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RESONANCE-BASED ACOUSTIC TECHNIQUE APPLIED TO THE DETERMINATION OF YOUNG'S MODULUS IN GRANITES

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ABSTRACT
The natural stone industry plays an important role in construction sector activity, and due to this fact the accurate knowledge of the physico-mechanical properties of this kind of materials is indispensable for the configuration of their quality standards. In this work we have optimized an acoustic technique, based on the measurement of the fundamental mode resonance frequency of the longitudinal wave, to determine Young's modulus in different granite variety specimens. The equipment employed was an Erudite MK3 test system (CNS Farnell) working in the range 1 to 100 kHz. The resonance frequencies obtained ranged from 4 to 10 kHz, depending on the granite variety. Based on these resonance frequencies, we obtained values of the longitudinal wave velocities ranging from 2500 to 6000 m/s and of the dynamic Young's modulus from 20 to 100 GPa. We also compared the dynamic modulus results with the static Young's modulus obtained by destructive techniques, and analysed its relationship with other significant mechanical properties.

INTRODUCTION
The worldwide market of ornamental stone is characterized by its high degree of concentration, since only 10 countries account for more than 80% of production. Spain ranks first in slate production, second in marble, and is the top European producer of granite. Within the national panorama of the ornamental stone sector, more than 40 varieties are produced in the region of Extremadura which is second ranked nationally in the extraction of this type of stone [1]. There is ever more widespread use of granites as construction materials. It is therefore indispensable to have the maximum of rigorous information about their physico-mechanical characteristics so that one can foresee their potential deterioration under the effects of external and internal agents. In this context, our teams of researchers from the University of Extremadura, from the Technological Institute of Ornamental Stone and Construction Materials (INTROMAC), and from the Institute of Acoustics (CSIC) have been studying a broad range of specimens corresponding to 39 varieties of granite from industrial quarries of different locations in Extremadura [2], [3]. Of these, 35 have been subjected to some type of acoustic analysis. The fundamental goal has been to analyze these granites using both destructive and non-destructive techniques for their petrological and physico-mechanical characterization.

Acoustic tests were conducted on six 30×10×5 (cm) prismatic cube specimens of each variety determining the resonance frequency of the acoustic waves in this pieces. The longitudinal wave velocity was determined by means of a resonance-based method using an Erudite MK3 test device of working frequency range 1 Hz to 100 kHz, with EMAT vibrator and piezoelectric receiver (Figure 1). The frequency range of the equipment covers sonic and ultrasonic frequencies, but due to the dimensions of the specimens the fundamental resonances obtained were in the sonic range. As one sees in the figure, the specimens are placed in a special device for their correct measurement, there being three support points on its base (at the ends and in the centre) to facilitate the formation of the fundamental resonance mode in the sample. The equipment automatically varies the emission frequency of the pulse until an amplitude maximum is detected indicative of resonance of the wave in the specimen. At least 6 readings of the resonance frequency were made in each sample, all in the direction of greatest length of the specimen (30 cm). From this value of the frequency and knowing that the longitudinal wavelength in the fundamental resonance mode is twice the length of the specimen, we get the value of the longitudinal ultrasound wave velocity.

Figure 1.- Acoustic resonance-based experimental device.

We used the value of the resonance frequency of 35 of the varieties to obtain their dynamic Young's modulus according to the expression [4]:

$$E_D(GPa) = 4 \times l^2 \times F_L^2 \times \rho \times T \qquad \text{(Eq. 1)}$$

where l is the length of the specimen in m, F_L the resonance frequency in Hz, ρ the density in kg/m^3, and T a factor that depends on the relative dimensions of the specimen.

Additionally, we performed a complete physico-mechanical analysis of the granite varieties studied, determining their compressive, flexural, and impact strength, Knoop hardness, breaking load, porosity, absorption, and density. Likewise, we made destructive tests of 12 of the varieties to determine their static Young's modulus. This test was carried out on at least 4 cylindrical specimens of 6 cm diameter and 120 cm height. The specimen is subjected to a continuously increasing load at the rate of 0.49 to 0.98 MPa/s until breakage, using a 150 ton press. All the tests were done according to the procedures stipulated in the norms established by Spanish legislation.

