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On the relevance of diapycnal mixing for the stability of frontal meanders*

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SUMMARY: This work examines the possible importance of shear-induced diapycnal mixing in controlling the evolution and stability of meanders in oceanic frontal jets. We first review the conditions necessary for vortex stability and investigate how these may be modified in the presence of diapycnal mixing. The procedure used is rather crude but provides a measure of the relative importance of diapycnal mixing. It consists in constructing a simplified equation for the radial velocity that retains the density tendency and examining under what circumstances this velocity may grow in time. Next, we use a simple two-dimensional isopycnic model to examine the intensity of diapycnal mixing in meanders. In the model the along-front velocity is in geostrophic balance and the ageostrophic contributions are an oscillating deformation field and diapycnal mass exchange. The horizontal deformation field increases the slope of the isopycnals in temporal scales typical of Gulf Stream meanders, causing a reduction of the gradient Richardson number, *Ri*. The diapycnal flux is calculated as the divergence of the density Reynolds flux, which is parameterized in terms of *Ri*. The results of the model show that diapycnal mixing increases during the frontogenetical stages, reaching density tendency values of the order of 10^{-4} kg m^{-3}s^{-1} and convergence/divergence values of the order of 10^{-3} s^{-1}. It turns out that diapycnal mixing in meanders may be intense enough to control the separation and slope of the isopycnals and to condition the possibility of barotropic instability.

Key words: diapycnal mixing, barotropic instability, meanders, oceanic jets, atmospheric jets.

INTRODUCTION

The existence of an inverse relation between diapycnal (across-isopycnal) and horizontal oceanic diffusion is coherent with our physical intuition that natural turbulence in stratified fluids is anisotropic, i.e. when inhibited in one direction it will tend to align itself along a perpendicular plane. This physical argument was long ago used by very intuitive researchers such as Rossby (1936), Parr (1936) and Montgomery (1938), and has since been invoked in a number of works (e.g. Turner, 1973; Armi, 1979; Gregg, 1987). It is based not only on mass conservation (an ellipsoid has to become very elongated to contain the same mass as a sphere) but also on energetic arguments and observational data (e.g. Turner, 1973).

The intrinsic relation between horizontal and diapycnal motions is well illustrated in a number of oceanographic examples, such as in the mechanism of double-diffusion and in quasi-geostrophic theory. In the former example the great horizontal coherence of relatively thin layers that have their origin in double diffusion is very striking (Schmitt, 1994). An important feature of the latter example is that horizontal geostrophic motions cannot maintain the geostrophic thermal wind balance. To maintain this

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balance it is necessary to have ageostrophic motions, which include both vertical motions and diapycnal mixing (Keyser and Shapiro, 1986; Vélez-Belchi and Tintore, 2001). One aspect to keep in mind hereafter is the difference between vertical and diapycnal motions. Because of the slope of isopycnals, vertical motions do not necessarily imply changes in density and may involve both epipycnal (along-isopycnal) and diapycnal motions. It sould also be noted that, strictly speaking, we should be concerned with dianeutral rather than diapycnal motions, since water parcels with constant composition moving adiabatically follow neutral surfaces rather than isopycnals (McDougall, 1987). The small difference between neutral surfaces and isopycnals in the upper thermocline of intense jets, however, allows us to use the isopycnic terminology hereafter (Pelegrí and Csanady, 1994).

Despite the above considerations, most instability analyses are systematically divided into horizontal (barotropic and baroclinic) and diapycnal analysis (mainly shear-induced and double-diffusion), with no or little interaction between them. In particular, the unusual character of horizontal instability phenomena in western boundary currents, manifested by intense meanders, is the probable reason why the possibility of diapycnal mixing within these currents has so far received little attention (however, see Pelegrí and Csanady, 1991, 1994; Rodríguez-Santana *et al.*, 1999).

Furthermore, it may be argued that diapycnal instabilities (of Kelvin-Helmholtz type) have temporal and spatial scales quite different from those of horizontal instabilities. The horizontal size and growing time of each individual Kelvin-Helmholtz set of disturbances, of the order of one hundred meters and just a few minutes in the thermocline, is certainly different from those of horizontal instabilities in intense geophysical flows, of the order of one day and one hundred kilometers. The important point, however, is that Kelvin-Helmholtz type instabilities will only appear after frontogenesis has caused sufficient compression of the density field. This means that in oceanic (and atmospheric) jets there will be one single spatial and temporal relevant scale, the one associated with the intensification of the embedding frontal system during certain phases of the horizontal instabilities, or meanders, that develop in the flow (Keyser and Shapiro, 1986). This feature has been confirmed by recent analysis of Gulf Stream data (Rodríguez-Santana *et al.*, 1999; see also Pelegrí and Csanady, 1994).

In this work we examine the interaction between diapycnal mixing and horizontal instability within meanders in oceanic jets (or waves in atmopheric jets) in a very simple manner. Here we are not interested in obtaining the exact condition for instability, but in examining the relative size of the contribution due to diapycnal mixing. An important aspect of this work is how to estimate the intensity of mixing. To this end, we use a two-dimensional isopycnic ocean model with the along-front velocity in geostrophic balance, but allowing the existence of both frontogenesis and diapycnal mixing. The intensity of diapycnal mixing is calculated as the divergence of the vertical density Reynolds flux, which is determined from the vertical stratification and the vertical diffusion coefficient, the latter parameterised in terms of the gradient Richardson number. Hence, as the front evolves, the stratification and the gradient Richardson number change, leading to an intensification of diapycnal mixing. To simulate the frontogenetical process that takes place between different phases of Gulf Stream meanders, we use a deformation field that lasts 28 hours, in accordance with observations by Rodríguez-Santana *et al.* (1999). It should be clear that the same type of analysis could also be applicable to waves in atmospheric jets, with adiabatic heating and clear air turbulence replacing diapycnal mixing.

