Multi-product firms and product variety*

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Abstract

The goal of this paper is to study the role of multi-product firms in the market provision of product variety. The analysis is conducted using the spokes model of non-localized competition proposed by Chen and Riordan (2007). Firstly, we show that multi-product firms are at a competitive disadvantage vis-a-vis single-product firms and can only emerge if economies of scope are sufficiently strong. Secondly, under duopoly product variety may be higher or lower with respect to both the first best and the monopolistically competitive equilibrium. However, within a relevant range of parameter values duopolists drastically restrict their product range in order to relax price competition, and as a result product variety is far below the efficient level.

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1 Introduction

Does the market provide too much or too little product variety? Is the supply of books, CD's, TV programs, furniture or cereals sufficiently diverse to efficiently match the preferences of heterogeneous consumers? Or, on the contrary, do profit maximizing firms tend to produce a disproportionate array of products and incur into excessive costs? The existing theoretical literature clearly suggests that anything can happen, i.e., product diversity may be excessive or insufficient depending on the relative strength of various effects. However, the literature has typically focused on the case of single-product firms (monopolistic competition). In contrast, in many markets individual firms produce a significant fraction of all varieties. These multi-product firms choose their product range as an additional strategic tool, which may potentially affect the overall provision of variety. Does the presence of multi-product firms reduce or expand product variety with respect to the case of single-product firms? And with respect to the first best? Can an incumbent firm use product proliferation to monopolize the market and prevent entry?

In this paper we address these issues by introducing multi-product firms into the spokes model of non-localized competition proposed by Chen and Riordan (2007). There are several reasons that justify the choice of model; in particular, the spokes model provides a tractable, intuitive and transparent framework to study competition and product variety when neighboring effects are absent, which is increasingly the case in many industries. By considering a continuous approximation of the original model we show that it is possible to study in the same framework alternative market structures, including monopolistic competition (large number of single-product firms, duopoly (two large multi-product firms) and even asymmetric competition (one large multi-product firm) competing against a competitive fringe: a large number of single-product firms).

The study of product diversity has been typically conducted using two alternative families of models. On the one hand, spatial models of localized competition, similar to those proposed by Hotelling (1929) and Salop (1979), have been extensively used. However, it is generally agreed that they are not well suited to study either the welfare implications of product variety or multi-product firms. On the other hand, a large literature has followed Spence (1976) and Dixit and Stiglitz (1977) (SDS) and assumed the existence of a representative consumer with well defined preferences over all possible varieties. In this set up neighboring effects are absent and each firm competes against 'the market'. Unfortunately, these models provide little guidance regarding the set of circumstances under which one would expect, from a social point of view, that equilibria would involve excessive or insufficient product variety.[1] In any case, these frameworks can easily accommodate multi-product firms.[2] The two papers which are most closely related to the theme of this paper are Ottaviano and Thisse (1999) and Anderson and de Palma (2006).

The spokes model (Chen and Riordan, 2007) extends the Hotelling model to an arbitrary number of varieties in a perfectly symmetric set up. A crucial feature of the model is that each consumer only cares about two varieties, which differ across consumers. The preference space consists of N spokes that start from the same central point. The producer of each potential variety is located in the extreme end of a different spoke. If $N = 2$ then we are in the standard Hotelling set up. As N goes to infinity, and if each variety is produced by a different firm, the model becomes an adequate representation of monopolistic competition. This set up provides new insights by identifying the set of circumstances under which we should expect excessive or insufficient product diversity.

In this paper we take the spokes model one step further by considering multi-product firms. It turns out that the spokes model can accommodate multi-product firms as easily as the SDS model and, moreover, it brings about new insights and useful welfare results.

Tractability of the spokes model is considerably enhanced by assuming that the number of varieties is sufficiently large. In the next section we present the finite model and formulate the continuous approximation. Thus, the product range of a multi-product firm can be treated as a continuous variable. This approach could also be very useful in other applications of the spokes framework.

In Section 3 we study the benchmark case (already examined in Chen and Riordan, 2007) in which each potential variety is produced by a different firm. The novelty here is that we use the continuous approximation, which allows us to represent in the same graph the fraction of active varieties as a function of the fixed cost, under both scenarios: the free entry equilibrium

[1] A notable exception is Ottaviano and Thisse (1999), which we discuss in more detail below.

[2] Some of the prominent and recent papers include Allanson and Montagna (2005), Nocke and Yeaple (2006), Eckel and Neary (2006), Bernard et al. (2006). These papers are mostly concerned with the role of multiproduct firms in a trade liberalization process.

and the first best. As a result we can visualize the regions of parameter values for which market forces deliver excessive or insufficient product variety.

The main goal of Section 4 is to understand the role of preferences, first-mover advantages, and economies of scope in the emergence of multi-product firms. We consider a game played between a single (large) multi-product firm and a large number of single-product (small) monopolistically competitive firms. We ask whether or not the large firm enjoys a competitive advantage vis-a-vis small firms in the absence of economies of scope. If we focus on the most interesting region of the parameter space, in the pricing stage the large firm behaves like a coalition formed by a fraction of single-product firms, internalizes some of the cross price effects, and sets higher prices than small firms. Thus, the presence of the large firm tends to relax price competition, which creates a positive externality to small firms. In fact, profits per variety are higher for single-product firms. The reason is that any small firm can always imitate the pricing behavior of the large firm and make the same level of profits per variety of the large firm, but in equilibrium small firms find it profitable to set a lower price, which implies that they make higher profits per variety. In free entry equilibrium small firms make zero profits, which implies that no multi-product firm will be active. It is only in a particular region of the parameter space where multi-product firms can survive simply because in equilibrium all firms set the same price and make zero profits. These results are independent of whether or not the large firm enjoys a first-mover advantage. In other words, in contrast to the case of localized competition, an incumbent firm cannot use product proliferation to monopolize the market and prevent entry. To summarize, in industries characterized by non-localized competition firm size (as measured by the length of its product range) is a competitive disadvantage. Therefore, multi-product firms can only emerge if economies of scope are sufficiently strong.

In Section 5, we assume that economies of scope are such that there can only exist two (multi-product) firms in the market. Thus, firms choose both product range and prices strategically. Product variety under duopoly can be higher or lower than under monopolistic competition depending on the relative strength of three different effects:

a) *Cannibalization:* a duopolist takes into account that introducing a new variety reduces the demand for the other varieties it produces. The cannibalization effect alone would tend to reduce product diversity.

b) *Appropriability*: For any total number of varieties, duopoly prices are higher or equal than monopolistically competitive prices. Higher expected prices induce firms to expand their product range.

c) *Strategic price effect*: a duopolist anticipates that its product range affects price competition. If consumers' reservation prices are sufficiently high then duopolists find it optimal to restrict product variety in order to relax price competition. For low reservation prices the price competition effect may be reversed and a duopolist may find it optimal to expand its product variety in order to raise its rival's prices.

As mentioned above product variety under monopolistic competition may be insufficient or excessive with respect to the first best. Since duopolists may expand or decrease product variety with respect to the level chosen by monopolistically competitive firms, it is not surprising that product variety under duopoly can also be excessive or insufficient depending on parameter values. The size of the discrepancy between the market provision of variety and the first best level is particularly large when consumers' reservation prices are not too small and the fixed cost per variety is relatively low. In the limit case of zero fixed costs, duopolists may choose to produce a relatively low fraction of potential varieties, as low as 50%, even though social efficiency calls for 100%. Thus, in contrast to the case of single-product firms, under duopoly product diversity may be inefficiently low for both low and high values of the fixed cost, and inefficiently high for intermediate values.

Some of the effects that are present in this paper are similar, or at least related, to those identified by the literature on multi-product firms in an SDS framework (Ottaviano and Thisse, 1999, and Anderson and de Palma, 2006).

A non-desirable feature of the framework used in Ottaviano and Thisse (1999) [3] is that the optimal prices of a multi-product firm are independent of the number of varieties produced by the firm. In other words, in their set up a firm cannot take advantage of producing a significant fraction of all potential varieties in order to raise its prices. In that model multi-product firms generate less product diversity than in the case of monopolistic competition, simply because of the cannibalization effect, which is roughly the same for all parameter values. As a result, under multi-product firms product diversity is socially excessive only if the fixed cost is sufficiently low (the threshold is lower than under monopolistic competition). In contrast, in the current set up when the fixed cost is low duopolists find it optimal (in the adequate parameter range) to restrict their product range in order to maintain a friendly price environment, which results in insufficient product variety.

[3] See also Ottaviano et al (2002).

The strategic price effect of our duopoly game is related to that of Anderson and de Palma (2006). In particular, in their model a broader product range also induces more aggressive pricing behavior, although the reasons are very different. In their set up consumers have preferences for both firms and varieties. Consumers' decisions are taken in two steps. First, they choose which firm to purchase from. Second, they choose which products to buy from the selected firm. As a result, a firm with a broader product range becomes more attractive to consumers, which induces a more aggressive price response by rival firms. In free entry equilibrium there are too many firms, each one producing too narrow a product range. In contrast (and consistent with Ottaviano and Thisse, 1999), we consider markets where consumers have preferences only for varieties and hence it is possible to discuss whether overall product variety under multi-product firms is excessive or insufficient from a social point of view.

The results of this paper also contrast with those obtained in standard spatial models (See, for instance, Schmalensee, 1978; and Bonanno, 1987). In these models, an incumbent firm may find it profitable to monopolize the market by crowding the product space or by choosing the appropriate location. Instead, the current set up suggests that the presence of neighboring effects in those models was crucial for their results. In fact, in the absence of neighboring effects proliferation cannot be an effective entry deterrence mechanism.

2 The spokes model

2.1 Finite number of varieties (Chen and Riordan, 2007)

Let us consider a market where there are N potential varieties, indexed by i, $i = 1, ..., N$. A particular variety may or may not be supplied. The preference space can be described in a way similar to the standard spatial models. There are N spokes of length $\frac{1}{2}$, also indexed by i, $i = 1, ..., N$, which start from the same central point. The producer of variety i is located in the extreme end of spoke i.

Demand is perfectly symmetric. There is a continuum of consumers with mass $\frac{N}{2}$ uniformly distributed over the N spokes. Each consumer has a taste for two varieties only and the pair of selected varieties differ across consumers. In fact, consumers are uniformly distributed over the $\frac{N(N-1)}{2}$ possible pairs.

Thus, the mass of consumers who have a taste for an arbitrary pair is $\frac{1}{N-1}$ and since there are $N-1$ pairs that contain a particular variety, the mass of consumers who have a taste for variety i is 1.

Consumers with a taste for varieties i and j, $i, j = 1, ..., N$, $j \neq i$, are uniformly distributed over the union of spokes i and j. Each consumer demands one unit of the good. As usual consumer location represents the relative valuation of the two varieties. In particular, a consumer who has a taste for varieties i and j and is located at a distance x, $x \in \left[0, \frac{1}{2}\right]$, from the extreme of the ith spoke, if she chooses to consume one unit of variety i then she obtains a utility equal to $R - tx$. Alternatively if she chooses to consume one unit of variety j then she obtains $R - t(1-x)$. A maintained hypothesis is $R > 2t$. Under this condition, for any given number of active varieties, single-product firms always have incentives to serve as many consumers as possible, which simplifies the presentation.[4]

Suppose all N varieties are active, each one supplied by an independent firm. If $N = 2$ then this is the standard Hotelling model. If $N > 2$ firm i competes symmetrically with the other $N-1$ firms. If N is very large the model captures Chamberlain (1933)'s definition of monopolistic competition, in the sense that each firm: (i) enjoys some market power, i.e., faces a downward sloping demand function, and (ii) is negligible, if ejected from the market then no other firm is significantly affected.

