

Dynamic modeling of the electric transportation network

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Abstract. – We introduce a model for the dynamic self-organization of the electric grid. The model is characterized by a conserved magnitude, *energy*, that can travel following the links of the network to satisfy nodes' *load*. The load fluctuates in time causing local overloads that drive the dynamic evolution of the network topology. Our model displays a transition from a fully connected network to a configuration with a non-trivial topology and where global failures are suppressed. The most efficient topology is characterized by an exponential degree distribution, in agreement with the topology of the real electric grid. The model intrinsically presents self-induced break-down events, which can be thought as representative of real black-outs.

Introduction. – The electric grid is a critical infrastructure for our economy and society. Recent events, ranging from the large-scale black-outs a few years ago to the California crisis today [1], highlight the need to enhance the insight into the electric grid, complementing the traditional technological analysis [2] with new transversal points of view. Our approach is to study the electric grid macroscopic behavior rather than to dissect individual events. At that macroscopic scale, the electric grid exhibits behaviors typical of complex systems. For instance, on the basis of 15 years time series of transmission system black-outs of the U.S. electric network [3, 4], it has been proposed that the electric grid may be a self-organized critical system, operating at or near a critical point.

A signature of the electric grid is that it can be represented as a complex network, where nodes are the generators and the links the transmission lines. Recent research in complex networks has shown that a detailed knowledge of the topology of a communication or transportation network is essential for the understanding of cascading failures [5–8]. While some of these studies have focused on the topological robustness of the underlying network to random failures and targeted attacks [9], other research have considered dynamic processes on static networks [10, 11]. However, these studies do not consider the network as a dynamic entity whose evolution is driven by the action of the nodes [12–17]. In this paper, we present a dynamic model aiming to describe the growth and evolution of a transportation network. The network growth relies on the need of resource distribution in a heterogenous environment.

The model. – We consider N dynamic elements located at the nodes of a two-dimensional square lattice. Each element i is characterized by its size s_i , drawn from a probability distribution $p(s)$. To each element i are associated two dynamic variables: the *load* (energy consumption) l_i^t , and the *supply* (available energy) f_i^t . We assume that i) the load of element

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i is described by $l_i^t = m_i + \sqrt{m_i} \xi_i^t$, where m_i is a constant value and ξ_i^t represents a fluctuation term; and ii) initially at $t = 0$, the available energy and the constant load at each element are proportional to its size s_i ,

$$f_i^0 = f_r s_i, \quad (1)$$

$$m_i = m_r s_i, \quad (2)$$

where f_r and m_r are constant values and in general $f_r > m_r$. Starting from an initial condition where each element is isolated, the network grows as follows. At each time step:

- 1) If the load overcomes the supply at element i ($f_i^t < l_i^t$), a *failure* occurs.

After a failure, the failing element i chooses a target neighbor through a wiring strategy. Following the empirical results observed in communication networks with spatial constraints [18], the target node j is chosen such that it maximizes the function

$$\pi(i, j) = \frac{s_j^\beta}{d_{ij}^\gamma}, \quad (3)$$

where d_{ij} represents the Euclidean distance between the two elements, and the exponents β and γ indicate the preference for size and proximity, respectively, in the wiring.

- 2) The supply at element i is updated according to f_i .

$$f_i^{t+1} = \begin{cases} f_i^t & \text{if } k_i = 0, \\ l_i^t + \sum_{j \in \mathcal{V}(i)} \frac{f_j^t - l_j^t}{k_j} & \text{if } k_i \neq 0, \end{cases} \quad (4)$$

where k_j is the number of links possessed by the element j , j running over the neighbors of the element i , $\mathcal{V}(i)$. The prescription given by eq. (4) allows the energy to be distributed through the links of the network as it is needed, depending on the instantaneous load of each linked element, making next overloads more unlikely.

Each N_{steps} time steps, the links are actually set and the network is consequently updated. The choice of the time step for the network construction is a further degree of freedom of our model. In real electric grids, the time scales of the fluctuating demand are much faster than the time scales at which the network is modified. Therefore, we choose to update the network each $N_{\text{steps}} \gg 1$ time steps of the local dynamics.

