 The moving G-frame method and generalized holonomy in supergravity

In supergravity, the k bosonic spinors ϵ_I^α of the supersymmetries preserved by a $\frac{k}{32}$ BPS solution (eq. (1a)) are Killing spinors obeying the equation,

$$\boxed{\mathcal{D}\epsilon_I^\alpha := D\epsilon_I^\alpha - \epsilon_I^\beta t_\beta^\alpha = 0}, \quad D\epsilon_I^\alpha := d\epsilon_I^\alpha - \epsilon_I^\beta \omega_\beta^\alpha, \quad \omega_\beta^\alpha := \frac{1}{4}\omega^{ab}\Gamma_{ab\beta}^\alpha, \quad (10)$$

where $D = e^a D_a$ is the standard (Lorentz) covariant derivative, $\omega^{ab} = -\omega^{ba} = e^c \omega_c^{ab}$ is the one-form spin connection and $t_\beta^\alpha := e^a t_{a\beta}^\alpha$ is the tensorial contribution to the *generalized* connection $w_\beta^\alpha = \omega_\beta^\alpha + t_\beta^\alpha$ defining the *generalized* covariant derivative $\mathcal{D} = d - w := D - t$. This tensorial contribution is constructed from the gauge field strength(s) and, in $D=10$, also includes the derivatives of the axion and dilaton. For instance, in the case of $D = 11$ supergravity the tensorial contribution reads

$$t_\beta^\alpha = \frac{i}{18}e^a (F_{a[3]}\Gamma^{[3]} + \frac{1}{8}F^{[4]}\Gamma_{a[4]})_\beta^\alpha; \quad \frac{i}{18}e^a F_{a[3]} := F_{ab_1 b_2 b_3}, \quad F^{[4]} := F^{b_1 b_2 b_3 b_4}. \quad (11)$$

The generalized covariant derivative is defined by the gravitino supersymmetry transformations which may be written as

$$\delta_\varepsilon \psi = \mathcal{D}\varepsilon^\alpha = D\varepsilon^\alpha - \varepsilon^\beta t_\beta^\alpha = d\varepsilon^\alpha - \varepsilon^\beta \omega_\beta^\alpha = d\varepsilon^\alpha - \varepsilon^\beta (\omega + t)_\beta^\alpha \quad (12)$$

(up to quadratic fermion terms for type II, $D=10$). The Killing spinor equation (10) expresses the supersymmetry invariance of a purely *bosonic* solution, since $\psi = 0$ requires $\delta_\varepsilon \psi = 0$. The preserved supersymmetries are determined by the Grassmann odd functions $\varepsilon^\alpha(x) = \kappa^I \epsilon_I^\alpha(x)$ constructed from k independent bosonic Killing spinors and the fermionic parameters κ^I , $I = 1, \dots, k$.

The selfconsistency (integrability) condition for the Killing spinor equation (10), $\mathcal{D}\mathcal{D}\epsilon_J^\alpha = 0$, can be written [14, 15, 16] in terms of the *generalized curvature* $\mathcal{R} = dw - w \wedge w$ of the generalized connection $w = \omega + t$ (eqs. (10) and (11) for $D=11$) as

$$\epsilon_J^\beta \mathcal{R}_\beta^\alpha = 0 \quad (a) \quad , \quad \mathcal{R}_\beta^\alpha = d\omega_\beta^\alpha - \omega_\beta^\gamma \wedge \omega_\gamma^\alpha \quad (b) \quad . \quad (13)$$

The generalized curvature \mathcal{R}_β^α takes its values in the Lie algebra \mathcal{H} of the *generalized holonomy group* H . This suggested to classify the partially supersymmetric supergravity solutions by their generalized holonomy [14, 15, 16], which turned out useful in the search for new solutions. It was found in [17, 18] for $D=11$ and type II supergravity that H is a subgroup of $SL(32, \mathbb{R})$ (rather than $GL(32, \mathbb{R})$).