MEASUREMENT RESULTS

The first columns of Table I summarize the measured resonance frequencies and the magnitudes (longitudinal wave velocity and Young's modulus) derived from those measurements. The longitudinal wave velocity values are comparable to those reported by other workers for stone materials, and for granites in particular: e.g., for granite, the value reported by Sheriff et al. is some 4500 m/s [6], and by the ASTM is 3470 m/s [7].

The dynamic Young's modulus values (E_D) given in Table I were also comparable to the literature values for rocks. Thus, for cylindrical specimens of argillite, Liao et al. [8] find values of around 50 GPa, Yale et al. [9] values of 15–55 GPa for sandstone, and Song et al. [10] values of 50–70 GPa for granites.

In addition, other physico-mechanical variables were measured by means of appropriate techniques. For these magnitudes, the results were comparable to those reported by other workers and to the systematic studies done by the INTROMAC for this type of stone [5].

Table I. Statistical summary of the physico-mechanical variables measured in the tests carried out on the set of granite varieties studied.

	Reson. Freq. F_L (Hz)	Reson.-Based Velocity V_{R-B} (m/s)	Dynamic Young's Modulus E_D (GPa)	Static Young's Modulus E_S (GPa)	Water Abs. Capillarity C (g/m^2 s$^{0.5}$)	Water Abs. Atm. Press. *Ab* (%)
Mean	7162	4297	50.3	33.8	0.48	0.21
Median	7124	4274	49.6	33.6	0.45	0.20
SD	991	594	14.6	13.2	0.24	0.10
Range	4265-9865	2559-5919	20.3-100.4	16.1-53.1	0.11-1.19	0.1-0.5
N	35	35	35	12	36	36

	Open Porosity *Po* (%)	App. Density ρb (kg/m^3)	Impact Strength *Is* (cm)	Knoop Hardness *Dk* (Mpa)	Comp. Strength *R* (MPa)	Flexural Strength *Rtf* (MPa)	Breaking Load *F*(N)
Mean	0.66	2679	47	2277	154	14.2	2052
Median	0.60	2630	46	2131	151	13.5	2050
SD	0.36	113	11	739	22	5.4	499
Range	0.10-1.93	2570-3030	31-80	1086-4486	108-218	6.3-39.2	1000-3200
N	36	37	34	37	36	37	33

CORRELATIONS BETWEEN PHYSICO-MECHANICAL PROPERTIES

Comparison of the resonance-based technique values of the dynamic Young's modulus with those we determined by T/T A-scan showed them to be very similar for each variety. Indeed, the linear regression coefficient for the correlation between the two was around 0.9 [11]. This similarity in the determinations of dynamic elastic constants by the two techniques has also been reported by Suárez-Rivera et al. for different types of rock fragments, with resonance frequencies from 15 to 30 kHz [12].

We also found linear correlations between the dynamic Young's modulus and some physico-mechanical parameters, obtaining linear regression coefficients of 0.475, 0.677, -0.493, -0.546, and 0.769 (p-values < 0.01) for the correlations with the compression strength, flexural strength, open porosity, water absorption capillarity, and apparent density, respectively. By way of example, Figure 2 shows the linear regression between the dynamic Young's modulus and the water absorption capillarity for the granite varieties tested.

For the static Young's modulus, however, we only found moderate statistically significant correlations with the open porosity (r = -0.621) and the water absorption capillarity (r = -0.619).

