Modeling of the Terminal Velocities of the Dust Ejected Material by the Impact

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1 Abstract

We compute the distribution of velocities of the particles ejected by the impact of the projectile released from NASA Deep Impact spacecraft on the nucleus of comet 9P/Tempel 1 on the successive 20 hours following the collision. This is performed by the development and use of an ill-conditioned inverse problem approach, whose main ingredients are a set of observations taken by the Narrow Angle Camera (NAC) of OSIRIS onboard the Rosetta spacecraft, and a set of simple models of the expansion of the dust ejecta plume for different velocities. Terminal velocities are derived using a maximum likelihood estimator.

We compare our results with published estimates of the expansion velocity of the dust cloud. Our approach and models reproduce well the velocity distribution of the ejected particles. We consider these successful comparisons of the velocities as an evidence for the appropriateness of the approach. This analysis provides a more thorough understanding of the properties of the Deep Impact dust cloud.

2 Introduction

In order to investigate the comet's interior, on 4 July 2004 the NASA mission Deep Impact fired a projectile of 364 kg into the nucleus of 9P/Tempel 1. A crater was formed and material was ejected from the comet [1]. OSIRIS, the scientific camera system on the Rosetta spacecraft [2], was activated five days before the Deep Impact event, and observed 9P/Tempel 1 near-continuously for more than two weeks [3]. Some properties of the cloud (e.g. structure, morphology, number of water molecules produced in the impact, abundance ratio between the CN parent molecules and water) are already reported in the literature.

The main aim of this work is to present the velocity distribution of the ejected dust computed with a discrete linear inverse approach. The Narrow Angle Camera (NAC) of OSIRIS monitored the cometary dust of 9P/Tempel

1, and by aperture photometry on the images, light curves for different circular fields provide the observational input to our approach. A model of the expansion of the dust ejecta plume for different velocities supplies the modeled light curves. Because of the good temporal parameter sampling that the OSIRIS data offer (into around an image per minute), this approach is very well suitable. The comparison between these results and those derived from other authors not only tests and refines the conclusions derived from other observations and methods, but also provides an evidence for the appropriateness of our models and of our approach, and furthermore, allow to test of additional production of the material.

3 Observational data and models

The observational data consist of 73 images of 9P/Tempel 1 taken by the NAC OSIRIS with the clear filter, from the impact time to around 20 hours after impact time (for a detailed description of the OSIRIS observations, see [4]). By cometocentric circular aperture photometry on the images of 9P/Tempel 1, the brightness of the comet is integrated over a set of nine apertures of different radii (from 3000 to 15000 km, 1 pixel = 1500 km). An observational data row matrix O is defined, and contains the collection of integrated brightness (73 images × 9 apertures = 657 data-points):

$$O = [\mathrm{O}_1,\ \mathrm{O}_2,...,\mathrm{O}_{73},...,\mathrm{O}_{657}].$$

The measurement error row matrix E of the observed brightness (or the known individual standard deviation matrix) is defined by:

$$E = [\sigma_1,\ \sigma_2,...,\sigma_{73},...,\sigma_{657}].$$

The models of the expansion of the dust ejecta plume are computed using the decay of the flux when the material leaves the different apertures. For early times (less than 20 hours), we assume: (1) that the expanding dust cloud moves with a constant velocity, (2) a neglected effect of the radiation pressure removing the material, (3) and that the dust reaches its final velocity instantly at the time of the impact. We compute the fluxes of the dust ejecta plume for different projected ejecta velocities v, from 1 to 600 ms^{-1}, with a steep i of 50 ms^{-1}. We adopt a maximum velocity of 600 ms^{-1}, although a small fraction of the dust might be actually be ejected at higher velocities [1]. A conservative minimum velocity of 1 ms^{-1} is adopted. The models are normalized in flux by multiplying each modeled flux by a normalization constant. The generated normalized models are stored in a matrix M:

$$M = \begin{bmatrix} M_{1,1} & M_{1,2} & ... & M_{1,657} \\ M_{2,1} & M_{2,2} & ... & M_{2,657} \\ & & & \\ M_{13,1} & M_{13,2} & ... & M_{13,657} \end{bmatrix}$$

where the first sub-index j of each element denotes the velocity bin i and the second sub-index k the modeled brightness point. As an illustration, Fig.1 shows the lightcurves of the cometary dust for nine circular fields from 2 to 10 pixels, and models with projected velocities of 110, 160 and 300 m s^{-1}.

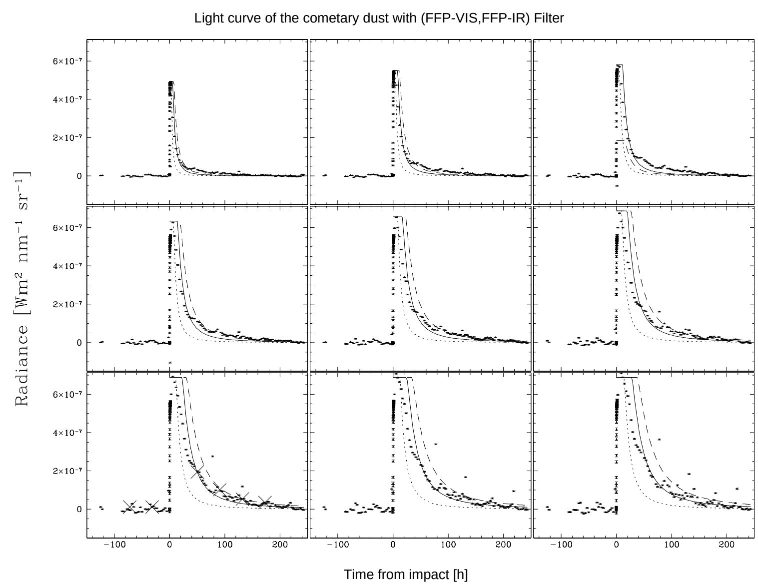

Fig. 1. Light curves of the cometary dust in the clear filter, for nine circular fields from 2 to 10 pixels (or from 3000 to 15000 km radius at the comet) from left to right. The dashed, solid and dotted lines show model profiles for velocities of 110, 160, and 300 m/s, respectively. The crosses on the bottom left panel are separated by 40.832 h, the rotation period of comet 9P/Tempel 1.