HORIZONTAL INSTABILITY IN MEANDERS

There are several reviews that examine either the theories of instability in stratified shear flow (Turner, 1973; Gregg, 1987) or the theories of barotropic and baroclinic instability (Pedlosky, 1979; Gill, 1982). The list of studies on the horizontal instability of meanders or vortices is also quite impressive. Early studies were concerned with the conditions leading to radial instability in atmospheric cyclones (Sawyer, 1946; Fjortoff, 1950; van Mieghan, 1951). More recent studies on barotropic instability have been made by Leibovich and Stewartson (1983), Eckhoff (1984), Gent and McWilliams (1986) and Paldor (1999) for a review see Hopfinger and van Heijst (1993). In these papers the necessary condition for instability of barotropic vortices is that the radial derivative of potential vorticity must change sign somewhere within the fluid domain. This result is consistent with results for parallel geophysical flows, which indicate that the necessary condition is the existence of a latitudinal inflection point of potential vorticity (e.g. Pedlosky, 1979).

While some of these studies indeed examine the interaction between barotropic and baroclinic instability, it seems surprising that little has been said about the effect of diapycnal mixing on radial instability. In this section we will investigate how large diapycnal mixing has to be in order to become significant in the radial instability analysis. The approach used here has several important simplifications and is aimed only at providing a tool for the identification of the relative importance of diapycnal mixing.

Analysis in cylindrical coordinates

For our analysis we use the mass and momentum isopycnic equations in cylindrical coordinates. These equations apply equally well for a vortex or a meander, the latter case being a portion of a vortex that is defined by its local curvature. We assume a basic flow model with streamlines flowing along lines of constant Montgomery potential, $\phi = p/\rho + gz$. Let us neglect all frictional terms and temporarily also ignore diapycnal mixing. The horizontal momentum equations in acceleration form (which have already made use of the isentropic continuity equation) are:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{v}{r}\frac{\partial u}{\partial \theta} = -\frac{\partial \phi}{\partial r} + fv + \frac{v^2}{r} \quad (1)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + \frac{v}{r}\frac{\partial v}{\partial \theta} = -\frac{1}{r}\frac{\partial \phi}{\partial \theta} - fu - \frac{uv}{r} \quad (2)$$

where f is the Coriolis parameter, and v and u are the tangential and radial velocities respectively. Let us scale the variables as $u=Uu'$, $v=Vv'$, $t=(1/f)(V/U)t'$, $r=(V/f)r'$, $\phi=V^2\phi'$, and further let $U/V=\varepsilon$ be a small quantity. Notice that in isopycnic coordinates, in the absence of diapycnal mixing, the density ρ is constant. The non-dimensional equations become (dropping primes):

$$\varepsilon^2\frac{\partial u}{\partial t} + \varepsilon^2 u\frac{\partial u}{\partial r} + \varepsilon\frac{v}{r}\frac{\partial u}{\partial \theta} = -\frac{\partial \phi}{\partial r} + v + \frac{v^2}{r} \quad (3)$$

$$\varepsilon\frac{\partial v}{\partial t} + \varepsilon u\frac{\partial v}{\partial r} + \frac{v}{r}\frac{\partial v}{\partial \theta} = -\frac{1}{r}\frac{\partial \phi}{\partial \theta} - \varepsilon u - \varepsilon\frac{vu}{r} \quad (4)$$

Let us now expand all dependent variables in terms of the small parameter ε, e.g. $v=v_0+\varepsilon v_1+\varepsilon^2 v_2 + O(\varepsilon^3)$. The O(1) equations are:

$$0 = -\frac{\partial \phi_0}{\partial r} + v_0 + \frac{v_0^2}{r} \quad (5)$$

$$\frac{dv_0}{dt} \equiv \frac{v_0}{r}\frac{\partial v_0}{\partial \theta} = -\frac{1}{r}\frac{\partial \phi_0}{\partial \theta} = 0 \quad (6)$$

The radial component of the momentum equation describes the behaviour of the azimuthal (tangential) velocity, while the tangential component is coherent with the basic hypothesis of flow along ϕ contours. The O(ε) equations are:

$$\frac{du_0}{dt} \equiv \frac{v_0}{r}\frac{\partial u_0}{\partial \theta} = v_1 + \frac{2v_0 v_1}{r} - \frac{\partial \phi_1}{\partial r} \quad (7)$$

$$\frac{dv_1}{dt} \equiv u_0\frac{\partial v_0}{\partial r} + \frac{v_0}{r}\frac{\partial v_1}{\partial \theta} = -\left(u_0 + \frac{v_0 u_0}{r} + u_0\frac{\partial v_0}{\partial r} + \frac{1}{r}\frac{\partial \phi_1}{\partial \theta}\right) \quad (8)$$

Taking the total time derivative of Equation (7), using Equation (8) and making use of the f-plane approximation leads to

$$\frac{d^2 u_0}{dt^2} = -u_0\left(1 + \frac{v_0}{r} + \frac{\partial v_0}{\partial r}\right)\left(1 + \frac{2v_0}{r}\right) - \frac{d}{dt}\left(\frac{\partial \phi_1}{\partial r}\right) + \frac{2v_0^2 v_1}{r^2} - \frac{1}{r}\frac{\partial \phi_1}{\partial \theta}\left(1 + \frac{2v_0}{r}\right) \quad (9)$$