The initial total supply E^0 and the total load L^0 in the system are given by

$$E^0 = \sum_i f_r s_i = N f_r \Lambda_1^P, \quad (5)$$

$$L^0 = \sum_i m_r s_i + \sqrt{m_r s_i} \xi_i^0 = N m_r \Lambda_1^P + N \sqrt{m_r} \Lambda_2^P \langle \xi \rangle, \quad (6)$$

where Λ_1^P and Λ_2^P are two coefficients that depend only on the geographic distribution of city sizes $P(s)$. Under the transformation given by eq. (4), the total energy E^0 is conserved. Thus in the remainder we will remove the superscript.

To model the actual geographic distribution of city sizes, the distribution of s_i follows a Zipf law [19] of the form $P(s) = A s^{-2}$ in the interval $(1, s_{\text{max}})$ and $A = s_{\text{max}} / (s_{\text{max}} - 1)$ is the normalization coefficient. Fluctuations ξ_i are described either by the logistic map in the chaotic regime, or by white noise [20].

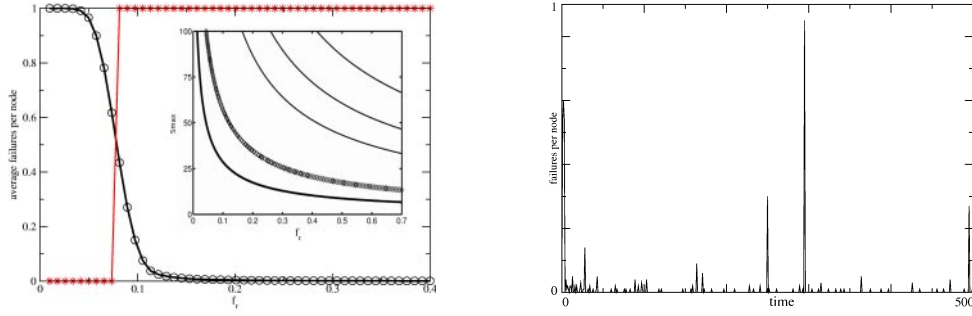


Fig. 1 – Left panel: (circles) the average failure events per node and (stars) the integer part of ϵ/λ vs. f_r . Parameter values $s_{\max} = 100$, $\mu = 1/2$, $N_{\text{step}} = 2500$, $N = 2500$. Inset: iso-energy lines for $\epsilon = \lambda$, $\epsilon_{\text{isolated}}$ (diamonds), and $\epsilon = 5, 10, 15$ from bottom to top. Right panel: temporal evolution of the total number of failures per node for $f_r = 0.3$ (other parameters as in the left panel).

The average energy ϵ and average load λ per node in the system are now given by

$$\epsilon = \frac{E}{N} = f_r \frac{s_{\max} \ln s_{\max}}{s_{\max} - 1} \equiv f_r \Theta(s_{\max}), \quad (7)$$

$$\lambda = \frac{L}{N} = m_r \Theta(s_{\max}) + \frac{1}{4} \sqrt{m_r} \frac{\sqrt{s_{\max}}}{\sqrt{s_{\max} + 1}} \equiv m_r \Theta(s_{\max}) + \frac{1}{2} \sqrt{m_r} \Phi(s_{\max}). \quad (8)$$

It is worth noting that at any given time the maximum possible load in the network is $\lambda_{\max} = m_r \Theta(s_{\max}) + \sqrt{m_r} \Phi(s_{\max})$. Thus for a system composed by isolated nodes following the same dynamics, the total energy that guarantees that all the nodes have access to the energy they need is

$$\epsilon_{\text{isolated}} = \lambda_{\max} = m_r \Theta(s_{\max}) + \sqrt{m_r} \Phi(s_{\max}). \quad (9)$$

