

Looking the void in the eyes - the kSZ effect in LTB models

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Abstract. As an alternative explanation of the dimming of distant supernovae it has recently been advocated that we live in a special place in the Universe near the centre of a large void described by a Lemaitre-Tolman-Bondi (LTB) metric. The Universe is no longer homogeneous and isotropic and the apparent late time acceleration is actually a consequence of spatial gradients in the metric. If we did not live close to the centre of the void, we would have observed a Cosmic Microwave Background (CMB) dipole much larger than that allowed by observations. Hence, until now it has been argued, for the model to be consistent with observations, that by coincidence we happen to live very close to the centre of the void or we are moving towards it. However, even if we are at the centre of the void, we can observe distant galaxy clusters, which are off-centre. In their frame of reference there should be a large CMB dipole, which manifests itself observationally for us as a kinematic Sunyaev-Zeldovich (kSZ) effect. kSZ observations give far stronger constraints on the LTB model compared to other observational probes such as Type Ia Supernovae, the CMB, and baryon acoustic oscillations. We show that current observations of only 9 clusters with large error bars already rule out LTB models with void sizes greater than ~ 1.5 Gpc and a significant underdensity, and that near future kSZ surveys like the Atacama Cosmology Telescope (ACT), South Pole Telescope (SPT), APEX telescope, or the Planck satellite will be able to strongly rule out or confirm LTB models with giga parsec sized voids. On the other hand, if the LTB model is confirmed by observations, a kSZ survey gives a unique possibility of directly reconstructing the expansion rate and underdensity profile of the void.

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1. Introduction

Distant supernovae appear dimmer than expected in a matter-dominated homogeneous and isotropic FRW universe. The currently favoured explanation of this dimming is the late time acceleration of the universe due to a mysterious energy component that acts like a repulsive force. The nature of the so-called Dark Energy responsible for the apparent acceleration is completely unknown. Observations seem to suggest that it is similar to Einstein's cosmological constant, but there is inconclusive evidence. There has been a tremendous effort in the last few years to try to pin down deviations from a cosmological constant, e.g. with deep galaxy catalogues like 2dFGRS [1] and SDSS [2], and extensive supernovae surveys like ESSENCE [3], SNLS [4], and SDSS-SN [5], and many more are planned for the near future e.g. DES [6], PAU [7, 8], BOSS [9] and JDEM [10].

In the meantime, our realisation that the universe around us is far from homogeneous and isotropic has triggered the study of alternatives to this mysterious energy. Since the end of the nineties it has been suggested by various groups [11, 12, 13, 14, 15, 16, 17, 18, 19] that an isotropic but inhomogeneous Lemaître-Tolman-Bondi universe could also induce an apparent dimming of the light of distant supernovae, in this case due to local spatial gradients in the expansion rate and matter density, rather than due to late acceleration. It is just a matter of interpretation which mechanism is responsible for the dimming of the light we receive from those supernovae. Certainly the homogeneous and isotropic FRW model is more appealing from a philosophical point of view, but so was the static universe and we had to abandon it when the recession of galaxies was discovered at the beginning of last century.

There is nothing wrong or inconsistent with the possibility that we live close to the centre of a giga parsec scale void. Such a void may indeed have been observed as the CMB cold spot [20, 21, 22] and smaller voids have been seen in the local galaxy distribution [23, 24]. The size and depth of the distant observed voids, i.e. $r_0 \sim 2$ Gpc and $\Omega_M \sim 0.2$ within a flat Einstein-de Sitter universe, seems to be consistent with that in which we may happen to live [25], and could account for the supernovae dimming, together with the observed baryon acoustic oscillations and CMB acoustic peaks, the age of the universe, local rate of expansion, etc. [19].

Moreover, according to the theory of eternal inflation [26], rare fluctuations at the Planck boundary may be responsible for the non-perturbative amplification of local inhomogeneities in the metric, which would look like local voids in the matter distribution [27]. In the eternal inflation approach one assumes to be a typical observer whose local patch comes directly from a rare fluctuation within an inflationary domain at the Planck scale. Moreover, since it is a rare fluctuation it should be highly spherically symmetric, and the theory predicts that we should live close to the centre of such a void [27]. The size and depth of those voids depends on the theory of inflation on very large scales and may be a probe (perhaps the only probe) of the global structure of the universe.

In fact, observations suggest that if there is such a large void, we should live close to the centre, otherwise our anisotropic position in the void would be seen as a large dipole in the CMB. Of course, we do observe a dipole, but it is normally assumed to be due to the combined gravitational pull of the Virgo cluster, and the Shapley super cluster. There is always the possibility that we live off-centre and we are moving towards the centre of the void, so that the two effects are partially cancelled, giving rise to the observed dipole. However, such a coincidence could not happen for all galaxies in the void and, in general, clusters that are off-centred should see, in their frame of reference, a large CMB dipole. Such a dipole would manifest itself observationally for us as an apparent kinematic Sunyaev-Zeldovich effect for the given cluster.

It is the purpose of this paper to study the very strong constraints that present observations of the kSZ effect already put on the LTB void models, and predict how near future observations from kSZ surveys like ACT or SPT will strongly rule out (or confirm) LTB models with giga parsec sized voids.

In section 2 we describe the general LTB void models, giving the corresponding Einstein-Friedmann equations, as well as parameterisations of their solutions. In a subsection we describe the GBH constrained model, where we assume the Big Bang is homogenous, and thus the model depends on a single function, the inhomogeneous matter ratio $\Omega_M(r)$. In section 3 we study the induced dipole for off-centred clusters and compute the size of the analogue velocity of those clusters depending on the parameters of the void model. In section 4 we analyse present observations and give constraints on the model from current observations. In section 5 we then explore the prospects that future experiments like ACT, SPT and Planck will provide for strongly constraining or even ruling out LTB models of the universe. Finally, in section 6 we give our conclusions.

2. Lemaître-Tolman-Bondi void models

The Lemaître-Tolman-Bondi model describes general radially symmetric space-times and can be used as a toy model for describing voids in the universe. The metric is

$$ds^2 = -dt^2 + X^2(r, t) dr^2 + A^2(r, t) d\Omega^2, \quad (1)$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$. Assuming a spherically symmetric matter source with negligible pressure,

$$T_\nu^\mu = -\rho_M(r, t) \delta_0^\mu \delta_\nu^0, \quad (2)$$

the $(0, r)$ component of the Einstein equations, $G_r^0 = 0$, implies $X(r, t) = A'(r, t)/\sqrt{1 - k(r)}$, with an arbitrary function $k(r)$ playing the role of the spatial curvature parameter. The other components of the Einstein equations read [28, 16, 19]

$$\frac{\dot{A}^2 + k}{A^2} + 2\frac{\dot{A}\dot{A}'}{AA'} + \frac{k'(r)}{AA'} = 8\pi G \rho_M, \quad (3)$$

$$\dot{A}^2 + 2A\ddot{A} + k(r) = 0. \quad (4)$$

Integrating the last equation, we get

$$\frac{\dot{A}^2}{A^2} = \frac{F(r)}{A^3} - \frac{k(r)}{A^2}, \quad (5)$$

with another arbitrary function $F(r)$, playing the role of effective matter content, which substituted into the first equation gives

$$\frac{F'(r)}{A'A^2(r,t)} = 8\pi G \rho_M(r,t). \quad (6)$$

We can also use Eq. (3) to define the critical density as

$$\frac{\dot{A}^2}{A^2} + 2\frac{\dot{A}\dot{A}'}{AA'} = 8\pi G \rho_C(r,t). \quad (7)$$