At this point we note that the last three terms in this equation depend on the perturbed variables and that, as we will see below, they are not modified by the presence of diapycnal mixing. Since we are only interested in examining the relative contribution of diapycnal mixing, we will ignore them, i.e. we will examine the following equation:

$$\frac{d^2 u_0}{dt^2} = -\left(1 + \frac{v_0}{r} + \frac{\partial v_0}{\partial r}\right)\left(1 + \frac{2v_0}{r}\right)u_0 \quad (10)$$

Equation (10) indicates that, to the lowest order, the radial velocity is unstable when $(1+v_0/r+\partial v_0/\partial r)(1+2v_0/r)<0$.

Contribution due to diapycnal mixing

In the presence of diapycnal mixing the dimensional isopycnic equations in the tangential and radial directions become (e.g. Pelegrí and Csanady, 1994):

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{v}{r}\frac{\partial u}{\partial \theta} + w_\rho\frac{\partial u}{\partial \rho} = -\frac{\partial \phi}{\partial r} + fv + \frac{v^2}{r} \quad (11)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + \frac{v}{r}\frac{\partial v}{\partial \theta} + w_\rho\frac{\partial v}{\partial \rho} = -\frac{1}{r}\frac{\partial \phi}{\partial \theta} - fu - \frac{uv}{r} \quad (12)$$

The density tendency is defined as $w_\rho = D\rho/Dt$. It is related to the diapycnal velocity, with units of distance over time, by $w_e = Jw_\rho$, where $J = \partial z/\partial\rho$ is called the Jacobian, as it stands for the Jacobian of the transformation from Cartesian to isopycnic coordinates. Following the above procedure, with the additional scaling of $z = D z'$, $\rho = \rho_0 \rho'$, and $w_\rho = \varepsilon f \rho_0 w_\rho'$, leads to (dropping primes and with w_0 as the order zero density tendency):

$$\frac{d^2 u_0}{dt^2} = -\left(1 + \frac{v_0}{r} + \frac{\partial v_0}{\partial r} + \frac{w_0}{u_0}\frac{\partial v_0}{\partial \rho}\right)\left(1 + \frac{2v_0}{r}\right)u_0 \quad (13)$$

One possible interpretation of this last equation is that instability occurs when $(1+v_0/r+\partial v_0/\partial r+(w_0/u_0)\partial v_0/\partial\rho)(1+2v_0/r)<0$. Another possible approach is to consider the case in which diapycnal mixing greatly exceeds all other contributions, so that Equation (13) is approximated by

$$\frac{d^2 u_0}{dt^2} \approx -w_0 \frac{\partial v_0}{\partial \rho}\left(1 + \frac{2v_0}{r}\right) \quad (14)$$

Under this approximation the radial velocity of a water parcel will decrease in time (i.e. instability will be controlled by diapycnal mixing) when the term on the right-hand side of Equation (14) is negative. Except in steep anticyclonic meanders this term will be negative if $w_0<0$, i.e. when the water parcel mixes towards the sea surface.

At this point we should discuss the meaning of the quantity within the first parenthesis of Equation (13). To do so let us consider the dimensional isopycnic vorticity equation in the presence of diapycnal terms:

$$\frac{d}{dt}(\zeta + f) + w_\rho \frac{\partial \zeta}{\partial \rho} = -\frac{\zeta + f}{r}\left(\frac{\partial (ur)}{\partial r} + \frac{\partial v}{\partial \theta}\right) + \left(\frac{1}{r}\frac{\partial w_\rho}{\partial \theta}\frac{\partial u}{\partial \rho} - \frac{\partial w_\rho}{\partial r}\frac{\partial v}{\partial \rho}\right) \quad (15)$$

This equation may be rewritten in non-dimensional form as follows (dropping primes):

$$\frac{d}{dt}\left(1 + \frac{\zeta}{f}\right) + \varepsilon w_\rho \frac{\partial}{\partial \rho}\left(1 + \frac{\zeta}{f}\right) = -\frac{1}{r}\left(1 + \frac{\zeta}{f}\right)\left(\varepsilon \frac{\partial (ur)}{\partial r} + \frac{\partial v}{\partial \theta}\right) + \frac{\varepsilon^2}{r}\frac{\partial w_\rho}{\partial \theta}\frac{\partial u}{\partial \rho} - \frac{d}{dt}\left(\frac{w_\rho}{u}\frac{\partial v}{\partial \rho}\right) + \frac{w_\rho}{u}\frac{d}{dt}\left(\frac{\partial v}{\partial \rho}\right) \quad (16)$$

where

$$\frac{\zeta}{f} = \frac{v}{r} + \frac{\partial v}{\partial r} - \frac{\varepsilon}{r}\frac{\partial u}{\partial \theta} \quad (16a)$$

$$\frac{d}{dt} = \varepsilon \frac{\partial}{\partial t} + \varepsilon u \frac{\partial}{\partial r} + \frac{v}{r}\frac{\partial}{\partial \theta} \quad (16b)$$

To obtain (16) we have used $w_\rho/u = d\rho/dr = \partial\rho/\partial r$ and $\partial w_\rho/\partial r = d/dt(\partial\rho/\partial r)$. Expanding the dependent variables in terms of the small parameter ε and substitution in this last equation leads, to the lowest order:

$$\frac{\partial}{\partial \theta}\left(1 + \frac{v_0}{r} + \frac{\partial v_0}{\partial r} + \frac{w_0}{u_0}\frac{\partial v_0}{\partial \rho}\right) = 0 \quad (17)$$

This result indicates that, to the lowest order, the absolute vorticity is given by $1+v_0/r+\partial w_0/\partial r+(w_0/u_0)(\partial w_0/\partial\rho)$ and it has to be conserved as a water parcel moves along the vortex. This suggests that if the motion is initially stable then the absolute vorticity cannot change, no matter how steep the vortex becomes, which is only possible if the term containing the density tendency changes accordingly.

