Numerical Modelling of some Problems in Nonlinear Acoustics

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Some recent developments in numerical nonlinear acoustics are presented. First a three dimensional perturbation approach based on the finite-element method is described. This procedure can predict the propagation of acoustic fields produced by sources of arbitrary geometry as well as the pressure distribution inside a three dimensional cavity including boundary layer absorption. Its main limitation is due to its range of validity limited to waves of finite but moderate amplitude. We then describe a second approach: a numerical model for nonlinear waves and weak shocks in thermoviscous fluids based in a time-domain finite-difference algorithm. This algorithm does not present any practical limitations about the amplitude of the wave but it is referred to one-dimensional problems. Some future trends are also commented.

INTRODUCTION

Nonlinear acoustic phenomena are of practical interest since the 1930's, and particularly today in applications such as: industrial use of high power ultrasound, sonar, acoustic microscopy, medical ultrasound and non-destructive testing. In these applications, the need to account simultaneously for the combined effects of nonlinearity with absorption and geometrical characteristics of the system creates a very hard analytical task. In the last years numerical modelling becomes an important and useful tool for solving this kind of problems [1]. Two recent numerical developments concerning other problems in nonlinear acoustics are presented in this paper.

NUMERICAL APPROXIMATIONS

Fundamental equations

We consider nonlinear waves in homogeneous thermoviscous fluids. In order to obtain the governing nonlinear wave equations in Lagrangian coordinates, the isentropic state equation of Tait-Kirkwood and equations expressing the conservation of mass and momentum are considered [2].

$$\frac{p+\pi}{p_0+\pi}=\left(\frac{\rho}{\rho_0}\right)^{\chi} \qquad (1)$$

$$\frac{\rho_0-\rho}{\rho}=\vec{\nabla}\cdot\vec{u} \qquad (2)$$

$$\rho_0\frac{\partial^2\vec{u}}{\partial t^2}=-\vec{\nabla}p+\mu\nabla^2\frac{\partial\vec{u}}{\partial t}+\left(\mu_B+\frac{1}{3}\mu\right)\vec{\nabla}\left(\vec{\nabla}\cdot\frac{\partial\vec{u}}{\partial t}\right) \qquad (3)$$

where p is the pressure, t the time, ρ is the density of the fluid, p_0 is the ambient pressure, ρ_0 is the ambient density, $\vec{u}$ is the displacement, μ and μ_B are the viscosity and bulk viscosity respectively, and π and χ are characteristic constants of the fluid. This state equation reduces to the ideal gas state equation for $\pi=0$ and $\chi=\gamma$, where γ is the specific heat ratio.

Second order three-dimensional solution

By combining Eqs. (1), (2) and (3) a second order three-dimensional equation for the pressure is obtained. To solve this equation a perturbative method is applied. The solution is assumed to be the addition of two terms, the linear solution, p_l, and a second order correction, p_2, being $p_2 << p_l$. All the terms of third or higher orders are neglected. Therefore, $\left(\frac{p-p_0}{\rho_0 c_0^2}\right)^2 \cong \left(\frac{p_l}{\rho_0 c_0^2}\right)^2$, where c_0 is the low-amplitude velocity of sound. Considering the particular case of an initially generated harmonic wave a linear spatial equation is obtained for the second order correction which can be solved by classical methods. A numerical solution based on the finite-element method is proposed to solve three-dimensional acoustical problems of arbitrary geometry [3]. The effect of the boundary layer has been modelled by considering a complex impedance at the wall of the cavity.

Time domain finite-difference solutions

From Eqs. (1) to (3) a fully nonlinear one-dimensional wave equation for the displacement is written without truncation. A time-domain numerical approach based on a new finite-difference algorithm is developed to solve this problem (SNOW-AC). The complete solution is carried out in the time domain, i.e., all the harmonic components are obtained by only one solution. Time periodic, pulsed and any other excitation conditions can be considered. Since the fluid is at complete rest at the outset, the transitory phase is completely modelled. The model can support any small attenuation parameters without any stability or convergence problem. Cases from linear to strongly nonlinear behaviour can be studied [4].