The boundary condition functions $F(r)$ and $k(r)$ are specified by the nature of the inhomogeneities through the local Hubble rate, the local total energy density and the local spatial curvature,

$$H(r,t) = \frac{\dot{A}(r,t)}{A(r,t)}, \quad (8)$$

$$F(r) = H_0^2(r) \Omega_M(r) A_0^3(r), \quad (9)$$

$$k(r) = H_0^2(r) (\Omega_M(r) - 1) A_0^2(r), \quad (10)$$

where functions with subscripts 0 correspond to present day values, $A_0(r) = A(r, t_0)$ and $H_0(r) = H(r, t_0)$. With these definitions, the r -dependent Hubble rate is written as [28, 16]

$$H^2(r,t) = H_0^2(r) \left[\Omega_M(r) \left(\frac{A_0(r)}{A(r,t)} \right)^3 + (1 - \Omega_M(r)) \left(\frac{A_0(r)}{A(r,t)} \right)^2 \right], \quad (11)$$

and we fix the gauge by setting $A_0(r) = r$.

For light travelling along radial null geodesics, $ds^2 = d\Omega^2 = 0$, we have

$$\frac{dt}{dr} = \mp \frac{A'(r,t)}{\sqrt{1-k(r)}} \quad (12)$$

which, together with the redshift equation,

$$\frac{d \log(1+z)}{dr} = \pm \frac{\dot{A}'(r,t)}{\sqrt{1-k(r)}} \quad (13)$$

can be written as a parametric set of differential equations, with $N = \log(1+z)$ being the effective number of e-folds before the present time,

$$\frac{dt}{dN} = -\frac{A'(r,t)}{\dot{A}'(r,t)}, \quad (14)$$

$$\frac{dr}{dN} = \pm \frac{\sqrt{1-k(r)}}{\dot{A}'(r,t)} \quad (15)$$

2.1. The constrained GBH model

In general LTB models are uniquely specified by the two functions $k(r)$ and $F(r)$ or equivalently by $H_0(r)$ and $\Omega_M(r)$, but to test them against data we have to parameterise the functions, to reduce the degrees of freedom to a discrete set of parameters. For simplicity in this paper we will use the constrained GBH model [19] to describe the void profile. First of all, it uses a minimum set of parameters to make a realistic void profile, and secondly, it is assumed that the time to the Big Bang is constant for spatial slices. The second condition gives a relation between $H_0(r)$ and $\Omega_M(r)$, and hence constrain the models to one free function, and a proportionality constant describing the overall expansion rate. Our chosen model is thus given by

$$\Omega_M(r) = \Omega_{\text{out}} + \left(\Omega_{\text{in}} - \Omega_{\text{out}} \right) \left(\frac{1 - \tanh[(r - r_0)/2\Delta r]}{1 + \tanh[r_0/2\Delta r]} \right) \quad (16)$$

$$H_0(r) = H_0 \left[\frac{1}{\Omega_K(r)} - \frac{\Omega_M(r)}{\sqrt{\Omega_K^3(r)}} \sinh^{-1} \sqrt{\frac{\Omega_K(r)}{\Omega_M(r)}} \right] = H_0 \sum_{n=0}^{\infty} \frac{2[\Omega_K(r)]^n}{(2n+1)(2n+3)}, \quad (17)$$

where $\Omega_K(r) = 1 - \Omega_M(r)$, and the second equation follows from the requirement of a constant time to a homogeneous Big Bang. We use an ‘‘inflationary prior’’, and assume that space is asymptotically flat, i.e. in the following we set $\Omega_{\text{out}} = 1$. The model has then only four free parameters: The overall expansion rate H_0 , the underdensity at the centre of the void Ω_{in} , the size of the void r_0 , and the transition width of the void profile Δr . For more details on the model see Ref. [19].

3. The CMB sky seen by off-centre observers

Imagine a cluster well embedded in a big void, but not exactly at the centre, and consider photons reaching the cluster from the last scattering surface (LSS). Because of the symmetry of the problem the smallest and largest redshifts are found along the radial direction. There are two effects contributing to the redshift of photons passing through the void: The dominant effect is caused by the higher expansion rate inside the void (see Eq. (14)). The photons coming from the farthest end of the void, crossing the centre, are inside the void for the longest time, and thus have the biggest redshift, while the photons arriving directly from the LSS along a radial geodesic will be affected for the shortest amount of time, and consequently suffer the least redshift. There is also a subdominant effect due to the change in the gravitational potential or the matter density: Photons coming from the farthest end of the void are first gravitationally redshifted when entering the void, and then gravitationally blueshifted after crossing the centre. On the other hand, photons arriving directly from infinity are only redshifted. There is a difference in the two redshifts, because of the time dependence of the underdensity, and hence the gravitational potential, with the subdominating effect leading to a small blueshift towards the centre. This effect is in some sense a large scale Rees-Sciama effect [29], see Fig. 1.

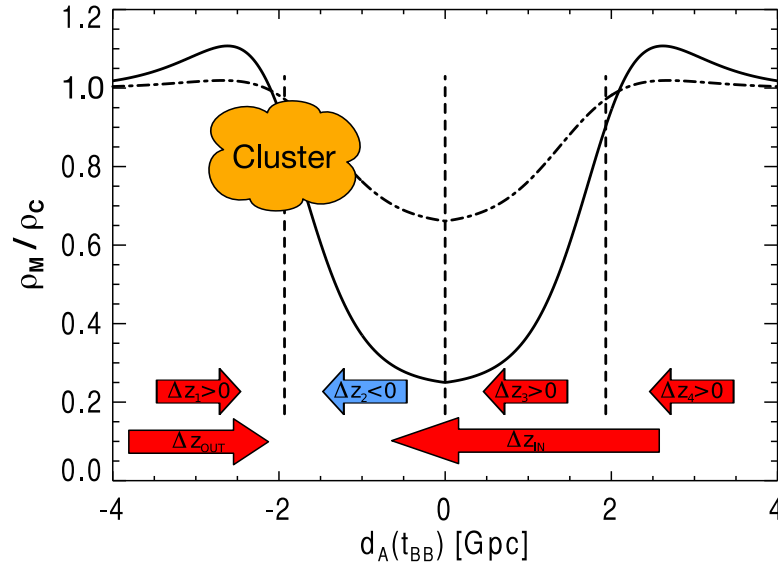


Figure 1. An off-centre cluster of galaxies in a void will “observe” CMB photons coming from the last scattering surface from all directions. Due to the higher expansion rate inside the void, photons arriving through the centre (from the right in the figure) will have a larger redshift (Δz_{in}), than photons arriving directly from the LSS (left, with Δz_{out}). There is a subdominant effect due to the time-dependent density profile (the solid line corresponds to the current time, while the dot-dashed line to one tenth of the present time). With a larger underdensity at later times, we have $\Delta z_1 > \Delta z_4$, and $\Delta z_2 + \Delta z_3 < 0$, giving an overall difference $\Delta z_1 > \Delta z_2 + \Delta z_3 + \Delta z_4$ or, equivalently, a subdominant dipole with a blueshift towards the centre of the void. The overall effect is a blueshift away from the centre.