4 Approach

We now consider an approach to retrieve the velocity distribution of particles. Instead to simultaneously fit synthetic images to a selected set of observations (e.g. [5]), we simultaneously fit synthetic light-curves to the observed ones. In the earlier method, in order to clean the images from eventual residuals of cosmic rays and to improve the S/N ratio, new images should be created by stacking a set of successive images. This would decrease the temporal parameter sampling. Because we are extracting the brightness from each image, our method keeps the good OSIRIS temporal parameter sampling. Given the matrices O, E, and M, our focus is to find the curve or the most likely model, among all combination of simple models, that approximately fits the sets of observations, i.e., the observed brightness distribution of the cometary dust. Distribution of terminal velocities reached by the dust are derived using a maximum likelihood estimator and a least squares solution.

We fit simultaneously the observed fluxes O by minimizing the sum of the chi-squared differences between all the data-points O and linear combinations of the modeled fluxes M:

$$\chi^2 = \sum_{k=1}^{657} \frac{1}{\sigma_k^2} \left(\sum_{j=1}^{13} [a(j)]_k - O_k \right)^2 \qquad (1)$$

where $[a(j)]_k$ is defined as

$$[a(j)]_k = c_j M_{jk} \qquad (2)$$

and c, a set of positive dimensionless coefficients to be determined, as:

$$c = (c_1, c_2, ..., c_{13}). \qquad (3)$$

The minimization of the expression (1) occurs where the derivate with respect to all parameters c_j vanishes. The so-called normal equations of the least-squares problem can be expressed as:

$$\frac{\partial \chi^2}{\partial c_j} = 2 \sum_{k=1}^{657} \frac{1}{\sigma_k^2} \left(\sum_{j=1}^{13} c_j M_{jk} - O_k \right) M_{jk} = 0 \qquad (4)$$

Eq. 4 yields a linear system of 13 equations which are solved using the LAPACK library [6]. The solution corresponds to the deepest minimum of the chi-square function (1). There may be other solutions besides it which corresponds to local minima of the function (1) (with a similar likelihood). This means that our derived parameters c may be affected by large errors.

5 Results

In Fig. 2 we show the velocity distribution . The dust particles have velocities of ∼225 ms^{-1}. This velocity is in a good agreement with the projected speed values derived from other authors (e.g. [7] (200 ms^{-1}), [8] (125 ms^{-1}), [9] (130-230 ms^{-1}), [5] (190 ms^{-1})). Nevertheless, [10], from HST images, derive lower velocities (70-80 ms^{-1}).

Dust with a velocity of 200 ms^{-1} reaches the edge of the smallest aperture (2-pixel radius) in slightly more than 4 h. The acceleration phase is not seen, meaning that not more than a few percent of the impact material was produced later than 1 h after the impact.

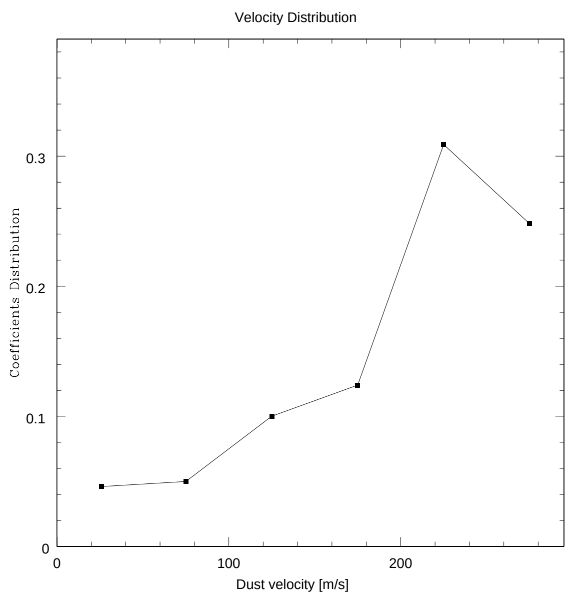

Fig. 2. Parameters c_j versus dust terminal velocity. The distribution peaks at 225 ms^{-1}.

6 Conclusion

We developed a discrete linear inverse problem approach which allows to simultaneously fit modeled light-curves to a set of observational light-curves of cometary dust of 9P/Tempel 1 acquired by NAC aboard Rosetta spacecraft. We derive a broad accurate velocity distribution of the dust particles (between 1 and 600 ms^{-1}). It peaks at 225 ms^{-1}. This value is in good agreement with already published estimates of the expansion velocity of the dust cloud. We consider these successful comparisons as an evidence of the appropriateness of the method.

The velocities of the dust seen in the light curve in the first minutes after the impact suggest early acceleration of the ejected material close to the cometary surface.

Despite the relative success of the models and approach presented in this work, a few points may be raised that need further improvement. In particular, the relative robustness of the method. Furthermore, because of the unstable character of the ill-conditioned solutions, further regularization algorithms for the stabilization of the solution should be investigated. Limits on late acceleration (and therefore in late production of material) also should be enquired, work is in progress.

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