Thus, a $k/32$ supersymmetric solution of the $D=11$ or of the type II $D=10$ supergravity equations can be characterized by a set of k Killing spinors ϵ_I^α , $I = 1, \dots, k$, obeying eq. (10). Alternatively, we may use the $\tilde{n} = 32 - k$ preonic spinors λ_α^r , $r = 1, \dots, \tilde{n}$, orthogonal to the Killing spinors, eq. (9). Together, they make $\tilde{n} + k = 32$ bosonic spinors that can be used to define a *moving G-frame* [6].

By using the moving G -frame method it was found [6] that the generalized curvature of a BPS preonic supergravity solution should have the form

$$\mathcal{R}_\beta^\alpha = dB^I \lambda_\beta \epsilon_I^\alpha, \quad \alpha = 1, \dots, 32, \quad I = 1, \dots, 31, \quad (14)$$

where $B^I = e^a B_a^I$ is a set of 31 one-forms which determines the non-trivial part of the $sl(32, \mathbb{R})$ -valued generalized connection w_β^α of the hypothetical preonic solution, $w_\beta^\alpha = \lambda_\beta B^I \epsilon_I^\alpha - (dg g^{-1})_\beta^\alpha$. The generalization of the $k = 31$ equation (14) to the $k < 31$ case [6], also recovers the general statement [20] that the generalized holonomy of a $k/32$ supersymmetric solution is the semidirect product $SL(32 - k, \mathbb{R}) \ltimes \mathbb{R}^{k(32-k)}$, which gives $H = \mathbb{R}^{31}$ for the preonic $k = 31$ case.

Due to the presence of λ_β , eq. (14) solves the selfconsistency conditions (13a) for the existence of 31 Killing spinors (10). The appearance of the Killing spinors ϵ_I^α in the preonic generalized curvature of eq. (14) follows from the fact [17, 18] that the generalized holonomy group H is a subgroup of $SL(32, \mathbb{R})$ both for $D=11$ and type IIB $D=10$ supergravities. This implies $\mathcal{R}_\alpha^\alpha = 0$ (zero trace), which is satisfied by (14) because of (9). Furthermore, the actual form of the generalized connection [17, 18] shows that they also are $sl(32, \mathbb{R})$ -valued, $w_\alpha^\alpha = 0$, so that the generalized structure group G is also a subgroup of $SL(32, \mathbb{R})$. This implies, after some algebra [6], that the preonic spinor of a preonic solution is G -covariantly constant,

$$\mathcal{D}\lambda_\beta := D\lambda_\beta + t_\beta^\alpha \lambda_\alpha := d\lambda_\beta + w_\beta^\alpha \lambda_\alpha = 0, \quad \alpha = 1, \dots, 32, \quad I = 1, \dots, 31. \quad (15)$$

Notice the difference in sign and position of indices between the generalized covariant derivatives in eqs. (10) and (15). When the charge conjugation matrix $C^{\alpha\beta}$ exists [IIA $D=10$, $D=11$], it may be used to rise the spinorial indices but, since it is not ‘ G -covariantly’ constant ($\mathcal{D}C^{\alpha\beta} := -2w^{[\alpha\beta]} \neq 0$ unless the flux F_{abcd} vanishes²), the two ‘ G -covariant’ derivatives in (12) and (15) are different in general.

²It follows from (11) that $w_a^{[\alpha\beta]} \propto F^{b_1 b_2 b_3 b_4} (C\Gamma_{ab_1 b_2 b_3 b_4})^{\alpha\beta}$, the vanishing of which implies $F^{b_1 b_2 b_3 b_4} = 0$.