Supplying a variety involves a fixed cost, F, and, for simplicity, zero marginal costs.

2.2 The continuous approximation

It will be very convenient to treat the number of active varieties as a continuous variable. In particular, let us denote by γ, $\gamma \in [0,1]$, the fraction of active varieties. If $0 < \gamma < 1$, consumers can be classified in three different groups. Some consumers will have access to the two varieties they have a taste for, some other consumers will only be able to buy one of the varieties, and finally the third group of consumers will drop out of the market since neither of the two selected varieties is available.

We can treat γ as a continuous variable by considering the limit model as N goes to infinity and expressing all relevant variables relative to the total mass of consumers. In particular, for a given value of γ and N, the number

[4] Chen and Riordan (2007) consider a larger parameter space, $R > t$.

of pairs of varieties for which two suppliers are active is:

$$\frac{\gamma N\left(\gamma N-1\right)}{2}$$

Since the fraction of consumers who have a taste for a particular pair is:

$$\frac{2}{N\left(N-1\right)}$$

then the fraction of consumers who have access to two varieties is:

$$\frac{\gamma N\left(\gamma N-1\right)}{N\left(N-1\right)}$$

If we take the limit as N goes to infinity the fraction of consumers with access to two varieties is γ^2.

Similarly, the fraction of consumers that have access to neither of the two preferred varieties is $(1-\gamma)^2$. Finally, the fraction of consumers with access to only one variety is $2\gamma\left(1-\gamma\right)$.

The total amount of fixed costs per consumer is:

$$\frac{\gamma NF}{\frac{N}{2}}=2\gamma F$$

From the point of view of supplier i, the fraction of consumers that demand variety i and have the opportunity of choosing between product i and their other selected variety is:

$$\frac{\gamma N-1}{N-1}$$

Hence, as N goes to infinity this fraction is equal to γ. Similarly, in the limit as N goes to infinity, the fraction of consumers that demand variety i and do not have access to the other selected variety is $1-\gamma$.

2.3 The first best

What is the fraction of active varieties that maximizes total surplus? For any fraction, γ, it is efficient to allocate those consumers who have access to their two selected varieties to the closest supplier. Thus, the average surplus obtained by consumers with access to two varieties is $R-\frac{t}{4}$, since the average transportation cost is $\frac{t}{4}$. Those consumers that only have access to one variety will incur on average higher transportation costs. Specifically, their surplus will be $R-\frac{t}{2}$. Finally, those consumers without access to any of their selected varieties will receive zero surplus. As mentioned above the amount of fixed costs per consumer is $2\gamma F$. Therefore, we can write total surplus as follows:

$$W(\gamma) = \gamma^2 \left(R - \frac{t}{4}\right) + 2\gamma(1-\gamma)\left(R - \frac{t}{2}\right) - 2\gamma F$$

Since this function is concave the optimal value of γ, denoted by γ^* , can be computed directly from the first order condition:

$$\frac{dW}{d\gamma} = 2\gamma\left(R - \frac{t}{4}\right) + 2(1-2\gamma)\left(R - \frac{t}{2}\right) - 2F = 0$$

which implies that (See Figure 1)

$$\gamma^* = \begin{cases} 0 \text{ if } F \geq R - \frac{t}{2} \\ \frac{R-\frac{t}{2}-F}{R-\frac{3}{4}t} \text{ if } \frac{t}{4} \leq F \leq R - \frac{t}{2} \\ 1 \text{ if } F \leq \frac{t}{4} \end{cases}$$

In order to obtain some intuition we can rewrite the first order condition as follows:

$$\frac{dW}{d\gamma} = 2\left\{\gamma\frac{t}{4} + (1-\gamma)\left(R - \frac{t}{2}\right) - F\right\} = 0$$

The first term represents the preference matching effect. A fraction γ of consumers who have a taste for the new variety already had access to one of their selected varieties and they will enjoy lower average transportation costs $\left(\frac{t}{4}\right)$, i.e., there is a better matching between available products and consumer preferences. The second term is the market expansion or aggregate demand effect. A fraction $1-\gamma$ of consumers did not have access to any of their selected varieties and once the new variety is introduced they will purchase one unit of the good and enjoy an average surplus of $\left(R - \frac{t}{2}\right)$ .The relative weights of these two effects depend on the fraction of existing varieties. Since $R - \frac{t}{2} > \frac{t}{4}$ then total surplus created by an additional variety decreases with γ. Finally, the third term is the cost of entry.

3 The benchmark: Monopolistic competition

In this section we consider the case in which the number of potential firms is equal to the number of potential varieties, N. Each firm can produce only one variety. If firm i decides to enter the market then it pays a fixed cost, F, and chooses a price, p_i. Firms maximize profits and enter the market only if net profits are positive. Since each firm is negligible and there is no uncertainty, then it does not matter whether entry and price decisions are taken sequentially or simultaneously. We focus exclusively on symmetric equilibria of this free entry game.

Let us first calculate the symmetric equilibrium price, for a given γ. In those market segments where consumers have access to two varieties, consumers will choose supplier exactly as in the Hotelling model. That is, the fraction of consumers that choose firm i is given by:

$$\frac{1}{2} + \frac{\overline{p} - p_i}{2t}$$

where $\overline{p}$ is the price charged by rival firms.[5] In those segments where firm $i's$ product is the consumers' only choice total demand is 1, provided all consumers obtain a positive surplus, i.e., $p_1 + t \leq R$. In fact, under the maintained hypothesis ($R > 2t$) firms never find it optimal to set a price above $R - t$ (See Appendix). In other words, firms have incentives to serve as many consumers as possible. Hence, firm i's optimization problem consists of choosing p_i in order to maximize:

$$\pi_i(\gamma, p_i, \overline{p}) = \left[\gamma\left(\frac{1}{2} + \frac{\overline{p} - p_i}{2t}\right) + 1 - \gamma\right] p_i - F \tag{1}$$

subject to $p_i \leq R - t$. If this constraint is not binding then the optimal price is given by:

$$p_i = \frac{t(2-\gamma)}{2\gamma} + \frac{\overline{p}}{2}$$

Hence, the symmetric equilibrium price, $p_i = \overline{p} \equiv p^{MC}$, is:

$$p^{MC} = t\frac{2-\gamma}{\gamma} \tag{2}$$

[5] And provided $p_i \in [\overline{p} - t, \overline{p} + t]$. See below.

As γ increases competition intensifies and the price falls. If all potential varieties are active ($\gamma = 1$) then $\overline{p} = t$, as in the standard Hotelling model. Equilibrium price is given by equation (2) provided its value is not above $R - t$, which is equivalent to $\gamma \geq \frac{2t}{R}$. Moreover, if $p^{MC} + t < R - t$, an individual firm may find it optimal to deviate from p^{MC} and set $p_i = R - t$. Such a deviation is not profitable provided:

$$\frac{R}{t} \leq \Psi(\gamma) \equiv 1 + \frac{(2-\gamma)^2}{2\gamma(1-\gamma)} \tag{3}$$

If condition (3) does not hold then a symmetric equilibrium does not exist.[6]

Hence, provided $\gamma \in \left[\frac{2t}{R}, 1\right]$ the fraction of active varieties in equilibrium, γ^{MC} , will be given by the zero profit condition:

$$\pi_i\left(\gamma^{MC}, p^{MC}, p^{MC}\right) = \frac{t}{2}\frac{\left(2-\gamma^{MC}\right)^2}{\gamma^{MC}} - F = 0$$

Equivalently, if $F \in \left[\frac{t}{2}, \frac{(R-t)^2}{R}\right]$ then (provided condition (3) holds):

$$\gamma^{MC}(F) = 2 + \frac{F}{t} - \sqrt{\left(2 + \frac{F}{t}\right)^2 - 4} \tag{4}$$

and if $F \leq \frac{t}{2}$ then $\gamma^{MC} = 1$.

If instead $\gamma < \frac{2t}{R}$, which occurs whenever $F \geq \frac{(R-t)^2}{R}$, then each individual firm faces little competition and finds it optimal to set a price equal to $R-t$, and serve all consumers that have no other choice. In this case, the zero profit condition is:

$$\pi_i\left(\gamma^{MC}, R-t, R-t\right) = \frac{2-\gamma^{MC}}{2}(R-t) - F = 0$$

Hence, if $F \in \left[\frac{(R-t)^2}{R}, R-t\right]$ then

$$\gamma^{MC}(F) = 2\frac{R-t-F}{R-t} \tag{5}$$

and if $F \geq R - t$ then $\gamma^{MC} = 0$.

[6]Note that $\Psi(\gamma)$ reaches a minimum at $\gamma = \frac{2}{3}$, and that $\Psi\left(\frac{2}{3}\right) = 5$.

We can compare product diversity under monopolistic competition with the first best. This comparison is simple for extreme values of F. In particular, if F is sufficiently low, such that γ^{MC} is close to 1, then there is excessive entry, i.e., $\gamma^{MC} > \gamma^*$. In contrast, if F is sufficiently high, such that γ^{MC} is close to zero, then there is insufficient entry, i.e., $\gamma^{MC} < \gamma^*$. However, the results for intermediate values of F are somewhat more complicated. We can distinguish between two cases. If $R < \left(\frac{5}{2} + \sqrt{2}\right) t$ (case A), then $\gamma^{MC}(F)$ and $\gamma^*(F)$ only cross each other once, at a threshold value F_a, $F_a \in \left[\frac{(R-t)^2}{R}, R - t\right]$. Thus, there is excessive entry for relatively low values of F $(F < F_a)$ and insufficient entry for relatively high values of F $(F > F_a)$. Figure 1a illustrates this case. However, if If $R > \left(\frac{5}{2} + \sqrt{2}\right) t$ (case B), then $\gamma^{MC}(F)$ and $\gamma^*(F)$ cross each other three times, at values F_b, F_c and F_a, such that $\frac{t}{2} < F_b < F_c < \frac{(R-t)^2}{R} < F_a < R - t$, and hence we obtain two intervals with excessive entry and two intervals of excessive entry which alternate. This case is illustrated in Figure 1b (See Appendix for the algebraic details.)

This discussion is summarized in the following proposition:

Proposition 1 *Under monopolistic competition: (i) if* $R < \left(\frac{5}{2} + \sqrt{2}\right) t$*, then there is excessive entry if* $F \in \left(\frac{t}{4}, F_a\right)$ *and insufficient entry if* $F \in \left(F_a, R - \frac{t}{2}\right)$*. (ii) if* $R > \left(\frac{5}{2} + \sqrt{2}\right) t$*, then there is excessive entry if* $F \in \left(\frac{t}{4}, F_b\right) \cup (F_c, F_a)$ *and insufficient entry if* $F \in (F_b, F_c) \cup \left(F_a, R - \frac{t}{2}\right)$

The number of varieties in a monopolistically competitive equilibrium can be excessive or insufficient from the social viewpoint depending on the relative weight of three different effects:

a) *No price discrimination*: firms cannot price discriminate among heterogeneous consumers. This is one of the reasons why firms cannot appropriate all the surplus they create by entering the market.

b) *Price competition*: Monopolistically competitive prices may be lower than under monopoly, provided the number of varieties is not too small.

c) *Business stealing*: A fraction of the customers of an entrant firm are stolen from existing firms. Profits made out of stolen customers are always higher than the reduction in transportation costs experienced by these reallocated consumers.

Thus, the first and second effects depress private incentives with respect to social incentives and hence work in favor of insufficient product variety. In contrast, the third effect exacerbates private incentives and works in favor of excessive product variety.

In Salop's model, if the entry cost is higher than a certain threshold then all firms are local monopolists. As a result, the second and third effect are non-operative and in equilibrium product variety is insufficient. However, if the entry cost is below the threshold (which is the usual maintained hypothesis) then the entire market is served. In this case, the business stealing effect dominates and in equilibrium product variety is excessive.