Consequently, in the ideal case of a spherical void, and a well embedded cluster, the cluster observer will see an almost perfect dipole in the CMB, aligned along the radial direction, and with the blueshift pointing away from the centre of the void, where the observer is (see Fig. 1). The detailed effect of a spherical void on the CMB sky of an off-centre observer has been calculated in [30], and it is shown that for small distances from the centre of the void the dominating term is a dipole. For simplicity, in this paper we will estimate the change on the CMB sky as a pure dipole. To find the amplitude it is then enough to integrate two radial light rays one going towards the centre, and the other away from the centre of the void. The calculation of the grid of parameters is done using our public `easyLTB` program [31], but we have also used the completely different approach of Taylor expansion around an Einstein de Sitter solution, as detailed in [19], and checked that we get the same result for the parameters in Fig. 2. It should be noted that this dipole approximation breaks down when the effective size of the void on the sky, as observed from the cluster, becomes too small (i.e. less than $\sim 2\pi$), or possibly when the density or Hubble expansion profiles have very contrived time dependence.

Motivated by the observed size of our CMB dipole, we assume that we are located at the centre of the void, and observe clusters in the light cone at different redshifts. Each cluster is then observed at a certain time $t_{cl}(z)$, related to its redshift. In order

to find the dipole as seen by the cluster, we integrate Eq. (14) along the radial axis in the positive and negative direction, and always backwards in time, with a starting time $t_{cl}(z)$, and an ending time $t_{LSS} \sim 10^5$ yr. The size of the dipole can be easily calculated from the temperature seen by observers in the cluster in different directions, $T(\theta) = T_*/(1+z(\theta))$, where T_* is the temperature of the LSS. If we now look in opposite directions, towards and away from the centre of the void, we find

$$\frac{\Delta T}{T}_{\text{dipole}} = \frac{T(\theta) - \hat{T}}{\hat{T}} = \frac{|T_{\text{in}} - T_{\text{out}}|}{T_{\text{in}} + T_{\text{out}}} = \frac{|z_{\text{in}} - z_{\text{out}}|}{2 + z_{\text{in}} + z_{\text{out}}}, \quad (18)$$

where $\hat{T} = (T_{\text{in}} + T_{\text{out}})/2$ is the mean temperature observed at the location of the cluster, and $z_{\text{in/out}}$ are the redshifts to the LSS for radially ingoing/outgoing light rays.

3.1. The kinematic Sunyaev-Zeldovich effect

The hot gas (mainly the electrons) inside a cluster inverse Compton scatters CMB photons, changing their frequency distribution. The thermal Sunyaev-Zeldovich effect [32, 33] is the main effect, redistributing low energy photons to higher energies due to upscattering by the hot cluster gas, but also the intra-cluster gas works as a mirror rescattering photons from all directions towards the observer. If the CMB sky observed by the cluster is different than the CMB sky observed by us, there will be an additional change in the spectrum, which is known as the kinematic Sunyaev-Zeldovich effect [34]. Because of rotational symmetry around the axis associated with the line of sight, only changes projected along this axis will have an observational impact. In the LTB model, because the observer is supposed to be at the centre, an off-centred cluster will observe a CMB dipole exactly along the line of sight, which to first order in the perturbation is indistinguishable from a kinematic dipole due to the peculiar velocity of the cluster. The change in the photon intensity is given as [34, 35, 36]

$$\frac{\Delta I_\nu}{I_\nu} = -\frac{x e^x}{e^x - 1} \int \sigma_T n_e \frac{\mathbf{v}_p}{c} \cdot d\mathbf{l}, \quad (19)$$

and thus

$$\frac{\Delta T_{\text{kSZ}}}{T_{\text{rad}}} = -\frac{x^2 e^x}{(e^x - 1)^2} \int \sigma_T n_e \frac{\mathbf{v}_p}{c} \cdot d\mathbf{l} \simeq -\beta_p \tau_e \frac{x^2 e^x}{(e^x - 1)^2}, \quad (20)$$

where $x = h\nu/kT$, τ_e is the cluster optical depth, and the apparent peculiar velocity is related to the temperature dipole as

$$\beta_p = \frac{v_p}{c} = \frac{\Delta T}{T}_{\text{dipole}}. \quad (21)$$

Even though the kSZ signal from a single cluster can be interpreted as a peculiar velocity, a large void would result in a systematic trend for all clusters (with a certain scatter given by the intrinsic peculiar velocities of the clusters), with an average redshift or positive apparent velocity (see Fig. 2). The average velocity as a function of the distance would give a direct and unique handle on the density and velocity profile of the void. At the same time, the absence of such a systematic average peculiar velocity in future kSZ surveys can strongly rule out a void of any appreciable size.

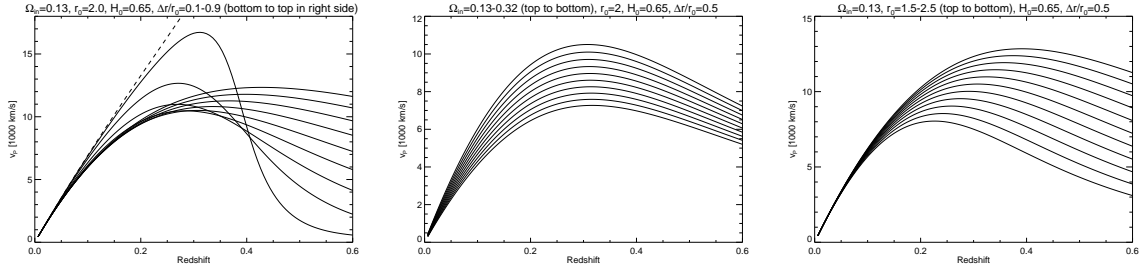


Figure 2. Examples of the size of the dipole for different parameters of the constrained GBH model [19]. The dashed line in the left figure is the first order approximation given in [30].