MODELING DIAPYCNAL MIXING IN MEANDERS

In order to assess the importance of diapycnal mixing in the Gulf Stream, we make use of Rodríguez-Santana's (1997) model, allowing frontogenesis to take place during time scales characteristic of Gulf Stream meanders (Rodríguez-Santana *et al.*, 1999). For this purpose we use the original model formulation in a Cartesian coordinate system, leaving its analysis in cylindrical coordinates for a posterior study.

Model equations and methodology

The two-dimensional model in isopycnic coordinates consists in a simple alongstream geostrophic balance, but allowing diapycnal mixing to regulate the separation between isopycnals through the mass conservation equation. The model is further simplified by assuming that both the Jacobian, J, and the baroclinic velocity, $\vec{v}_d$, are independent of the alongstream coordinate. The initial frontal depth-density field (Fig. 1a), is driven with an externally imposed horizontal deformation field (Hoskins, 1982), which results in the compression of the frontal system (Fig. 1b-c).

The velocity field is the result of a deformation

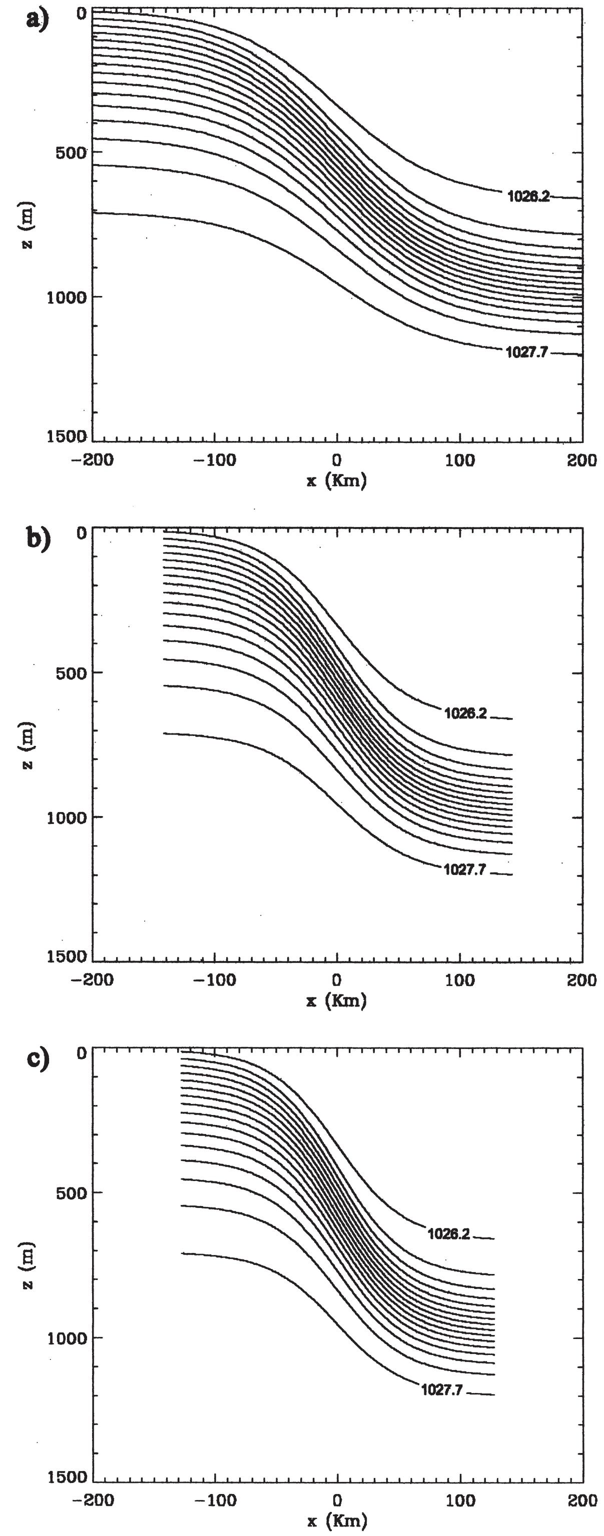

FIG. 1. – Depth distribution of the isopycnals between 1026.2 and 1027.7 kg m^{-3}, every 0.1 kg m^{-3}, at times (a) 0 hours, (b) 19 hours, and (c) 25 hours.

velocity field, $\vec{v}_d = (u_d, v_d, 0)$, and a baroclinic velocity field, $\vec{v}_d = (0, v_b, w_\rho)$. The components of the deformation velocity field are $u_d = -\gamma x$ in the direction normal to the front and $v_d = \gamma y$ in the direction parallel to the front, γ being a constant. This definition produces a non-divergent deformation field:

$$\frac{\partial u_d}{\partial x} + \frac{\partial v_d}{\partial y} = 0 \tag{18}$$