4 No preonic solutions without α' corrections in type II supergravities

In $D = 11$ the only fermionic field in the supergravity multiplet is the gravitino $\psi = dx^\mu \psi_\mu(x)$, and the k Killing spinors describing supersymmetries of a $k/32$ bosonic solution of the supergravity equations obey the differential equation (10) only. The $D=10$ type II supergravity theories include, in addition, the 32-component dilatino field $\tilde{\chi}(x)$, which carries a reducible representation of $Spin(1,9)$, and can be split into two Majorana-Weyl spinors with different (IIA) or equal (IIB) chirality,

$$\text{IIA } \tilde{\chi}^{\tilde{\alpha}} := (\chi^{\alpha 1}, \chi_\alpha^2), \quad \text{IIB } \tilde{\chi}_{\tilde{\alpha}} := (\chi_\alpha^1, \chi_\alpha^2), \quad \tilde{\alpha} = 1, \dots, 32, \quad \alpha = 1, \dots, 16. \quad (16)$$

The IIA and IIB dilatino supersymmetry transformation is algebraic, $\delta_{susy} \tilde{\chi} = \tilde{\epsilon} M$, where the matrix M is expressed, respectively, through the type IIA, IIB supergravity fluxes.

Due to the presence of $\tilde{\chi}$, the Killing spinors $\check{\epsilon}_I$ associated with the preserved supersymmetries of a type II purely bosonic solution ($\check{\psi}_a^{\tilde{\alpha}} = 0$ and $\check{\chi}^{\tilde{\alpha}} = 0$) have to obey not only the differential equation (10), but also the *algebraic* equation $\check{\epsilon}_I M = 0$ that follows from $\delta_{susy} \tilde{\chi}^{\tilde{\alpha}} = 0$,

$$\text{type II : } \quad \mathcal{D}\check{\epsilon}_I := D\check{\epsilon}_I - \check{\epsilon}_I \check{t} = 0 \quad (a) \quad \check{\epsilon}_I M = 0 \quad (b) \quad (I = 1, \dots, k). \quad (17)$$

Actually, the study of the algebraic Killing spinor equation (17b) led to establishing the absence of preonic solutions, both in type IIB [9] and type IIA supergravities [10]. Applying the spinor moving G -frame method of [6], one finds that eq. (17b) is solved by

$$\boxed{M = \check{\lambda} \otimes \check{s}}, \quad \text{namely} \quad \text{IIA : } M_{\tilde{\beta}}^{\tilde{\alpha}} = \check{\lambda}_{\tilde{\beta}} \check{s}^{\tilde{\alpha}}, \quad \text{IIB : } M_{\tilde{\beta}\tilde{\alpha}} = \check{\lambda}_{\tilde{\beta}} \check{s}_{\tilde{\alpha}}, \quad (18)$$

where $\check{\lambda}$ is the 32-component *preonic spinor* and \check{s} is a generic 32-component spinor which, at the final stage, has to be expressed through the (IIA or IIB) fluxes (on-shell field strengths) defining the M matrix. For instance, for IIA supergravity

$$\text{IIA : } \quad \check{\lambda}_{\tilde{\alpha}} := (\lambda_\alpha^1, \lambda^{\alpha 2}), \quad \check{s}^{\tilde{\alpha}} := (s^{\alpha 1}, s_\alpha^2), \quad \alpha = 1, \dots, 16, \quad (19)$$

and the 32×32 matrix M of eq. (17b) reads

$$\text{IIA : } \quad M_{\tilde{\beta}}^{\tilde{\alpha}} = \frac{1}{2} \partial \Phi \otimes \sigma_1 - \frac{1}{4} \mathcal{H}^{(3)} \otimes i\sigma_2 + \frac{3}{8} e^\Phi \mathcal{R}^{(2)} \otimes \sigma_3 + \frac{1}{8} \mathcal{R}^{(4)} \otimes \mathbb{I}_2, \quad (20)$$

where $\sigma_1, \sigma_2, \sigma_3$ are the standard 2×2 Pauli matrices and the slashed variables, which involve the $D = 10, 16 \times 16$ Pauli matrices³ (also denoted by the letter σ, σ^a) are given by