For extreme values of F the spokes model behaves like Salop's. If F is sufficiently low to support equilibrium values of γ close to one, then the business stealing effect dominates (excessive product variety). Similarly, if F is sufficiently high to support equilibrium values of γ close to zero, then firms are close to being local monopolists and the no price discrimination effect dominates (insufficient product variety).

For intermediate values of F it is not so easy to track the relative strength of these three different effects. In the spokes model, the strength of both the price competition and the business stealing effects increase with the number of varieties. Since these two effects work in opposite directions it is not obvious what is the net effect of a change in γ on the difference between the private and social incentives to enter. As F decreases from levels that support equilibrium values of γ close to zero, then γ increases and the business stealing effect intensifies (equilibrium prices are constant provided $\gamma < \frac{2t}{R}$). At a certain point the business stealing dominates the no price discrimination effect and private incentives to enter are larger than social incentives (excessive product variety). If F falls so much that γ increases above $\frac{2t}{R}$, then the price competition effect starts biting and private incentives to enter are moderated. If R is sufficiently low then the price competition effect is relatively weak (in other words, monopoly price is relatively low) and the price competition effect cannot overturn the business stealing effect, and as a result product variety is excessive for all values of γ above $\frac{2t}{R}$. In this case the parameter space is divided into two intervals, with excessive product variety for relatively low values of F and insufficient product variety for relatively high values of F. However, if R is sufficiently large then the competition effect, together with the no price discrimination effect may dominate the business stealing effect for intermediate values of γ. In this case, the parameter space is divided into four intervals. We still obtain excessive product variety for extreme low values of F, insufficient product variety for extreme high values of F, but the sign of the inefficiency alternates for intermediate values of F.

Obviously, these results are equivalent to those obtained in Chen and Riordan (2007). Working directly in the limiting case of a large number of firms, plus focusing on a slightly smaller parameter space ($R > 2t$), simplifies the presentation (the reader should compare our Figure 1 with their Figure 3 and Tables 1 and 2) and, more importantly, sets the stage for the introduction of multi-product firms.

4 Asymmetric competition

The main goal of this section is to determine whether or not, in the current set up, size (as measured by the number of varieties) is a source of competitive advantage and whether product proliferation is an effective entry deterrence mechanism. In order to address these issues we consider an incumbent monopolist who anticipates potential competition from a large number of small firms, each one of them able to supply a single variety (a competitive fringe). In order to focus on the potential strategic effect of committing to a large product range, we assume there are no economies of scope. Can the incumbent firm crowd the product space and make further entry unprofitable? What is the outcome of such preemption efforts in terms of product selection and prices? These questions have been examined in the context of standard spatial models (See Schmalensee, 1978; and Bonanno, 1987).

Consider the following three stage game. In the first stage firm L chooses the fraction of varieties, γ_L, and pays the fixed costs associated to activating new varieties, $F\gamma_L$. In the second stage, small firms decide whether or not to enter, and hence the mass of small active firms, γ_C, is determined. Fixed costs are the same for small and large firms, so that each small firm that chooses to enter pays F. In the third stage, firms simultaneously set the prices for those varieties that have been activated. For simplicity all firms face zero marginal costs.

We will disregard the trivial case where F is sufficiently low such that a large firm may find it optimal to set $\gamma_L = 1$ and hence make further entry physically impossible.[7]

The analysis of this game provides very transparent and powerful intuitions that suggest that the main qualitative results of the model are likely to hold in a large family of models of non-localized competition. However, working out all the details takes time and effort. For these reasons we have moved

[7] Thus, we focus on the case that if $\gamma_L = 1$ then $\pi^L < 0$. That is, $F > \frac{1}{2}\left(R - \frac{t}{2}\right)$.

the formal analysis to the Appendix. Before discussing the main intuitions we first state the main result of this section.

Proposition 2 *If* $F \in \left[\frac{(R-t)^2}{R}, R-t\right]$ *then* $\gamma_C + \gamma_L = \gamma^{MC}$ *and the values of* γ_C *and* γ_L *are undetermined (all firms make zero profits). If* $F \in \left(\frac{2R-t}{4}, \frac{(R-t)^2}{R}\right)$ *then* $\gamma_L = 0$ *and* $\gamma_C = \gamma^{MC}$. *In other words, the large firm cannot exploit its size and first mover advantages and make strictly positive profits.*

When neighboring effects are absent, a large multi-product firm behaves as a coalition of a subset of single-product firms in the pricing stage. Thus, it internalizes some of the cross-price effects and tends to set a higher price than single-product firms. More specifically, given that firms face kinked demand curves there are parameter values (relatively high fixed cost) for which the multi-product firm sets the same prices as single-product firms, but for other parameter values (relatively low fixed cost) its prices are strictly higher. In the first case, profits per variety are the same for all firms independent of their size. Since there is free entry in equilibrium all firms make zero profits. In the latter case, the presence of a large multi-product firm creates a positive externality to single-product firms by relaxing price competition, which creates further incentives to enter. In fact, such "collusive behavior" benefits single-product firms more than the multi-product firm. In other words, profits per variety of the large firm are lower than those of small firms. The reason is the following. A single-product firm can always imitate the pricing behavior of the large firm and make the same profits per variety. If these firms choose not to do so and set a lower price it is because their profits are strictly higher. In free entry equilibrium small firms make zero profits, which implies that (in this region of parameters) the large firm finds it optimal to set $\gamma_L = 0$.

To summarize, in the absence of neighboring effects and economies of scope, the size of a firm's product range cannot be a source of competitive advantage. Moreover, it can be a source of competitive disadvantage. Therefore, multi-product firms will emerge only in the presence of sufficiently strong economies of scope, which compensate for the disadvantage associated with the pricing behavior anticipated by small firms.[8]

[8] If we let the large firm precommit to both product range and prices then a large firm

In standard location models (see, in particular, Schamalensee, 1978, and Bonanno, 1987) it was shown that an incumbent monopolist may find it optimal to prevent further entry by either crowding out the product space or by choosing the right location pattern. Our results indicate that this is only possible when competition is localized and the firm producing the new brand competes only with one or two of the existing brands. In this case, the entrant correctly anticipates that the incumbent firm will react to the entry decision by cutting the price of the competing brands. In contrast, in our set up those neighboring effects are absent and a new brand does not change the prices of existing brands, which implies that, for a given number of established brands, incentives to enter increase with concentration.

5 Symmetric duopoly

The presence of economies of scope creates incentives to form large multi-product firms, each one producing a significant fraction of the total number of varieties. The aim of this section is to investigate how the strategic incentives of large multi-product firms affect prices and, specially, product diversity. In order to simplify the presentation, instead of allowing for an endogenous number of firms, we restrict attention to the duopoly case.[9]

Let us consider the following game. There are two firms, A and B. In the first stage firms simultaneously choose the fraction of potential varieties they wish to supply, γ_A and $\gamma_{B.}$ In the second stage, after observing γ_A and γ_B, firms simultaneously set prices for all the active varieties. We focus on symmetric subgame perfect equilibria, where $\gamma_A = \gamma_B = \frac{1}{2}\gamma^D$, and all varieties are sold at the same price.

In the second stage, given γ_A and γ_B, a fraction γ_A^2 of consumers will have access to two varieties supplied by firm A, a fraction $2\gamma_A(1-\gamma_A-\gamma_B)$ will have access only to one of the varieties supplied by firm A, and a fraction $2\gamma_A\gamma_B$ will have access to one variety supplied by firm A and one variety

can survive, but only by setting the prices prevailing in a monopolistically competitive equilibrium, which implies zero profits.

[9] We could have followed Ottaviano and Thisse (1999) or Anderson and de Palma (2006) and assumed that there is an arbitrary number of potential firms, each one of them must pay an entry cost, G, and a fixed cost $F\gamma$, which is proportional to the fraction of varieties produced by the firm, γ. Clearly, there would be values of G for which the equilibrium configuration is a duopoly.

supplied by firm B. Hence, firm A enjoys absolute monopoly power with the first two groups of consumers and competes with firm B for the third group.

In the first part of this section we focus on the case $R \geq 3t$. At the end of the section we discuss the case $2t < R < 3t$, where some of the effects are reversed and therefore requires separate consideration.

5.1 The case of high reservation prices

In this subsection we restrict ourselves to the case $R > 3t$. In this case no firm has incentives to set a price above $R - t$ (See Appendix). Hence, we can write firms' optimization problem in the second stage as follows. Firm A chooses the price of its varieties, p_A, in order to maximize:

$$\pi_A = \left[\gamma_A^2 + 2\gamma_A (1 - \gamma_A - \gamma_B) + 2\gamma_A\gamma_B \left(\frac{1}{2} + \frac{p_B - p_A}{2t}\right)\right] p_A - 2\gamma_A F$$

subject to $p_A + t \leq R$.[10] If the constraint is not binding then firm A's reaction function is:

$$p_A = \frac{t}{2\gamma_B}(2 - \gamma_A - \gamma_B) + \frac{p_B}{2}$$

As usual reaction functions are upward sloping (prices are strategic complements). More interesting is the effect of the fraction of varieties supplied by each firm on the optimal price. First, p_A decreases with $\gamma_{A.}$ The reason is that the fraction of total consumers that have to choose between two varieties supplied by different firms, $\frac{2\gamma_A\gamma_B}{\gamma_A^2+2\gamma_A(1-\gamma_A-\gamma_B)+2\gamma_A\gamma_B}$, increases with γ_A. Second, p_A decreases with γ_B. A higher γ_B reduces the fraction of consumers that can only buy from firm A and raises the fraction of consumers that have two options.

Firm $B's$ reaction function is symmetric. Thus, for a given pair (γ_A, γ_B), in equilibrium firm A sets the following price:

$$p_A = \frac{t}{3}\frac{(2 - \gamma_A - \gamma_B)(2\gamma_A + \gamma_B)}{\gamma_A\gamma_B} \tag{6}$$

[10] It will be apparent that we need not worry about deviations such that $p_A \notin [p_B - t, p_B + t]$.

Note that if $\gamma_A = \gamma_B = \frac{1}{2}$, then $p_A = p_B = 2t$. That is, if all potential varieties are supplied (and firms hold a symmetric position), then prices are higher under duopoly $(2t)$ than under monopolistic competition (t), but lower than $R - t$. In fact, a duopolistic firm can be interpreted as a coalition of one half of monopolistically competitive firms. By raising the price above the monopolistically competitive level a duopolistic firm can raise its profits, since two thirds of its potential consumers are able to choose between two varieties supplied by different firms, but one third are trapped and can only choose between two varieties supplied by the same firm. It is important to note that p_A decreases with both γ_A and γ_B. That is, the higher the fraction of varieties supplied by either firm the lower the prices.

