Conditions for large non-Gaussianity in two-field slow-roll inflation

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We study the level of primordial non-Gaussianity in slow-roll two-field inflation. Using an analytic formula for the nonlinear parameter $f_{\rm NL}$ in the case of a sum or product separable potential, we find that it is possible to generate significant non-Gaussianity even during slow-roll inflation with Gaussian perturbations at Hubble exit. In this paper we give the general conditions to obtain large non-Gaussianity and calculate the level of fine-tuning required to obtain this. We present explicit models in which the non-Gaussianity at the end of inflation can exceed the current observational bound of $|f_{\rm NL}| \lesssim 100$.

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I. INTRODUCTION

Standard slow-roll single-field inflation generates a quasi scale invariant spectrum of Gaussian, adiabatic perturbations. While this is in agreement with current data, string theory and SUSY generically contain many scalar fields and it is important to test how these extra fields may change the predictions of the simplest models. In particular, the simplest models predict a level of non-Gaussianity which is too small to observe in the foreseeable future [1, 2, 3], so any detection would be of great significance.

In this paper we focus on the possibility to obtain a large level of non-Gaussianity during slow-roll inflation and derive general conditions for when it may be large. We present simple, explicit two-field models which can saturate the observational bound. Although there are many models of inflation which generate a large non-Gaussianity [4, 5, 6, 7, 8, 9], few of them do so during inflation except by breaking slow-roll, e.g. with a kink in the potential [10] or having a non-standard kinetic term, e.g. DBI inflation [11]. Other widely considered methods to generate a large non-Gaussianity include the curvaton scenario [12], modulated reheating [13] or an inhomogeneous end to inflation [14, 15, 16, 17]. All these scenarios require an additional light scalar field that doesn't affect the dynamics during inflation but that becomes important either at the end of inflation or later.

Several authors have considered the possibility of generating non-Gaussianity during inflation for a separable potential. In particular Vernizzi and Wands calculated a general formula for the non linearity parameter f_{NL} which parameterises the bispectrum in the case of a sum separable potential [18], this was later extended to the case of a product separable potential including non-canonical kinetic terms [19] and a sum separable potential for an arbitrary number of fields [20]. This has been further extended to the trispectrum (4-point function) [21]. For other approaches, see for example [22, 23].

Rather less work, however, has been done on analysing the formulas which have been calculated and are present in the literature. The results are rather long and depend on many parameters. In general they appear to give a slow-roll suppressed non-Gaussianity, subject to several simplifying assumptions that are additional to the slow-roll assumption and that are not always valid. Here we carefully consider several explicit models of two-field inflation and scan the parameter space more generally than has been done before. We show that it is possible to generate an extremely large non-Gaussianity during slow-roll inflation even when the field perturbations at Hubble exit are

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Gaussian. However this is only possible for specific values of the model parameters and initial conditions. We consider under which conditions a general model can generate a large non-Gaussianity and show that this always requires some fine tuning of initial conditions.

Although previous papers have shown that it is possible to generate a narrow "spike" of large non-Gaussianity while the inflaton trajectory turns a corner, the non-Gaussianity decays again very quickly after the corner [18, 22]. This spike of non-Gaussianity is associated with a temporary "jump" in the slow roll parameters [23]. In the models we consider the non-Gaussianity is large over many e-foldings of inflation and all of the slow roll parameters remain smaller than unity.

The paper is organised as follows: First we define relevant quantities and introduce some notation (Section II). Then, in the following two sections we derive the general conditions to generate a large non-Gaussianity during inflation, in Section III for a product separable potential and in Section IV for a sum separable potential. In Section V we give specific examples for product and sum potentials which can generate large non-Gaussianity. This includes a two-field model of hybrid inflation in Section V C. In Section VI we extend the previous results to a generalised action with non-canonical kinetic terms. We conclude in Section VII.

II. BACKGROUND THEORY

In this paper, we consider an inflationary epoch driven by two scalar fields, whose dynamics are governed by the action

$$S = \int d^4x \sqrt{-g} \left[M_P^2 \frac{R}{2} - \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - W(\varphi, \chi) \right]. \tag{1}$$

Here $M_P = 1/\sqrt{8\pi G}$ is the reduced Planck mass. We consider slow-roll inflation, during which all of the slow-roll parameters defined below are less than unity.

$$\epsilon_\varphi = \frac{M_P^2}{2} \left(\frac{W_\varphi}{W} \right)^2, \qquad \epsilon_\chi = \frac{M_P^2}{2} \left(\frac{W_\chi}{W} \right)^2, \qquad \epsilon = \epsilon_\varphi + \epsilon_\chi,$$

$$\eta_{\varphi\varphi} = M_P^2 \frac{W_{\varphi\varphi}}{W}, \qquad \eta_{\varphi\chi} = M_P^2 \frac{W_{\varphi\chi}}{W}, \qquad \eta_{\chi\chi} = M_P^2 \frac{W_{\chi\chi}}{W}. \tag{2}$$

While it is physically interesting to consider the slow-roll parameters, it will be useful to use the definition of angles along or perpendicular to the background trajectory of the two inflationary fields [24]:

$$\cos\theta = \frac{\dot\varphi}{\sqrt{\dot\varphi^2 + \dot\chi^2}}, \qquad \sin\theta = \frac{\dot\chi}{\sqrt{\dot\varphi^2 + \dot\chi^2}}, \tag{3}$$

such that, in the slow roll approximation,

$$\frac{\epsilon_\varphi}{\epsilon} = \cos^2\theta, \qquad \frac{\epsilon_\chi}{\epsilon} = \sin^2\theta. \tag{4}$$

Throughout this paper we use formulae given in [19] and we refer the reader to this paper and references therein for more details. In summary we define several observable quantities in terms of the primordial curvature perturbation ζ, which may be calculated using the δN formalism [25, 26, 27, 28, 29]. The number of e–foldings, N, given by

$$N = \int_{t_*}^{t_{\rm fin}} H(t) dt, \tag{5}$$

is evaluated from an initial flat hypersurface to a final uniform density hypersurface. The perturbation in the number of e–foldings, δN, is the difference between the curvature perturbations on the initial and final hypersurfaces. In this paper we take the initial time to be Hubble exit during inflation, denoted by t_*, and the final time to be the end of inflation. The curvature perturbation is given by [29]

$$\zeta = \delta N = \sum_I N_{,I} \delta\varphi_{I*} + \sum_{IJ} N_{,IJ} \delta\varphi_{I*} \delta\varphi_{J*} + \cdots, \tag{6}$$

where $N_{,I} = \partial N/(\partial\varphi_*^I)$ and the index I runs over all of the fields. We will consider the power spectrum and bispectrum

defined (in Fourier space) by

$$\langle \zeta_{\mathbf{k_1}} \zeta_{\mathbf{k_2}} \rangle \equiv (2\pi)^3 \delta^3(\mathbf{k_1} + \mathbf{k_2}) \frac{2\pi^2}{k_1^3} \mathcal{P}_\zeta(k_1), \tag{7}$$

$$\langle \zeta_{\mathbf{k_1}} \zeta_{\mathbf{k_2}} \zeta_{\mathbf{k_3}} \rangle \equiv (2\pi)^3 \delta^3(\mathbf{k_1} + \mathbf{k_2} + \mathbf{k_3}) B_\zeta(k_1, k_2, k_3). \tag{8}$$

¿From this we can define three quantities of key observational interest, respectively the spectral index, the tensor-to-scalar ratio and the non-linearity parameter

$$n_\zeta - 1 \equiv \frac{\partial \log \mathcal{P}_\zeta}{\partial \log k} \tag{9}$$

$$r = \frac{\mathcal{P}_T}{\mathcal{P}_\zeta} = \frac{8\mathcal{P}_*}{M_P^2 \mathcal{P}_\zeta}, \tag{10}$$

$$f_{\mathrm{NL}} = \frac{5}{6} \frac{k_1^3 k_2^3 k_3^3}{k_1^3 + k_2^3 + k_3^3} \frac{B_\zeta(k_1, k_2, k_3)}{4\pi^4 \mathcal{P}_\zeta^2}. \tag{11}$$