The component v_b of the baroclinic velocity field, parallel to the front, is in geostrophic balance:

$$f v_b = \frac{\partial \phi}{\partial x} \tag{19}$$

where ϕ is the Montgomery potential defined in the last section. The density tendency, $w_\rho = D\rho/Dt$, satisfies the mass conservation equation in isopycnic coordinates:

$$\frac{\partial j}{\partial t} = -u_d \frac{\partial j}{\partial x} - \frac{\partial (wj)}{\partial \rho} \tag{20}$$

with $j \equiv \rho J$. In a reference system moving with the deformation field (x', y, ρ), where $x' = xe^{-\gamma t}$, Equation (20) reduces to:

$$\frac{\partial j}{\partial t} = -\frac{\partial (wj)}{\partial \rho} \tag{21}$$

We integrate Equation (21) through recalculation of the frontal depth-density structure at each time step. To carry this out we estimate w_ρ as follows:

$$w_\rho = \frac{\partial F_z}{\partial z} = \frac{\partial}{\partial z}\left(\frac{K_t}{J}\right) = \frac{1}{J}\frac{\partial}{\partial \rho}\left(\frac{K_t}{J}\right) \tag{22}$$

where F_z is the vertical density Reynolds flux and K_t is the vertical eddy diffusivity. To calculate K_t we use a Langevin type equation:

$$\frac{\partial K_t}{\partial t}\tau = -K_t + K_f \tag{23}$$

where τ is the characteristic temporal scale of the turbulence, K_t is the effective vertical density diffusivity, and K_f is the vertical density diffusivity forced by the instantaneous conditions. The temporal scale of turbulence is taken as $\tau = N^{-1} = (-j/g)^{1/2}$, N being the buoyancy frequency (Pelegrí and Sangrà, 1998). The vertical density diffusivity K_f depends on the gradient Richardson number, Ri, according to the following expressions (Pelegrí and Sangrà, 1998, Pelegrí *et al.*, 1998):

$$K_f = \begin{Bmatrix} 1.110^{-8}\, Ri^{-9.2}\, Ri \le 0.33 \\ 2.610^{-3}(1 + 10Ri)^{-3/2}\, Ri \ge 0.33 \end{Bmatrix} \tag{24}$$

and

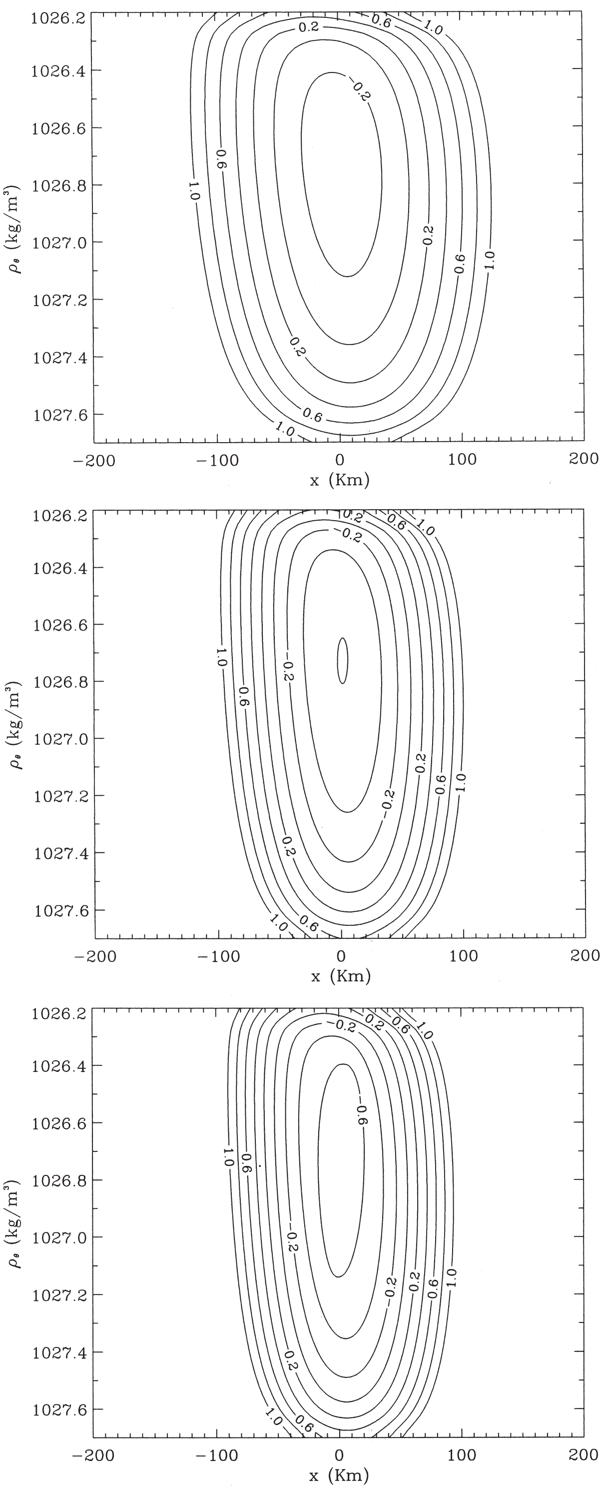

FIG. 2. – Density distribution of *log (Ri)* at times (a) 0 hours, (b) 19 hours, and (c) 25 hours.