Along a symmetric equilibrium path, $\gamma_A = \gamma_B = \frac{1}{2}\gamma^D$. Thus, the candidate to equilibrium price can be obtained rewriting equation (6):

$$p^D = \frac{2t\left(2 - \gamma^D\right)}{\gamma^D}$$

provided $p^D \leq R - t$, i.e.:

$$\gamma^D \geq \frac{4t}{R+t} \tag{7}$$

Let us begin the search for equilibrium candidates in the region of parameter values where condition (7) does not hold. In this case, firms are expected to set prices equal to $R - t$ and make profits:

$$\pi_A = \gamma_A \left(2 - \gamma_A - \gamma_B\right)\left(R - t\right) - 2\gamma_A F$$

This is firm A's objective function in the first stage, which is concave in γ_A. From the first order condition with respect to γ_A, evaluated at $\gamma_A = \gamma_B = \frac{1}{2}\gamma^D$, we obtain the level of product variety in the candidate equilibrium:

$$\gamma^D = \frac{4}{3}\frac{R - t - F}{R - t} \tag{8}$$

provided $\gamma^D \leq \frac{4t}{R+t}$, i.e. provided,

$$F \geq \frac{\left(R - t\right)\left(R - 2t\right)}{R + t}$$

Let us turn our attention to the case condition (7) holds. By plugging equilibrium prices in the profit function we obtain firm A's payoff as seen from the first stage, i.e., profits as a function of γ_A and γ_B :

$$\pi_A(\gamma_A, \gamma_B) = \frac{t}{9\gamma_A\gamma_B}(2-\gamma_A-\gamma_B)^2(2\gamma_A+\gamma_B)^2 - 2\gamma_A F$$

The first derivative evaluated at $\gamma_A = \gamma_B = \frac{1}{2}\gamma^D$ is:

$$\frac{d\pi_A}{d\gamma_A} = \frac{4t\left(2-\gamma^D\right)\left(1-2\gamma^D\right)}{3\gamma^D} - 2F \tag{9}$$

Note that if $\gamma^D > \frac{1}{2}$ then the first derivative is negative. Thus, if $7t > R > 3t$, then $\frac{1}{2} < \frac{4t}{R+t} < 1$, and hence there is no symmetric equilibrium where $\gamma^D > \frac{4t}{R+t}$. Moreover, $\gamma_A = \gamma_B = \frac{2t}{R+t}$ is in fact the unique symmetric equilibrium (In the Appendix we show that no firm wants to deviate from the proposed behavior). Summarizing, if $7t \geq R > 3t$ and $F \geq \frac{(R-t)(R-2t)}{R+t}$, γ^D is given by equation (8), and if $F \leq \frac{(R-t)(R-2t)}{R+t}$, $\gamma^D = \frac{4t}{R+t}$. Note that for all F, firms never choose values of γ_A and γ_B that induce prices below $R-t$.

In case $R > 7t$, existence of symmetric equilibria is not guaranteed. In the Appendix we show that there is no symmetric equilibrium with $\gamma^D \geq \frac{4t}{R+t}$.[11] Thus, the only equilibrium candidate is once again given by equation (8) if $F \geq \frac{(R-t)(R-2t)}{R+t}$, and $\gamma^D = \frac{4t}{R+t}$ if $F \leq \frac{(R-t)(R-2t)}{R+t}$. Unfortunately, if F is sufficiently low firms find it profitable to deviate and hence a symmetric equilibrium does not exist.

All this discussion is summarized in the next Lemma:

Lemma 1 *Whenever a symmetric equilibrium exists, γ^D is given by equation (8) if $F \geq \frac{(R-t)(R-2t)}{R+t}$, and $\gamma^D = \frac{4t}{R+t}$ if $F \leq \frac{(R-t)(R-2t)}{R+t}$. A symmetric equilibrium exists for all F if $7t \geq R > 3t$, but it does not exist if $R \geq 7t$ and F sufficiently low.*

If we compare γ^D with γ^{MC} then they cross each other twice at F_d and F_e, such that $\frac{t}{2} < F_d < \frac{(R-t)(R-2t)}{R+t} < F_e < \frac{(R-t)^2}{R}$. (See Figure 2 and Appendix for details.)

Proposition 3 *Under duopoly, if $R > 3t$, the fraction of varieties supplied in a symmetric equilibrium is lower than under monopolistic competition, if $F \in [0, F_d) \cup (F_e, R-t)$, and higher if $F \in (F_d, F_e)$.*

[11] It turns out that even asymmetric (pure strategy) equilibria do not exist either.

The differential incentives to introduce product variety by duopolistic and monopolistically competitive firms depend on the relative weight of three effects:

a) Cannibalization: if a multi-product firm introduces a new variety it anticipates that some of the buyers are already customers of the firm.

b) Strategic price effect: a multi-product firm anticipates that introducing a higher number of varieties beyond a certain threshold triggers lower equilibrium prices.

c) Appropriability: for any total number of varieties equilibrium prices are higher or equal under duopoly than under monopolistic competition.

The first two effects work in favor of lower product diversity under duopoly. The third effect works in the opposite direction as firms can appropriate a larger fraction of the surplus created.

For relatively high values of F so that γ^{MC} is very low, both the strategic price effect and the competition effect are non-operative since prices are equal to the monopoly level under both market structures. In this case the cannibalization effect dominates and product variety is lower under duopoly. In contrast, if F is relatively low so that γ^{MC} is close to 1, the strategic price effect dominates, since duopolistic firms are not willing to expand their product range at the cost of triggering a price war. However, for intermediate values of F, the competition effect dominates and duopolistic firms introduce more product variety than monopolistic firms because they can charge higher prices and appropriate a large fraction of total surplus.

Let us now compare γ^D and γ^* (See Figure 3 and Appendix for details). They cross each other twice at F_f and F_g, such that $F_f < \frac{(R-t)(R-2t)}{R+t} < F_e < F_g$ with $F_d \gtreqless F_f$ as $R \gtreqless 4t$.

Proposition 4 *Under duopoly, if $R > 3t$, the fraction of varieties supplied in a symmetric equilibrium is insufficient if $F \in [0, F_f) \cup \left(F_g, R - \frac{t}{2}\right)$, and excessive if $F \in (F_f, F_g)$.*

Duopolistic firms may also produce excessive product variety, but this is only possible for intermediate values of F. Unlike monopolistic firms, if F is low multi-product firms refrain from expanding their product range in order to relax price competition, and as a result product variety is insufficient.

It is important to note that the strategic price effect may cause large inefficiencies. Consider the case $R = 7t$ and $F = 0$. In this case, social efficiency requires all potential varieties being produced. In contrast, in equilibrium duopolists only produce one half of all potential varieties. Hence, the strategic price effect may cause a substantial underprovision of product diversity.

5.2 The case of low reservation prices

Finally, we turn our attention to a region of the parameter space, $2t < R < 3t$, where some of the effects analyzed above are reversed. In this region firms may choose to set prices above $R - t$, and not to serve some potential customers in submarkets where consumers can buy either one of the firms' varieties or nothing. However, firms will never choose to set prices above $R - \frac{t}{2}$ (See Appendix). In the second stage, firm A chooses p_A in order to maximize:

$$\pi_A = \left[\gamma_A^2 + 2\gamma_A (1 - \gamma_A - \gamma_B) \frac{R - p_A}{t} + 2\gamma_A\gamma_B \left(\frac{1}{2} + \frac{p_B - p_A}{2t}\right)\right] p_A - 2\gamma_A F$$

subject to $p_A \in \left[R - t, R - \frac{t}{2}\right]$. If these constraints are not binding then firm A's optimal price is given by:

$$p_A = \frac{(\gamma_A + \gamma_B)\, t + 2(1 - \gamma_A - \gamma_B)\, R + \gamma_B p_B}{2(2 - 2\gamma_A - \gamma_B)}$$

If we let $\gamma \equiv \gamma_A + \gamma_B$ and taking into account firm B's reaction function, then we can write firm A's equilibrium price as follows:

$$p_A = \frac{[\gamma t + (1 - \gamma)\, 2R]\, [2(2 - \gamma) - \gamma_B]}{8(2 - \gamma)(1 - \gamma) + 3\gamma_A\gamma_B}$$

Note that in this case p_A increases with both γ_A and γ_B. In other words, in this region of the parameter space the sign of the strategic price effect is reversed. An expansion of a firm's product range induces its rival to set a higher price. That is, more product variety implies a more relaxed price environment. The intuition is the following. If γ_B is very low, then firm A pays a great of deal of attention to those submarkets where consumers can either buy one of its varieties or nothing. Since R is relatively low with respect to t then firm A has incentives to moderate its pricing (set a price equal to $R - t$) and serve all these consumers. As γ_B increases then firm A puts less weight on these submarkets and more weight on submarkets where consumers can choose between varieties supplied by different firms. In those submarkets firm A does not have to attract consumers located at the other end of the segment, but only consumers located in the middle. As a result firm A finds it optimal to set a higher price.[12]

Thus, in the first stage, firms have incentives to expand their product range in order to relax price competition in the second stage. As a result, the strategic price effect, together with the competition effect, may dominate the cannibalization effect. As a result, even in the case of low values of F product variety under duopoly may be higher than under monopolistic competition (and hence excessive from a social viewpoint).

In order to illustrate this point let us determine the set of values of F that give rise to a symmetric equilibrium with $\gamma_A = \gamma_B = \frac{1}{2}$. In this case $p_A = p_B = 2t > R - t$. Also, provided $R > \frac{5}{2}t$, then $p_A = p_B < R - \frac{t}{2}$. It can be shown (See Appendix) that this is an equilibrium provided $F \leq t$. That is, if $F \in \left(\frac{t}{2}, t\right]$ then $\gamma^D = 1 > \gamma^{MC} > \gamma^*$.

6 Concluding remarks

In this paper we have examined the role of multi-product firms in the market provision of product variety. The spokes model provides a very useful set up to compare the product diversity generated by single-product as well as multi-product firms in industries where neighboring effects can be neglected.

We have shown that multi-product firms are in a competitive disadvantage vis-a-vis single-product firms and they will emerge only if economies of scope are sufficiently strong. This result is independent of whether or not a multi-product firm enjoys a first-mover advantage. In other words, in the absence of neighboring effects product proliferation is not an effective entry deterrence mechanism.

If economies of scope are such that only two firms can supply all possible varieties, then it turns out that product variety may be higher or lower than in the case of monopolistic competition; moreover, duopolists may also provide too little or too much variety with respect to the first level. However, for a relevant range of parameter values, duopolists drastically restrict their product range in order to relax price competition. As a result, product

[12] The result that more product variety may imply higher prices is analogous to "price rising entry" discussed by Chen and Riordan (2006) in the case of a finite number of single-product firms. However, those related phenomena occur for different parameter values because the presence of multiproduct firms tends to raise average prices.

diversity may be significantly lower than the efficient level.

These results contribute to a better understanding of the impact of multi-product firms and nicely complement those obtained in the SDS framework. Moreover, on a more methodological spirit, the analysis indicates that the spokes set up is sufficiently flexible to accommodate multi-product firms and hence it reinforces the idea that the model proposed by Chen and Riordan (2007) is indeed a significant development within the family of spatial models.

The market structures considered in this paper are extreme and somewhat arbitrary. In some real world markets we do observe firms producing a broad product range competing with firms producing a much more limited product range. Equilibria with asymmetric firms may be caused by first mover advantages, but also by the existence of alternative technologies. Perhaps, firms must incur a large sunk cost in order to reduce the fixed cost associated with the production of each variety. In any case, to endogenize the market structure seems a natural step forward. However, if we were to follow this avenue we would be discussing not only the optimal number of varieties but also the optimal number of firms.

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8 Appendix

8.1 Proof of Proposition 1

Let us first consider the region where $F \in \left[\frac{(R-t)^2}{R}, R-t\right]$, i.e., $\gamma < \frac{2t}{R}$. We need to check that in this region the only symmetric equilibrium involves $p = R - t$. A representative firm i chooses p_i in order to maximize:

$$\pi_i = \left[\gamma\left(\frac{1}{2} + \frac{\overline{p} - p_i}{2t}\right) + (1-\gamma)\frac{R - p_i}{t}\right] p_i$$

subject to $p_i \geq R - t$. The first order condition of an interior solution can be written as:

$$p_i = \frac{\gamma(t + \overline{p}) + (1-\gamma) 2R}{2(2-\gamma)}$$

If a symmetric equilibrium exists, then the price is given by:

$$p(\gamma) = \frac{\gamma t + 2(1-\gamma) R}{4 - 3\gamma}$$

It turns out that $p(0) = \frac{R}{2} < R - t$, and $p'(\gamma) < 0$. We reach a contradiction. Moreover, if other firms set $\overline{p} = R - t$ according to the first order condition the best response is $p_i = \frac{R}{2} < R - t$. Hence, the only symmetric equilibrium involves $p = R - t$, and the equilibrium value of γ is given by equation (5). Hence, in this region both γ^* and γ^{MC} are linear functions of F. Next, we compute the value of F, denoted F_a, at which $\gamma^* = \gamma^{MC}$:

$$F_a = \frac{(R-t)^2}{R - \frac{t}{2}}$$

Note that $\frac{(R-t)^2}{R} < F_a < R - t$.