$$\partial \Phi := D_a \Phi \sigma_{\alpha\beta}^a, \quad \mathcal{H}^{(3)} = \frac{1}{3!} H_{abc} \sigma^{abc}, \quad \tilde{\partial} \Phi := D_a \Phi \tilde{\sigma}^{a\alpha\beta}, \quad \tilde{\mathcal{H}}^{(3)} = \frac{1}{3!} H_{abc} \tilde{\sigma}^{abca\alpha\beta}, \quad (21)$$

$$\mathcal{R}^{(2)} := \frac{1}{2!} R_{ab} (\sigma^{ab})_{\alpha\beta} =: -\tilde{\mathcal{R}}^{(2)\beta}{}_{\alpha}, \quad \mathcal{R}^{(4)} := \frac{1}{4!} R_{abcd} \sigma^{abcd}{}_{\alpha\beta} =: \tilde{\mathcal{R}}^{(4)\beta}{}_{\alpha}. \quad (22)$$

³In $D = 10$, $\sigma^a = \sigma_{\alpha\beta}^a$, $\tilde{\sigma}^a = \sigma^{a\alpha\beta}$, $a = 0, 1, \dots, 9$. They obey $\sigma^a \tilde{\sigma}^b + \sigma^b \tilde{\sigma}^a = 2\eta^{ab} := \text{diag}(+, -, \dots, -)$ and $\tilde{\sigma}^a \sigma^b + \tilde{\sigma}^b \sigma^a = 2\eta^{ab}$. Then $\sigma^{ab\dots} := (\sigma^{[a} \tilde{\sigma}^{b\dots]})$, $\tilde{\sigma}^{ab\dots} := (\tilde{\sigma}^{[a} \sigma^{b\dots]})$, e.g. $\sigma^{ab} := (\sigma^{[a} \tilde{\sigma}^{b]})_{\alpha\beta}$ etc.

Thus, the IIA matrix M includes all the possible IIA fluxes, namely,

$$R_2 := dC_1, \quad R_4 := dC_3 - C_1 \wedge H_3, \quad H_3 := dB_2 \quad \text{and} \quad d\Phi. \quad (23)$$

Eq. (18) restricts strongly both the fluxes in M and the preonic spinor $\check{\lambda}$. For IIB, it may be seen [10] that, when the quantum stringy corrections to the supergravity equations and supersymmetry transformations are ignored, the equation $M = \check{\lambda} \otimes \check{s}$ requires either $\check{\lambda} = 0$ (no preonic spinor), which implies a fully supersymmetric solution with 32 rather than just 31 supersymmetries, *or* $\check{s}=0$ and, hence, $M = 0$, in which case all IIB fluxes but R_5 are zero. In this second case ($\lambda \neq 0$), it may be seen that the differential condition (15) (or the Killing spinor equations (10)), which involves the only non-zero flux R_5 , implies that only an *even* number of supersymmetries may be broken and thus no preonic solutions, as previously found in [9].

For type IIA the consequences of the algebraic equation (17b) are the same [10], but the condition $M = 0$ now implies that *all* fluxes are zero, in which case it is known [16] that the only solutions with $k > 16$ supersymmetries are the fully supersymmetric ones. Indeed, after some algebra, one finds that eq. (18) implies that the auxiliary spinor $\check{s}^{\check{\alpha}} = (s^{\alpha^1}, s_{\alpha}^2)$ is related to the type IIA preonic spinor ($D=10$ Majorana spinor) $\check{\lambda}_{\check{\alpha}} := (\lambda_{\alpha}^1, \lambda^{\alpha^2})$ in (19) by $s^{\alpha^1} = a\lambda^{\alpha^2}$, $s_{\alpha}^2 = a\lambda_{\alpha}^1$ for some real constant a , and that the two Majorana-Weyl parts λ_{α}^1 and λ^{α^2} of $\check{\lambda}_{\check{\alpha}}$ are restricted by a set of equations including

$$\partial\Phi := \sigma_{\alpha\beta}^a D_a \Phi = 2a\lambda_{\alpha}^1 \lambda_{\beta}^1, \quad \tilde{\partial}\Phi := \tilde{\sigma}^{a\alpha\beta} D_a \Phi = 2a\lambda^{\alpha^2} \lambda^{\beta^2}. \quad (24)$$