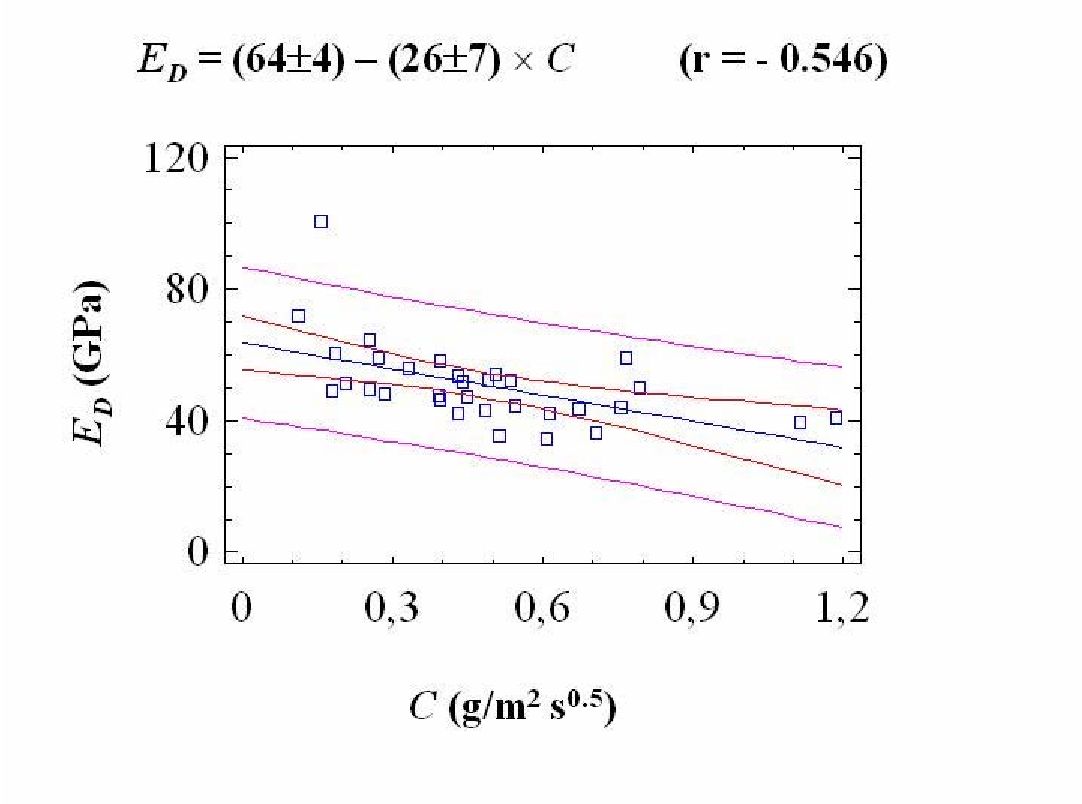

Figure 2. Linear regression between the dynamic Young’s modulus and water absorption capillarity, for the granite varieties tested.

In general, the acoustic technique gave values greater than those obtained applying destructive techniques to determine the static Young's modulus (E_S) [5], with the ratio between the two ranging from 0.9 to 3.5 according to the variety. This result is also coherent with the literature, according to which, due to the nonlinear deformation response of the rock to applied stress, the dynamic elastic constants in rocks in general, and in granites in particular, may be significantly greater than the static values, depending on the type and characteristics of the rock [9], [13].

To illustrate the relationship between the two variables, Figure 3 shows the fit obtained correlating the ratio between the dynamic and static Young's modulus with the static modulus.

In performing the fit, we took into account that two of the varieties studied presented values could be considered anomalous. Thus, for the variety Amarillo (Yellow) Jara (AJA), the resonance frequency and hence the wave velocity obtained were significantly lower (2800 m/s) than the velocity determined by T/T techniques (4400 m/s) [11]. This could be because this variety is extracted from a batholith of heavily and heterogeneously deteriorated granite, so that specimens may differ significantly in their physico-mechanical characteristics. Indeed, we observed that the specimens used for the static Young's modulus corresponded to the group of specimens used to determine the ultrasound velocity by means of the T/T technique. In the fit we therefore used the value of the Young's modulus obtained from this technique [11] instead of from the resonance-based technique.