Let us now turn to the region where $\frac{t}{2} \leq F \leq \frac{(R-t)^2}{R}$. In this region γ^{MC} is given by equation (4). As noted in the text $\gamma^{MC}\left(F = \frac{t}{2}\right) > \gamma^*\left(F = \frac{t}{2}\right)$ and $\gamma^{MC}\left(\frac{(R-t)^2}{R}\right) > \gamma^*\left(\frac{(R-t)^2}{R}\right)$. In fact, $\frac{d\gamma^{MC}}{dF} < 0$ and $\frac{d^2\gamma^{MC}}{dF^2} > 0$. Thus, in principle γ^{MC} and γ^* could cross twice in this region. If these two functions cross in this interval, then they will do so for those values of γ that satisfy:

$$\frac{t}{2}\frac{(2-\gamma)^2}{\gamma} = \gamma\left(R - \frac{t}{4}\right) + (1 - 2\gamma)\left(R - \frac{t}{2}\right)$$

i.e.,

$$-\left(2\frac{R}{t} - \frac{1}{2}\right)\gamma^2 - \left(2\frac{R}{t} + 3\right)\gamma + 4 = 0$$

The solutions of this quadratic equation are given by:

$$\gamma = \frac{3 + 2\frac{R}{t} \pm \sqrt{4\left(\frac{R}{t}\right)^2 - 20\frac{R}{t} + 17}}{4\frac{R}{t} - 1}$$

Real solutions are obtained if and only if $4\left(\frac{R}{t}\right)^2 - 20\frac{R}{t} + 17 \geq 0$, i.e., $\frac{R}{t} \geq \frac{5}{2} + \sqrt{2}$. If such a condition is satisfied then both real solutions belong to the interval $\left[\frac{2t}{R}, 1\right]$. In particular, the highest solution is lower than 1, that is:

$$\gamma = \frac{3 + 2\frac{R}{t} + \sqrt{4\left(\frac{R}{t}\right)^2 - 20\frac{R}{t} + 17}}{4\frac{R}{t} - 1} < 1$$

which is equivalent to $4R > t$. The lowest solution is higher than $\frac{2t}{R}$:

$$\frac{3+2\frac{R}{t}-\sqrt{4\left(\frac{R}{t}\right)^2-20\frac{R}{t}+17}}{4\frac{R}{t}-1}>\frac{2t}{R}$$

which holds provided $R>t$. Let us denote by F_b and F_c the two values of F at which γ^* and γ^{MC} cross each other in the interval where $1>\gamma>\frac{2t}{R}$. In fact, it is possible to compute explicit expressions for F_b and F_c (available upon request.)

8.2 Proof of Proposition 2

It is useful to start by examining the optimal pricing policy of a monopolist, i.e., the optimal p_L in case $\gamma_C=0$. It is immediate that a monopolist never sets a price below $R-t$ or above $R-\frac{t}{2}$. Thus, the optimal price maximizes:

$$\pi_L=\gamma_L\left[(1-\gamma_L)\frac{R-p_L}{t}+\frac{\gamma_L}{2}\right]p_L-\gamma_L F$$

subject to $p_L\in\left[R-t,R-\frac{t}{2}\right]$. Thus, the monopoly price is:

$$p_L=\begin{cases} R-t \text{ if } \gamma_L\leq\frac{2(R-2t)}{2R-3t} \\ \frac{\gamma_L t+(1-\gamma_L)2R}{4(1-\gamma_L)} \text{ if } \frac{2(R-2t)}{2R-3t}\leq\gamma_L\leq\frac{2(R-t)}{2R-t} \\ R-\frac{t}{2} \text{ if } \gamma_L\geq\frac{2(R-t)}{2R-t}\end{cases} \tag{10}$$

Note that p_L weakly increases with γ_L. In submarkets where consumers have access to two varieties the price elasticity is zero in the relevant interval. As γ_L increases the fraction of submarkets where consumers have access to two varieties is higher and hence the price elasticity of total demand is lower. As a result the monopolist finds it optimal to charge a higher price.

Let us now turn our attention to the third stage of the game between the multi-product firm and the competitive fringe. Given (γ_L,γ_C) and the price set by the competitive fringe, p_C, firm L chooses p_L in order to maximize[13]:

$$\pi_L=\begin{cases}\gamma_L\left[1-\gamma_L-\gamma_C+\frac{\gamma_L}{2}+\gamma_C\left(\frac{1}{2}+\frac{p_C-p_L}{2t}\right)\right]p_L-\gamma_L F \text{ if } p_L\leq R-t \\ \gamma_L\left[(1-\gamma_L-\gamma_C)\frac{R-p_L}{t}+\frac{\gamma_L}{2}+\gamma_C\left(\frac{1}{2}+\frac{p_C-p_L}{2t}\right)\right]p_L-\gamma_L F \text{ if } R-t\leq p_L\leq R-\frac{t}{2}\end{cases}$$

[13]Since a monopolist never has incentives to set a price above $R-\frac{t}{2}$, it is immediate to check that a large firm facing competition from a competitive fringe does not find it optimal to set a price above $R-\frac{t}{2}$ either.

The reaction function is given by :

$$p_L = \begin{cases} t\frac{2-\gamma}{2\gamma_C} + \frac{p_C}{2} \text{ if } p_C \leq \underline{p}_C \\ R-t \text{ if } \underline{p}_C \leq p_C \leq \widehat{p}_C \\ \frac{\gamma t+(1-\gamma)2R+\gamma_C p_C}{2(2-\gamma-\gamma_L)} \text{ if } \widehat{p}_C \leq p_C \leq \overline{p}_C \\ R-\frac{t}{2} \text{ if } p_C \geq \overline{p}_C \end{cases} \tag{11}$$

where $\gamma \equiv \gamma_L + \gamma_C$, $\underline{p}_C \equiv \frac{2R\gamma_C - t(2-\gamma+2\gamma_C)}{\gamma_C}$ and $\widehat{p}_C \equiv \frac{2R(1-\gamma_L)-t(4-\gamma-2\gamma_L)}{\gamma_C}$ and $\overline{p}_C \equiv \frac{2R(1-\gamma_L)-t(2-\gamma_L)}{\gamma_C}$.

A small firm i chooses p_i in order to maximize:[14]

$$\pi_i = \begin{cases} \left[1-\gamma_L-\gamma_C+\gamma_C\left(\frac{1}{2}+\frac{p_C-p_i}{2t}\right)+\gamma_L\left(\frac{1}{2}+\frac{p_L-p_i}{2t}\right)\right]p_i - F \\ \text{if } p_i \leq R-t \\ \left[(1-\gamma_L-\gamma_C)\left(\frac{R-p_i}{t}\right)+\gamma_C\left(\frac{1}{2}+\frac{p_C-p_i}{2t}\right)+\gamma_L\left(\frac{1}{2}+\frac{p_L-p_i}{2t}\right)\right]p_i - F \\ \text{if } p_i > R-t \end{cases}$$

Assuming symmetry ($p_i = p_C$) the joint reaction function of small firms can be written as:

$$p_C = \begin{cases} \frac{(2-\gamma_L-\gamma_C)t+\gamma_L p_L}{2\gamma_L+\gamma_C} \text{ if } p_L \leq \underline{p}_L \\ R-t, \text{ if } \underline{p}_L \leq p_L \leq \overline{p}_L \\ \frac{(1-\gamma)R+\frac{\gamma}{2}t+\frac{\gamma_L}{2}p_L}{2-\gamma-\frac{\gamma_C}{2}}, \text{ if } p_L > \overline{p}_L \end{cases}$$

where $\underline{p}_L \equiv \frac{R(\gamma+\gamma_L)-t(2+\gamma_L)}{\gamma_L}$ and $\overline{p}_L = \frac{R(2-\gamma_C)-t(4-3\gamma-\gamma_C)}{\gamma_L}$.

It is important to note that, given reaction functions (10) and (11), for any (γ_L, γ_C) such that $\gamma_L + \gamma_C < 1$ there exists a unique pure strategy equilibrium in prices (p_L, p_C). Depending on (γ_L, γ_C), there are three possible configurations of equilibrium prices:

(i) Region A: $p_C < p_L \leq R-t$.
(ii) Region B : $p_C = p_L = R-t$.
(iii) Region C : $R-t < p_L, p_C < p_L$.

[14] This expression has been written for the case that $p_L, p_C \in [p_i - t, p_i + t]$. As in the case of monopolistic competition, and under some parameter values, a small firm may find it optimal to deviate and set $p_i = R - t > p_C + t$. Thus, the existence of an equilibrium where all small firms set the same price requires an additional restriction on parameter values analogous to the one specified in Section 3.

Unsurprisingly, small firms never set a price which is strictly higher than that of a large firm.

In the third stage any small firm can imitate the behavior of the large firm and set $p_i = p_L$. In this case, the profits of such a small firm are equal to the profits per variety of the large firm. Since in equilibrium small firms must make zero profits, then the large firm never chooses a value of $\gamma_L > 0$, that induces an equilibrium price in a region with $p_C < p_L$. The reason is that in this case: $\frac{1}{\gamma_L}\pi^L(p_L) = \pi^i(p_L) < \pi^i(p_C) = 0$. Therefore, the large firm only chooses $\gamma_L > 0$ if it anticipates that the subsequent equilibrium price is in Region B, in which case both the large and the small firms make zero profits.

Region B is determined by the following constraints: (i) $R-t \geq \underline{p}_L$ (small firms do not want to set a price below $R-t$), (ii) $\underline{p}_C \leq R-t$ (large firm does not want to set a price below $R-t$), and (iii) $R-t \leq \widehat{p}_C$ (large firm does want to set a price above $R-t$). Condition (i) is equivalent to:

$$\gamma_L + \gamma_C \leq \frac{2t}{R} \tag{12}$$

Similarly, conditions (ii) and (iii) can be respectively written as:

$$\gamma_C R + \gamma_L R \leq 2t \tag{13}$$

$$(2R - 3t)\,\gamma_L + (R - 2t)\,\gamma_C \leq 2\,(R - 2t) \tag{14}$$

If $R \geq 3t$ then conditions (13) and (14) are redundant, and Region B is defined as the set of (γ_L, γ_C) that satisfy condition (12). If $2t < R < 3t$ then condition (13) is also redundant but condition (14) is not, and hence (γ_L, γ_C) must satisfy both conditions (12) and (14).

Let us consider the case that F takes values in the interval $\left[\frac{(R-t)^2}{R}, R-t\right]$. Under monopolistic competition, prices are equal to $R-t$, and product variety, γ^{MC}, is given by the zero profit condition (5), where $\gamma^{MC} \leq \frac{2t}{R}$. In the current game, if all firms set prices equal to $R-t$ then all active firms make zero profits if and only if $\gamma_C + \gamma_L = \gamma^{MC}$. If firm L sets $\gamma_L \in \left(\gamma^{MC}, \frac{2(R-2t)}{2R-3t}\right)$, then there are two possibilities: either no small firm enters and the large firm sets $p_L = R-t$, or $\gamma_C > 0$ and equilibrium prices correspond to Region C. In both cases $\pi_L < 0$. If $\gamma_L > \frac{2(R-2t)}{2R-3t}$, then if $\gamma_C = 0$ the large firm sets a price above $R-t$ and nevertheless $\pi_L < 0$, since small firms can always set $p_i = p_L$ and make at least as much profits as $\frac{\pi_L}{\gamma_L}$.Alternatively, if $\gamma_C > 0$ then equilibrium prices correspond to Region C and hence $\pi_L < 0$. Consequently, the large firm never sets $\gamma_L > \gamma^{MC}$.