Dynamics of failures. – The transportation network evolves as a consequence of the overload. As more links are added, sets of nodes will become connected forming clusters sharing their energy. We introduce the time average of the number of failures per node as an order parameter of the system. The order parameter shows a transition that depends on the available energy in the system, f_r . For low values of f_r , $\epsilon < \lambda$, the system evolves towards a fully connected network in which the total number of failures is of the order of the system size (fig. 1 left). The total supply in the system is not able to sustain the needs of the nodes. If $\epsilon > \lambda$, a statistically failure-free network is formed, accounting for a cooperative behavior in which the network is able to effectively redistribute the energy throughout the system. Therefore the condition $\epsilon = \lambda$ represents a transition point. We remark that, since the transition occurs at a value of ϵ lower than $\epsilon_{\text{isolated}}$ (see fig. 1 left, inset), in our model a dynamic sharing of the resources in a heterogeneous environment is energetically favorable. However, isolated break-down events (a huge amount of elements fails at once, see fig. 1 right) are still present in the stable regime. These events represent global cascade failures induced by local fluctuation, which are amplified by propagation through the network. Global cascades are not suppressed, as it is not possible to satisfy the total energy needs. However, as one further enters the stable regime, global cascades are less likely to occur, representing rare events that do not prevent the average number of failure to approximate to zero.

We find interesting to analyze the transition depending on m_r . If m_r and f_r are independent, *i.e.*, the load and the initially available energy at each element are independent, the transition $\epsilon = \lambda$ is simply given by $f_r = m_r + \frac{1}{2} \sqrt{m_r} \Phi(s_{\max}) / \Theta(s_{\max})$. However, we should

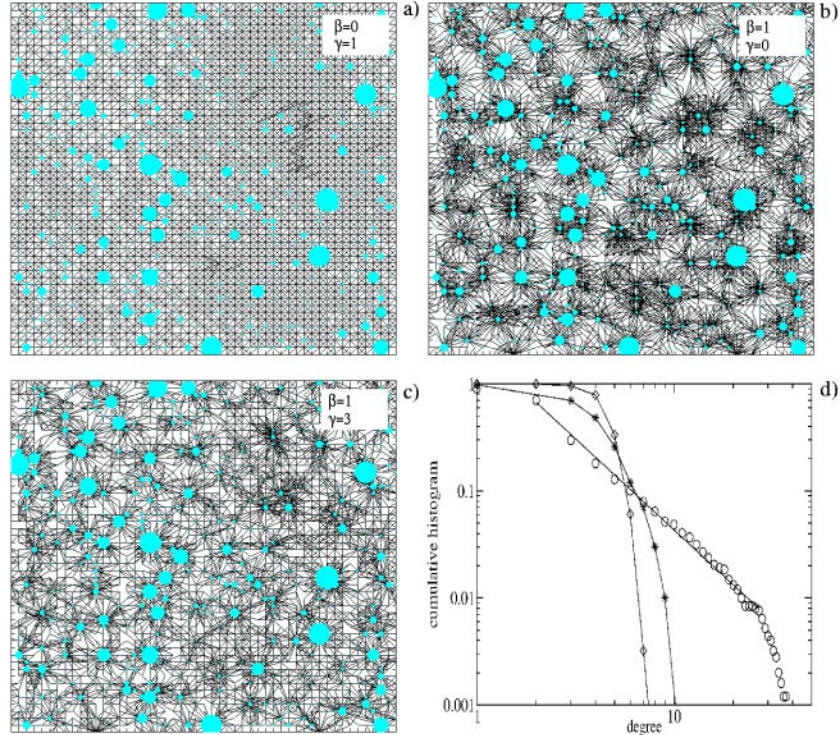


Fig. 2 – (a-c) Network structure; d) degree distribution corresponding to the network displayed in a) (diamonds), b) (circles), and c) (stars). Lines correspond to fitted curves.

expect that the available energy and the load are related: elements with a high load will also have large amount of available energy. In this case, m_r depends on f_r accounting for the fact that the energy availability in each node is meant to supply the local need. If, for simplicity, we assume that they are proportional $m_r = \mu f_r$, with $\mu \in [0, 1]$ the transition is given by $\varepsilon = [\frac{\sqrt{\mu}}{2(1-\mu)}\Phi(s_{\max})/\Theta(s_{\max})]^2$. In the remainder we will assume that $\mu = 1/2$.