Here $\mathcal{P}_*$ is the power spectrum of the scalar field fluctuations and $\mathcal{P}_T = 8\mathcal{P}_* = 8H_*^2/(4\pi^2)$ is the power spectrum of the tensor fluctuations. Both the spectra are calculated at the end of inflation and we ignore any evolution after this time. For full evolution after inflation, see [19, 30]. As defined above, f_{NL} is shape dependent, but it has been shown that the shape dependent part is much less than unity [18, 31]. The ideal CMB experiment is only expected to reach a precision of f_{NL} around unity [32], so we will calculate the shape independent part of f_{NL}, denoted by $f_{\mathrm{NL}}^{(4)}$ in [18, 19]. Whenever the non-Gaussianity is large, $|f_{\mathrm{NL}}| > 1$, we can associate $f_{\mathrm{NL}}^{(4)} \simeq f_{\mathrm{NL}}$. This ($k$ independent) part of f_{NL} and the spectral index can be calculated by the δN formalism,

$$\mathcal{P}_\zeta = \sum_I N_{,I}^2 \mathcal{P}_*, \tag{12}$$

$$n_\zeta - 1 = -2\epsilon^* + \frac{2}{H} \frac{\sum_{IJ} \dot{\varphi}_J N_{,JI} N_{,I}}{\sum_K N_{,K}^2}, \tag{13}$$

$$f_{\mathrm{NL}}^{(4)} = \frac{5}{6} \frac{\sum_{IJ} N_{,IJ} N_{,I} N_{,J}}{\left(\sum_I N_{,I}^2\right)^2}. \tag{14}$$

There is no universal agreement over the sign of f_{NL} in the literature. We use the opposite sign convention to [18, 19], in order to be in agreement with the WMAP sign convention [33]. The latest observations from 5 years of WMAP data are

$$n_\zeta = 0.96^{+0.014}_{-0.013}, \qquad (\text{assuming } r = 0), \tag{15}$$

$$r < 0.2 \ (95\% \ \mathrm{CL}), \tag{16}$$

$$-9 < f_{\mathrm{NL}}^{\mathrm{local}} < 111 \ (95\% \ \mathrm{CL}). \tag{17}$$

The "local" bound on f_{NL} is based on the definition $\zeta = \zeta_G + 3f_{\mathrm{NL}}\zeta_G^2/5$, where ζ_G is the Gaussian part of ζ [32, 33]. When more than one field direction during inflation contributes to ζ in Eqn. (6) then this definition of f_{NL} is not equivalent to the very commonly used definition Eqn. (11), even though they have the same shape dependence (see e.g. [34]). The bound on $f_{\mathrm{NL}}^{\mathrm{local}}$ may therefore not be applicable to the f_{NL} that we calculate. In this paper we will focus on the non-linearity parameter f_{NL}. Constraints on this parameter are expected to improve by nearly an order of magnitude with the PLANCK satellite [35].

III.   LARGE NON-GAUSSIANITY IN PRODUCT POTENTIALS, $W(\varphi, \chi) = U(\varphi)V(\chi)$

We follow the terminology of [18] and use the definitions,

$$u \equiv \frac{\epsilon_\varphi^e}{\epsilon^e} = \cos^2 \theta^e, \quad v \equiv \frac{\epsilon_\chi^e}{\epsilon^e} = \sin^2 \theta^e. \tag{18}$$

We will use a sub- or superscript "*" to denote values evaluated at the time of Hubble exit and a subscripted "e" will denote the time at t_e. From Eqns. (12) and (13), the power spectrum and spectral index for a product potential with canonical kinetic terms are

$$\mathcal{P}_\zeta = \frac{W_*}{24\pi^2 M_P^4} \left(\frac{u^2}{\epsilon_\varphi^*} + \frac{v^2}{\epsilon_\chi^*} \right),$$

(19)

$$n_\zeta - 1 = -2\epsilon^* - 4 \frac{u^2 \left(1 - \frac{\eta_{\varphi\varphi}^*}{2\epsilon_\varphi^*} \right) + v^2 \left(1 - \frac{\eta_{\chi\chi}^*}{2\epsilon_\chi^*} \right)}{u^2/\epsilon_\varphi^* + v^2/\epsilon_\chi^*}.$$

(20)

The non-linear parameter $f_{\rm NL}^{(4)}$ becomes [19]

$$f_{\rm NL}^{(4)} = \frac{5}{6} \frac{2}{\left(\frac{u^2}{\epsilon_\varphi^*} + \frac{v^2}{\epsilon_\chi^*} \right)^2} \left[\frac{u^3}{\epsilon_\varphi^*} \left(1 - \frac{\eta_{\varphi\varphi}^*}{2\epsilon_\varphi^*} \right) + \frac{v^3}{\epsilon_\chi^*} \left(1 - \frac{\eta_{\chi\chi}^*}{2\epsilon_\chi^*} \right) - \left(\frac{u}{\epsilon_\varphi^*} - \frac{v}{\epsilon_\chi^*} \right)^2 \mathcal{A}_P \right],$$

(21)

where

$$\hat{\eta} \equiv \frac{\epsilon_\chi \eta_{\varphi\varphi} + \epsilon_\varphi \eta_{\chi\chi}}{\epsilon},$$

(22)

$$\mathcal{A}_P \equiv -\frac{\epsilon_\varphi^e \epsilon_\chi^e}{(\epsilon^e)^2} \left[\hat{\eta}^e - 4 \frac{\epsilon_\varphi^e \epsilon_\chi^e}{\epsilon^e} \right].$$

(23)

The slow-roll parameters, defined by Eq. (2), are

$$\epsilon_\varphi = \frac{M_P^2}{2} \left(\frac{U_\varphi}{U} \right)^2 = \epsilon \cos^2 \theta, \qquad \epsilon_\chi = \frac{M_P^2}{2} \left(\frac{V_\chi}{V} \right)^2 = \epsilon \sin^2 \theta,$$

(24)

and

$$\eta_{\varphi\varphi} = M_P^2 \frac{U_{\varphi\varphi}}{U}, \qquad \eta_{\varphi\chi} = M_P^2 \frac{U_\varphi V_\chi}{W}, \qquad \eta_{\chi\chi} = M_P^2 \frac{V_{\chi\chi}}{V}.$$

(25)

Using a few auxiliary functions (involving only θ^* and θ^e), we may write Eqn. (21) as

$$f_{\rm NL}^{(4)} = \frac{5}{6} \left[2 j_p(\theta^*, \theta^e) \epsilon^* - f_p(\theta^*, \theta^e) \eta_{\varphi\varphi}^* - g_p(\theta^*, \theta^e) \eta_{\chi\chi}^* + 2 h_p(\theta^*, \theta^e, X) \left(\hat{\eta}^e - 4 \sin^2 \theta^e \cos^2 \theta^e \epsilon^e \right) \right],$$

(26)

where the auxiliary functions are defined as

$$\begin{aligned}
f_p(\theta^*, \theta^e) &\equiv \frac{u^3 \sin^4 \theta^*}{\left(u^2 \sin^2 \theta^* + v^2 \cos^2 \theta^* \right)^2} = \frac{\tan^4 \theta^*}{\left(\tan^2 \theta^* + \tan^4 \theta^e \right)^2 \cos^2 \theta^e}, \\
g_p(\theta^*, \theta^e) &\equiv \frac{v^3 \cos^4 \theta^*}{\left(u^2 \sin^2 \theta^* + v^2 \cos^2 \theta^* \right)^2} = \frac{\tan^8 \theta^e}{\left(\tan^2 \theta^* + \tan^4 \theta^e \right)^2 \sin^2 \theta^e}, \\
h_p(\theta^*, \theta^e) &\equiv \sin^2 \theta^e \cos^2 \theta^e \frac{\left(u \sin^2 \theta^* - v \cos^2 \theta^* \right)^2}{\left(u^2 \sin^2 \theta^* + v^2 \cos^2 \theta^* \right)^2} = \tan^2 \theta^e \frac{\left(\tan^2 \theta^* - \tan^2 \theta^e \right)^2}{\left(\tan^2 \theta^* + \tan^4 \theta^e \right)^2}, \\
j_p(\theta^*, \theta^e) &\equiv \frac{\left(u^3 \sin^4 \theta^* \cos^2 \theta^* + v^3 \sin^2 \theta^* \cos^4 \theta^* \right)}{\left(u^2 \sin^2 \theta^* + v^2 \cos^2 \theta^* \right)^2} = \frac{\sin^2 \theta^*}{\cos^2 \theta^e} \frac{\left(\tan^2 \theta^* + \tan^6 \theta^e \right)}{\left(\tan^2 \theta^* + \tan^4 \theta^e \right)^2}.
\end{aligned}$$

(27)

By analysing the functions j_p, f_p, g_p and h_p over the range of allowed values for θ^* and θ^e, it is possible to locate regions of parameter space which gives large $f_{\rm NL}^{(4)}$. We find that the function $j_p(\theta^*, \theta^e)$ satisfies $0 \leq j_p \leq 1$ and therefore the 'j'-term can never lead to significant values of $f_{\rm NL}^{(4)}$. We therefore ignore this term in the analysis that follows.