If the large firm sets $\gamma_L \leq \gamma^{MC}$ then $\gamma_C = \gamma^{MC} - \gamma_L$ (provided γ_L and γ_C satisfy restriction (14)) and in equilibrium all firms make zero profits (Region B). If $\gamma_L \leq \gamma^{MC}$ and $\gamma_C = \gamma^{MC} - \gamma_L$ do not satisfy restriction (14) then the resulting price equilibrium lies in Region C and hence $\pi^L < 0$.

Let us now turn to the case $F < \frac{(R-t)^2}{R}$.Under monopolistic competition firms charge a price below $R-t$ and $1 > \gamma^{MC} > \frac{2t}{R}$. In this case all the combinations of γ_C and γ_L for which small firms make zero profits lie outside Region B and therefore for any value of $\gamma_L > 0$ the price equilibrium lies in Regions A or C and hence $\pi^L < 0$.

8.3 Proof of Lemma 1

Consider first $7t \geq R > 3t$. In the region where $F \in [\frac{(R-t)(R-2t)}{R+t}, R-t]$, we have $\gamma^D < \frac{4t}{R+t}$. Note that since we focus on the case that $R > 3t$ then $\frac{4t}{R+t} < 1$. We need to check that in this region the only symmetric equilibrium involves $p = R - t$. Let us first consider prices above $R-t$. Firm A chooses p_A in order to maximize:

$$\pi_A = \left[\gamma_A^2 + 2\gamma_A (1 - \gamma_A - \gamma_B) \frac{R - p_A}{t} + 2\gamma_A\gamma_B \left(\frac{1}{2} + \frac{p_B - p_A}{2t}\right)\right] p_A$$

subject to $R - \frac{t}{2} \geq p_i \geq R - t$. Suppose these constraints are not binding. Then, firm A's optimal price is given by:

$$p_A = \frac{(\gamma_A + \gamma_B) t + 2 (1 - \gamma_A - \gamma_B) R + \gamma_B p_B}{2 (2 - 2\gamma_A - \gamma_B)}$$

If $p_B = R - t$, then the optimal price, p_A, will be given by:

$$p_A = \frac{R}{2} + \frac{\gamma_A t}{2 (2 - 2\gamma_A - \gamma_B)} < R - t$$

and the slope of the reaction function is less than one. Hence, there is no symmetric equilibrium with prices above $R-t$.

Let us consider the case $3t < R \leq 7t$. As discussed in the text the only candidate to a symmetric equilibrium includes $p_A = p_B = R - t$, and

$\gamma^A = \gamma^B = \frac{1}{2}\gamma^D$ where γ^D is given by equation (8) if $F \geq \frac{(R-t)(R-2t)}{R+t}$, and by $\gamma^D = \frac{4t}{R+t}$,otherwise. In the second stage there are no incentives to deviate, and the arguments given in the main text are sufficient. However, in the first stage there is a potential deviation that we need to check. In particular, say firm A might find it optimal to set γ^A sufficiently above $\frac{2t}{R+t}$ such that it induces in the second stage prices below $R-t$. Profits after this deviation are:

$$\pi_A^{dev} = \frac{2(R+t)}{9\gamma_A}\left(\frac{2R}{R+t} - \gamma_A\right)^2\left(\gamma_A + \frac{t}{R+t}\right)^2 - 2\gamma_A F$$

whereas its profits under no deviation (i.e., for $\gamma_A = \gamma_B = \frac{1}{2}\gamma^D = \frac{2t}{R+t}$) are

$$\pi_A^D = \frac{4t}{R+t}\left(\frac{(R-t)^2}{R+t} - F\right)$$

Based on these expressions, we can write

$$\pi_A^D - \pi_A^{dev} = \frac{2}{R+t}g(\gamma_A, R, t) + \left(\gamma_A - \frac{2t}{R+t}\right)2F$$

where

$$g(\gamma_A, R, t) \equiv \frac{2t(R-t)^2}{(R+t)^2} - \frac{(R+t)}{9\gamma_A}\left(\frac{2R}{R+t} - \gamma_A\right)^2\left(\gamma_A + \frac{t}{R+t}\right)^2$$

Since $\gamma_A + \gamma_B \leq 1$, upward deviation with $\gamma_A > \frac{2t}{R+t}$ yields an upper bound for γ_A such that $\gamma_A \leq 1 - \gamma_B = 1 - \frac{2t}{R+t} = \frac{R-t}{R+t}$.

The function $g(\gamma_A, R, t)$ is such that $g(\frac{1}{2}\gamma^D, R, t) = 0$. Additionally, $g(\gamma_A, R, t)$ is increasing in γ_A for all $R \in (3t, 6.5t)$. Therefore, $g(\gamma_A, R, t) > 0$ for all $\gamma_A > \frac{2t}{R+t}$ provided $R < 6.5t$. When $6.5t < R < 7t$ we have that $g(\gamma_A, R, t)$ is decreasing in γ_A for relatively small deviations. Due to that, we perform a numerical analysis to conclude that $g(\gamma_A, R, t) > 0$ for all $\gamma_A > \frac{2t}{R+t}$ also when $6.5t < R < 7t$ (see Appendix on numerical simulations for details). Thus, $\pi_A^D > \pi_A^{dev}$ under $3t < R < 7t$ for all F, which means that this upward global deviation cannot be profitable for $3t < R < 7t$. This shows that a symmetric equilibrium exists for all F if $3t < R < 7t$.

If $R \geq 7t$ then there is another candidate for a symmetric equilibrium, which is given by equalizing equation (9) to zero:

$$\frac{4t\left(2-\gamma^D\right)\left(1-2\gamma^D\right)}{3\gamma^D} = 2F$$

Unfortunately, second order conditions are not satisfied. More specifically, under $p_A, p_B \leq R - t$, first order conditions of firm A's profit maximization in the second stage yields

$$\frac{\partial \pi_A}{\partial p_A} = \frac{\gamma_A}{t}(2t - t\gamma_A - t\gamma_B - 2p_A\gamma_B + p_B\gamma_B) = 0$$

and a similar expression for firm B. Solving these two equations:

$$p_A = \frac{1}{3\gamma_A\gamma_B}(4t\gamma_A + 2t\gamma_B - 3t\gamma_A\gamma_B - 2t\gamma_A^2 - t\gamma_B^2)$$

$$p_B = \frac{1}{3\gamma_A\gamma_B}(4t\gamma_B + 2t\gamma_A - 3t\gamma_A\gamma_B - 2t\gamma_B^2 - t\gamma_A^2)$$

Inserting these prices into each firm's profit function and maximizing with respect to the firm's range of varieties yields

$$\begin{aligned}\frac{\partial \pi_A}{\partial \gamma_A} &= \frac{t}{9\gamma_A^2\gamma_B}(16\gamma_A^2 - 32\gamma_A^3 - 4\gamma_B^2 + 12\gamma_A^4 + 4\gamma_B^3 - \gamma_B^4 - 32\gamma_A^2\gamma_B + \\ &\quad +24\gamma_A^3\gamma_B + 13\gamma_A^2\gamma_B^2) - 2F\end{aligned}$$

$$\frac{\partial^2 \pi_A}{\partial \gamma_A^2} = \frac{2t}{9\gamma_A^3\gamma_B}(4\gamma_B^2 - 16\gamma_A^3 + 12\gamma_A^4 - 4\gamma_B^3 + \gamma_B^4 + 12\gamma_A^3\gamma_B)$$

Evaluating the second derivative at $\gamma_A = \gamma_B = \frac{1}{2}\gamma^D$ we have $\frac{\partial^2 \pi_A}{\partial \gamma_A^2} = \frac{2t}{9(\gamma^D)^2}(16 - 40\gamma^D + 25(\gamma^D)^2) > 0$ for all $\gamma^D \neq \frac{4}{5}$.

The other candidate is the same as in the case $R < 7t$. However, if F is sufficiently low then firms have incentives to deviate and hence a symmetric equilibrium does not exist. Below we discuss some numerical simulations that provide a better idea of the extent of the existence problem (See also the Appendix on numerical simulations.)

8.4 Proof of Proposition 3

We now compare γ^D with γ^{MC}. For $F \leq \frac{t}{2}$ we have $\gamma^D = \frac{4t}{R+t}$ and $\gamma^{MC} = 1$ so that $\gamma^D < \gamma^{MC}$; and for $F \geq \frac{(R-t)^2}{R}$ we have $\gamma^D = \frac{4}{3}\frac{R-t-F}{R-t}$ and $\gamma^{MC} = 2\frac{R-t-F}{R-t}$ so that $\gamma^D < \gamma^{MC}$ as well. For intermediate values of F such that $\frac{t}{2} < F < \frac{(R-t)^2}{R}$ it turns out that γ^D and γ^{MC} cross twice. Specifically, for

$\frac{t}{2} < F < \frac{(R-t)(R-2t)}{R+t}$ we have $\gamma^D = \frac{4t}{R+t}$ and $\frac{t}{2}\frac{(2-\gamma^{MC})^2}{\gamma^{MC}} = F$, and then γ^D and γ^{MC} are equal at $F = F_d$, which is given by

$$F_d = \frac{(R-t)^2}{2(R+t)}$$

where $\frac{t}{2} < F_d < \frac{(R-t)(R-2t)}{R+t}$ for all $R > 3t$.

For $\frac{(R-t)(R-2t)}{R+t} < F < \frac{(R-t)^2}{R}$ we have $\gamma^D = \frac{4}{3}\frac{R-t-F}{R-t}$ and $\frac{t}{2}\frac{(2-\gamma^{MC})^2}{\gamma^{MC}} = F$. Denote here the lower and the upper roots to $\gamma^D(F) = \gamma^{MC}(F)$ by F^- and F^+, respectively. Given that $F^- < \frac{3R^2-8Rt+5t^2}{2(3R-t)} < \frac{(R-t)(R-2t)}{R+t}$, we can focus on the upper root and thus write $F_e = F^+$ as given by

$$F_e = \frac{3(\frac{R}{t})^2 - 8\frac{R}{t} + 5 + 3\sqrt{\eta(\frac{R}{t})}}{\frac{2}{t}(3\frac{R}{t} - 1)}$$

where $\eta(\frac{R}{t}) \equiv (\frac{R}{t})^4 - 6(\frac{R}{t})^3 + 12(\frac{R}{t})^2 - 10\frac{R}{t} + 3$. We now check that F_e is real valued. Since $\eta'(3) > 0$ and $\eta''(\frac{R}{t}) > 0$ for all $R > 2t$, we have $\eta'(\frac{R}{t}) > 0$ for all $R \geq 3t$. In consequence, $\eta(\frac{R}{t}) > 0$ for all $R > 3t$ due to the fact that $\eta(3) = 0$. Thus, F_e is real valued for all $R > 3t$. Finally, we can write $\frac{(R-t)(R-2t)}{R+t} < F_e$ as $18\left(3\frac{R}{t} - 1\right)\left(\frac{R}{t} - 3\right)\left(\frac{R}{t} - 1\right)^3 > 0$, which holds for all $R > 3t$; and we can also write $F_e < \frac{(R-t)^2}{R}$ as $2\left(3\frac{R}{t} - 1\right)\left(\frac{R}{t} - 1\right)^2[3(\frac{R}{t})^2 - 2] > 0$, which holds for all $R > t$.