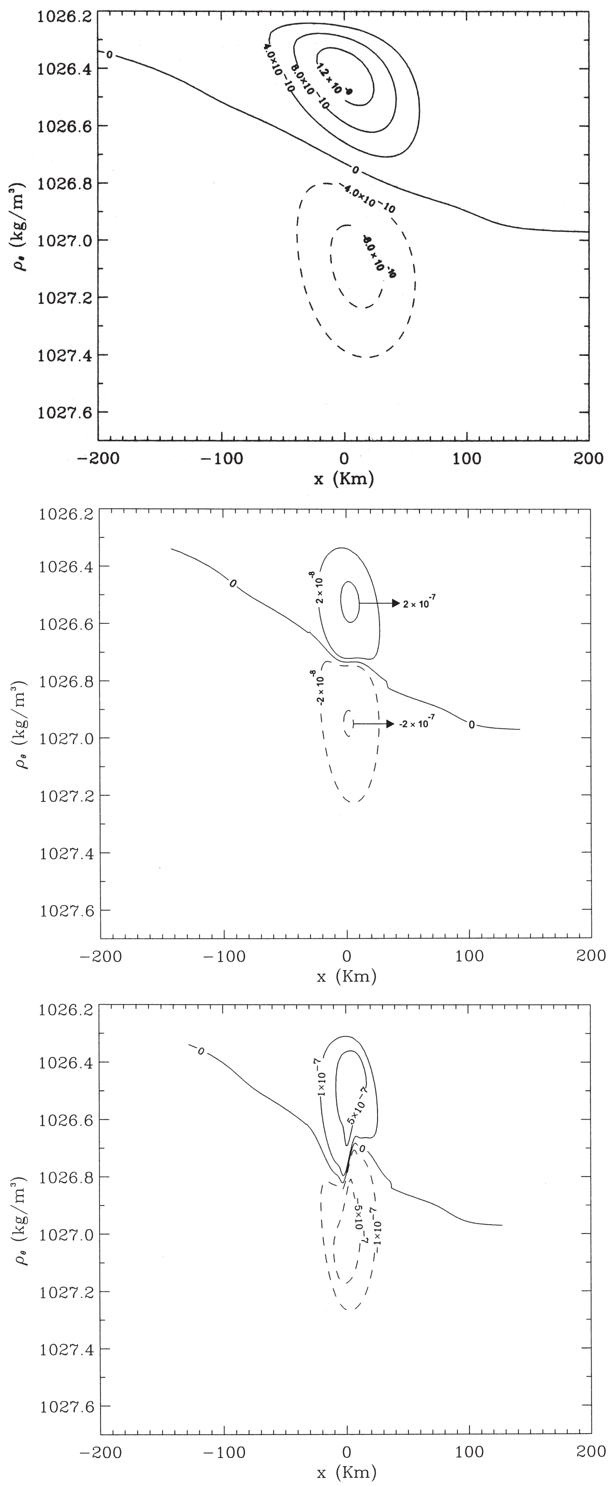

FIG. 3. – Density distribution of w_ρ at times (a) 0 hours, (b) 19 hours, and (c) 25 hours. Negative values are represented with dashed lines and positive values with solid lines.

$$Ri = \frac{-g\dfrac{\partial \rho}{\partial z}}{\rho\left(\dfrac{\partial v}{\partial z}\right)^2} = \frac{-g\dfrac{\partial z}{\partial \rho}}{\rho\left(\dfrac{\partial v}{\partial \rho}\right)^2} \tag{25}$$

In the next subsection we show the application of this model to a deformation field acting for 28 hours with a deformation constant $\gamma = 5 \times 10^{-4} s^{-1}$. The deformation constant is chosen as characteristic of oceanic deformation fields (Bleck *et al.*, 1988), while the simulation period corresponds to the time required by different phases of Gulf Stream meanders to pass through a fixed position in the northern Blake Plateau (Rodríguez-Santana *et al.*, 1999).

Model results

In order to illustrate the frontogenetical process we have produced Figure 1, which shows the evolution of selected isopycnals at three different times, $t = 0$, 19 and 25 hours. Figures 2 and 3 illustrate the concurrent gradient Richardson number, *Ri*, and density tendency, w_ρ fields, respectively. The initial conditions correspond to rather low diapycnal shear, and have minimum *Ri* values of about 0.6 and maximum w_ρ values of about 1.2×10^{-9} kg m^{-3} s^{-1}. At $t = 19$ hours, the frontal system is quite compressed (Fig. 1b) and causes an increase in diapycnal shear near the central zone. This leads to low *Ri* values (Fig. 2b), subcritical (< 0.25) near $x = 0$ and $\rho = 1026.7$ kg m^{-3}. This in turn produces relatively large density tendencies, of about 2×10^{-7} kg m^{-3} s^{-1}, in the central subcritical zone (Fig. 3b), and concurrent diapycnal convergence values of about -1.6×10^{-6} s^{-1}. At $t = 25$ hours, the frontal system has compressed even further (Fig. 1c) and causes very high diapycnal shear. This creates a rather large central region with subcritical *Ri* values and very high w_ρ values, of about 5×10^{-7} kg m^{-3} s^{-1} (Figs. 2c and 3c respectively). The concurrent diapycnal convergence in the central zone reaches maximum values of about -10^{-5} s^{-1}.