These 16×16 matrix equations have only the trivial solution $d\Phi = 0$, $a\lambda_{\alpha}^1 = 0$, $a\lambda^{\alpha^2} = 0$, and this implies the absence of preonic supergravity solutions. To check that the solution of each equation in (24) is trivial, it is sufficient to notice that the rank of the matrices in their *r.h.* sides is obviously one or zero, while the the rank of those in their *l.h* sides is 32 ($D_a \Phi$ timelike or spacelike), 16 (lightlike) or zero. The only consistent choice is rank zero, which implies the trivial solution. Thus, there is also a BPS preon conspiracy in classical type IIA supergravity.

5 Remarks on D=11, stringy corrections and preon conspiracy

As far as the D=11 supergravity is concerned, a negative result on the existence of simply connected preonic solutions was recently reported in [11]. Eq. (14) [6], expressing the selfconsistency condition for a preonic solution of $D=11$ supergravity, was solved by the methods of spinorial geometry (see refs. in [9, 11]) with the result that such a 31/32 solution of the *free* supergravity field equations actually has trivial generalized holonomy, $\mathcal{R}_{\beta}^{\alpha} = 0$, and hence is locally isomorphic to a 32/32 supersymmetric solution. Nevertheless, these results still allow [11] for 31/32 solutions among not simply connected manifolds⁴. Further,

⁴*Note added.* This possibility has just been ruled out [24].

since the field equations and the Bianchi identities were also used to reach the above negative result, the presence of brane sources or of stringy corrections to the supergravity equations might change the conclusion.

To illustrate this let us notice [10] that, in the light of the results in [21], one may expect that the stringy corrections to the supersymmetry transformations may modify the algebraic structure of eqs. (24). The essential point is the appearance of some Q_{abcde}^\pm contributions changing (24) to

$$\sigma_{\alpha\beta}^a D_a \Phi + Q_{abcde}^- \sigma_{\alpha\beta}^{abcde} = 2a\lambda_\alpha^1 \lambda_\beta^1, \quad \tilde{\sigma}^{a\alpha\beta} D_a \Phi + Q_{abcde}^+ \tilde{\sigma}^{abcde\alpha\beta} = 2a\lambda^{2\alpha} \lambda^{2\beta}. \quad (25)$$

With a general Q_{abcde}^\pm tensor, the algebraic structure of the *l.h.* sides of these equations correspond to the decompositions of general symmetric 16×16 matrices into irreducible $SO(1,9)$ tensors. Thus, there are no restrictions on their rank and, hence, these equations may have nontrivial solutions.

The BPS preon conspiracy, *i.e.* the absence of 31/32 supersymmetric solutions for the $D=11$ and $D=10$ type II classical supergravities [9, 13, 11] discussed here, is consistent with the preonic conjecture (one might think, by way of an analogy, of quark confinement, which does not preclude the existence of quarks). Introduced as M-theoretical objects [1], the ‘observation’ of BPS preons might require considering M-theoretic/stringy corrections to the supergravity equations; they could appear already at the next $\propto \alpha'^3$ approximation. This would imply that BPS preons are intrinsically quantum objects which cannot be ‘seen’ in a classical supergravity description.

The present challenge for the preonic hypothesis is to exhibit its real usefulness in the study of String/M-theory. At present, it provides an algebraic classification of all BPS states, and the preon-inspired moving G -frame method has been useful for studying supergravity solutions. To conclude, it is also interesting to note that dynamical models for the $D=4,6,10$ counterparts of (pointlike) BPS preons ($n=4, 8$ and 16 , see below (5)) describe massless conformal higher spin theories (see [12, 22, 23, 13] and refs. therein).

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