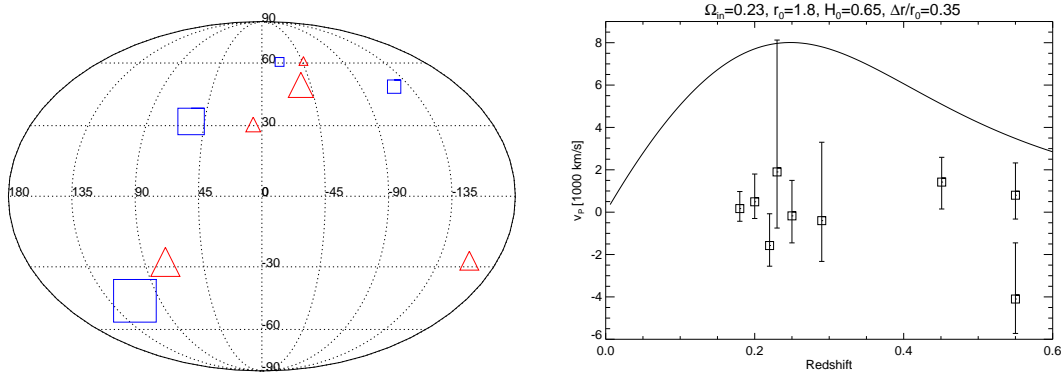


Figure 3. The angular and redshift distribution of current observations together with the predicted dipole distribution for a giga parsec sized void model. Red triangles and blue squares represent positive and negative peculiar velocities respectively, with the size of the symbol indicating the magnitude of the velocities.

4. Constraints from current observations

We have compiled a set of 9 clusters with kSZ measurements from [37, 38, 39] (see table 1 and Fig. 3). Currently both systematic and random errors are very large, but even though observations are scarce they already give very interesting bounds on the size of a possible void.

We have used the observations to compare with the constrained GBH model, making a grid of models with different parameters (see table 3), and calculated the likelihood of each model. From the literature we only have the (asymmetric) 68% confidence limits for the observations, and not the full probability distributions. To respect the asymmetric error bars, while keeping the probability distribution close to Gaussian, we use the ϵ -skew-normal (ϵ SN) distribution [40], which has the probability density function (PDF)

$$f(v; v_i, \sigma^-, \sigma^+) = \frac{1}{\sqrt{2\pi} [\sigma^- + \sigma^+]} \exp \left[-\frac{[v - v_i]^2}{2 \sigma^{\pm 2}} \right], \quad (22)$$

where $\sigma^\pm = \sigma^-$ when $v \leq v_i$, and $\sigma^\pm = \sigma^+$ when $v > v_i$, to model the individual data points. The ϵ SN model is a reasonable model for the data, if we do not have knowledge about the detailed likelihood for each data point, since the probability distribution is continuous up to the first derivative, the maximum likelihood is at v_i , and 68% of the PDF is in the interval $[v_i - \sigma^-, v_i + \sigma^+]$, as required by the data. The mean value is not at v_i reflecting the fact that the distribution is skewed.

When comparing our models to the observations we not only have to take into account measurement errors as given in the literature but also the fact that, if the model is correct, the apparent peculiar velocities of the clusters are the sums of the apparent dipole velocities from the void, and the intrinsic peculiar velocities. In the following we assume that the clusters, due to their high redshift, have uncorrelated intrinsic velocities, though for future observations of small fields with many clusters this may not be true (see e.g. [41, 42, 43]). The peculiar velocities are assumed to be normal distributed, and are added in squares to the measurement errors.

For the full sample (see table 2 for sub samples) the average cluster velocity, excluding any systematic shift, and correctly weighted[‡] is $\bar{v} = 320_{-400}^{+440}$ km s⁻¹, while the standard deviation among the cluster velocities is $\sigma_v = 1630_{-350}^{+400}$ km s⁻¹. This is a fairly large velocity scatter; much larger than expected from linear theory. Given that the clusters in current kSZ surveys are among the most massive clusters known, we would expect radial peculiar velocities on the order of ~ 400 km s⁻¹ from linear theory [43], which is a factor of 4 lower than what is found from the internal scatter in our data set.

To estimate the typical peculiar velocity scatter σ_{pv} we will use two different values: The average velocity as expected in a Λ CDM model, fixed at 400 km s⁻¹, and the scatter of the current data set, fixed at 1600 km s⁻¹. We could include evolution in the velocities – according to linear theory they grow with the growth factor – but since current data have very large errors, it would not make much of a difference.

Summarising, the total log-likelihood for a given model is

$$-2 \ln \mathcal{L} = \sum_i \left[\frac{v_i - \alpha v(\vec{p}, z_i) + v_{\text{sys}}}{[\sigma_i^{\pm 2} + \sigma_{pv}^2]^{1/2}} \right]^2, \quad (23)$$

where v_i is the observed value of the velocity, $v(\vec{p}, z_i)$ is the value according to the model, with parameters \vec{p} , and σ^\pm is σ^+ if $v > v_i$ and else σ^- . According to [38] there can be significant systematic errors in the data, and we allow for this by adding a systematic shift in the values: v_{sys} . Below we use the systematic shifts $v_{\text{sys}} = 0$, and $v_{\text{sys}} = 750$ km s⁻¹. The size of the dipole is slightly overestimated by our model. First of all because the effect of the void on the CMB sky is not a pure dipole, and second because at large distances the projected size of the void on the sky of the cluster becomes less than 2π ,

[‡] We use the ϵ SN distribution for each data point, and weight the data according to $w_i = 1/\sigma^2$, where the average standard deviation is the harmonic mean of the error bars: $2/\sigma = 1/\sigma^- + 1/\sigma^+$. Using these weights the PDF of the average velocity is found as $f_{\bar{v}}(v) = \sum_i w_i f_i(v) / \sum_i w_i$, while the standard deviation is found by Monte-Carlo sampling of the data distributions: $\sigma_v = \langle \sum_i w_i (v_i - \bar{v})^2 / \sum_i w_i \rangle$, with v_i drawn on random from $f_i(v)$.

Name	Redshift	Pec vel [km s ⁻¹]	Syst. Error	Galactic l	Galactic b [$^{\circ}$]
A1689 [37]	0.18	+170 ⁺⁸¹⁵ ₋₆₃₀	750	313.39	61.10
A2163 [37]	0.20	+490 ⁺¹³⁷⁰ ₋₈₈₀	750	6.75	30.52
A2261 [38]	0.22	-1575 ⁺¹⁵⁰⁰ ₋₉₇₅	750	55.61	31.86
A2390 [38]	0.23	+1900 ⁺⁶²²⁵ ₋₂₆₅₀	750	73.93	-27.83
A1835 [38]	0.25	-175 ⁺¹⁶⁷⁵ ₋₁₂₇₅	750	340.38	60.59
Zw 3146 [38]	0.29	-400 ⁺³⁷⁰⁰ ₋₁₉₂₅	750	239.39	47.96
RX J1347-1145 [39]	0.45	1420 ⁺¹¹⁷⁰ ₋₁₂₇₀	?	324.04	48.81
Cl 0016+16 [38]	0.55	-4100 ⁺²⁶⁵⁰ ₋₁₆₂₅	750	201.5	-27.32
MS 0451 [38]	0.55	490 ⁺¹³⁷⁰ ₋₈₈₀	750	112.55	-45.54

Table 1. Clusters with observed velocities from the literature. The given errors are at the 1- σ or 68% confidence level. The systematic error is estimated in [38], and is mostly due to confusion with primary CMB anisotropies, and contributions from sub-mm point sources. These error sources are not instrument dependent, and we add the same error to the observations in [37] and [39] too.