The functions f_p, g_p and h_p (as functions of θ^* and θ^e) are plotted in Fig. 1. The main point to note is that these prefactors are large (and can be very large) in two regions:
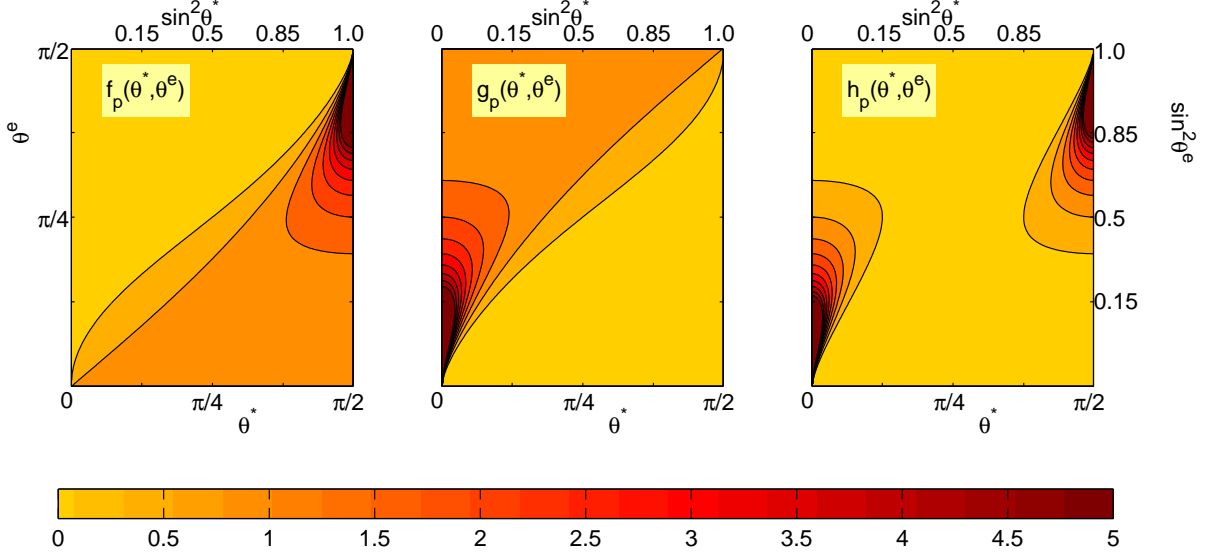
FIG. 1: The contour plot of the functions, f_p, g_p and h_p, in the plane of θ^*, and θ^e. The bottom and left-hand axes show the angles, θ^* and θ^e respectively. The top and right-hand axes show $\sin^2\theta^*$ and $\sin^2\theta^e$.

A $\cos^2\theta^* \ll 1$ and $\cos^2\theta^e \ll 1$ ($\epsilon_\chi \gg \epsilon_\varphi$). In this region, $f_p \gg g_p$ and $h_p \sim f_p \sin^2\theta^e$.

B $\sin^2\theta^* \ll 1$ and $\sin^2\theta^e \ll 1$ ($\epsilon_\varphi \gg \epsilon_\chi$). In this region, $g_p \gg f_p$ and $h_p \sim g_p \cos^2\theta^e$.

In both these regions, these prefactors can account for a significant level of $f_{\rm NL}^{(4)}$, despite the relevant terms in Eqn. (26) having slow-roll factors. Each of these regions describes one of the fields dominating over the other in kinetic energy. Note, however, that single-field inflation (i.e. with one field static) will not lead to a large value of non-Gaussianity, since these fractions vanish exactly for $\dot{\chi}^e = 0$ or $\dot{\varphi}^e = 0$ ($\theta^e = 0$ or $\pi/2$ respectively).

Due to the symmetry, we shall focus on Region B and explicitly write down full conditions for both Regions A and B in Section III A. To find the condition to give large non-Gaussianity, we will concentrate on the terms including the functions of $g_p(\theta^*, \theta^e)$ and $h_p(\theta^*, \theta^e)$, since the other terms cannot give large non-Gaussianity as discussed above. The large $f_{\rm NL}^{(4)}$ is given by

$$
\begin{aligned}
f_{\rm NL}^{(4)} &\simeq \frac{5}{6} g_p(\theta^*, \theta^e) \left[-\eta_{\chi\chi}^* + 2\cos^2\theta^e \left(\hat{\eta}^e - 4\sin^2\theta^e \cos^2\theta^e \epsilon^e \right) \right], \\
&\simeq \frac{5}{6} \frac{\sin^6\theta^e}{\left(\sin^2\theta^* + \sin^4\theta^e \right)^2} \left[-\eta_{\chi\chi}^* + 2\eta_{\chi\chi}^e \right].
\end{aligned}
\tag{28}
$$

Here we assumed that $|\eta_{\chi\chi}| \gg \sin^2\theta^e |\eta_{\varphi\varphi}^e - 4\epsilon^e|$. For $|f_{\rm NL}|$ to be larger than unity, the function $g_p(\theta^*, \theta^e)$ must be bigger than the inverse of the slow-roll parameters in the square bracket. In this way from Eqn. (28) we find the general way to obtain large non-Gaussianity, $|f_{\rm NL}^{(4)}| \gtrsim 1$,

$$
\sin^2\theta^* \lesssim \sin^4\theta^e \left(\frac{1}{\sqrt{\sin^2\theta^e \mathcal{G}_p}} - 1 \right), \qquad \mathcal{G}_p = \frac{6}{5} \left| -\eta_{\chi\chi}^* + 2\eta_{\chi\chi}^e \right|^{-1}.
\tag{29}
$$

¿From this condition we can derive some corollaries. Firstly, from $\partial g_p / \partial(\sin^2\theta^e) = 0$ and the definition of g_p, the initial and final angle of the trajectory are constrained:

$$
\sin^2\theta^* < \frac{1}{3} \left(\frac{3}{4} \right)^4 \frac{1}{\mathcal{G}_p^2}, \quad \text{and} \quad \sin^2\theta^e < \frac{1}{\mathcal{G}_p},
\tag{30}
$$

and secondly we see that $\cos^2\theta$ must typically grow by at least two orders of magnitude during inflation, since we
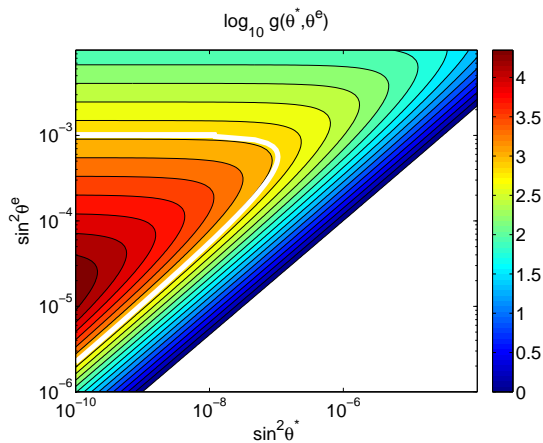
FIG. 2: A blown-up graph of Region B. The conditions for g_p in Eqn. (29) is plotted for $\mathcal{G}_p = 1000$ with a white line. It can be seen that this condition encloses the contour for g_p larger than some constant, $\mathcal{G}_p$.

require

$$\frac{\sin^2 \theta^e}{\sin^2 \theta^*} > 4\mathcal{G}_p. \tag{31}$$

In this region of large non-Gaussianity, the power spectrum from Eqn. (19), the spectral index from Eqn. (20) and the tensor-to-scalar ratio in Region B are:

$$\mathcal{P}_\zeta \simeq \frac{W_*}{24\pi^2 M_P^4 \epsilon^*} (1 + \tilde{r}), \tag{32}$$

$$n_\zeta - 1 \simeq -2\epsilon_* + 2\frac{-2\epsilon^* + \eta_{\varphi\varphi}^* + \tilde{r}\eta_{\chi\chi}^*}{1 + \tilde{r}}, \tag{33}$$

$$r \simeq 16\epsilon^* (1 + \tilde{r})^{-1}, \tag{34}$$

where

$$\tilde{r} = \frac{\sin^4 \theta^e}{\sin^2 \theta^*} > 0. \tag{35}$$

The spectrum of curvature perturbations can be dominated by the fluctuations in the slow-rolling field φ or χ depending on $\tilde{r}$, which can be smaller or bigger than unity. We note, however, that the tensor-to-scalar ratio can only be suppressed in multiple field inflation, since the tensor power spectrum is unchanged from the single field value and $\mathcal{P}_\zeta$ is enhanced. The spectral index is changed by the presence of $\tilde{r}$, but remains slow roll suppressed.

A. Condition for Large Non-Gaussianity in Product Potentials

If we summarise, the general conditions to obtain large $f_{\text{NL}}^{(4)}$ can be written simply. Ultimately, the two-field system must have one field which dominates the evolution throughout. This corresponds to being in Region A or B (depending on which field dominates). Hence, one field must dominate the evolution almost fully at the beginning, while the other field gains a relatively large percentage of kinetic energy by the end of inflation. Specifically, the slow-roll parameters and the value of f_p or g_p must be such to give $|f_{\text{NL}}^{(4)}| \gtrsim 1$ in Eqn. (26).