Neither did the value of the static Young's modulus of the variety Negro (Black) Villar (NVI) correspond to the tendency shown in Figure 3 by the rest of the varieties. The abnormally low value of this static elastic constant (28.8 GPa) don’t reflects the variety's general physico-mechanical properties: it has the lowest open porosity (0.10 ± 0.01%) and highest compressive strength (220 ± 40 MPa), flexural strength (38 ± 3 MPa), apparent density (3030 ± 9 kg/m^3), breaking load (3200 ± 600 N), resonance frequency (9865 Hz), and velocity (5919 m/s), and

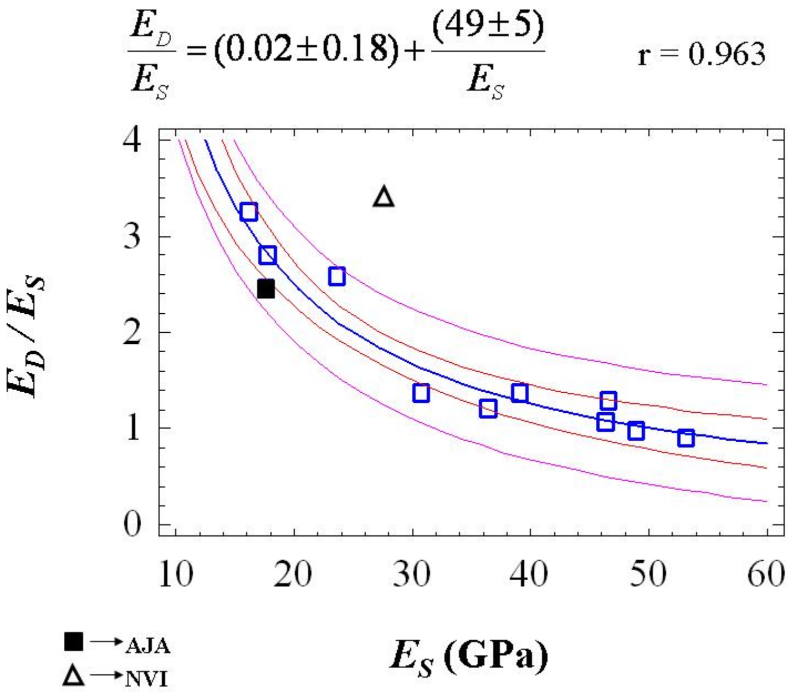

Figure 3.- The ratio of the dynamic to static Young's modulus versus static Young's modulus, for the granite varieties tested.

therefore the greatest value of the dynamic Young's modulus. These extreme values are because this variety is in reality a diabase, but we have not found clear explanation for the low value of the static modulus that we obtained. For these reasons, the fit of Figure 3 was made without using the E_D and E_S values of this variety.

The fitting equation used was a linear in the reciprocal of X, which gave correlation coefficient r = 0.963, indicating that the proposed model explains 93% of the system's variability. This type of equation was also proposed by Wang in a work on reservoir rocks [13], with separate fits for soft rocks (E_S < 15 GPa) and hard rocks (E_S > 15 Gpa), of respective equations: (E_D/ E_S) = 2.413 + (2.556/ E_S) and (E_D/E_S) = 0.867 + (13.180/ E_S). The coefficients of our fit are of the order of those obtained by Wang for hard rocks, as can be seen in Figure 3.

CONCLUSIONS

We have made a complete physico-mechanical analysis of a broad set of granite varieties, determining their acoustic resonance frequencies, the longitudinal wave propagation velocities, and the Young's moduli.

The velocities obtained from the resonance frequencies ranged approximately from 2500 to 6000 m/s, with dynamic Young's moduli (E_D) from 20 to 100 GPa, and the static Young's moduli (E_S) from 16 to 53 Gpa; all these estimated values are comparable to those reported by other workers in similar studies.

Also, we found moderate correlations between these elastic constants and other physico-mechanical variables – the porosity, absorption, density, and strength.

Finally, we were able to establish a relationship of the ratio E_D/E_S with E_S in the form of a fit to a function linear in the reciprocal of X, with coefficients of the order of those reported in the literature for hard rocks, mainly granites.

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