Sample	\bar{v} [km s ⁻¹]	σ_v	#clusters
All clusters	320 ⁺⁴⁴⁰ ₋₄₀₀	1630 ⁺⁴⁰⁰ ₋₃₅₀	9
Cluster in [37, 38]	190 ⁺⁴⁸⁰ ₋₄₃₀	1630 ⁺⁴⁴⁰ ₋₃₇₀	8
Clusters in [38]	-250 ⁺⁷⁵⁰ ₋₆₆₀	2210 ⁺⁷¹⁰ ₋₅₈₀	6

Table 2. Statistical properties of sub-samples, and the full sample of clusters. The quoted values are in good agreement with what is found in [38].

and then the dipole part is even less. To model it we include an empirical rescaling factor $0 < \alpha \leq 1$, and to estimate the effect error we use $\alpha = 1$ and $\alpha = 0.8$.

The two choices of each of three free parameters leave us with 8 different likelihoods to explore. The two dimensional contours are shown in Fig. 4. As can be seen in the lower right panel, a void with a central underdensity of $\Omega_{\text{in}} = 0.23$ (just inside the 3- σ limit allowed by other observations, see Fig. 9 and Ref. [19]), cannot be bigger than 1.5 Gpc at the 3- σ limit. This is the best-case, allowing for generous errors in our modelling, an unrealistic large scatter in the peculiar velocities, and a favourable systematic shift due to an incomplete analysis of observations. Incidentally, the 3- σ limit for the size of the void, allowed by other data, is 1.45 Gpc [19]. For the constrained LTB model, interpreting the current kSZ cluster data in the most favourable way, we therefore still have at least a 3- σ inconsistency between observations of the geometry of the universe (Supernovae, BAO, and 1st peak in the CMB), and observations of the kSZ in clusters of galaxies.

On the other hand, if we use the strictest interpretation of the model (upper left

H_0	H_{in}	H_{out}	Ω_{in}	r_0	Δr
100 km s ⁻¹ Mpc ⁻¹				Gpc	r_0
0.65	0.44 – 0.57	0.43	0.13 – 0.93	0.1 – 2.5	0.1 – 0.9

Table 3. Priors used when scanning the parameters of the GBH models. H_0 is a pre factor for $H_0(r)$ and H_{in} and H_{out} are derived from the priors on Ω_{in} and H_0 .

panel in Fig. 4), then even a void with a slight central underdensity of $\Omega_{\text{in}} = 0.7$ is limited to 1.5 Gpc at the 3- σ level, or equivalently a void with a central underdensity of $\Omega_{\text{in}} < 0.5$ have to be less than 1.25 Gpc in size.

For completeness, in Fig. 5 we also show the equivalent one-dimensional likelihoods. However, we caution against overinterpreting them: current data gives a non-detection of a local void with four clusters moving towards us and five moving away, and the two-dimensional likelihood contours in the r_0 - Ω_{in} plane are strongly degenerate. Hence, the reduced one-dimensional contours give unreasonably large confidence limits, and we refrain from quoting any general bounds on single parameters, but conclude that measurements of the kSZ effect in clusters is by far the most constraining way to limit the size of a local void described by an LTB metric.

5. Future Experiments

In this section we explore the limits on the LTB models placed by future experiments like the South Pole Telescope [44], the Atacama Cosmology Telescope [45], the APEX telescope [46], or the Planck satellite [47]. Below we use parameters for the ACT, but our results apply equally well to any of the four surveys. Our basic hypothesis in this section is that the Universe is homogeneous, and we show that giga parsec sized voids with a significant underdensity are strongly ruled out, even with a few observed galaxy clusters.

The first generation of large scale kSZ survey is characterised by a limited number of frequency bands (ACT, APEX and SPT), or by low angular resolution (Planck), and even though the resulting cluster catalogues will be a gold mine for studying the thermal SZ signal, it is much harder to exploit the full potential of the kinematic SZ for a number of reasons: the intensity dependence is similar to that of the primordial CMB, point sources can contaminate the fluxes, giving a need for excellent resolution, and finally what is really measured is the relative dip (or bump) in the frequency due to the integrated (Thomson) scattering of photons. That is, what is measured directly is the momentum of the cluster gas with respect to the CMB, not the peculiar velocity itself, see Eq. (19). If only three (ACT) or four (SPT) frequency bands are available then an estimate of the total gas mass, and the internal velocity dispersion, are needed in order to extract the peculiar velocity. The gas density can be estimated using X-rays together with thermal SZ measurements, or alternatively using weak lensing together with extensive modelling, while many precise redshifts of cluster galaxies in each cluster