For Region A ($\cos^2 \theta \ll 1$), the condition is

$$\cos^2 \theta^* \lesssim \cos^4 \theta^e \left(\frac{1}{\sqrt{\cos^2 \theta^e \mathcal{F}_p}} - 1 \right), \qquad \mathcal{F}_p = \frac{6}{5} \left| -\eta_{\varphi\varphi}^* + 2\eta_{\varphi\varphi}^e \right|^{-1} \qquad \text{(Region A)}. \tag{36}$$

where we have assumed that $|\eta_{\varphi\varphi}| \gg \cos^2 \theta^e |\eta_{\chi\chi} - 4\epsilon^e|$.

For Region B ($\sin^2 \theta \ll 1$), the condition is

$$\sin^2 \theta^* \lesssim \sin^4 \theta^e \left(\frac{1}{\sqrt{\sin^2 \theta^e \mathcal{G}_p}} - 1 \right), \qquad \mathcal{G}_p = \frac{6}{5} \left| -\eta_{\chi\chi}^* + 2\eta_{\chi\chi}^e \right|^{-1} \qquad \text{(Region B)}. \tag{37}$$

where we have assumed that $|\eta_{\chi\chi}| \gg \sin^2 \theta^e |\eta_{\varphi\varphi} - 4\epsilon^e|$.

This condition also leads to a large growth of the subdominant slow-roll parameter. For Region A and B respectively:

$$\frac{\cos^2 \theta^e}{\cos^2 \theta^*} = \frac{\epsilon_\varphi^e}{\epsilon^e} \frac{\epsilon^*}{\epsilon_\varphi^*} \gtrsim 4\mathcal{F}_p \quad \text{(Region A)}, \qquad \frac{\sin^2 \theta^e}{\sin^2 \theta^*} = \frac{\epsilon_\chi^e}{\epsilon^e} \frac{\epsilon^*}{\epsilon_\chi^*} \gtrsim 4\mathcal{G}_p \quad \text{(Region B)}. \tag{38}$$

B. Direct Observation of $f_{\rm NL}^{(4)}$ and Fine-Tuning

Eqns. (36) and (37) encode the level of fine-tuning required on the initial and final slow-roll parameters to obtain an observable level of non-Gaussianity. As an example, if observations find $f_{\rm NL}^{(4)} \sim 10$, for standard order of slow-roll parameters ($\eta \sim 0.01$), one requires f_p or $g_p \sim 1000$. In the case of Region B, this corresponds to

$$\frac{\epsilon_\chi^*}{\epsilon^*} = \sin^2 \theta^* \lesssim 10^{-7}, \qquad \text{and} \qquad \frac{\epsilon_\chi^e}{\epsilon^e} = \sin^2 \theta^e \lesssim 10^{-3}. \tag{39}$$

This requires very special initial values for the fields.

IV. LARGE NON-GAUSSIANITY IN SUM POTENTIALS, $W(\varphi, \chi) = U(\varphi) + V(\chi)$

In this section we find the general condition for large non-Gaussianity with a sum separable potential, $W(\varphi, \chi) = U(\varphi) + V(\chi)$. Defining

$$u \equiv \frac{U^* + Z^e}{W^*}, \qquad v \equiv \frac{V^* - Z^e}{W^*}, \tag{40}$$

with

$$Z^e = \frac{(V^e \epsilon_\varphi^e - U^e \epsilon_\chi^e)}{\epsilon^e} = V^e \cos^2 \theta^e - U^e \sin^2 \theta^e, \tag{41}$$

the power spectrum and spectral index are given by [18]:

$$\mathcal{P}_\zeta = \frac{W_*}{24\pi^2 M_P^4} \left(\frac{u^2}{\epsilon_\varphi^*} + \frac{v^2}{\epsilon_\chi^*} \right), \tag{42}$$

$$n_\zeta - 1 = -2\epsilon^* - 4 \frac{u \left(1 - \frac{\eta_{\varphi\varphi}^*}{2\epsilon_\varphi^*} u \right) + v \left(1 - \frac{\eta_{\chi\chi}^*}{2\epsilon_\chi^*} v \right)}{u^2/\epsilon_\varphi^* + v^2/\epsilon_\chi^*}. \tag{43}$$

The nonlinear parameter $f_{\rm NL}^{(4)}$ is [18]:

$$f_{\rm NL}^{(4)} = \frac{5}{6} \frac{2}{\left(\frac{u^2}{\epsilon_\varphi^*} + \frac{v^2}{\epsilon_\chi^*}\right)^2} \left[\frac{u^2}{\epsilon_\varphi^*}\left(1 - \frac{\eta_{\varphi\varphi}^*}{2\epsilon_\varphi^*}u\right) + \frac{v^2}{\epsilon_\chi^*}\left(1 - \frac{\eta_{\chi\chi}^*}{2\epsilon_\chi^*}v\right) + \left(\frac{u}{\epsilon_\varphi^*} - \frac{v}{\epsilon_\chi^*}\right)^2 \mathcal{A}_S\right], \tag{44}$$

where we define

$$\hat\eta \equiv \frac{(\epsilon_\chi \eta_{\varphi\varphi} + \epsilon_\varphi \eta_{\chi\chi})}{\epsilon} = \eta_{\varphi\varphi}\sin^2\theta + \eta_{\chi\chi}\cos^2\theta, \tag{45}$$

$$\mathcal{A}_S \equiv -\frac{W_e^2}{W_*^2}\frac{\epsilon_\varphi^e \epsilon_\chi^e}{(\epsilon^e)^2}[\epsilon^e - \hat\eta^e] = -\frac{W_e^2}{W_*^2}\cos^2\theta^e \sin^2\theta^e [\epsilon^e - \hat\eta^e]. \tag{46}$$

The slow-roll parameters, defined by Eqn. (2), are

$$\epsilon_\varphi = \frac{M_P^2}{2}\left(\frac{U_\varphi}{U + V}\right)^2 = \epsilon\cos^2\theta, \qquad \epsilon_\chi = \frac{M_P^2}{2}\left(\frac{V_\chi}{U + V}\right)^2 = \epsilon\sin^2\theta, \tag{47}$$

and

$$\eta_{\varphi\varphi} = M_P^2 \frac{V_{\varphi\varphi}}{U + V}, \qquad \eta_{\varphi\chi} = 0, \qquad \eta_{\chi\chi} = M_P^2 \frac{V_{\chi\chi}}{U + V}, \tag{48}$$

Similar to the analysis of a product potential, it is possible to re-write Eqn. (44):

$$f_{\rm NL}^{(4)} = \frac{5}{6}\left[2j_s\epsilon^* - f_s\eta_{\varphi\varphi}^* - g_s\eta_{\chi\chi}^* - 2h_s\frac{W_e^2}{W_*^2}(\epsilon^e - \hat\eta^e)\right], \tag{49}$$

where the following auxiliary functions have been used:

$$\begin{aligned}
f_s(u, \sin^2\theta^*) &\equiv \frac{u^3 \sin^4\theta^*}{(u^2\sin^2\theta^* + v^2\cos^2\theta^*)^2}, \\
g_s(u, \sin^2\theta^*) &\equiv \frac{v^3 \cos^4\theta^*}{(u^2\sin^2\theta^* + v^2\cos^2\theta^*)^2}, \\
h_s(u, \sin^2\theta^*) &\equiv \sin^2\theta^e \cos^2\theta^e \frac{(u\sin^2\theta^* - v\cos^2\theta^*)^2}{(u^2\sin^2\theta^* + v^2\cos^2\theta^*)^2}, \\
j_s(u, \sin^2\theta^*) &\equiv \frac{(u^2\sin^4\theta^*\cos^2\theta^* + v^2\cos^4\theta^*\sin^2\theta^*)}{(u^2\sin^2\theta^* + v^2\cos^2\theta^*)^2}.
\end{aligned} \tag{50}$$

Note that $u + v = 1$ and hence these functions depend on just two variables, u and θ^* (or v and θ^*), and that $0 \leq u \leq 1$ and $0 \leq v \leq 1$. The first term of $f_{\rm NL}^{(4)}$ in Eqn. (49) is always smaller than unity, since $0 \leq j_s \leq 1$.

Once again there are two regions with potentially large non-Gaussianity:

C $u \ll 1$, and $\cos^2\theta^* \ll 1$

D $v \ll 1$, and $\sin^2\theta^* \ll 1$.