8.5 Proof of Proposition 4

Let us compare γ^D with γ^* under $R > 3t$. For $F \in [\frac{t}{4}, \frac{(R-t)(R-2t)}{R+t}]$ we have $\gamma^D = \frac{4t}{R+t}$, which equals $\gamma^*(F)$ at the value of F given by

$$F_f = \frac{(R-t)(2R-5t)}{2(R+t)}$$

where $\frac{t}{4} < F_f < \frac{(R-t)(R-2t)}{R+t}$ for all $R > 3t$, and $F_f \gtreqless F_d$ as $R \gtreqless 4t$.

For $F \in [\frac{(R-t)(R-2t)}{R+t}, R - \frac{t}{2}]$ we have $\gamma^D = \frac{4}{3}\frac{R-t-F}{R-t}$, which equals $\gamma^*(F)$ at the value of F given by

$$F_g = \frac{(R-t)(2R-3t)}{2R}$$

where $\frac{(R-t)(R-2t)}{R+t} < F_g < R - \frac{t}{2}$ for all $R > t$. In addition, we can write $F_e < F_g$ as $\left(\frac{R}{t} - 1\right)^3 \left(3\frac{R}{t} - 1\right) > 0$, which holds for all $R > 3t$. This leads to the intervals in the result.

8.6 Low reservation prices: $R < 3t$

With $\gamma_A = \gamma_B = \frac{1}{2}$ we have $p_A = p_B = 2t > R - t$ for all $R < 3t$, and $p_A = p_B < R - \frac{t}{2}$ for all $R > \frac{5}{2}t$. Then, at $\gamma_A = \gamma_B = \frac{1}{2}$ it follows that

$$\left.\frac{\partial \pi_A}{\partial \gamma_A}\right|_{\gamma_A = \gamma_B = \frac{1}{2}} = \frac{2}{3}(22t - 6R) - 2F$$

which is non-negative as long as $F \leq \frac{2}{3}(11t - 3R)$; and

$$\left.\frac{\partial^2 \pi_A}{\partial \gamma_A^2}\right|_{\gamma_A = \gamma_B = \frac{1}{2}} = -\frac{2}{81}\left(2282044\left(\frac{R}{t}\right)^2 - 2277940\frac{R}{t} - 12033\right)$$

which is negative for $\frac{5}{2}t < R < 3t$. In these circumstances, the firms' profits are $\pi_A = \pi_B = t - F \geq 0$ as $F \leq t$. Since $t < \frac{2}{3}(11t - 3R)$ for all $R < 3t$, the symmetric equilibrium where $p_A = p_B = 2t$ and $\gamma_A = \gamma_B = \frac{1}{2}$ exists for $\frac{t}{2} < F < t$ under $\frac{5}{2}t < R < 3t$, and it yields $\gamma^D = 1 > \gamma^{MC} > \gamma^*$.

8.7 Numerical analysis

We have solved the model numerically for $R > 6.5t$. More specifically, we have considered values of R/t from 6.5 upwards in steps of 0.05. We report the results of these numerical simulations only up to $R/t = 20$, but the same pattern has been found for higher values. Given R/t, we then examine firm A's profits as a function of both F and γ_A. We consider upward deviations from the equilibrium candidate, $\gamma_A = \frac{2t}{R+t}$. Since $\gamma_A + \gamma_B \leq 1$, and $\gamma_B = \frac{2t}{R+t}$, then the relevant range for γ_A is $\left[\frac{2t}{R+t}, \frac{R-t}{R+t}\right]$. Note that as γ_A increases above $\frac{2t}{R+t}$, first prices are still equal to $R - t$, next p_B falls below $R - t$, and eventually both prices fall below $R - t$.

The main result is that if $R/t \leq 7$ then for all values of F it is never profitable to deviate from the equilibrium candidate. For values of $R/t > 7$ then there exist a threshold value of F, $\overline{F}$, such that if $F \geq \overline{F}$ then there is no profitable deviation from the equilibrium candidate, but if $F < \overline{F}$ then a profitable deviation does exist and hence a symmetric equilibrium in pure strategies does not exist. Moreover, $\overline{F} = 0$ if $R/t = 7$, $\overline{F}$ increases with R/t, and $\overline{F} < \frac{(R-t)(R-2t)}{R+t}$. See Figure 4.[15]

[15] The program and detailed results are available upon request.

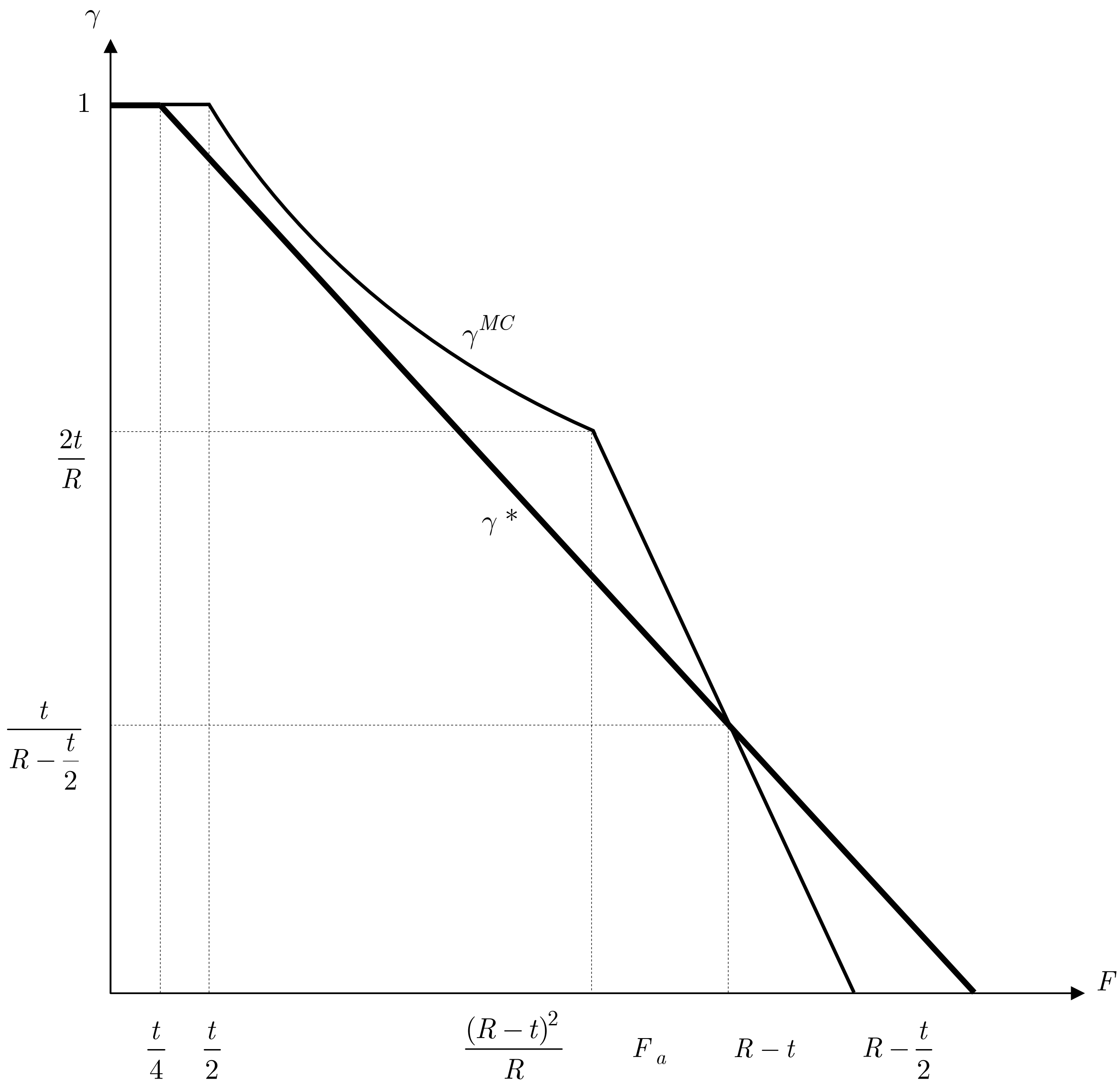

Figure 1a

First best and monopolistic competition: $R < \left(\frac{5}{2} + \sqrt{2}\right) t$.

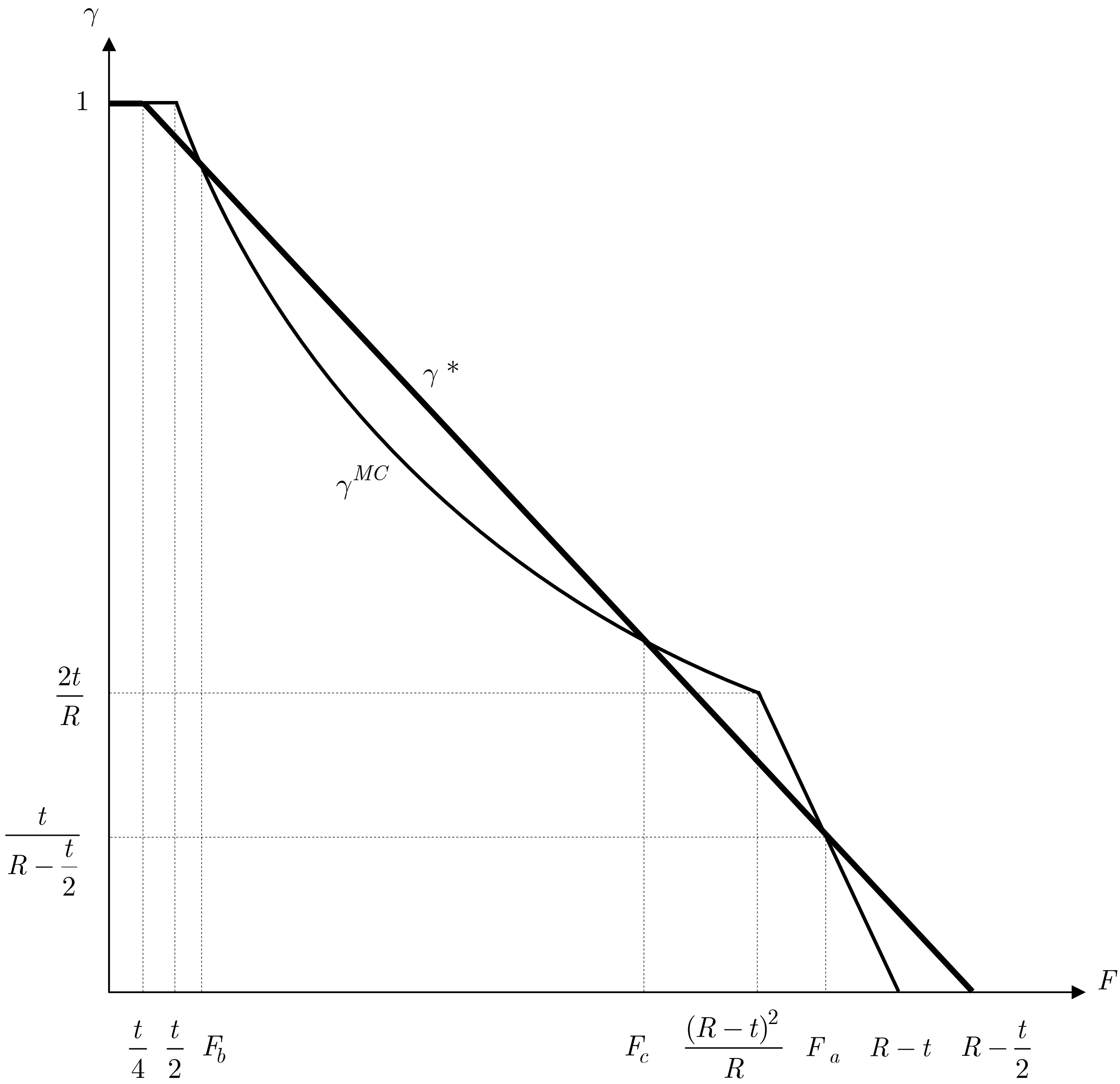

Figure 1b

First best and monopolistic competition: $R > \left(\dfrac{5}{2}+\sqrt{2}\right)t$.