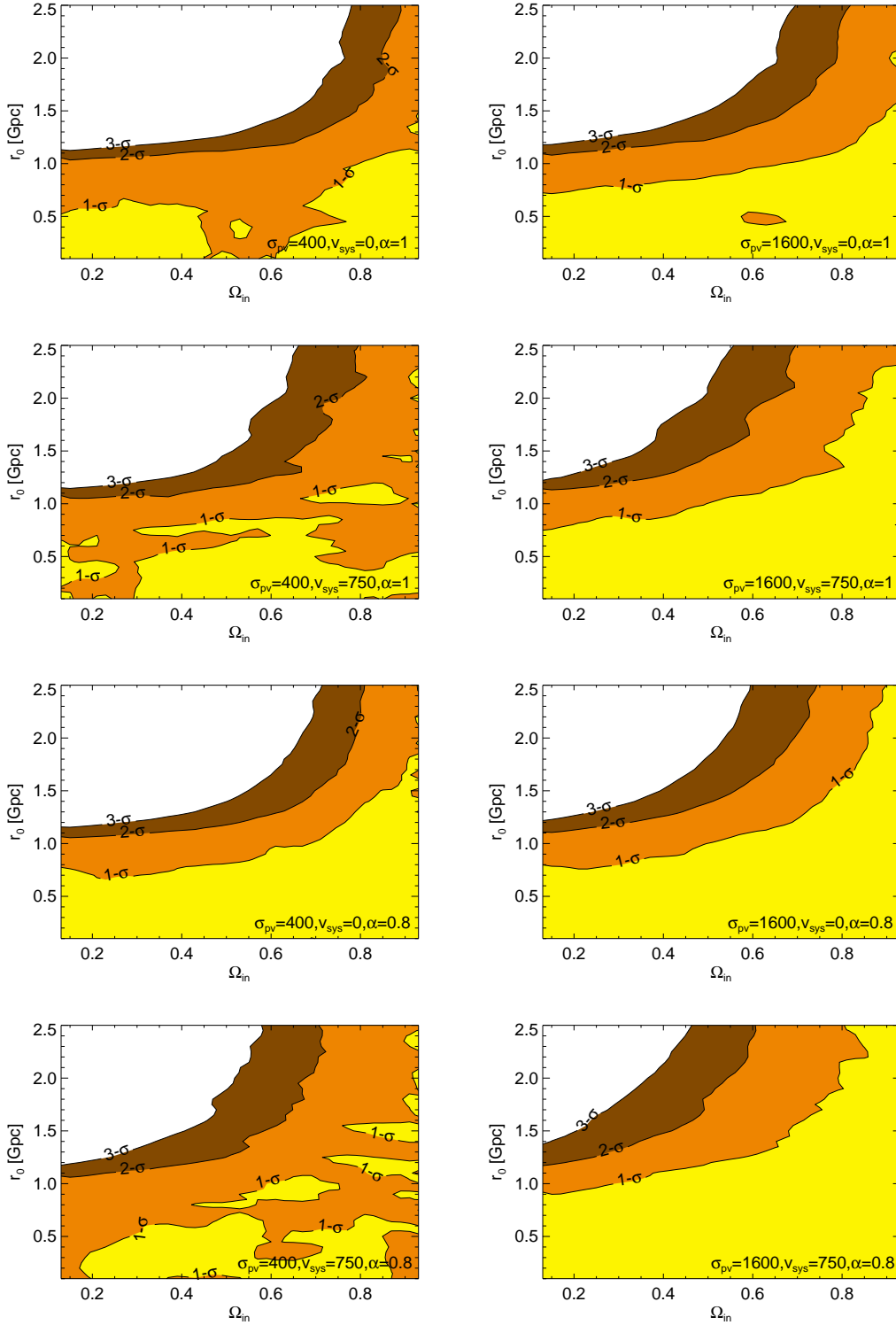


Figure 4. Likelihood contours for current observed clusters with 1- σ to 3- σ regions given by bright to dark yellow. From left to right, top to bottom we have $\sigma_{pv} = (400, 1600)$ km s $^{-1}$, $v_{sys} = (0, -750)$ km s $^{-1}$, and $\alpha = (1, 0.8)$.

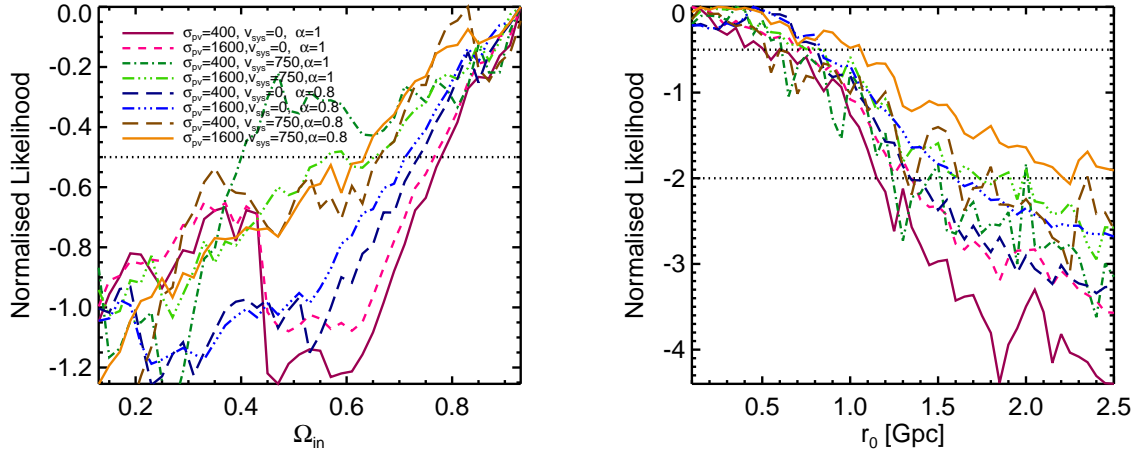


Figure 5. One dimensional likelihoods for current observed clusters with 1- σ and 2- σ levels indicated by dotted lines. Velocities are in km/s.

are needed to constrain the internal velocity dispersion [48]. The bottom line is that even though 10^3 - 10^4 clusters will be detected in the first generation of large scale SZ surveys we can only constrain the peculiar velocities of those clusters that are well measured by other means, severely limiting the clusters that can be explored, to those at lower redshifts ($z \lesssim 0.4$). Hence the number of clusters with kSZ measurements will be close to the number of clusters which are well observed in X-rays, i.e. 10^2 - 10^3 clusters.

To construct a mock survey we model the cluster PDF as

$$n(z) \propto \left(\frac{z}{z_{\max}}\right)^2 \exp\left[-\left(\frac{z}{z_{\max}}\right)^{5/3}\right] \exp\left[-\left(\frac{z}{z_X}\right)^4\right], \quad (24)$$

in accordance with the cluster density given in [44] if we set $z_{\max} = 0.6$, see Fig. 6. The last term is an exponential redshift cut-off, due to the limited reach of X-ray observations, and we set $z_X = 0.4$. Notice that the detailed form of the cluster PDF is not so important because X-ray observations mostly sample redshifts lower than the maximum where the volume density is nearly constant, and the redshift density then goes like z^2 .

We use a similar likelihood function for our mock survey as the one in Eq. (23)

$$-2 \ln \mathcal{L} = \sum_i \left[\frac{v_i - \alpha v(\vec{p}, z_i) + v_{\text{sys}}}{[\sigma_i^2 + \sigma_{\text{pv}}^2]^{1/2}} \right]^2, \quad (25)$$

where the only difference is that we assume Gaussian errors for our observations. Due to uncertainties in the internal velocity scatter in the cluster gas, confusion with the primordial CMB and confusion with point sources in any future experiment it will be very difficult to reduce the uncertainty in the peculiar velocity σ_i to less than ~ 100 km s^{-1} [48]. In the first generation experiments it will be hard to get an uncertainty $\sigma_i \lesssim 500$ km s^{-1} , due to the limited number of frequency bands, uncertainties in measuring the

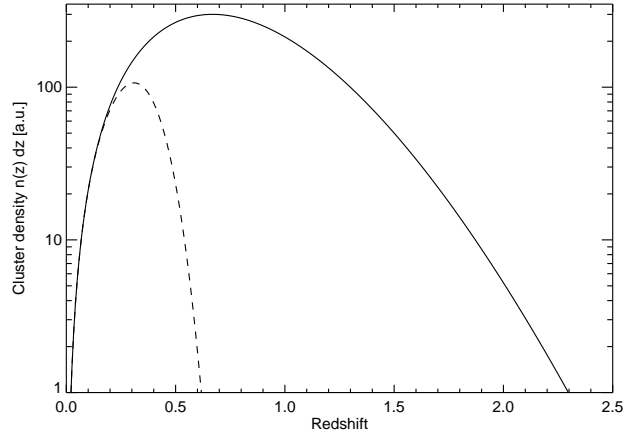


Figure 6. Cluster redshift number density used to select the redshift distribution for forecasting future surveys. The dashed line is the number density folded with the X-ray cut-off.