We temporarily analyse this second region, Region D, in which $v \ll 1, \sin^2\theta^* \ll 1$ and g_s can be large. In this region, we can approximate

$$g_s(v, \sin^2\theta^*) \simeq \frac{v^3}{(\sin^2\theta^* + v^2)^2}, \qquad h_s(v, \sin^2\theta^*) \simeq \frac{g_s(v, \sin^2\theta^*)\sin^2\theta^e \cos^2\theta^e}{v}. \tag{51}$$

In the special type of potential when $V^e \simeq V^*$, such as hybrid inflation, $v \sim \sin^2\theta^e W^e/W^*$ and hence

$$h_s(v, \sin^2\theta^*) \simeq g_s(v, \sin^2\theta^*)\cos^2\theta^e \frac{W_*}{W_e}. \tag{52}$$

The function g_s has exactly the same form as the function g_p for the product potential in the region where it is

large, see Eqn. (28) replacing $\sin^2 \theta^e$ with v. Hence it is large in exactly the same areas, and for $g_s \gtrsim \mathcal{G}_s$ we require

$$\sin^2 \theta^* \lesssim v^2 \left(\frac{1}{\sqrt{v\,\mathcal{G}_s}} - 1 \right), \qquad \mathcal{G}_s = \frac{6}{5} \left| -\eta^*_{\chi\chi} + 2\eta^e_{\chi\chi} \right|^{-1} \quad \text{(Region B)}. \tag{53}$$

Finally, we can approximate $f_{\rm NL}^{(4)}$ in Region D as

$$f_{\rm NL}^{(4)} \simeq \frac{5}{6} g_s \left[-\eta^*_{\chi\chi} - 2\frac{W_e^2}{W_*^2} \sin^2 \theta^e \cos^2 \theta^e \frac{(\epsilon^e - \hat{\eta}^e)}{v} \right], \tag{54}$$

where we have used $u \simeq 1$ and $\cos^2 \theta^* \simeq 1$. We may analyse this equation for the specific case when $V^e \simeq V^*$, which can be true in many models because $\sin^2 \theta^* \simeq (V_\chi/U_\varphi)^2 \ll 1$, so at least initially the potential V is extremely flat. In this scenario, $v \sim \sin^2 \theta^e W^e/W^*$ and hence

$$f_{\rm NL}^{(4)} \simeq \frac{5}{6} g_s \left[-\eta^*_{\chi\chi} - 2\frac{W_e}{W_*} \cos^2 \theta^e (\epsilon^e - \hat{\eta}^e) \right]. \tag{55}$$

Indeed, the form of power spectrum and spectral index in this large non-Gaussianity region, are same as that of product potential, Eqn. (32) and (33).

A. Condition for Large Non-Gaussianity in Sum Potentials

¿From the previous section, it is clear that, to get large non-Gaussianity, we need a constraint on the function g_s, for Region C,

$$\cos^2 \theta^* \lesssim u^2 \left(\frac{1}{\sqrt{u\,\mathcal{F}_s}} - 1 \right), \qquad \mathcal{F}_s = \frac{6}{5} \left| -\eta^*_{\varphi\varphi} + 2\eta^e_{\varphi\varphi} \right|^{-1} \quad \text{(Region C)}. \tag{56}$$

In the same way, for Region D, we obtain

$$\sin^2 \theta^* \lesssim v^2 \left(\frac{1}{\sqrt{v\,\mathcal{G}_s}} - 1 \right), \qquad \mathcal{G}_s = \frac{6}{5} \left| -\eta^*_{\chi\chi} + 2\eta^e_{\chi\chi} \right|^{-1} \quad \text{(Region D)}. \tag{57}$$

As in Eqn. (30), we find exact upper limits on the parameters:

$$\cos^2 \theta^* < \frac{1}{3} \left(\frac{3}{4} \right)^4 \frac{1}{\mathcal{F}_s^2}, \quad u < \frac{1}{\mathcal{F}_s} \quad \text{and} \quad \frac{u}{\cos^2 \theta^*} > 4\mathcal{F}_p \quad \text{(Region C)} \tag{58}$$

$$\sin^2 \theta^* < \frac{1}{3} \left(\frac{3}{4} \right)^4 \frac{1}{\mathcal{G}_s^2}, \quad v < \frac{1}{\mathcal{G}_s} \quad \text{and} \quad \frac{v}{\sin^2 \theta^*} > 4\mathcal{G}_p. \quad \text{(Region D)} \tag{59}$$

V. EXAMPLES

In this section we give specific examples, two of which can give large non-Gaussianity and one which cannot. We note that, in all the following examples, we have one parameter that acts as a normalisation factor in W_*. This parameter can be fixed to normalise the amplitude of the perturbations.

A. Quadratic times exponential potential, $W(\varphi, \chi) = \frac{1}{2} e^{-\lambda \varphi^2/M_P^2} m^2 \chi^2$

We consider a product potential, with $U(\varphi) = e^{-\lambda \varphi^2/M_P^2}$ and $V(\chi) = m^2 \chi^2/2$, for which the slow-roll parameters are:

$$\epsilon_\varphi = 2\lambda^2 \frac{\varphi^2}{M_P^2}, \qquad \eta_{\varphi\varphi} = -2\lambda + 4\lambda^2 \frac{\varphi^2}{M_P^2}, \qquad \epsilon_\chi = \eta_{\chi\chi} = \frac{2M_P^2}{\chi^2}, \qquad \eta_{\varphi\chi} = -\frac{4\lambda\,\varphi}{\chi}. \tag{60}$$
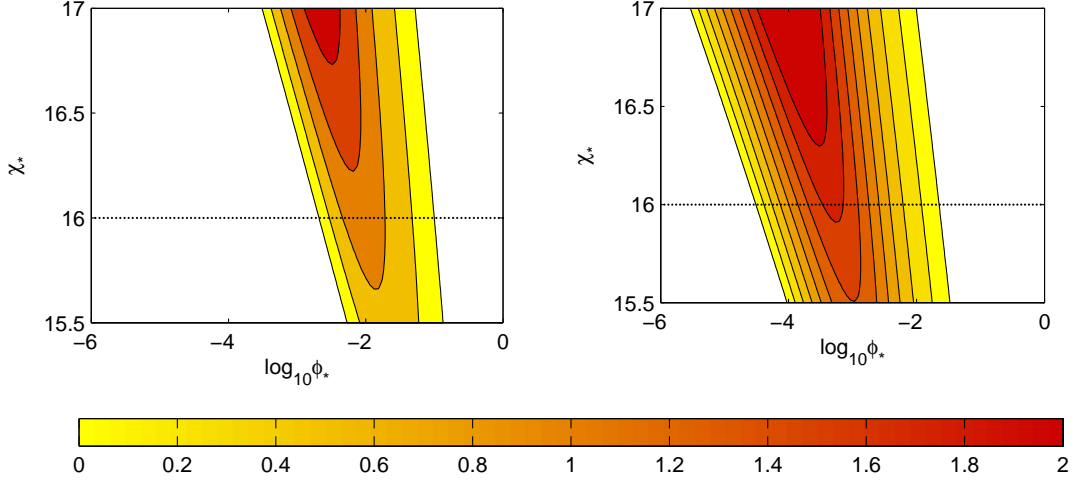
FIG. 3: The contour plot of $\log_{10} |f_{\rm NL}^{(4)}|$ for Example A, $W(\varphi, \chi) = \frac{1}{2} e^{-\lambda \varphi^2 / M_P^2} m^2 \chi^2$. Here we used $\lambda = 0.04$ (left) and 0.05 (right). White regions indicate when $|f_{\rm NL}^{(4)}| < 1$ and the dark centre-most region indicates $|f_{\rm NL}^{(4)}| > 100$.

We consider $\lambda > 0$ and slow-roll throughout inflation, so that the φ-field increases during inflation while the χ-field decreases. In this slow-roll limit, the exponential function can be expanded so that the potential is dominated by the quadratic potential of χ field, $\frac{1}{2} m^2 \chi^2$, and corrected by the integration between χ and φ field, $-\frac{1}{2} \lambda m^2 \varphi^2 \chi^2$, which might be important for preheating [36, 37]. Since inflation ends when the χ field rolls close to its minimum, i.e. when $\epsilon = 1 \simeq \epsilon_\chi$, the only way to generate large non-Gaussianity is to start with small field of φ (which corresponds to Region A).