To obtain a better perception of the last phases in the compression process, we present the evolution of the Jacobian, the diapycnal shear, the gradient Richardson number, and the density tendency at $x = 0$ km, from $t = 25$ hours to $t = 27.2$ hours (Fig. 4). We may appreciate how stratification increases or decreases at different vertical positions, corresponding to the shrinking or stretching of the isopycnal layers. Such shrinking or stretching is caused by the distribution of diapycnal divergence or convergence, e.g. the isopycnals in the central region thicken because of diapycnal mass convergence. The diapycnal convergence/divergence is driven by

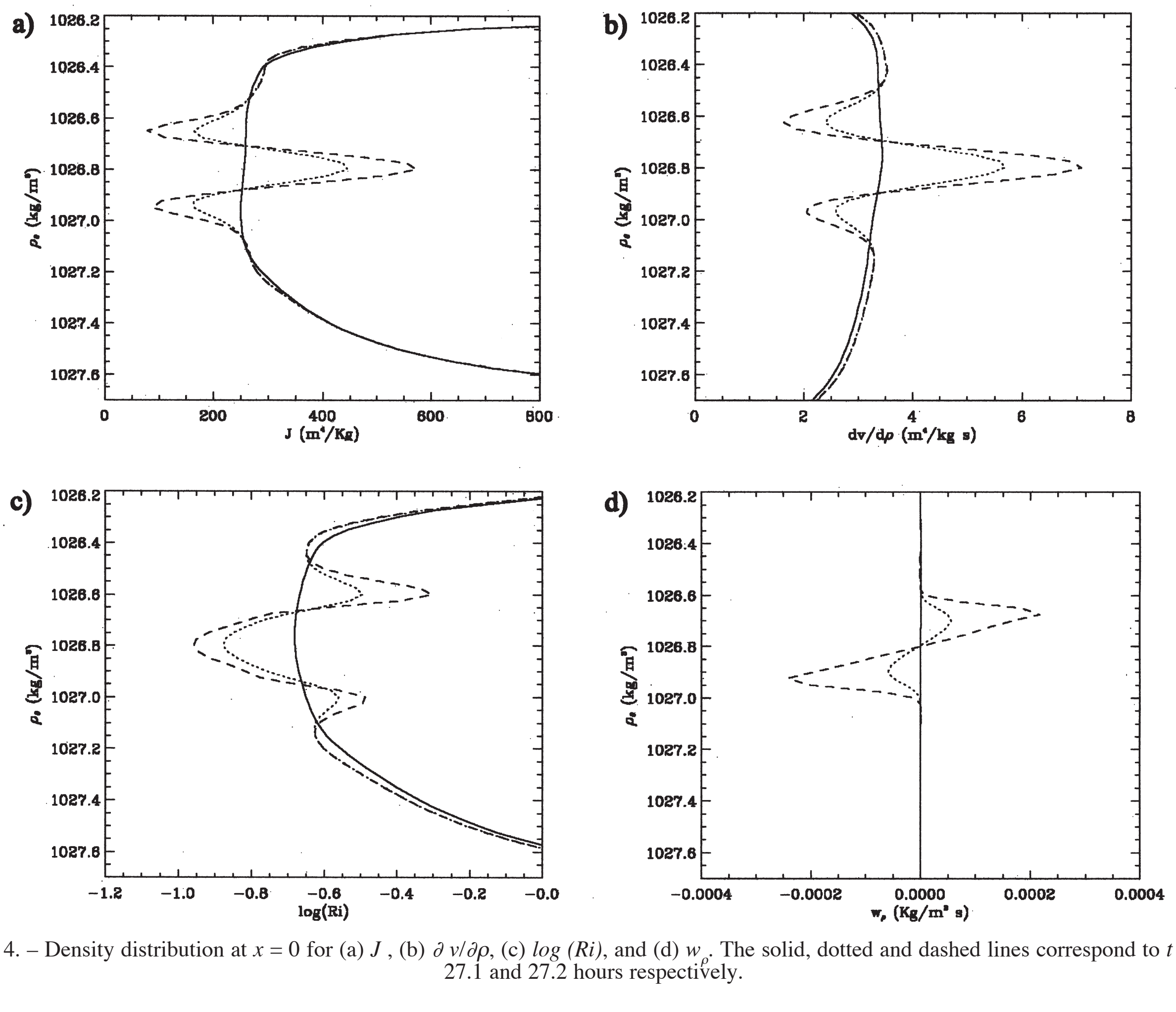

FIG. 4. – Density distribution at $x = 0$ for (a) J, (b) $\partial v/\partial\rho$, (c) *log (Ri)*, and (d) w_ρ. The solid, dotted and dashed lines correspond to $t = 25$, 27.1 and 27.2 hours respectively.

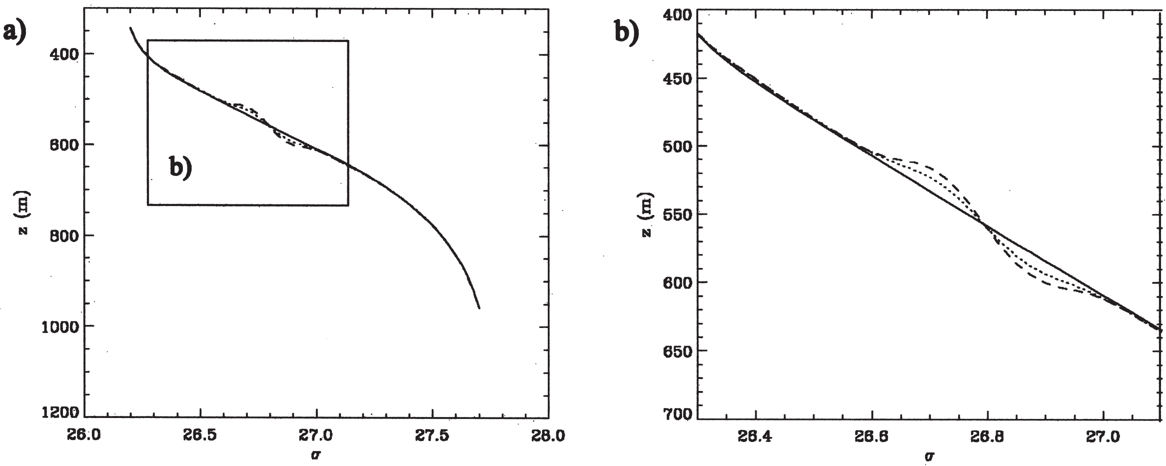

FIG. 5. – (a) Density profile at x = 0. (b) Detail of the area of interest. The solid, dotted and dashed lines correspond to t = 25, 27.1 and 27.2 hours respectively.