gas mass by other means, and incomplete knowledge of point sources in the clusters [48]. Below we will consider a best-case scenario of $\sigma_i = 400 \text{ km s}^{-1}$, and a worst case of $\sigma_i = 800 \text{ km s}^{-1}$. For simplicity we will fix the rescaling of the modelled velocities due to uncertainties in the theoretical model to $\alpha = 0.8$, and we will consider systematic errors of $v_{\text{sys}} = 0$ and 400 km s^{-1} , while the scatter in the peculiar velocities is fixed as above to $\sigma_{\text{pv}} = 400$ and 1600 km s^{-1} . These are reasonable choices that covers the probable spread in both the observable and the theoretically motivated parameters in Eq. (25).

We use Monte Carlo modelling to sample the PDF of the cluster redshift distribution Eq. (24), and we assume that the clusters have normal distributed peculiar velocities $v_i \sim \mathcal{N}(0, \sigma_{\text{pv}})$. In Figs. 7 and 8 are shown the 2D likelihoods in the cases were we have 10 and 100 clusters with good kSZ measurements. As can be seen already with 10 clusters, strong bounds can be put on the LTB model, while with 100 clusters, assuming the FRW model is correct, giga parsec sized voids are essentially ruled out. Complementarily we have shown that, if the measurements have systematic errors at the 400 km s^{-1} level, a spurious positive detection of a very shallow underdensity ($\Omega_{\text{in}} \lesssim 1$) could be inferred. Conversely, care should be taken when constraining LTB models with kSZ observations where systematic errors have been included self-consistently in the analysis, under the assumption of a homogeneous universe, by fixing the average peculiar velocity to zero, as advocated in Ref. [48].

6. Discussion and conclusions

As an alternative explanation of the dimming of distant supernovae it has recently been advocated that we live in a special place in the Universe near the centre of a large void. The universe is no longer homogeneous and isotropic and the apparent late time

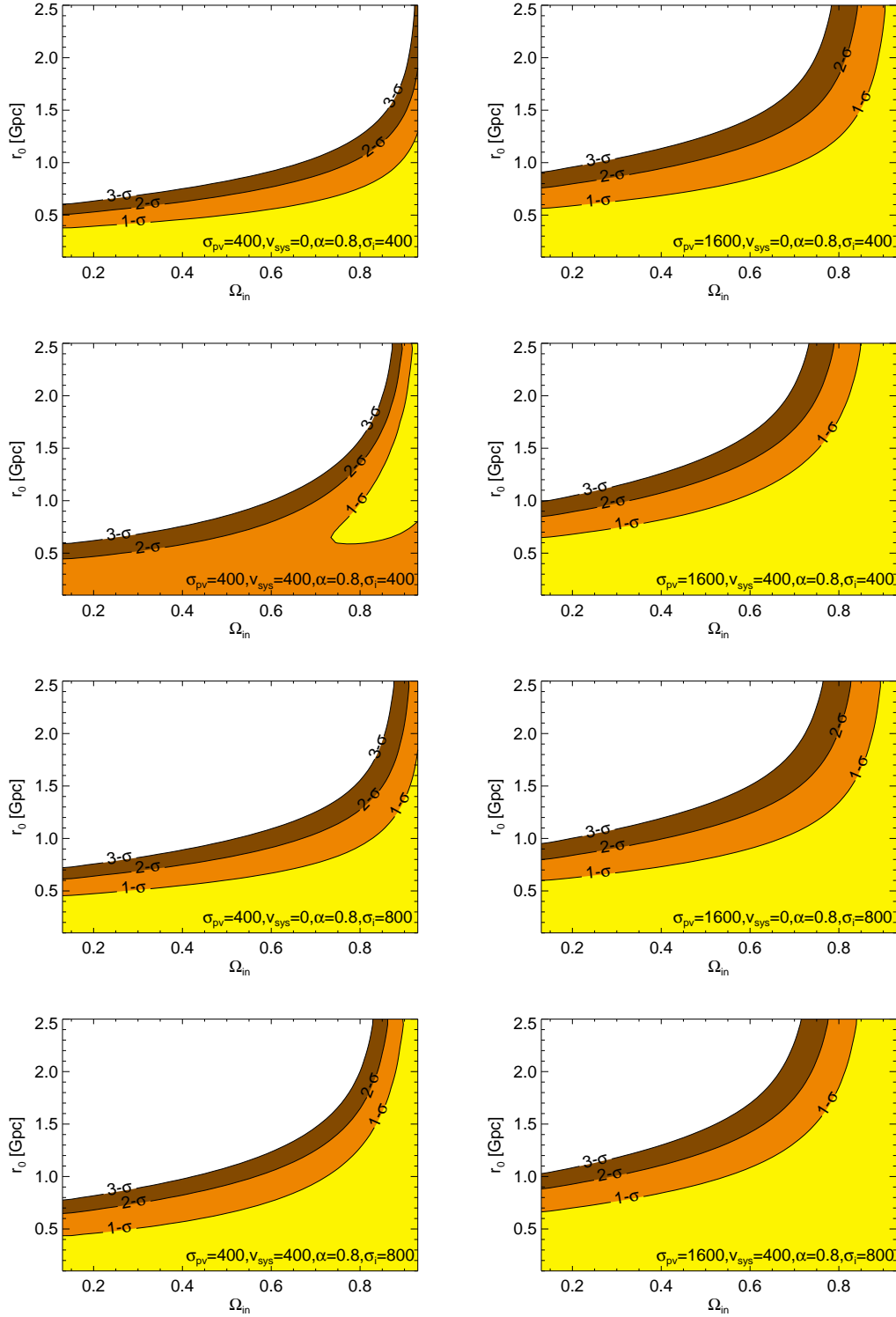


Figure 7. Likelihood contours for a mock survey with 10 well observed clusters with 1- σ to 3- σ regions given by bright to dark yellow. Also indicated in the individual plots are the parameters used for the simulation. Velocities are given in km s^{-1} .