RESULTS

Second order three-dimensional solution

The numerical development is applied to several cases and results are compared with experimental data. The model was tested in free-field conditions and for three-dimensional standing waves. Results referred to the near field of a directional transducer showed a very good agreement with experimental data [3] confirming the validity of the model for progressive waves. In Fig. 1 an axisymmetric three dimensional cavity excited by a transducer of complex structure is simulated.

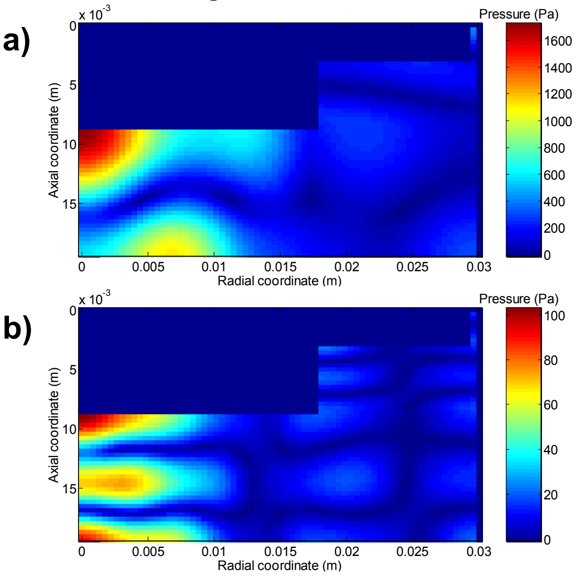

FIGURE 1. Pressure field distribution in an axisymmetric 3-D cavity. a) Fundamental b) Second harmonic

Time domain finite-difference solutions

The numerical model for nonlinear waves and weak shocks in thermoviscous fluids is applied to the analysis of one-dimensional cavities and plane waves propagation. Fig. 2 shows the steady state pressure amplitude (function of time and space) for an air-filled rigid-walled tube of length $c_0/(2f)$ (f is the excitation frequency) and an acoustic Mach number M=0.1. The wave is strongly distorted: the changes are very abrupt; an asymmetry between rarefaction and compression zones appears. The pressure nulls vary within a zone that takes almost all the tube: there is not a real node for the pressure; it is a “quasistanding” wave. In fact, the pressure wave has formed a shock that propagates from the emitter to the reflector (from $t=0$ to 0.5 times a period) and vice versa (from $t=0.5$ to 1 times a period).

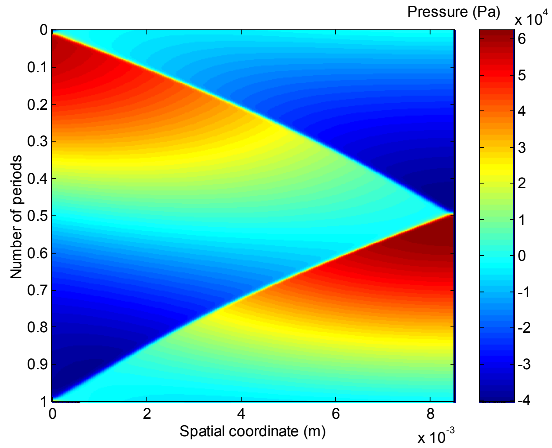

FIGURE 2. The steady state pressure amplitude as a function of time and space at the resonance frequency

CONCLUSIONS

Modelling the nonlinear field of actual transducers and real cavities opens up new possibilities in design for industrial processing where high intensity effects are important and nonlinear behaviour cannot be neglected. However, the propagation of finite amplitude waves through fluids involves, besides the nonlinear distortion of the waveform, well described by the algorithms presented here, other associated nonlinear effects such as acoustic radiation pressure, acoustic streaming and cavitation in liquids. Future studies should focus to solve these problems.

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