The dynamic network. – A key dynamic ingredient is given by the dynamics of the network. The evolution of the dynamic network is driven by the failures of the nodes and the attachment function given by eq. (3). In the model, failures are induced by local fluctuations in the load, afterward propagated by the transportation network. During the transient regime, several failures occur simultaneously leading to the initial development of the network. After the transient, the network settles to a configuration where the average number of failures drops to zero. We characterize the emerging topologies in the stationary configurations. Two factors determine the network topology: the wiring strategy and the available energy ε . A first characterization of the network topology is given by the degree distribution and the average shortest path length. By varying the parameters β and γ in the wiring strategy given by eq. (3) we obtain different classes of networks. In fig. 2 we show the degree distribution for three well-known topologies exhibited by our system for a fixed value of ε . The fitted curves individuate a Gaussian degree distribution when the selection of the target element is merely based on its relative Euclidean distance to the failing element ($\beta = 0$ and $\gamma = 1$); a power

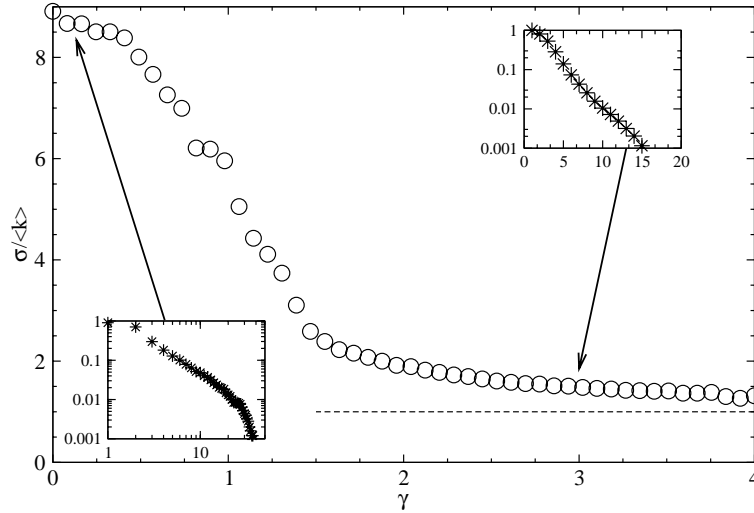


Fig. 3 – The normalized standard deviation $\sigma/\langle k \rangle$ of the degree distribution *vs.* γ for $\beta = 1$. Insets: degree distribution corresponding to $\gamma = 0$ and $\gamma = 3$, with fixed $\beta = 1$.

law link distribution when the selection of the target element j is based on its size α_j ($\beta = 1$ and $\gamma = 0$); and, finally, an exponential degree distribution when the selection of the target element j is based both on its size α_j and relative distance to the failing element ($\beta = 1$ and $\gamma = 3$). To analyze the parameter range in which different network topologies are observed, we have measured the normalized standard deviation $\sigma/\langle k \rangle$ of the degree distribution for different parameter values $\gamma \in [0, 4]$, for $\beta = 1$. An exponential degree distribution is characterized by $\sigma/\langle k \rangle = 1$, while it increases for a power law degree distribution. Figure 3 shows that exponential degree distributions are obtained for $\gamma > 2$.

Increasing the energy above its transition value ($\varepsilon = \lambda$), the degree distribution remains qualitatively unchanged. However, a higher number of links is required to stabilize the network as ε approaches the transition value (fig. 4). This increment in the total number of links is due to the higher number of failures that occur during the formation of the network closer to the transition, as is shown by the cumulative failure histograms in fig. 4d.

A further topological characterization is given by the average shortest path length l [21]

$$l = \frac{1}{N(N-1)} \sum_{i,j} \delta_{ij}, \quad (10)$$

where δ_{ij} is the shortest path between the elements i and j .

During the dynamic growth of the network, l decreases from infinity to a stable finite value. For a fixed E value, the minimum l is obtained for the network characterized by a power law degree distribution (fig. 5a). However, this does not mean that the network possessing a power law degree distribution is the most efficient transporting the energy across the system. Indeed, a possible measure of the efficiency in the energy transport is the following:

$$\eta = \frac{1}{N(N-1)} \sum_{i,j} \frac{s_j}{\delta_{ij}}. \quad (11)$$

Since in our system the largest elements act as energy reservoirs, the definition of the efficiency η states that the shorter is the path to the larger elements, the more efficient is the network.