vertical density Reynolds flux, which ultimately depends both on the vertical stratification and the diapycnal shear. In the central region, for example, both the stratification and the diapycnal shear initially attain their maximum values, causing maximum positive and negative density tendencies around it and inducing the observed mass convergence. At $t = 27.2$ hours the density tendency attains positive and negative maxima of absolute value $\pm 2 \times 10^{-4}$ kg m^{-3} s^{-1}. At this time the diapycnal shear reaches a maximum value of 7 m^4 kg^{-1} s^{-1}.

In zones with high diapycnal convergence and divergence the vertical profile of the density is modified. In Figure 5 we present the evolution of the vertical density profile from t = 25 hours to t = 27.2 hours. Figure 5b clearly illustrates the formation of a low stratified region at about 1026.8 kg m^{-3} and highly stratified regions at 1026.65 kg m^{-3} and 1026.95 kg m^{-3}, resembling a staircase type structure in the density profile (Pelegrí and Sangrá, 1998).

DISCUSSION

One result derived from the first part of this paper is that the absolute vorticity of a water parcel is, to lowest order, conserved as it moves along a vortex. This imposes $(w_\rho/u)(\partial v/\partial \rho)+\partial v/\partial r+v/r+f=f_0$, with f_0 being the planetary vorticity of the straight unperturbed flow. If the changes in f are small, such as when the Gulf Stream flows eastward, we obtain that diapycnal mixing will become relevant when the absolute value of $(w_\rho/u)(\partial v/\partial \rho)$ is of the same order as $\partial v/\partial \rho+v/r$. This result is coherent with the one derived following Equation (13), on the relative size that diapycnal mixing has to have to become important in controlling the growth of radial velocities in vortices.

Satellite pictures in Lee and Atkinson (1983) allow us to estimate some of the above numbers for meanders near Charleston Bump. For anticyclonic meanders, for example, using a tangential velocity of 1.5 m/s and a radius of 35 km, we obtain that both $\partial v/\partial r$ and $-f/2$ are about -4×10^{-5} s^{-1}. The above condition, however, applies for epipycnal gradients that cannot be directly estimated from usual density and velocity cross-sections. Using Pelegrí and Csanady's (1994) isopycnic cross-sections we may estimate the epipycnal velocity gradient to be probably much larger (smaller in absolute value), about -10^{-6} s^{-1}. Hence, a gross condition for diapycnal mixing to become important in the development of horizontal instabilities is that $(w_\rho/u)(\partial v/\partial \rho)$ has to be of the same order as the total vorticity, i.e. of the order 10^{-4} s^{-1}.

A second result from the first part of the paper comes after Equation (14), and may be used to set a limit on the characteristic time scale over which strong diapycnal mixing must act in order for the radial velocity to decrease. This time scale is given by

$$\tau = \left[\frac{w_\rho}{u} \frac{\partial v}{\partial \rho} \left(1 + \frac{2v}{r} \right) \right]^{-1/2}$$

or, using the above result, by

$$\tau \approx \left[\left(\frac{\partial v}{\partial r} + \frac{v}{r} \right) \left(1 + \frac{2v}{r} \right) \right]^{-1/2} .$$

In western boundary currents this time scale is of the order of 10^4 s, hours to days, and coincides with the characteristic time for the development of meanders in oceanic jets.

In the second part of the paper we have proposed a very simple model for a frontal system characteristic of intense ocean currents, such as those associated with the Gulf Stream. Using this model we have simulated a frontogenetical process that develops at a rate, and during a time, similar to those that characterise Gulf Stream meanders (Rodríguez-Santana, *et al.,* 1999; Pelegrí and Csanady, 1994). In our simulation we find that the density tendency reaches values of $\pm 2 \times 10^{-4}$ kg m^{-3} s^{-1} and that diapycnal shear reaches a maximum value of 7 m^{4} kg^{-1} s^{-1}. Even if these figures are one order of magnitude too large, and using radial velocities ranging between 10^{-1} of 10^{-4} m s^{-1}, this implies that $(w_\rho/u)(\partial v/\partial \rho)$ reaches values larger than the total vorticity, of the order 10^{-4} s^{-1}. This supports the idea that the intensity of diapycnal mixing during frontogenesis may indeed be high enough to control the development of barotropic instabilities.

One important limitation in our study is due to the several approximations used in the instability analysis. Another limitation is that the stability condition we have obtained is characteristic of barotropic type instabilities, while diapycnal mixing results from increased diapycnal shear (change in alongstream velocity with density) in some phases of meanders, which is a baroclinic process. Nevertheless, we believe that our results indeed endorse the potential role played by diapycnal mixing in the inhibition or generation of horizontal instabilities in strong oceanic currents. This agrees with the simple idea that diapycnal mixing, if intense enough, will redistribute momentum and vorticity in such a way as to significantly influence the development of horizontal instabilities.

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