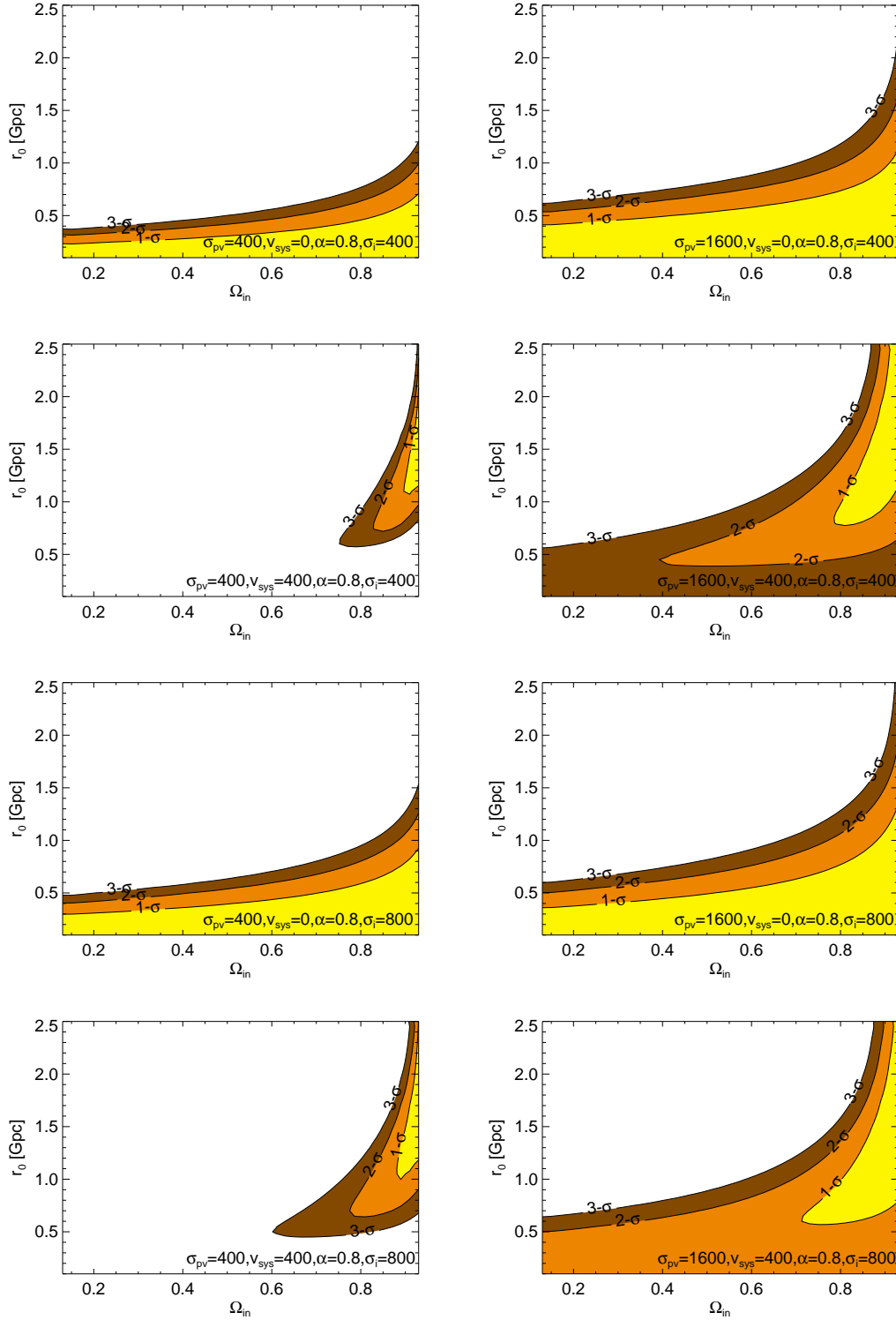


Figure 8. Same as in Fig. 7 but for a mock survey with 100 well observed clusters.

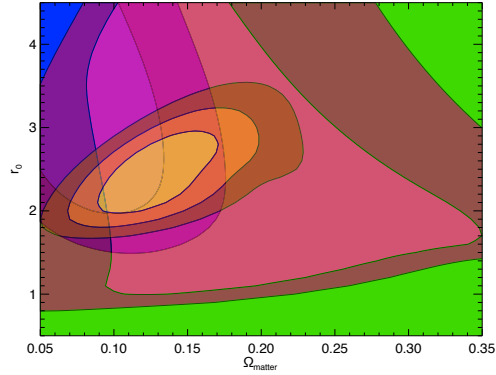


Figure 9. Likelihoods for the constrained model for complementary data. The combined data sets of Supernovae Type Ia (SNIa), BAO and WMAP are shown in yellow with 1-, 2-, and 3- σ contours, while the individual SNIa, BAO, and CMB data sets are shown in blue, purple, and green respectively with 1- and 2- σ contours. The figure is taken from [19].

acceleration is actually a consequence of spatial gradients in the metric. If we did not live close to the centre of the void, we would have observed a CMB dipole much larger than that allowed by observations. Hence, until now it has been argued, for the model to be consistent with observations, that by coincidence we happen to live very close to the centre of the void. However, even if we are at the centre of the void, we can observe distant galaxy clusters, which are off-centre. In their frame of reference there should be a large CMB dipole, which manifests itself observationally for us as a kinematic Sunyaev-Zeldovich effect.

In this paper we have studied the induced dipole for off-centred clusters due to the different trajectories of photons from the last scattering surface, and computed the size of the corresponding apparent velocity of those clusters with respect to the rest frame of the CMB LSS, depending on the parameters of the void model. We then analysed the present observations of the kSZ effect in a handful of clusters and gave very strong constraints on the size of the void in LTB models. In fact, for our specific constrained-GBH model the bounds are in conflict with other constraints from supernovae, baryon acoustic oscillations and CMB, at least at the 3- σ level, see Fig. 9, and therefore we conclude that the constrained-GBH void model is practically ruled out if the current interpretation of kSZ observations are correct.

At present, kSZ data leave small voids ($r_0 \lesssim 800$ Mpc) unconstrained, independently of the inner density contrast (Ω_{in}) used, simply because the radius is so small that the void does not impact the cluster with the lowest redshift in the sample, see Table 1. In order to put limits on small voids with large density contrasts, or sudden transitions, we would need to use local large scale structure data that can be checked for homogeneity, or alternatively low redshift supernovae.

Current kSZ observations are limited in numbers, and are still not precise enough

to make a single positive detection of the peculiar velocity of a cluster at the $2\text{-}\sigma$ level, see Fig. 3. Hence, one cannot rule out that they are plagued by large systematic and/or random errors. This is about to change in the coming years, and we have made predictions of how systematic near-future observations from kSZ surveys like ACT or SPT could strongly rule out all LTB models with giga parsec sized voids. In the case that the LTB models were confirmed, the average apparent velocity profile as a function of distance would give a direct handle on the density and expansion rate of the void. This unique relationship makes by far kSZ observations the most powerful data for constraining LTB models.

If the sensitivity of experiments like ACT, SPT and Planck are as planned, and the present systematic errors are under control, it is expected that large voids will definitely be ruled out at many sigma. If these experiments yield 10 (100) well observed clusters we could reasonably expect to rule out voids of 800 (500) Mpc radius at the $3\text{-}\sigma$ level. We hope to come back to this severe challenge to LTB models and redo the analysis with the new data in the near future.

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