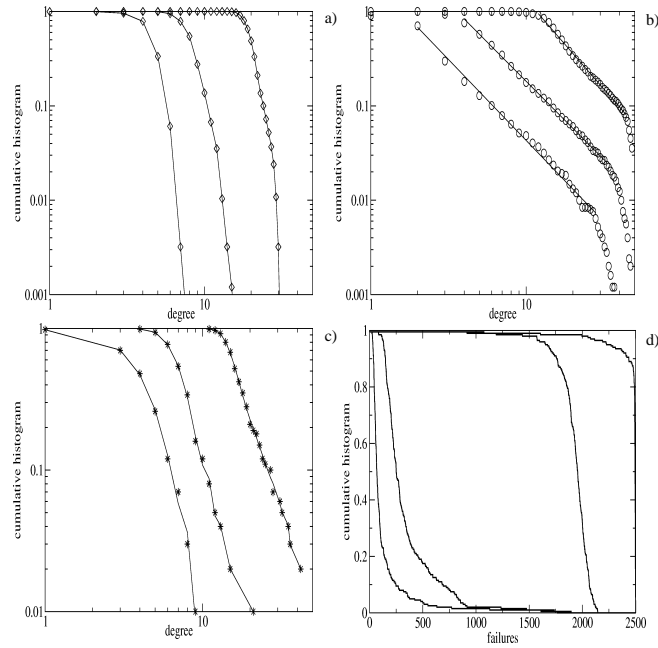


Fig. 4 – Degree distributions for three values of $f_r = 0.3, 0.4, 0.7$. Other parameter values in (a-c) as in fig. 2. d) Cumulative failure histograms varying f_r across the transition point.

Figure 5b shows that the network possessing an exponential degree distribution is the most efficient in our model.

Discussion. – In summary, we have introduced a model for the dynamic self-organization of a transportation network, in terms of its growth and evolution. Our model contains several features which are peculiar of the electric grid. The electric grid is a dynamic network that evolves based on local needs, failures of local elements, and energy sharing criteria. If the total energy is below the average consumption, the system reaches a fully connected network;

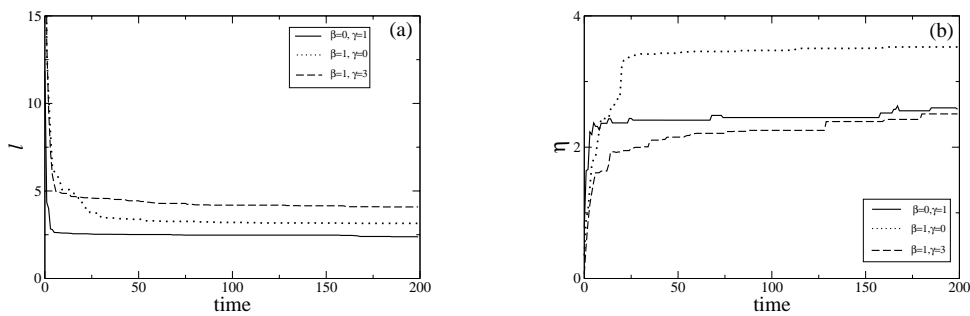


Fig. 5 – (a) Average shortest path length. (b) Network efficiency. Parameter values as in fig. 2.

whereas if the system has enough energy, the emerging network reflects the cooperative behavior in which the resources are globally shared. When the cooperative behavior is achieved, the network stops growing and reaches a configuration with a non-trivial topology shaped by the wiring strategy and the available total energy. Nevertheless, local fluctuations are still present once the network is formed and keep affecting its functionality. Our model intrinsically presents self-induced break-down events, which can be thought as representative of real black-outs. Another remarkable feature of our model is the heterogeneity in the initial energy distribution, modeling the geographic inhomogeneity in the energy availability. In a heterogeneous environment, the dynamic sharing of the resources is energetically favorable. Heterogeneity also induces the selection of a preferred topology as the most efficient in the energy transport, because it displays the shortest path to the bigger elements. The most efficient topology is characterized by an exponential degree distribution, in agreement with the topology of the real electric grid [22].

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