For this potential, the slow-roll solutions for φ and χ lead to

$$\varphi = \varphi_* e^{2\lambda N} \qquad \chi^2 = \chi_*^2 - 4N M_P^2. \tag{61}$$

Since $\lambda^2 \varphi^2 / M_P^2 < 1$, we find $\eta_{\varphi\varphi}^* \sim \eta_{\varphi\varphi}^e \sim -2\lambda$. ¿From Eqn. (36), the constraints on the initial values of the field φ can be obtained:

$$\frac{M_P^2 \chi_*^4 e^{-12\lambda N_e}}{2\lambda^3 \chi_e^6} \lesssim \frac{\varphi_*^2}{M_P^2} \lesssim \frac{2 M_P^2 e^{-4\lambda N_e}}{\lambda \chi_e^2}. \tag{62}$$

where $N_e \simeq (\chi_*^2 - \chi_e^2) / 4 M_P^2$ and $\chi_e^2 \simeq 2$. For the specific values of $\chi_* = 16 M_P$ and $\lambda = \{0.03, 0.04, 0.05\}$, the constraints lead to

$$\{0.134, 2 \times 10^{-3}, 3 \times 10^{-5}\} \lesssim \frac{|\varphi_*|}{M_P} \lesssim \{0.128, 0.031, 0.0078\},$$

In this range, the non linearity parameter is well approximated as

$$f_{\rm NL}^{(4)} \simeq \frac{5}{6} \frac{\cos^6 \theta^e}{(\cos^2 \theta^* + \cos^4 \theta^e)^2} \eta_{\varphi\varphi}. \tag{63}$$

The contour plots of $f_{\rm NL}^{(4)}$ for two values of λ ($\lambda = 0.04, 0.05$) are given in Fig. 3 using the full formula Eqn. (44), which is almost same as Eqn. (63). Here it can be seen that $f_{\rm NL}^{(4)}$ can be very large when $\varphi^* \sim 10^{-3} M_P$ ($\lambda = 0.04$) or $\varphi^* \sim 10^{-4} M_P$ ($\lambda = 0.05$). We note that the analytic constraints differ slightly from the plots, due to the value of $\eta_{\varphi\varphi}$ used. In the analytics, we assume $\eta_{\varphi\varphi} \sim -2\lambda$, but use the exact form (Eqn. (60)) in the plots.

The time evolution of $f_{\rm NL}^{(4)}$ from the δN formula is given in Fig. 4, where the final point is identified as some time during inflation. This method is also used in Vernizzi and Wands [18] for analytic evolution. We can see that $|f_{\rm NL}^{(4)}|$ increases sharply around $N = 30$. This can be understood from the evolution of two fields: from this time, the velocity of φ (or $\cos^2 \theta$) increases (as seen in the third and fourth plot of Fig. 4) to satisfy the condition Eqn. (36) to obtain large non-Gaussianity. Then $f_{\rm NL}^{(4)}$ begins to decrease around $N = 45$ as $\cos^4 \theta^e$ becomes bigger than $\cos^2 \theta^*$. Note that,
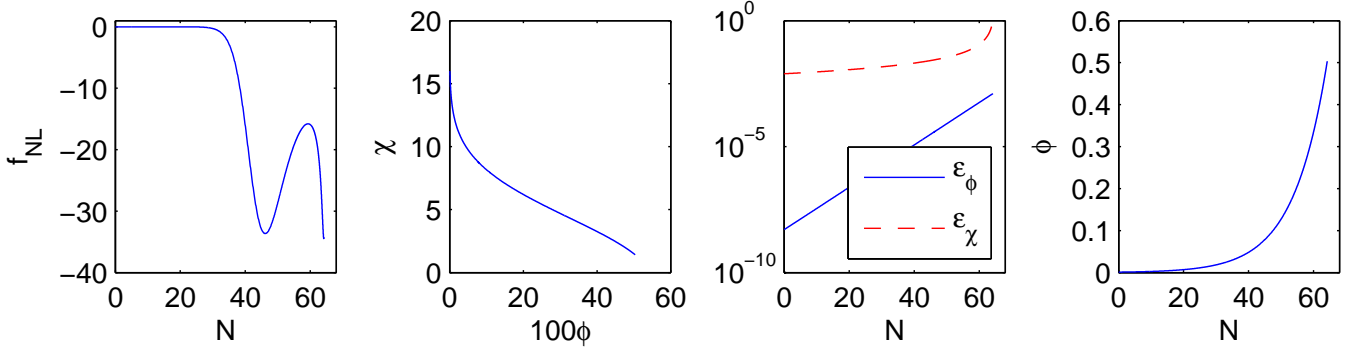
FIG. 4: The analytic evolution of $f_{\rm NL}^{(4)}$ and ϵ_χ and ϵ_φ for Example A. The evolution of both fields, φ and χ are also shown. We used $\lambda = 0.05$ and $\varphi_* = 10^{-3}M_P$, $\chi_* = 16M_P$. We numerically solved the full equations of motion until $\epsilon = 1$.

around the region for large $|f_{\rm NL}|$, the curvature perturbation of φ-field becomes comparable to, and then dominates, that of the χ-field. Around $N = 58$, the χ-field begins to roll so fast that $\cos^2\theta$ decreases and $|f_{\rm NL}^{(4)}|$ increases once more. However during this fast-roll stage, slow-roll breaks down and our analytic formula is not valid any more. Thus we calculate final values when $\epsilon = 1$.

B.   Quadratic product potential

In this section we show that a direct coupling between φ and χ, leaving to the product potential

$$W = \frac{\lambda}{2}\varphi^2\chi^2, \tag{64}$$

cannot generate a large non-Gaussianity during inflation. From the equation $N = \int H\,dt$ it follows that the fields evolve according to

$$\varphi_e^2 = \varphi_*^2 - 4NM_P^2 \tag{65}$$

and similarly for χ. Hence we have in Region B, where $\sin^2\theta^* \ll 1$ and $4NM_P^2 < \varphi_*^2 \ll \chi_*^2$,

$$\sin^2\theta^e \simeq \frac{\epsilon_\chi^e}{\epsilon^e} = \frac{\varphi_*^2 - 4NM_P^2}{\chi_*^2 - 4NM_P^2} < \frac{\varphi_*^2}{\chi_*^2} = \sin^2\theta^*. \tag{66}$$

Hence $\sin^2\theta$ decreases during inflation and this model cannot generate $|f_{\rm NL}| > 1$.

C.   Hybrid inflation

We consider a model of 2 field hybrid inflation,

$$W(\varphi,\chi) = W_0\left(1 + \alpha\frac{\varphi^2}{M_P^2} + \beta\frac{\chi^2}{M_P^2}\right), \tag{67}$$

which is vacuum dominated, i.e. which satisfies $\left|\alpha\varphi^2\right| \ll M_P^2$ and $\left|\beta\chi^2\right| \ll M_P^2$. Here we assume that inflation ends abruptly by another waterfall field which we don't write down in the potential above. We note that $f_{\rm NL}$ may change depending on the details of how the waterfall field is coupled to the two inflaton fields [14, 15, 17], but in general this is unlikely to generate a large contribution to $f_{\rm NL}$ without fine-tuning [16].

In this regime the slow-roll solutions are [5], (we can identify $\eta_\varphi = 2\alpha$ and $\eta_\sigma = 2\beta$ in the notation of that paper)

$$\varphi = \varphi_* e^{-2\alpha N}, \qquad \chi = \chi_* e^{-2\beta N}, \tag{68}$$

α	β	φ_*	χ_*	f_{NL}	$n_\zeta - 1$	r
0.018	-0.018	1	0.00018	42	0	0.006
0.04	0.005	1	0.0018	-9.27	0.09	0.10
0.01	-0.02	1	0.00037	11.1	-0.02	0.026

TABLE I: Table showing some initial conditions for the hybrid inflation model that lead to large levels of non-Gaussianity. The spectral index, calculated from Eqn. (43), and the tensor-to-scalar ratio are also shown.

and the slow-roll parameters are

$$\eta_{\varphi\varphi} = 2\alpha, \qquad \eta_{\chi\chi} = 2\beta, \qquad \eta_{\varphi\chi} = 0,$$
$$\epsilon_\varphi = 2\alpha^2 \frac{\varphi^2}{M_P^2} \ll |\eta_{\varphi\varphi}|, \qquad \epsilon_\chi = 2\beta^2 \frac{\chi^2}{M_P^2} \ll |\eta_{\chi\chi}|. \tag{69}$$

We note that the dominant slow-roll parameters $\eta_{\varphi\varphi}$ and $\eta_{\chi\chi}$ are constant in this model.

For this case, since $W^e \simeq W^*$, from Eqn. (54) (or equivalently Eqn. (55)) in Region D, where $\sin^2\theta^* \ll 1$ and $v \simeq \sin^2\theta^e \ll 1$, it follows that

$$f_{\mathrm{NL}}^{(4)} \simeq \frac{5}{6} g_s \left[-\eta_{\chi\chi} - 2\left(\epsilon^e - \hat{\eta}^e\right) \right] \simeq \frac{5}{6} \frac{\sin^6 \theta^e}{(\sin^2\theta^* + \sin^4\theta^e)^2} \eta_{\chi\chi}. \tag{70}$$

In the above we have used Eqn. (51) for g_s in Region D. We hence require the condition, Eqn. (57),

$$\sin^2\theta^* \lesssim v^2 \left(\frac{1}{\sqrt{v\,\mathcal{G}_s}} - 1 \right), \qquad \mathcal{G}_s = \frac{6}{5} |\eta_{\chi\chi}|^{-1}, \tag{71}$$

and find the exact upper limits on the parameters:

$$\sin^2\theta^* < \frac{1}{3} \left(\frac{5}{6}\right)^2 \left(\frac{3}{4}\right)^4 |\eta_{\chi\chi}|^2, \quad \text{and} \quad v = \sin^2\theta^e < \frac{5}{6} |\eta_{\chi\chi}|. \tag{72}$$

Noting that in Region D $\sin\theta = \beta\chi/(\alpha\varphi)$, from Eqn. (68) we require $N(\alpha - \beta) > 1$ so that $\sin^2\theta$ grows significantly during inflation.