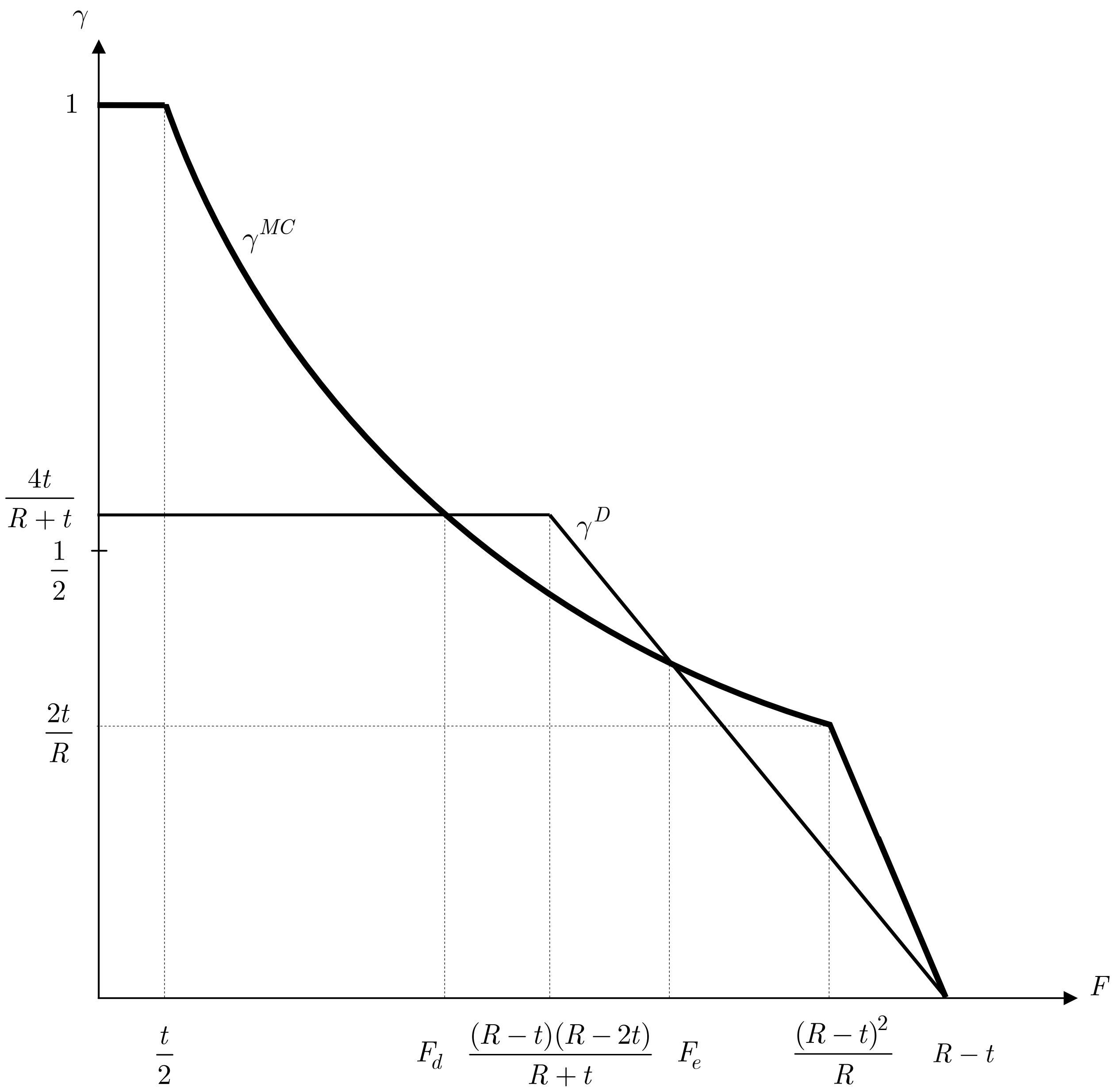

Figure 2

Duopoly and monopolistic competition.

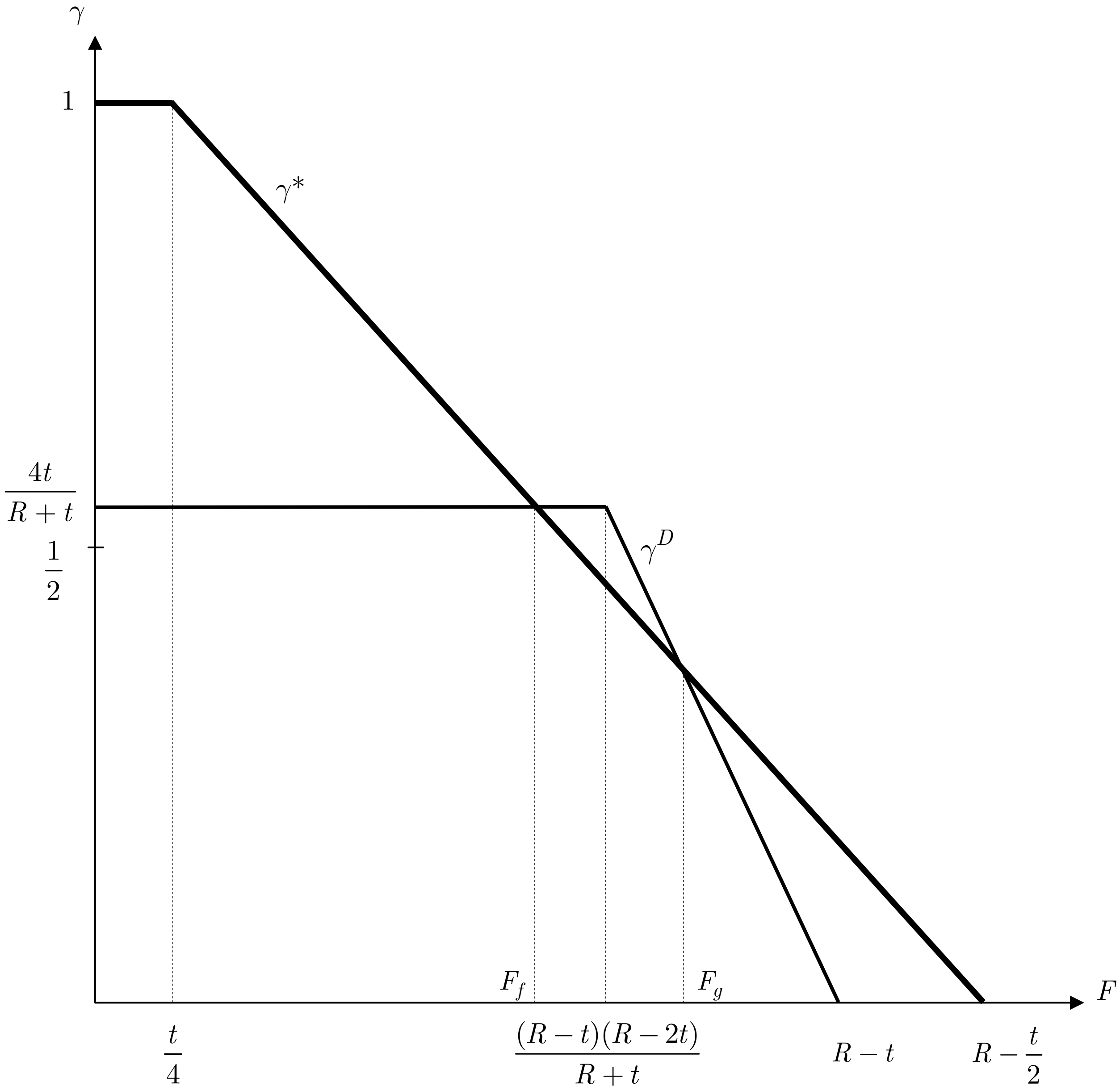

Figure 3

Duopoly and first best.

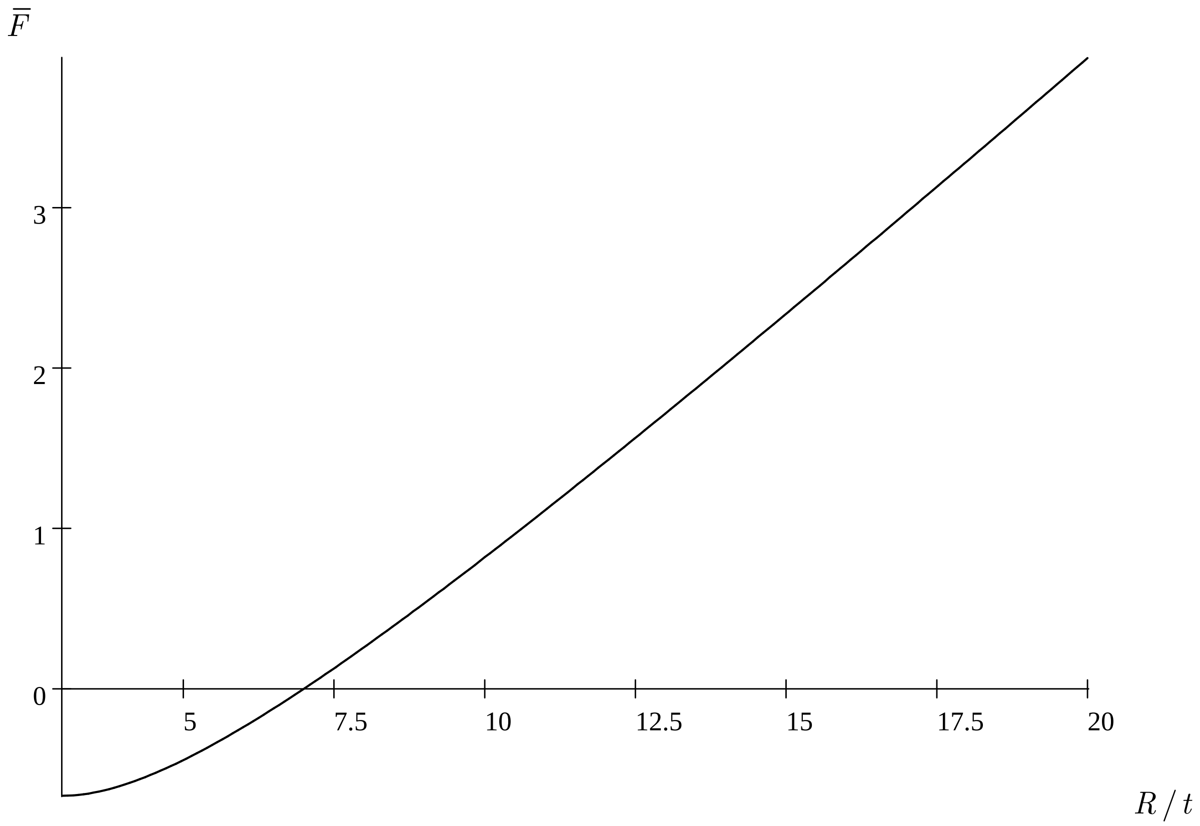

Figure 4