In Table I, we give some explicit examples of values of α, β, φ_* and χ_* which lead to a large non-Gaussianity. Using Eqn. (43) we also calculate the spectral index. The contours of $f_{\mathrm{NL}}^{(4)}$ of this sum potential for a specific choice of α and β is given in Fig. 5 The first example in the Table I shows that it is possible to have $f_{\mathrm{NL}} \simeq 50$ and a scale invariant spectrum. The tensor-to-scalar ratio is also small in this example.

Our formula for $f_{\mathrm{NL}}^{(4)}$ is also consistent with the calculation of [5] but our analysis of the parameter space is more extensive and in particular it allows for very small initial field values. We therefore find a region with much larger non-Gaussianity than was found in [5].

As a further check on the algebra, we note that it is also possible to analyse this model using the formalism of a product potential with $W(\varphi, \chi) = W_0 \exp(\alpha\varphi^2/M_P^2) \exp(\beta\chi^2/M_P^2)$. This is equivalent to the sum potential Eqn. (67) in the limit of vacuum domination ($\alpha\varphi^2/M_P^2 \ll 1$ and $\beta\chi^2/M_P^2 \ll 1$). We have checked that this gives the same results. We note from Table I that we require a very small value of χ_* in Region D. If the value becomes too small then the motion of the χ field will become dominated by quantum fluctuations rather than the classical drift down the potential, $3H\dot{\chi} \simeq -W_\chi = -W_0\beta\chi$, which we have assumed. In order that we can neglect the effect of the quantum fluctuations, the condition we require on the background trajectory is $|\dot{\chi}|\pi/H^2 > \sqrt{3/2}$ [38]. Using $\mathcal{P}_\zeta \simeq 10^{-10}$, $H^2 \simeq W_0$ and Eqn. (42) the condition that the classical trajectory is valid can be rewritten as $(1 + \tilde{r})\beta^2\chi_*^2/(\alpha^2\varphi_*^2) > 6\mathcal{P}_\zeta$, where $\tilde{r} = \sin^4\theta^e/\sin^2\theta^*$. We have checked that this is satisfied for all of the examples given in the Table I, but it does provide a significant constraint on the total parameter space.

Very recently Cogollo et $al.$ have calculated the effect of the loop correction to the primordial power spectrum and bispectrum [39]. This loop correction arises from taking into account the contribution to the power spectrum and bispectrum arising from terms in the δN expansion which are non-leading in the expansion of the field perturbation $\delta\varphi$, see e.g. [40]. However they can still be significant if the coefficient to the term given by the derivative of N is extremely large, e.g. [41]. These "higher order" terms are usually neglected, but [39] has shown the first explicit example of an inflation model where they cannot be neglected. They consider a 2-field hybrid model with the same potential as Eqn. (67) in the special case of an unstable straight trajectory along one of the axes, with α and β
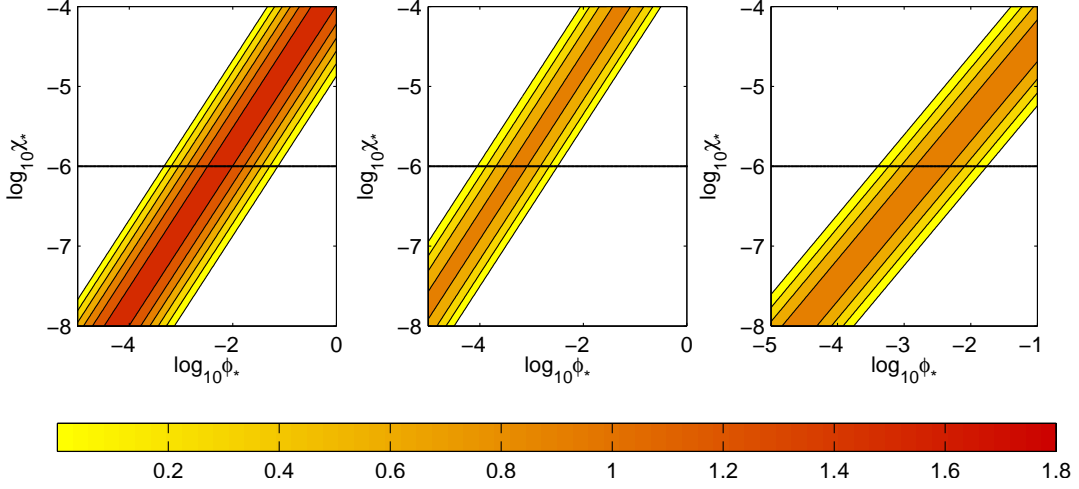
FIG. 5: The contour plot of $\log_{10}|f_{\mathrm{NL}}^{(4)}|$ with the sum potential, $W(\varphi, \chi) = V_0(1 + \alpha\varphi^2 + \beta\chi^2)$ (Example C). The parameters match the examples in Table I and the contours for $\log_{10}|f_{\mathrm{NL}}^{(4)}|$ are shown. White regions indicate when $|f_{\mathrm{NL}}^{(4)}| < 1$.

both negative. In this case, they find (for certain initial values) the loop correction is dominant and can generate an observable f_{NL}. We have checked that for the explicit values given in the table the loop correction does not dominate (under the assumption that the same loop correction is dominant here as the one in [39]). However this does provide a further restriction on the allowed parameter space of the "tree level" calculation of the model, which is assumed in Eqn. (70). We plan to return to this issue, and make a more thorough investigation of the hybrid model in a future publication.

VI. NON-GAUSSIANITY WITH NON-CANONICAL KINETIC TERMS (PRODUCT POTENTIALS)

Generally, the scalar fields need not only couple through the potential, but may also couple kinetically [42, 43, 44, 45]. In this section, therefore, we consider inflation governed by the following generalised action:

$$S = \int d^4x \sqrt{-g} \left[M_P^2 \frac{R}{2} - \frac{1}{2} g^{\mu\nu} \partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2} e^{2b(\varphi)} g^{\mu\nu} \partial_\mu\chi\partial_\nu\chi - W(\varphi, \chi) \right]. \tag{73}$$

When the kinetic energies are non-canonical, the local non-Gaussianity is altered from the previous sections. In the case of the sum potential, the modifications are non-trivial and are difficult to analyse. When the potential is of product form, however, we may at least partially use the previous analysis. We therefore concentrate only on a product potential, but our findings relate also to sum potentials.

With a product potential, the local non-Gaussianity is given by [19]:

$$\frac{6}{5} f_{\mathrm{NL}}^{(4)} = \frac{2e^{-2b_e+2b_*}}{\left(\frac{u^2\alpha^2}{\epsilon_\varphi^*} + \frac{v^2}{\epsilon_\chi^*} \right)^2} \left[\frac{u^3\alpha^3}{\epsilon_\varphi^*} \left(1 - \frac{\eta_{\varphi\varphi}^*}{2\epsilon_\varphi^*} \right) + \frac{v^3}{\epsilon_\chi^*} \left(1 - \frac{\eta_{\chi\chi}^*}{2\epsilon_\chi^*} \right) \right.$$
$$\left. + \frac{1}{2} \mathrm{sign}\,(b_\varphi)\, \mathrm{sign}\left(\frac{U_\varphi}{U} \right) \frac{vu^2\alpha^2}{(\epsilon_\varphi^*)^2} \sqrt{\epsilon_b^*\epsilon_\varphi^*} - \left(\frac{u\alpha}{\epsilon_\varphi^*} - \frac{v}{\epsilon_\chi^*} \right)^2 e^{2b_e-2b_*} \mathcal{A}_P \right], \tag{74}$$

where

$$\alpha \equiv e^{-2b_e+2b_*}\left[1+\frac{\epsilon_\chi^e}{\epsilon_\varphi^e}\left(1-e^{2b_e-2b_*}\right)\right] \tag{75}$$

$$= e^{-X}\left[1+\tan^2\theta^e\left(1-e^X\right)\right], \tag{76}$$

$$\mathcal{A}_P \equiv -\frac{\epsilon_\varphi^e\epsilon_\chi^e}{(\epsilon^e)^2}\left[\eta_{ss}^e-\frac{1}{2}\mathrm{sign}\left(b_\varphi\right)\mathrm{sign}\left(\frac{U_\varphi}{U}\right)\frac{(\epsilon_\chi^e)^2}{\epsilon^e}\sqrt{\frac{\epsilon_b^*}{\epsilon_\varphi^*}}-4\frac{\epsilon_\varphi^e\epsilon_\chi^e}{\epsilon^e}\right]. \tag{77}$$

Using new auxiliary functions, Eqn. (74) can be re-written as:

$$\frac{6}{5}f_{\mathrm{NL}}^{(4)} = \left[2J(\theta^*,\theta^e,X)\epsilon^*-F(\theta^*,\theta^e,X)\eta_{\varphi\varphi}^*-G(\theta^*,\theta^e,X)\eta_{\chi\chi}^*+\mathrm{sign}\left(b_\varphi\right)\mathrm{sign}\left(\frac{U_\varphi}{U}\right)K(\theta^*,\theta^e,X)\sqrt{\epsilon_b^*\epsilon_\varphi^*}\right.$$

$$\left.-2H(\theta^*,\theta^e,X)\left(\hat\eta^e-\frac{1}{2}\mathrm{sign}\left(b_\varphi\right)\mathrm{sign}\left(\frac{U_\varphi}{U}\right)\frac{\sin^4\theta^e}{\cos\theta^e}\epsilon^e\sqrt{\frac{\epsilon_b^*}{\epsilon^*}}-4\sin^2\theta^e\cos^2\theta^e\epsilon^e\right)\right], \tag{78}$$

where

$$F_p(\theta^*,\theta^e,X) \equiv \frac{e^{-X}\alpha^3\tan^4\theta^*}{\left(\alpha^2\tan^2\theta^*+\tan^4\theta^e\right)^2\cos^2\theta^e},$$

$$G_p(\theta^*,\theta^e,X) \equiv \frac{e^{-X}\tan^8\theta^e}{\left(\alpha^2\tan^2\theta^*+\tan^4\theta^e\right)^2\sin^2\theta^e},$$

$$H_p(\theta^*,\theta^e,X) \equiv \tan^2\theta^e\frac{\left(\alpha\tan^2\theta^*-\tan^2\theta^e\right)^2}{\left(\alpha^2\tan^2\theta^*+\tan^4\theta^e\right)^2},$$

$$J_p(\theta^*,\theta^e,X) \equiv e^{-X}\frac{\sin^2\theta^*}{\cos^2\theta^e}\frac{\left(\alpha^3\tan^2\theta^*+\tan^6\theta^e\right)}{\left(\alpha^2\tan^2\theta^*+\tan^4\theta^e\right)^2}.$$

$$K_p(\theta^*,\theta^e,X) \equiv e^{-X}\alpha^2\frac{\cos^4\theta^e\sin^2\theta^e}{\sin^2\theta^*}. \tag{79}$$

If $X=0$, then $\alpha=1$ and $F_p=f_p$, $G_p=g_p$, $H_p=h_p$ and $J_p=j_p$. The function F_p is plotted for various values of X in Figs. 6. ¿From this definition, it is clear that allowing $\alpha\neq0$, the range of θ^e for which we can obtain large $f_{NL}^{(4)}$ opens up. We also note that the symmetry between two fields is broken, due to $b(\varphi)$, which is apparent in Figs. 6. Note, though, that we require $\theta^*\sim0,\frac{\pi}{2}$ as before. Furthermore, when $X>0$ (or $b_e>b_*$), we no longer require $\cos^2\theta^e\ll1$.

Expressions for the power spectra and spectral index with a non canonical kinetic term are given in [19, 45].

VII.   CONCLUSION

We have made an in depth investigation into the large level of non-Gaussianity during two-field slow-roll inflation. We have shown that it is possible to generate a large level of non-Gaussianity during inflation without violating slow roll and when the inflaton field perturbations are Gaussian at Hubble exit.

The general conditions for generating a large non-Gaussianity show that the inflaton potential must have a specific shape so that the angle of the background trajectory can grow by about two orders of magnitude or more during inflation. In the case of an inflation potential made of a product of two quadratic potentials, this condition is not possible so that we conclude this model cannot generate a large non-Gaussianity during slow roll inflation. When the angle of the background trajectory grows sufficiently (in relative terms), we still need one of the fields to dominate throughout inflation, yet the remaining field can not remain full insignificant. For typical values of the slow roll parameters, we require that initially the subdominant field, say φ, must satisfy $\cos^2\theta^*=\epsilon_\varphi^*/\epsilon^*\lesssim10^{-7}$. This means that the field trajectory is almost exactly parallel to the χ axes initially and typically this requires a finely tuned initial condition. In the case of a product separable potential we then also require that the final value of the angle of the background trajectory lies in a narrow range much greater than the initial value (in relative terms) but still nearly parallel to the χ axis. The analysis for a sum separable potential is very similar but more complicated to analyse. The initial background trajectory must be similarly fine-tuned to be nearly parallel to one of the axes of the inflaton
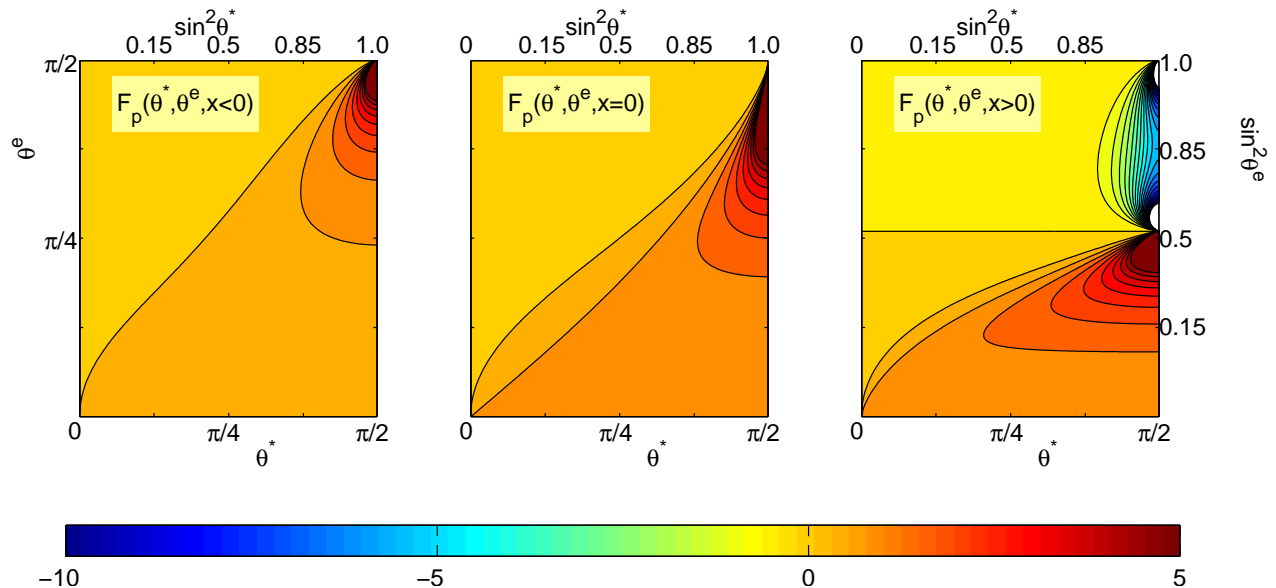
FIG. 6: The function F as defined in Eqn. (79) is plotted for three values of X: (a) $X = -0.001$ (left), (b) $X = 0$ (middle) and (c) $X = 0.001$ (right). The central graph corresponds to f_p since $X = 0$.

fields, but the tuning at the end of inflation is not written so simply. We have also shown how the constraints on generating a large non-Gaussianity may be eased if we generalise the inflaton model to allow a non-canonical kinetic term.

We have presented two explicit models where a large non-Gaussianity can be generated during slow roll inflation. One is a product potential driven by a field with a quadratic potential, which ends inflation when it approaches the minimum of the potential. If the second field with an exponential potential has a sufficiently small initial value then for certain values of the model parameter, λ, this field grows by the right amount to generate a large non-Gaussianity, $f_{NL} \sim 100$. We also consider the sum separable model of hybrid inflation. We provide a few specific choices of the model parameters, which are effectively $\eta_{\varphi\varphi}$ and $\eta_{\chi\chi}$, for a suitably large ratio of the initial field values this model generates a large non-Gaussianity. We find this is possible either if the η's have the opposite sign (so that inflation takes place near a saddle point) or when both of the η's have the same sign. However we have also found that if one of the field values is too small then quantum fluctuations may perturb the background trajectory to an extent that the classical trajectory in field space is no longer valid. There is also the possibility that the large scale loop correction which we have not generally considered in this paper may not be negligible. We intend to return to these issues and make a more thorough investigation of the hybrid model in a future publication.

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